Cables

Theory of Structure - I

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- Assumptions for the derivation of relation b/w force in cable and its slope:
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Introduction

- Cables are pure tension members, are used for support to transfer the load from one member to another.
- Used as
  - Supports to suspension roofs
  - Suspension bridges
  - Trolley wheels
- Self weight of cable is neglected in analysis of above structures
- When used as guys for radio antennas or transmission lines, derricks(loading crane);cable weight is considered.

Assumptions for the derivation of relation b/w force in cable and its slope:

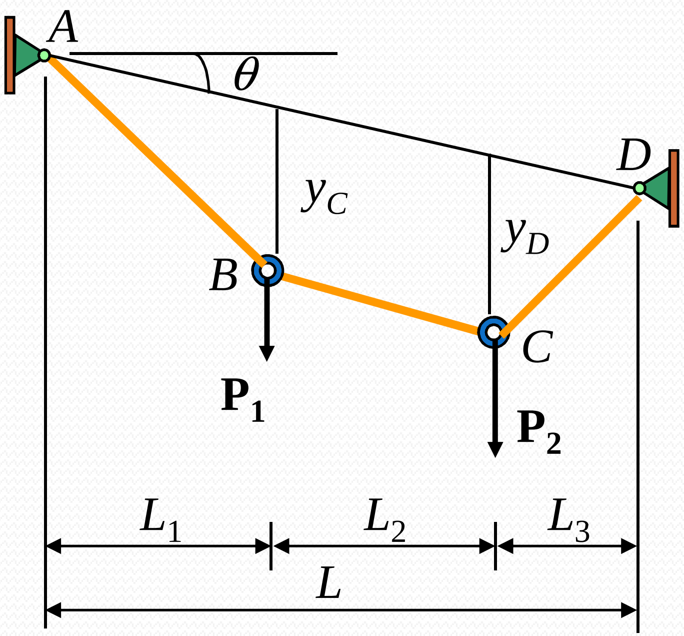
- The cable is perfectly flexible.
- The cable is perfectly inextensible.
- Due to flexibility cables offer no resistance to shear & bending, therefore force acting on cable is always tangent to the cable at points along its path.
- Due to inextensibility, cable has constant length before and after the application of load, geometry remain fixed, the cable is treated as a rigid body.

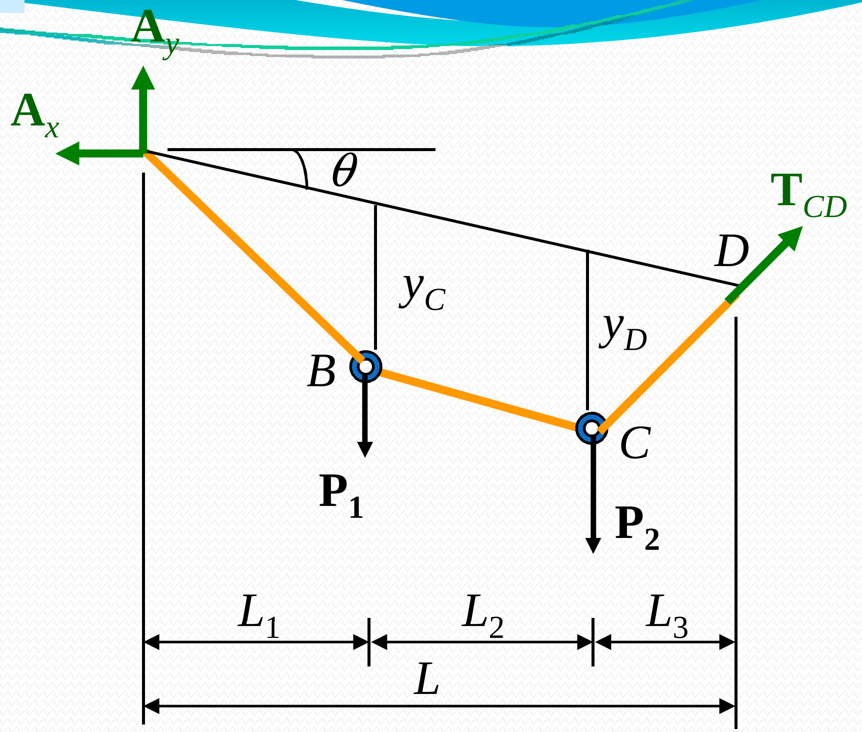
Cable subjected to concentrated loads

- When cables of negligible weight supports several concentrated loads, the cables takes the form of several straight –line segments, each of which is subjected to a constant tensile force.
- L_1,L_2,L_3,P_1 & P_2 are known,
- There are nine unknown which we have to determine.
- Where 'Θ' is the angle of chord A and B.
- From nine unknown, three segments are in tension.
- We can write two equilibrium equation at each point A,B,C & D.So total of eight equations.

- Where yc and yd are the sag at points C & D.

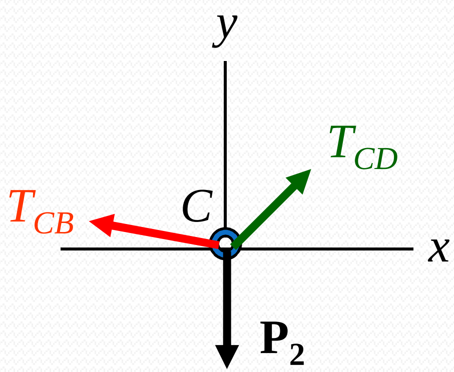
Cable Subjected to Concentrated Loads

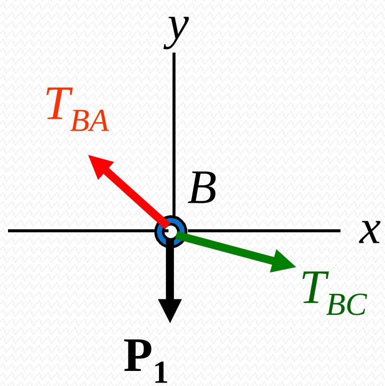




$\circlearrowleft + \; \Sigma M_A = 0$:

Obtain T_{CD}



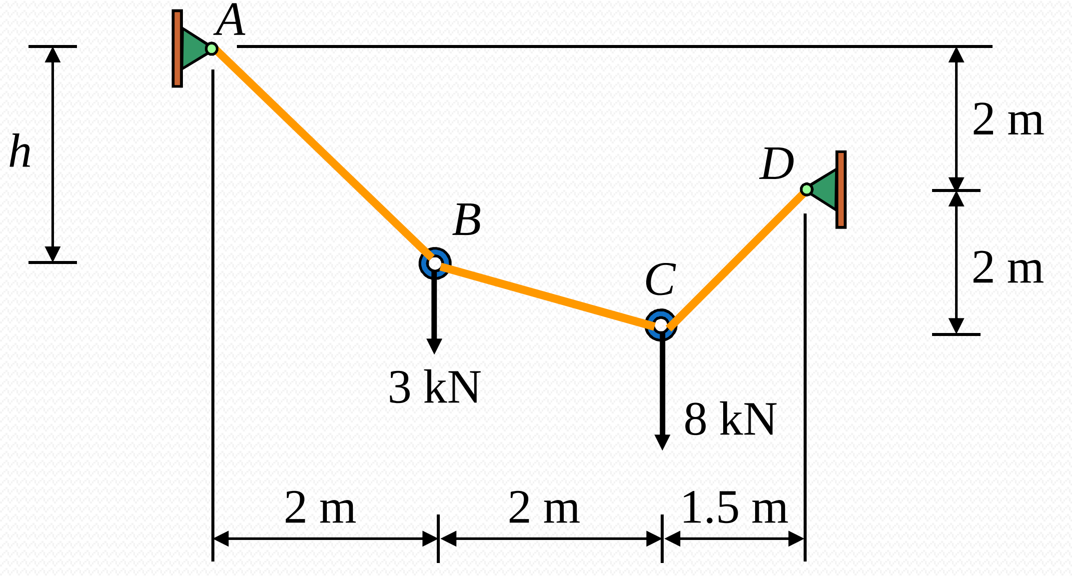


$\overset{+}{\rightarrow} \; \Sigma F_x = 0$:

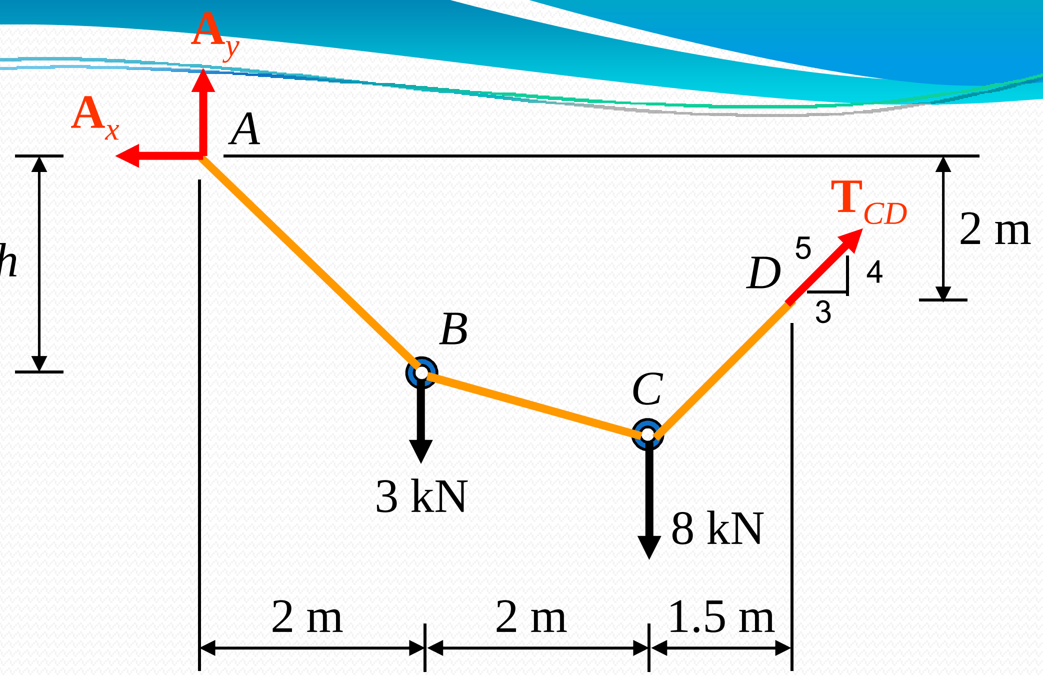
$+\uparrow \; \Sigma F_y = 0$:

Example 5-1

Determine the tension in each segment of the cable shown in the figure below. Also, what is the dimension *h* ?



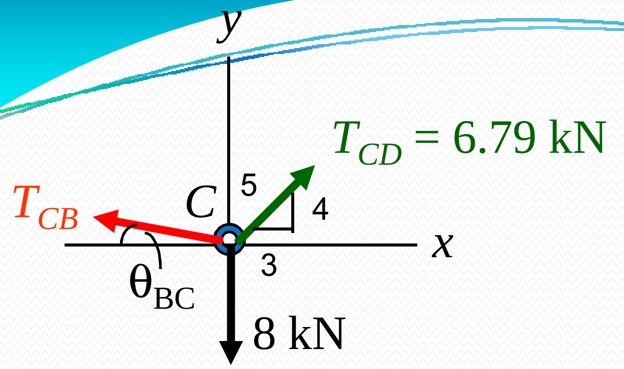
SOLUTION



$\circlearrowleft + \; \Sigma M_A = 0:$

$$T_{CD}(3/5)(2\text{ m}) + T_{CD}(4/5)(5.5\text{ m}) - 3\text{kN}(2\text{ m}) - 8\text{ kN}(4\text{ m}) = 0$$

$$T_{CD} = 6.79\text{ kN}$$

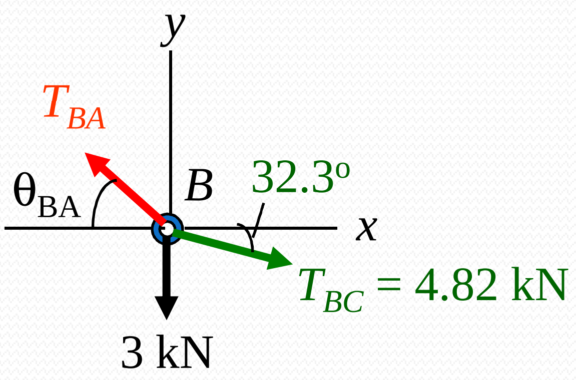


Joint C

$\xrightarrow{+} \Sigma F_x = 0: \quad 6.79(3/5) - T_{CB} \cos\theta_{BC} = 0$

$+\uparrow \Sigma F_y = 0: \quad 6.79(4/5) - 8 + T_{CB} \sin\theta_{CB} = 0$

$\theta_{BC} = 32.3^o \qquad T_{CB} = 4.82 \text{ kN}$



Joint B

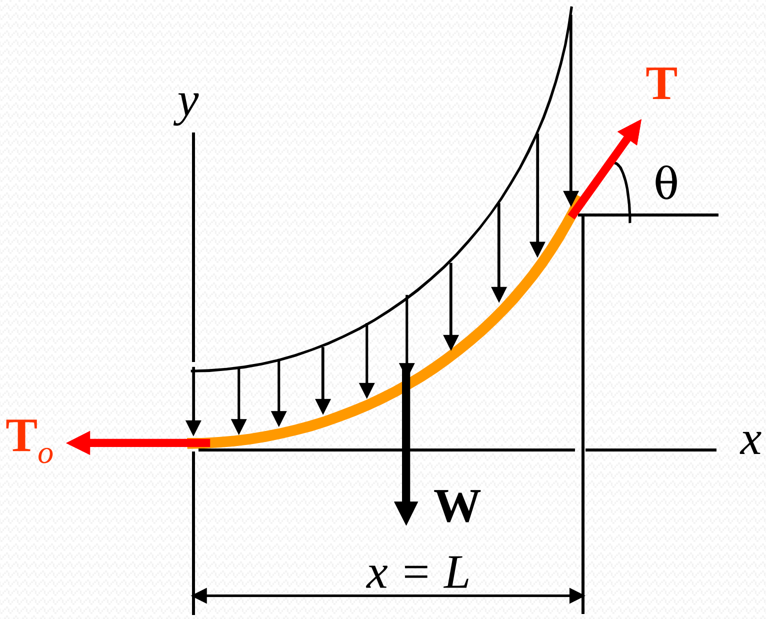
$\xrightarrow{+} \Sigma F_x = 0: \quad - T_{BA} \cos\theta_{BA} + 4.82\cos 32.3^o = 0$

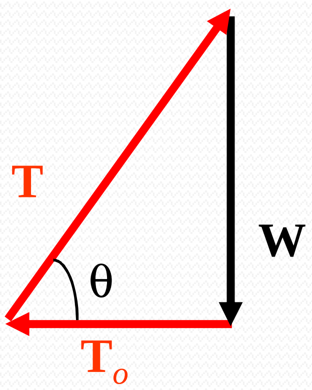
$+\uparrow \Sigma F_y = 0: \quad T_{BA} \sin\theta_{BA} - 4.82\sin 32.3^o - 3 = 0$

$\theta_{BA} = 53.8^o \qquad T_{BA} = 6.90 \text{ kN}$

$h = 2\tan\theta_{BA} = 2\tan 53.8^o = 2.74 \text{ m}$

Cable Subjected to Distributed Load





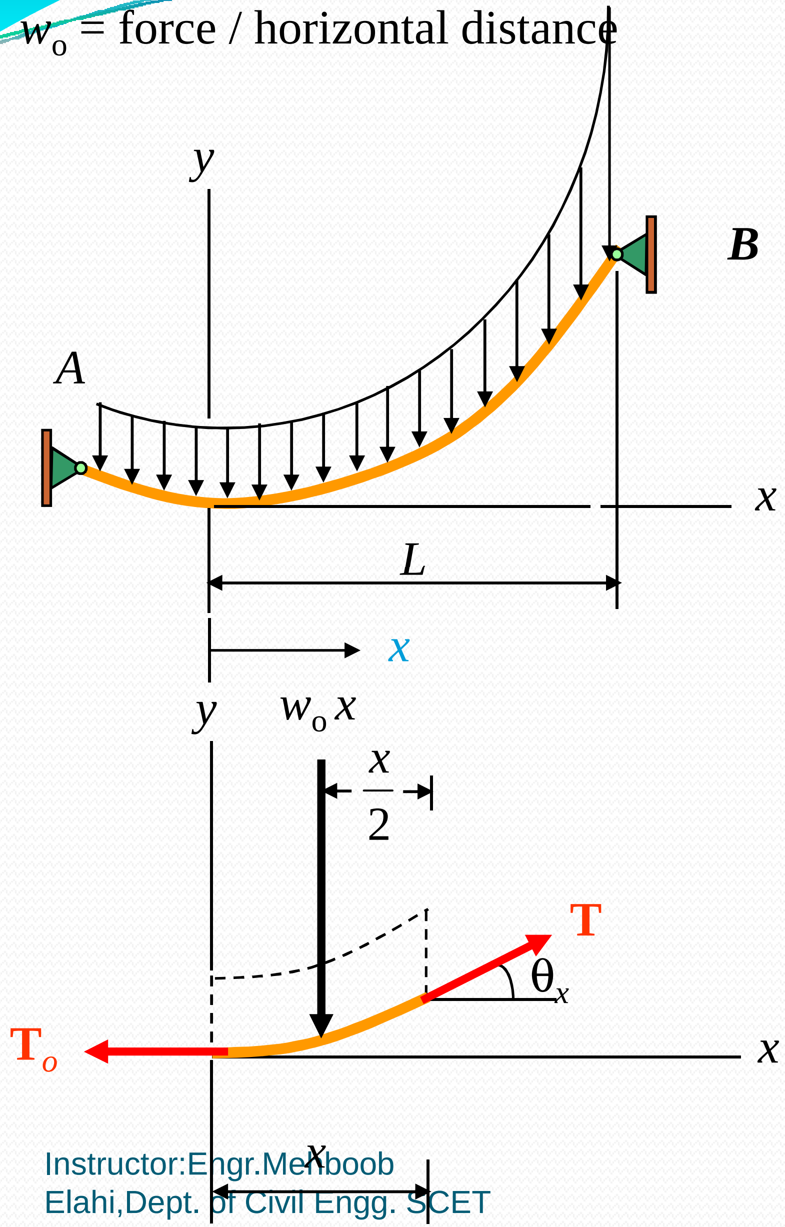
$$T \cos \theta = T_o = F_H = \text{Constant}$$

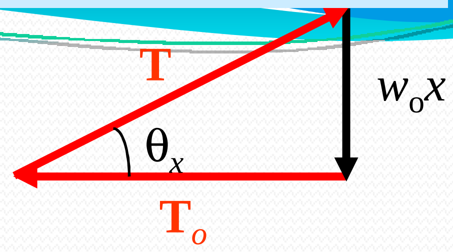
$$T \sin \theta = W$$

$$\frac{dy}{dx} = \tan \theta = \frac{W}{T_o}$$

Parabolic Cable: Subjected to Linear Uniform distributed Load

w_o = force / horizontal distance





$$\frac{dy}{dx} = \tan\theta = \frac{w_o x}{T_o}$$

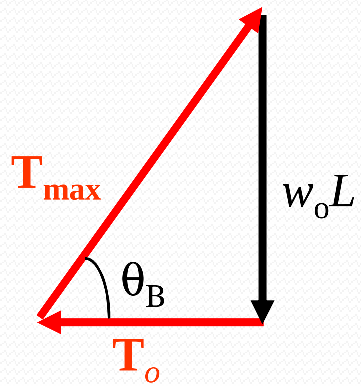
$$y = \int \frac{w_o x}{T_o} dx$$

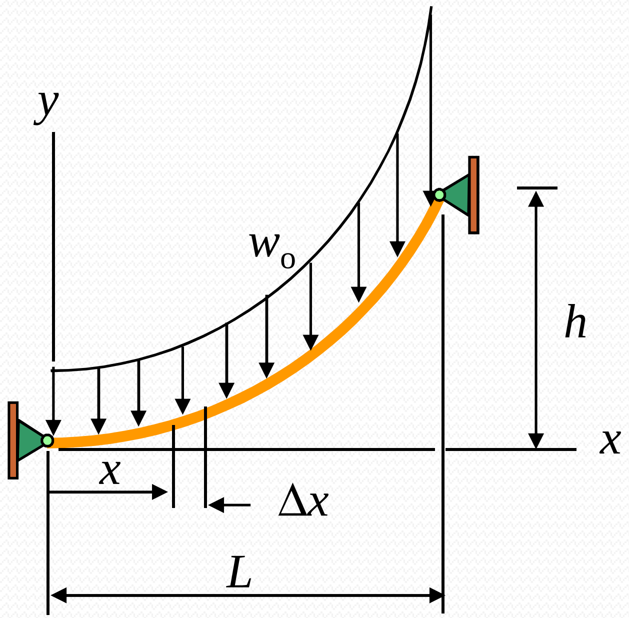
$$y = \frac{w_o x^2}{2T_o} + C_1 \quad (C_1 = 0)$$

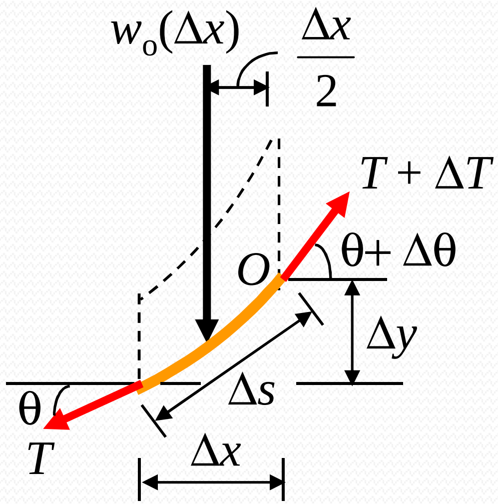
$$T_o = \frac{w_o x^2}{2y}$$

at $x = L$, $T = T_B = T_{\max}$

$$T_{\max} = \sqrt{T_o^{\,2} + (w_o L)^2}$$







$\xrightarrow{+}$ $\Sigma F_x = 0$: $\quad -T\cos\theta + (T + \Delta T)\cos(\theta + \Delta\theta) = 0$

$+\uparrow$ $\Sigma F_y = 0$: $\quad -T\sin\theta + w_o(\Delta x) + (T + \Delta T)\sin(\theta + \Delta\theta) = 0$

$+\circlearrowleft$ $\Sigma M_O = 0$: $\quad w_o(\Delta x)(\Delta x/2) - T\cos\theta\Delta y - T\sin\theta(\Delta x) = 0$

Dividing each of these equations by Δx and taking the limit as $\Delta x \rightarrow 0$, and hence $\Delta y \rightarrow 0$, $\Delta\theta \rightarrow 0$, and $\Delta T \rightarrow 0$, we obtain

$$\frac{d(T\cos\theta)}{dx} = 0 \qquad \text{----------(5-1)}$$

$$\frac{d(T\sin\theta)}{dx} = w_o \qquad \text{----------(5-2)}$$

$$\frac{dy}{dx} = \tan\theta \qquad \text{----------(5-3)}$$

Integrating Eq. 5-1, where $T = F_H$ at $x = 0$, we have:

$$T\cos\theta = F_H \qquad \text{----------(5-4)}$$

Integrating Eq. 5-2, where $T\, sin\ \theta = 0$ at $x = 0$, gives

$$T\sin\theta = w_o x \qquad \text{----------(5-5)}$$

Dividing Eq. 5-5 Eq. 5-4 eliminates *T*. Then using Eq. 5-3, we can obtain the slope at any point,

$$\tan\theta = \frac{dy}{dx} = \frac{w_o x}{F_H} \qquad \text{----------(5-6)}$$

Performing a second integration with $y = 0$ at $x = 0$ yields

$$y = \frac{w_o}{2F_H} x^2$$ ----------(5-7)

This is the equation of a *parabola*. The constant F_H may be obtained by using the boundary condition $y = h$ at $x = L$. Thus,

$$F_H = \frac{w_o L^2}{2h}$$ ----------(5-8)

Finally, substituting into Eq. 5-7 yeilds

$$y = \frac{h}{L^2} x^2$$ ----------(5-9)

From Eq. 5-4, the maximum tension in the cable occurs when θ is maximum; i.e., at $x = L$. Hence, from Eqs. 5-4 and 5-5,

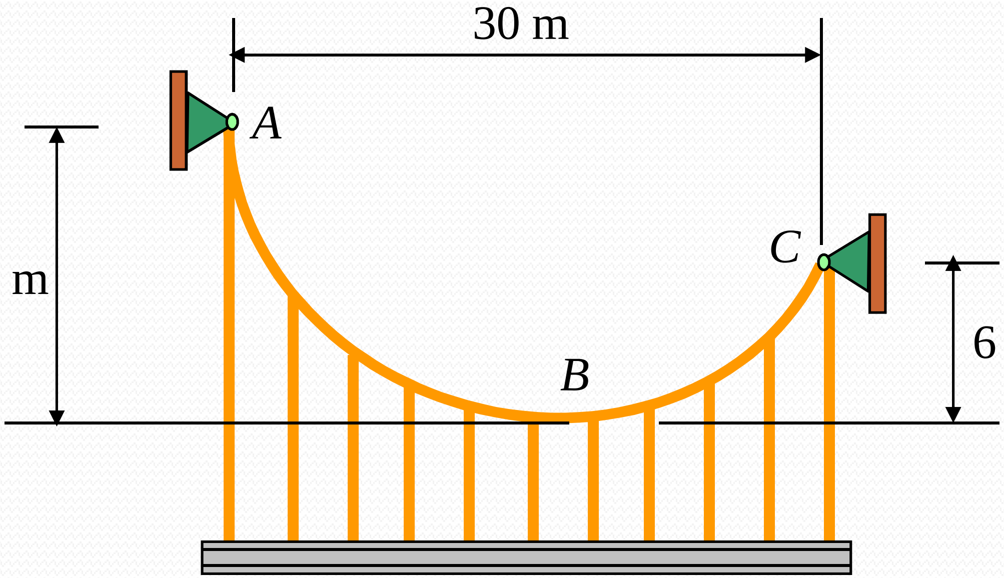
$$T_{\max} = \sqrt{F_H{}^2 + (w_o L)^2}$$ ----------(5-10)

Or, using Eq. 5-8, we can express $T_{\max}$ in terms of w_o, i.e.,

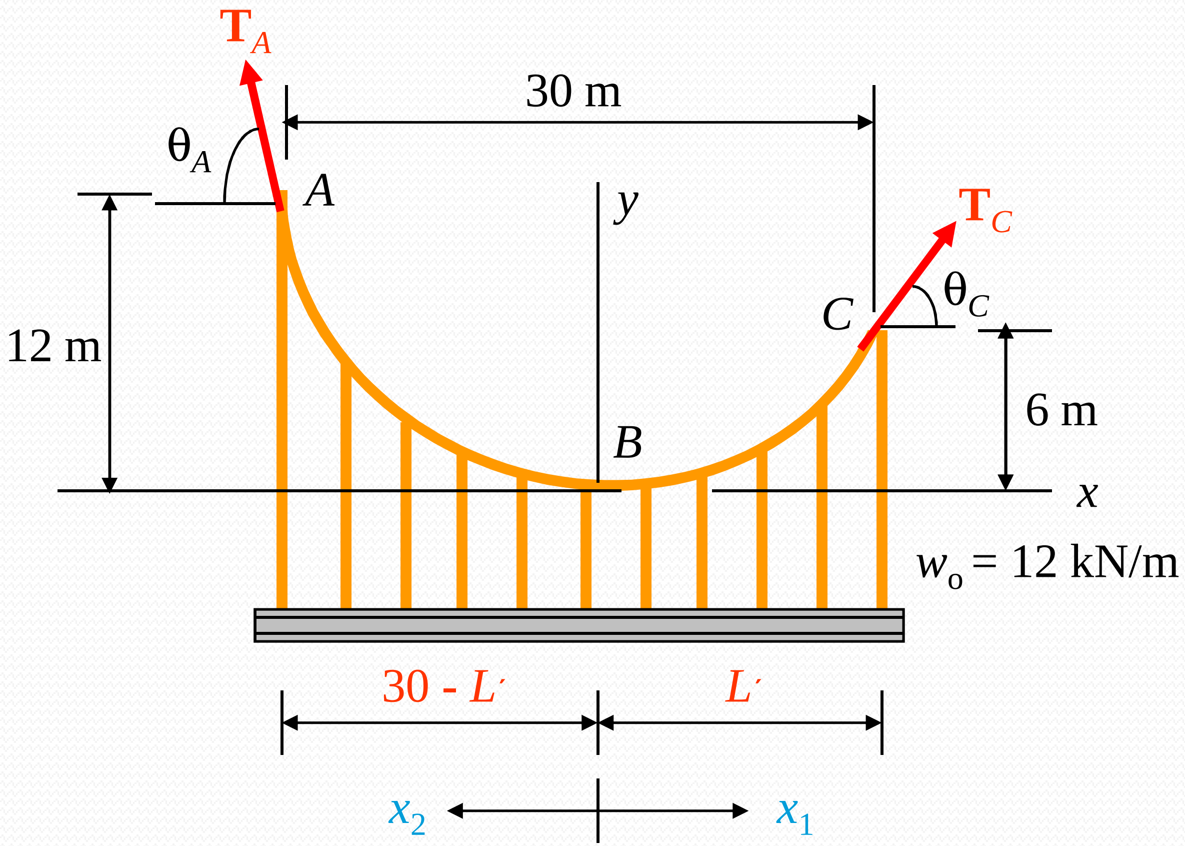
$$T_{\max} = w_o L \sqrt{1 + (L/2h)^2}$$ ----------(5-11)

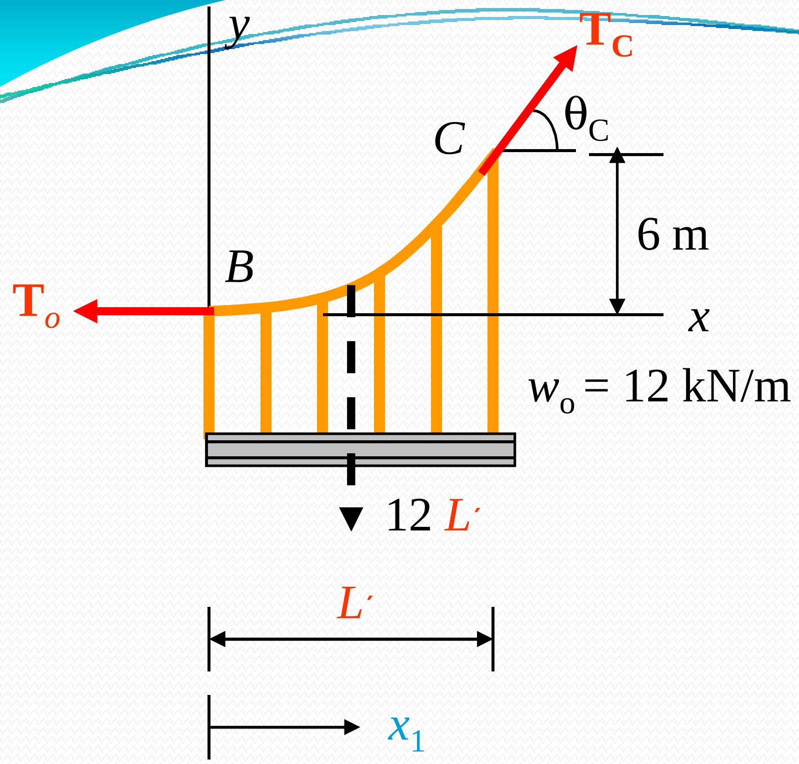
Example 5-2

The cable shown supports a girder which weighs 12kN/m. Determine the tension in the cable at points *A*, *B*, and *C*.



SOLUTION





$$\frac{dy_1}{dx_1} = \tan\theta = \frac{12x_1}{T_o}$$

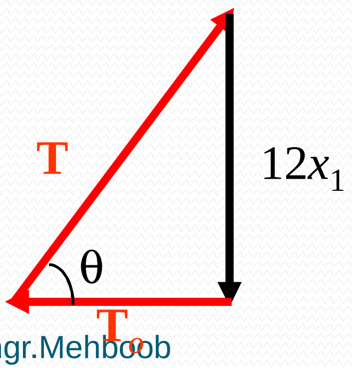
$$y_1 = \int \frac{12x_1}{T_o}\,dx_1$$

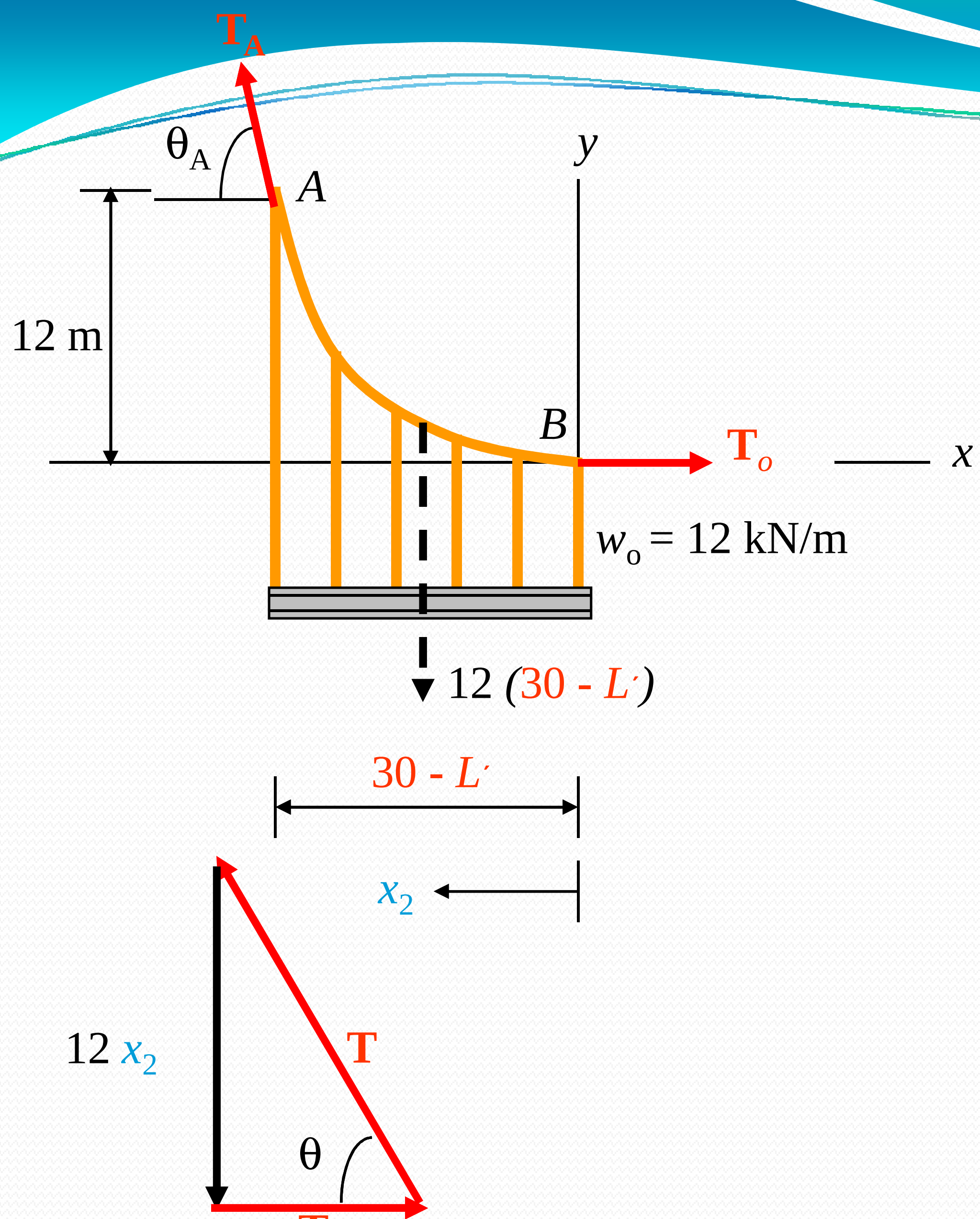
$$6 = \int_0^{L'} \frac{12x_1}{T_o}\,dx_1$$

$$6 = \left.\frac{12{x_1}^2}{2T_o}\right|_0^{L'} + C_1 \quad (C_1 = 0)$$

$$6 = \frac{12L'^2}{2T_o}$$

$$T_o = L'^2 \qquad \text{----------(1)}$$





$$\frac{dy_2}{dx_2} = \tan\theta = \frac{12x_2}{T_o}$$

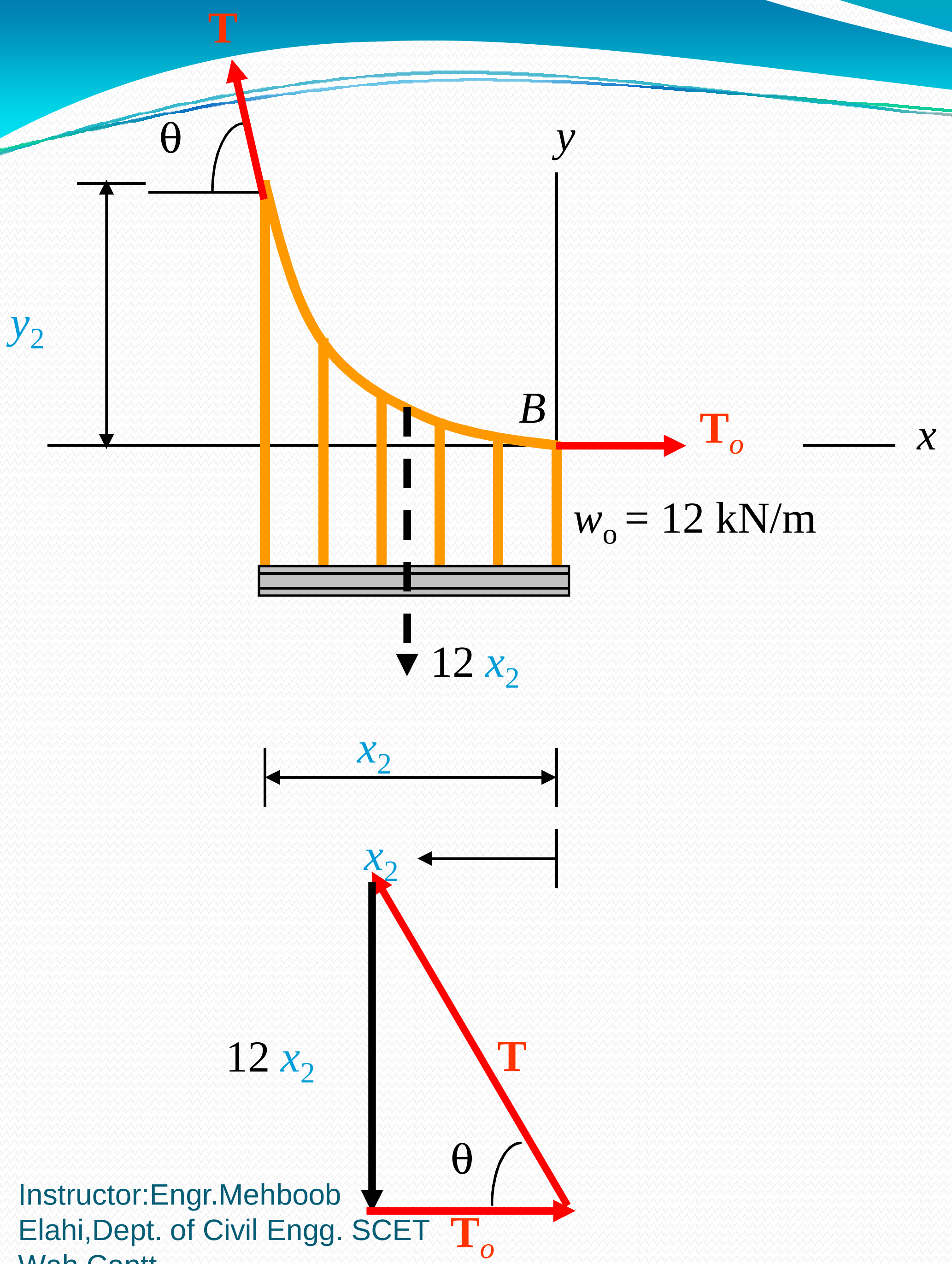
$$y_2 = \int \frac{12x_2}{T_o}\,dx_2$$

$$12 = \int_0^{(30-L')} \frac{12x_2}{T_o}\,dx_2$$

$$12 = \left.\frac{12x_2^{\,2}}{2T_o}\right|_0^{(30-L')} + C_2 \quad (C_2 = 0)$$

$$12 = \frac{12(30-L')^2}{2T_o}$$

$$1 = \frac{(30-L')^2}{2T_o} \qquad \text{----------(2)}$$

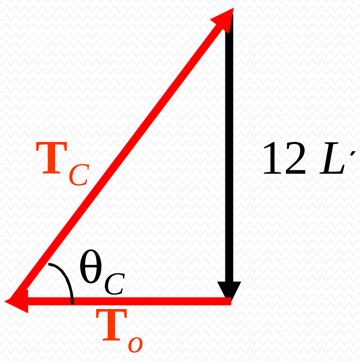

T
θ
y
y_2
B
T_o
x
$w_o = 12$ kN/m
12 x_2
x_2
x_2
12 x_2
T
θ
T_o

$$T_o = L'^2 \qquad \text{----------(1)}$$

$$1 = \frac{(30 - L')^2}{2T_o} \qquad \text{----------(2)}$$

From (1) and (2), $L' = 12.43$ m, $T_o = 154.50$ kN

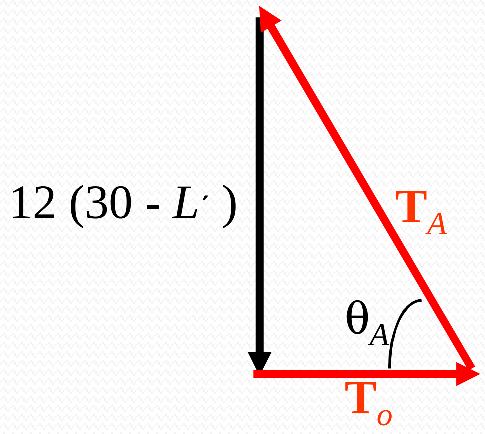
$$T_B = T_o = 154.50 \text{ kN}$$



$$T_C = \sqrt{T_o^{\,2} + (12L')^2}$$

$$= \sqrt{(154.50)^2 + (12 \times 12.43)^2}$$

= 214.75 kN



$$T_A = \sqrt{T_o^{\,2} + [12(30 - L')]^2}$$

$$= \sqrt{(154.50)^2 + [12(30 - 12.43)]^2}$$

= 261.39 kN

Practice Problems

- Chapter 5
- Structural Analysis by R. C. Hibbeler
- Examples and Exercise

Thank You