1 INTRODUCTION

1.1 CONCRETE, REINFORCED CONCRETE, AND PRESTRESSED CONCRETE

Concrete is a stonelike material obtained by permitting a carefully proportioned mixture of cement, sand and gravel or other aggregate, and water to harden in forms of the shape and dimensions of the desired structure. The bulk of the material consists of fine and coarse aggregate. Cement and water interact chemically to bind the aggregate particles into a solid mass. Additional water, over and above that needed for this chemical reaction, is necessary to give the mixture the workability that enables it to fill the forms and surround the embedded reinforcing steel prior to hardening. Concretes with a wide range of properties can be obtained by appropriate adjustment of the proportions of the constituent materials. Special cements (such as high early strength cements), special aggregates (such as various lightweight or heavyweight aggregates), admixtures (such as plasticizers, air-entraining agents, silica fume, and fly ash), and special curing methods (such as steam-curing) permit an even wider variety of properties to be obtained.

These properties depend to a very substantial degree on the proportions of the mix, on the thoroughness with which the various constituents are intermixed, and on the conditions of humidity and temperature in which the mix is maintained from the moment it is placed in the forms until it is fully hardened. The process of controlling conditions after placement is known as *curing*. To protect against the unintentional production of substandard concrete, a high degree of skillful control and supervision is necessary throughout the process, from the proportioning by weight of the individual components, through mixing and placing, until the completion of curing.

The factors that make concrete a universal building material are so pronounced that it has been used, in more primitive kinds and ways than at present, for thousands of years, starting with lime mortars from 12,000 to 6000 B.C. in Crete, Cyprus, Greece, and the Middle East. The facility with which, while plastic, it can be deposited and made to fill forms or molds of almost any practical shape is one of these factors. Its high fire and weather resistance are evident advantages. Most of the constituent materials, with the exception of cement and additives, are usually available at low cost locally or at small distances from the construction site. Its compressive strength, like that of natural stones, is high, which makes it suitable for members primarily subject to compression, such as columns and arches. On the other hand, again as in natural stones, it is a relatively brittle material whose tensile strength is small compared with its compressive strength. This prevents its economical use in structural members that are subject to tension either entirely (such as in tie rods) or over part of their cross sections (such as in beams or other flexural members).

To offset this limitation, it was found possible, in the second half of the nineteenth century, to use steel with its high tensile strength to reinforce concrete, chiefly in those places where its low tensile strength would limit the carrying capacity of the member. The reinforcement, usually round steel rods with appropriate surface deformations to provide interlocking, is placed in the forms in advance of the concrete. When completely surrounded by the hardened concrete mass, it forms an integral part of the member. The resulting combination of two materials, known as *reinforced concrete,* combines many of the advantages of each: the relatively low cost, good weather and fire resistance, good compressive strength, and excellent formability of concrete and the high tensile strength and much greater ductility and toughness of steel. It is this combination that allows the almost unlimited range of uses and possibilities of reinforced concrete in the construction of buildings, bridges, dams, tanks, reservoirs, and a host of other structures.

In more recent times, it has been found possible to produce steels, at relatively low cost, whose yield strength is 3 to 4 times and more that of ordinary reinforcing steels. Likewise, it is possible to produce concrete 4 to 5 times as strong in compression as the more ordinary concretes. These high-strength materials offer many advantages, including smaller member cross sections, reduced dead load, and longer spans. However, there are limits to the strengths of the constituent materials beyond which certain problems arise. To be sure, the strength of such a member would increase roughly in proportion to those of the materials. However, the high strains that result from the high stresses that would otherwise be permissible would lead to large deformations and consequently large deflections of such members under ordinary loading conditions. Equally important, the large strains in such high-strength reinforcing steel would induce large cracks in the surrounding low tensile strength concrete, cracks that would not only be unsightly but that could significantly reduce the durability of the structure. This limits the useful yield strength of high-strength reinforcing steel to 80 ksi[†] according to many codes and specifications; 60 ksi steel is most commonly used.

A special way has been found, however, to use steels and concretes of very high strength in combination. This type of construction is known as *prestressed concrete.* The steel, in the form of wires, strands, or bars, is embedded in the concrete under high tension that is held in equilibrium by compressive stresses in the concrete after hardening. Because of this precompression, the concrete in a flexural member will crack on the tension side at a much larger load than when not so precompressed. Prestressing greatly reduces both the deflections and the tensile cracks at ordinary loads in such structures, and thereby enables these high-strength materials to be used effectively. Prestressed concrete has extended, to a very significant extent, the range of spans of structural concrete and the types of structures for which it is suited.

1.2 Structural Forms

The figures that follow show some of the principal structural forms of reinforced concrete. Pertinent design methods for many of them are discussed later in this volume.

Floor-support systems for buildings include the monolithic slab-and-beam floor shown in Fig. 1.1, the one-way joist system of Fig. 1.2, and the flat plate floor, without beams or girders, shown in Fig. 1.3. The flat slab floor of Fig. 1.4, frequently used for more heavily loaded buildings such as warehouses, is similar to the flat plate floor, but makes use of increased slab thickness in the vicinity of the columns, as well as

[†] Abbreviation for kips per square inch, or thousands of pounds per square inch.

FIGURE 1.1
One-way reinforced concrete floor slab with monolithic supporting beams. (*Portland Cement Association.*)

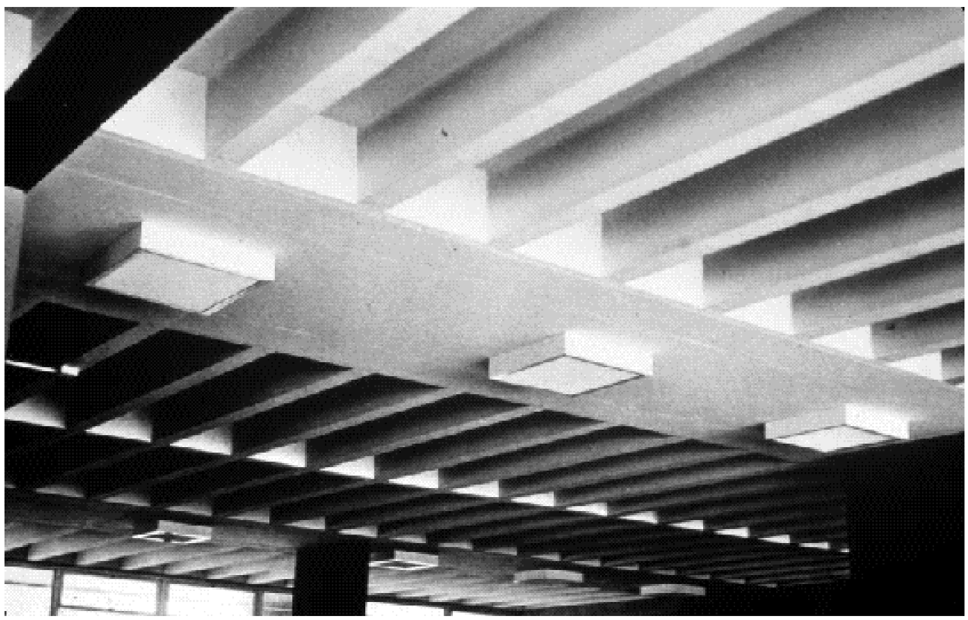


FIGURE 1.2
One-way joist floor system, with closely spaced ribs supported by monolithic concrete beams; transverse ribs provide for lateral distribution of localized loads. (*Portland Cement Association.*)

flared column tops, to reduce stresses and increase strength in the support region. The choice among these and other systems for floors and roofs depends upon functional requirements, loads, spans, and permissible member depths, as well as on cost and esthetic factors.

Where long clear spans are required for roofs, concrete shells permit use of extremely thin surfaces, often thinner, relatively, than an eggshell. The folded plate roof of Fig. 1.5 is simple to form because it is composed of flat surfaces; such roofs have been employed for spans of 200 ft and more. The cylindrical shell of Fig. 1.6 is also relatively easy to form because it has only a single curvature; it is similar to the folded plate in its structural behavior and range of spans and loads. Shells of this type were once quite popular in the United States and remain popular in other parts of the world.

Doubly curved shell surfaces may be generated by simple mathematical curves such as circular arcs, parabolas, and hyperbolas, or they may be composed of complex combinations of shapes. The hyperbolic paraboloid shape, defined by a concave downward parabola moving along a concave upward parabolic path, has been widely used.

FIGURE 1.3
Flat plate floor slab, carried directly by columns without beams or girders. (*Portland Cement Association.*)

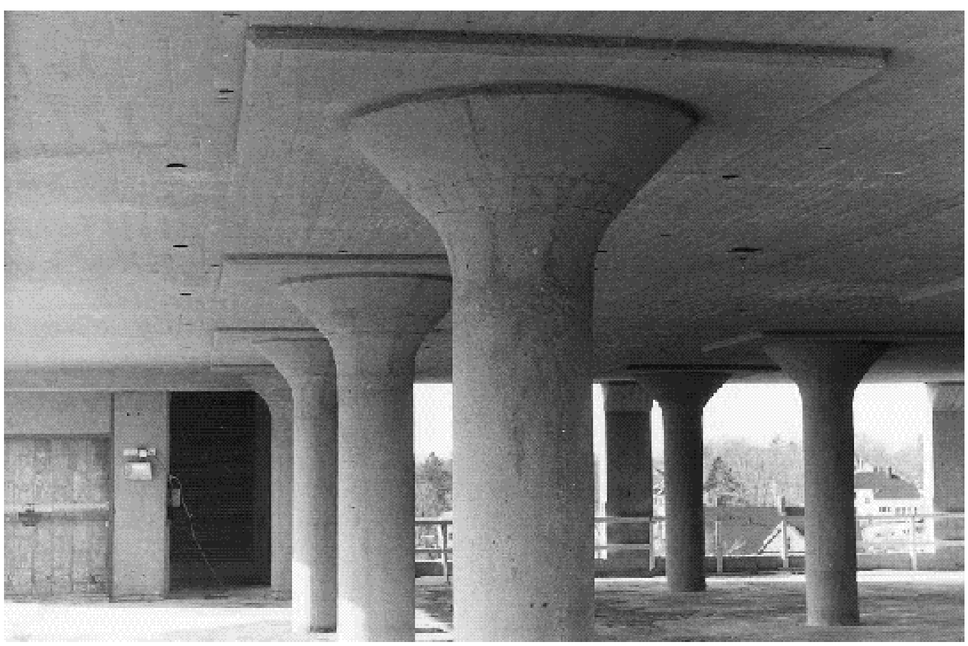


FIGURE 1.4
Flat slab floor, without beams but with slab thickness increased at the columns and with flared column tops to provide for local concentration of forces. (*University of Southern Maine.*)

It has the interesting property that the doubly curved surface contains two systems of straight-line generators, permitting straight form lumber to be used. The complex dome of Fig. 1.7, which provides shelter for performing arts events, consists essentially of a circular dome but includes monolithic, upwardly curved edge surfaces to provide stiffening and strengthening in that critical region.

FIGURE 1.5
Folded plate roof of 125 ft span that, in addition to carrying ordinary roof loads, carries the second floor as well from a system of cable hangers; the ground floor is kept free of columns.

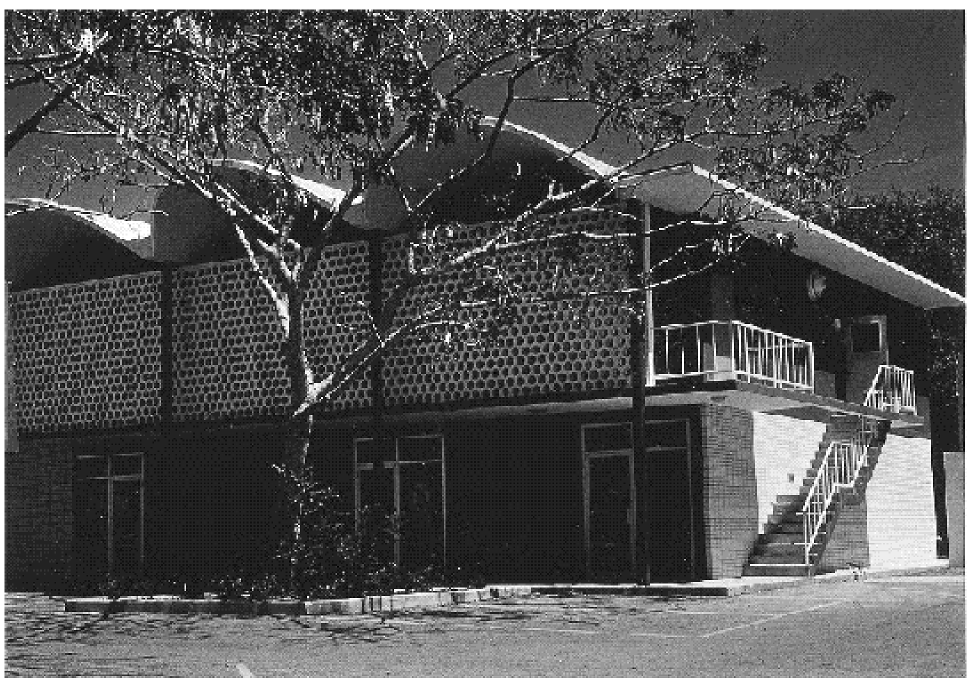


FIGURE 1.6
Cylindrical shell roof providing column-free interior space.

Bridge design has provided the opportunity for some of the most challenging and creative applications of structural engineering. The award-winning Napoleon Bonaparte Broward Bridge, shown in Fig. 1.8, is a six-lane, cable-stayed structure that spans the St. John's River at Dame Point, Jacksonville, Florida. Its 1300 ft center span is the

FIGURE 1.7
Spherical shell in Lausanne, Switzerland. Upwardly curved edges provide stiffening for the central dome.

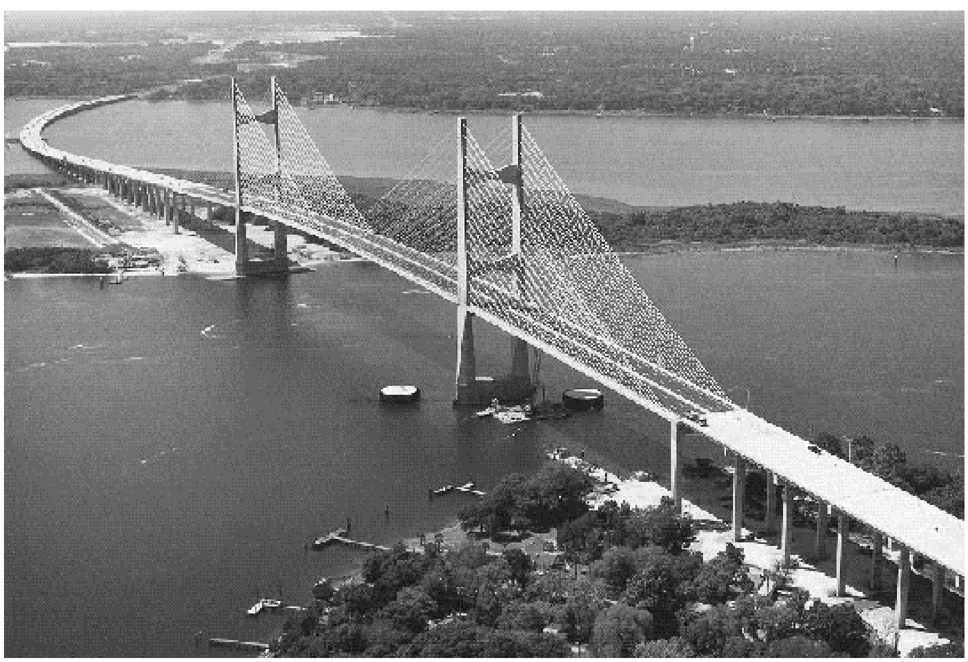


FIGURE 1.8
Napoleon Bonaparte Broward Bridge, with a 1300 ft center span at Dame Point, Jacksonville, Florida. *(HNTB Corporation, Kansas City, Missouri.)*

longest of its type in the United States. Figure 1.9 shows the Bennett Bay Centennial Bridge, a four-span continuous, segmentally cast-in-place box girder structure. Special attention was given to esthetics in this award-winning design. The spectacular Natchez Trace Parkway Bridge in Fig. 1.10, a two-span arch structure using hollow precast concrete elements, carries a two-lane highway 155 ft above the valley floor. This structure

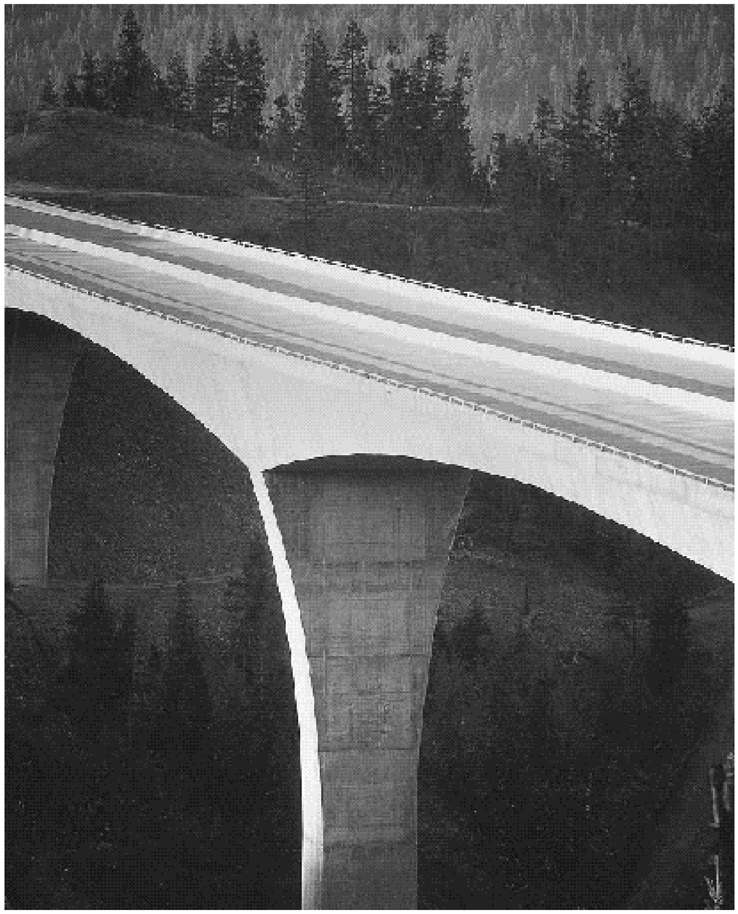


FIGURE 1.9
Bennett Bay Centennial Bridge, Coeur d'Alene, Idaho, a four-span continuous concrete box girder structure of length 1730 ft. (*HNTB Corporation, Kansas City, Missouri.*)

FIGURE 1.10
Natchez Trace Parkway Bridge near Franklin, Tennessee, an award-winning two-span concrete arch structure rising 155 ft above the valley floor. (*Figg Engineering Group, Tallahassee, Florida.*)

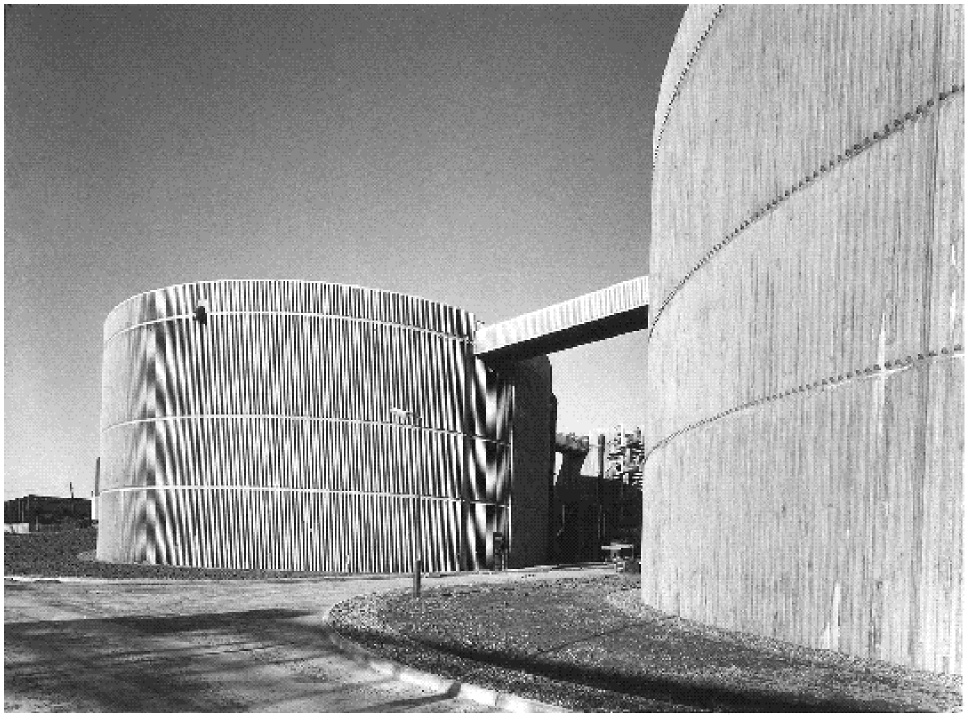


FIGURE 1.11
Circular concrete tanks used as a part of the wastewater purification facility at Howden, England. *(Northumbrian Water Authority with Luder and Jones, Architects.)*

has won many honors, including awards from the American Society of Civil Engineers and the National Endowment for the Arts.

Cylindrical concrete tanks are widely used for storage of water or in waste purification plants. The design shown in Fig. 1.11 is proof that a sanitary engineering facility can be esthetically pleasing as well as functional. Cylindrical tanks are often prestressed circumferentially to maintain compression in the concrete and eliminate the cracking that would otherwise result from internal pressure.

Concrete structures may be designed to provide a wide array of surface textures, colors, and structural forms. Figure 1.12 shows a precast concrete building containing both color changes and architectural finishes.

The forms shown in Figs. 1.1 to 1.12 hardly constitute a complete inventory but are illustrative of the shapes appropriate to the properties of reinforced or prestressed concrete. They illustrate the adaptability of the material to a great variety of one-dimensional (beams, girders, columns), two-dimensional (slabs, arches, rigid frames), and three-dimensional (shells, tanks) structures and structural components. This variability allows the shape of the structure to be adapted to its function in an economical manner, and furnishes the architect and design engineer with a wide variety of possibilities for esthetically satisfying structural solutions.

1.3 Loads

Loads that act on structures can be divided into three broad categories: dead loads, live loads, and environmental loads.

Dead loads are those that are constant in magnitude and fixed in location throughout the lifetime of the structure. Usually the major part of the dead load is the weight of the structure itself. This can be calculated with good accuracy from the design configuration, dimensions of the structure, and density of the material. For buildings, floor

FIGURE 1.12
Concrete structures can be produced in a wide range of colors, finishes, and architectural detailing. (*Courtesy of Rocky Mountain Prestress Corp.*)

fill, finish floors, and plastered ceilings are usually included as dead loads, and an allowance is made for suspended loads such as piping and lighting fixtures. For bridges, dead loads may include wearing surfaces, sidewalks, and curbs, and an allowance is made for piping and other suspended loads.

Live loads consist chiefly of occupancy loads in buildings and traffic loads on bridges. They may be either fully or partially in place or not present at all, and may also change in location. Their magnitude and distribution at any given time are uncertain, and even their maximum intensities throughout the lifetime of the structure are not known with precision. The minimum live loads for which the floors and roof of a building should be designed are usually specified in the building code that governs at the site of construction. Representative values of minimum live loads to be used in a wide variety of buildings are found in *Minimum Design Loads for Buildings and Other Structures* (Ref. 1.1), a portion of which is reprinted in Table 1.1. The table gives uniformly distributed live loads for various types of occupancies; these include impact provisions where necessary. These loads are expected maxima and considerably exceed average values.

In addition to these uniformly distributed loads, it is recommended that, as an alternative to the uniform load, floors be designed to support safely certain concentrated loads if these produce a greater stress. For example, according to Ref. 1.1, office floors are to be designed to carry a load of 2000 lb distributed over an area 2.5 ft^2, to allow for the weight of a safe or other heavy equipment, and stair treads must safely support a 300 lb load applied on the center of the tread. Certain reductions are often permitted in live loads for members supporting large areas, on the premise that it is not likely that the entire area would be fully loaded at one time (Refs. 1.1 and 1.2).

TABLE 1.1
Minimum uniformly distributed live loads

Occupancy or Use	Live Load, psf	Occupancy or Use	Live Load, psf
Apartments (see residential)		Dining rooms and restaurants	100
Access floor systems		Dwellings (see residential)	
Office use	50	Fire escapes	100
Computer use	100	On single-family dwellings only	40
Armories and drill rooms	150	Garages (passenger cars only)	40
Assembly areas and theaters		Trucks and buses[b]	
Fixed seats (fastened to floor)	60	Grandstands (see stadium and arena bleachers)	
Lobbies	100	Gymnasiums, main floors, and balconies[c]	100
Movable seats	100	Hospitals	
Platforms (assembly)	100	Operating rooms, laboratories	60
Stage floors	150	Private rooms	40
Balconies (exterior)	100	Wards	40
On one and two-family residences only, and not exceeding 100 ft^2	60	Corridors above first floor	80
		Hotels (see residential)	
Bowling alleys, poolrooms, and similar		Libraries	
recreational areas	75	Reading rooms	60
Catwalks for maintenance access	40	Stack rooms[d]	150
Corridors		Corridors above first floor	80
First floor	100	Manufacturing	
Other floors, same as occupancy served except as indicated		Light	125
		Heavy	250
Dance halls and ballrooms	100		
Decks (patio and roof)			
Same as area served, or for the type of occupancy accommodated			

(*continued*)

Tabulated live loads cannot always be used. The type of occupancy should be considered and the probable loads computed as accurately as possible. Warehouses for heavy storage may be designed for loads as high as 500 psf or more; unusually heavy operations in manufacturing buildings may require an increase in the 250 psf value specified in Table 1.1; special provisions must be made for all definitely located heavy concentrated loads.

Live loads for highway bridges are specified by the American Association of State Highway and Transportation Officials (AASHTO) in its *LRFD Bridge Design Specifications* (Ref. 1.3). For railway bridges, the American Railway Engineering and Maintenance-of-Way Association (AREMA) has published the *Manual of Railway Engineering* (Ref. 1.4), which specifies traffic loads.

Environmental loads consist mainly of snow loads, wind pressure and suction, earthquake loads (i.e., inertia forces caused by earthquake motions), soil pressures on subsurface portions of structures, loads from possible ponding of rainwater on flat surfaces, and forces caused by temperature differentials. Like live loads, environmental loads at any given time are uncertain both in magnitude and distribution. Reference 1.1 contains much information on environmental loads, which is often modified locally depending, for instance, on local climatic or seismic conditions.

Figure 1.13, from the 1972 edition of Ref. 1.1, gives snow loads for the continental United States, and is included here for illustration only. The 2002 edition of

TABLE 1.1 (Continued)

Occupancy or Use	Live Load, psf	Occupancy or Use	Live Load, psf
Marquees and Canopies	75	Sidewalks, vehicular driveways, and yards, subject to trucking[e]	250
Office Buildings		Stadiums and arenas	
File and computer rooms shall be designed for heavier loads based on anticipated occupancy		Bleachers[c]	100
Lobbies and first-floor corridors	100	Fixed seats (fastened to floor)[c]	60
Offices	50	Stairs and exitways	100
Corridors above first floor	80	One and two-family residences only	40
Penal institutions		Storage areas above ceilings	20
Cell blocks	40	Storage warehouses (shall be designed for heavier loads if required for anticipated storage)	
Corridors	100	Light	125
Residential		Heavy	250
Dwellings (one and two-family)		Stores	
Uninhabitable attics without storage	10	Retail	
Uninhabitable attics with storage	20	First floor	100
Habitable attics and sleeping areas	30	Upper floors	73
All other areas except stairs and balconies	40	Wholesale, all floors	125
Hotels and multifamily houses		Walkways and elevated platforms (other than exitways)	60
Private rooms and corridors serving them	40	Yards and terraces, pedestrians	100
Public rooms and corridors serving them	100		
Reviewing stands, grandstands, and bleachers[c]	100		
Schools			
Classrooms	40		
Corridors above first floor	80		
First-floor corridors	100		

[a] Pounds per square foot.

[b] Garages accommodating trucks and buses shall be designed in accordance with an approved method that contains provisions for truck and bus loadings.

[c] In addition to the vertical live loads, the design shall include horizontal swaying forces applied to each row of seats as follows: 24 lb per linear ft of seat applied in the direction parallel to each row of seats and 10 lb per linear ft of seat applied in the direction perpendicular to each row of seats. The parallel and perpendicular horizontal swaying forces need not be applied simultaneously.

[d] The loading applies to stack room floors that support nonmobile, double-faced library bookstacks subject to the following limitations: (1) the nominal bookstack unit height shall not exceed 90 in.; (2) the nominal shelf depth shall not exceed 12 in. for each face; and (3) parallel rows of double-faced bookstacks shall be separated by aisles not less than 36 in. wide.

[e] Other uniform loads in accordance with an approved method that contains provisions for truck loadings shall also be considered where appropriate.

Source: From Ref. 1.1. Used by permission of the American Society of Civil Engineers.

Ref. 1.1 gives much more detailed information. In either case, specified values represent not average values, but expected upper limits. A minimum roof load of 20 psf is often specified to provide for construction and repair loads and to ensure reasonable stiffness.

Much progress has been made in recent years in developing rational methods for predicting horizontal forces on structures due to wind and seismic action. Reference 1.1 summarizes current thinking regarding wind forces, and has much information pertaining to earthquake loads as well. Reference 1.5 presents detailed recommendations for lateral forces from earthquakes.

Reference 1.1 specifies design wind pressures per square foot of vertical wall surface. Depending upon locality, these equivalent static forces vary from about 10 to 50 psf.

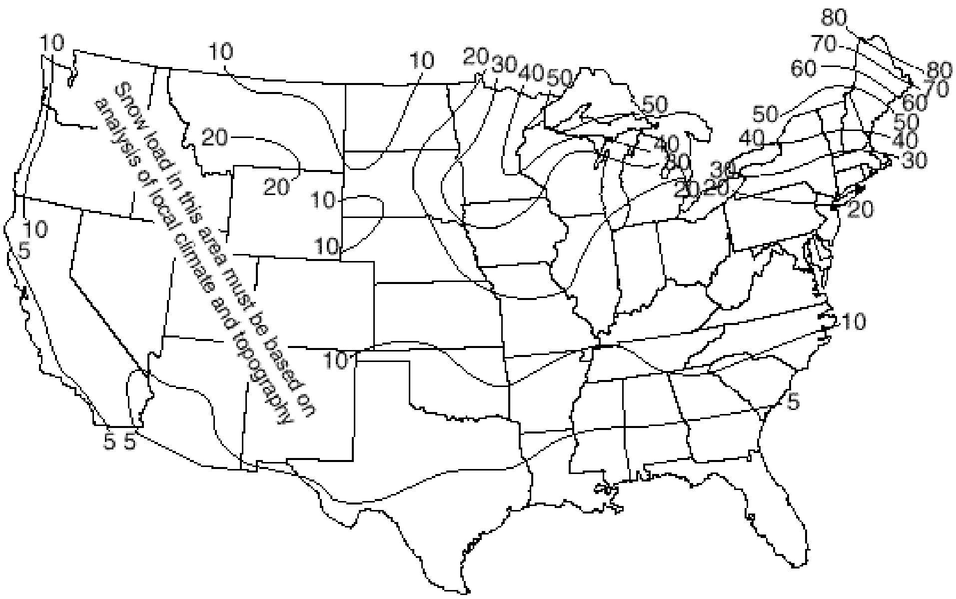


FIGURE 1.13
Snow load in pounds per square foot (psf) on the ground, 50-year mean recurrence interval. (*From* Minimum Design Loads for Buildings and Other Structures, *ANSI A58.1–1972, American National Standards Institute, New York, NY, 1972.*)

Factors include basic wind speed, exposure (urban vs. open terrain, for example), height of the structure, the importance of the structure (i.e., consequences of failure), and gust-effect factors to account for the fluctuating nature of the wind and its interaction with the structure.

Seismic forces may be found for a particular structure by elastic or inelastic dynamic analysis, considering expected ground accelerations and the mass, stiffness, and damping characteristics of the construction. However, often the design is based on equivalent static forces calculated from provisions such as those of Refs. 1.1 and 1.5. The base shear is found by considering such factors as location, type of structure and its occupancy, total dead load, and the particular soil condition. The total lateral force is distributed to floors over the entire height of the structure in such a way as to approximate the distribution of forces obtained from a dynamic analysis.

1.4 Serviceability, Strength, and Structural Safety

To serve its purpose, a structure must be safe against collapse and serviceable in use. Serviceability requires that deflections be adequately small; that cracks, if any, be kept to tolerable limits; that vibrations be minimized; etc. Safety requires that the strength of the structure be adequate for all loads that may foreseeably act on it. If the strength of a structure, built as designed, could be predicted accurately, and if the loads and their internal effects (moments, shears, axial forces) were known accurately, safety could be ensured by providing a carrying capacity just barely in excess of the known loads. However, there are a number of sources of uncertainty in the analysis, design, and construction of reinforced concrete structures. These sources of uncertainty, which require a definite margin of safety, may be listed as follows:

1. Actual loads may differ from those assumed.
2. Actual loads may be distributed in a manner different from that assumed.

3. The assumptions and simplifications inherent in any analysis may result in calculated load effects—moments, shears, etc.—different from those that, in fact, act in the structure.
4. The actual structural behavior may differ from that assumed, owing to imperfect knowledge.
5. Actual member dimensions may differ from those specified.
6. Reinforcement may not be in its proper position.
7. Actual material strength may be different from that specified.

In addition, in the establishment of a safety specification, consideration must be given to the consequences of failure. In some cases, a failure would merely be an inconvenience. In other cases, loss of life and significant loss of property may be involved. A further consideration should be the nature of the failure, should it occur. A gradual failure with ample warning permitting remedial measures is preferable to a sudden, unexpected collapse.

It is evident that the selection of an appropriate margin of safety is not a simple matter. However, progress has been made toward rational safety provisions in design codes (Refs. 1.6 to 1.9).

a. Variability of Loads

Since the maximum load that will occur during the life of a structure is uncertain, it can be considered a random variable. In spite of this uncertainty, the engineer must provide an adequate structure. A probability model for the maximum load can be devised by means of a probability density function for loads, as represented by the frequency curve of Fig. 1.14*a*. The exact form of this distribution curve, for any particular type of loading such as office loads, can be determined only on the basis of statistical data obtained from large-scale load surveys. A number of such surveys have been completed. For types of loads for which such data are scarce, fairly reliable information can be obtained from experience, observation, and judgment.

In such a frequency curve (Fig. 1.14*a*), the area under the curve between two abscissas, such as loads $Q_1$and Q_2, represents the probability of occurrence of loads Q of magnitude $Q_1 < Q < Q_2$. A specified service load Q_d for design is selected conservatively in the upper region of Q in the distribution curve, as shown. The probability of occurrence of loads larger than Q_d is then given by the shaded area to the right of Q_d. It is seen that this specified service load is considerably larger than the mean load $\bar{Q}$ acting on the structure. This mean load is much more typical of average load conditions than the design load Q_d.

b. Strength

The strength of a structure depends on the strength of the materials from which it is made. For this purpose, minimum material strengths are specified in standardized ways. Actual material strengths cannot be known precisely and therefore also constitute random variables (see Section 2.6). Structural strength depends, furthermore, on the care with which a structure is built, which in turn reflects the quality of supervision and inspection. Member sizes may differ from specified dimensions, reinforcement may be out of position, poorly placed concrete may show voids, etc.

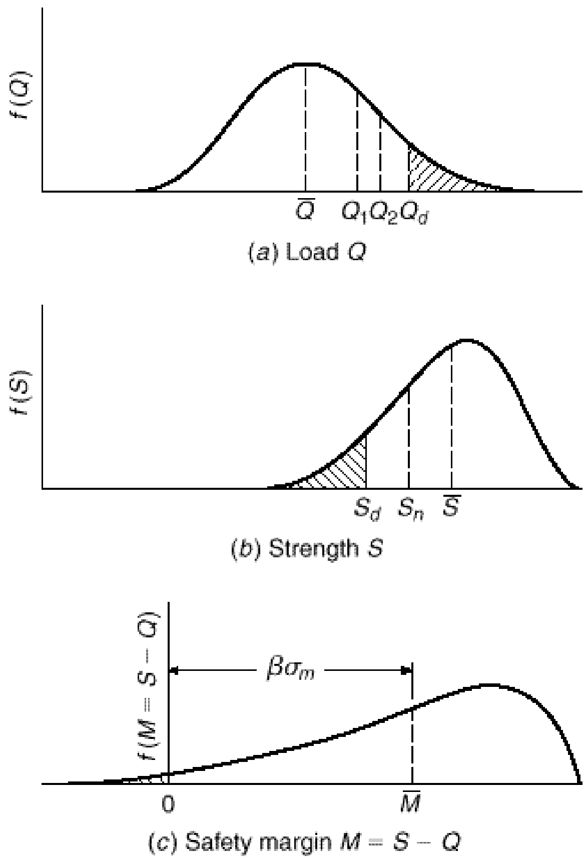


FIGURE 1.14
Frequency curves for (*a*) loads, *Q*; (*b*) strengths, *S*; and (*c*) safety margin, *M*.

Strength of the entire structure or of a population of repetitive structures, e.g., highway overpasses, can also be considered a random variable with a probability density function of the type shown in Fig. 1.14*b*. As in the case of loads, the exact form of this function cannot be known but can be approximated from known data, such as statistics of actual, measured materials and member strengths and similar information. Considerable information of this type has been, or is being, developed and used.

c. Structural Safety

A given structure has a *safety margin M* if

$$M = S - Q > 0 \tag{1.1}$$

i.e., if the strength of the structure is larger than the load acting on it. Since S and Q are random variables, the safety margin $M = S - Q$ is also a random variable. A plot of the probability function of M may appear as in Fig. 1.14*c*. Failure occurs when M is less than zero. Thus, the probability of failure is represented by the shaded area in the figure.

Even though the precise form of the probability density functions for S and Q, and therefore for M, is not known, much can be achieved in the way of a rational approach to structural safety. One such approach is to require that the mean safety margin M be a specified number · of standard deviations $\cdot_m$ above zero. It can be demonstrated that this results in the requirement that

$$\cdot_s \overline{S} \geq \cdot_L \overline{Q} \tag{1.2}$$

where $\cdot_s$ is a partial safety coefficient smaller than one applied to the mean strength $\overline{S}$, and $\cdot_L$ is a partial safety coefficient larger than one applied to the mean load $\overline{Q}$. The magnitude of each partial safety coefficient depends on the variance of the quantity to which it applies, S or Q, and on the chosen value of · , the reliability index of the structure. As a general guide, a value of the safety index · between 3 and 4 corresponds to a probability of failure of the order of 1:100,000 (Ref. 1.8). The value of · is often established by calibration against well-proved and established designs.

In practice, it is more convenient to introduce partial safety coefficients with respect to code-specified loads which, as already noted, considerably exceed average values, rather than with respect to mean loads as in Eq. (1.2); similarly, the partial safety coefficient for strength is applied to nominal strength generally computed somewhat conservatively, rather than to mean strengths as in Eq. (1.2). A restatement of the safety requirement in these terms is

$$\cdot S_n \geq \cdot Q_d \tag{1.3a}$$

in which · is a strength reduction factor applied to nominal strength S_n and · is a load factor applied to calculated or code-specified design loads Q_d. Furthermore, recognizing the differences in variability between, say, dead loads D and live loads L, it is both reasonable and easy to introduce different load factors for different types of loads. The preceding equation can thus be written

$$\cdot S_n \geq \cdot_d D + \cdot_l L \tag{1.3b}$$

in which $\cdot_d$ is a load factor somewhat greater than one applied to the calculated dead load D, and $\cdot_l$ is a larger load factor applied to the code-specified live load L. When additional loads, such as the wind load W, are to be considered, the reduced probability that maximum dead, live, and wind or other loads will act simultaneously can be incorporated by including a factor · less than 1 such that

$$\cdot S_n \geq \cdot(\cdot_d D + \cdot_l L + \cdot_w W + \cdots) \tag{1.3c}$$

Present U.S. design specifications follow the format of Eqs. (1.3*b*) and (1.3*c*).

1.5 Design Basis

The single most important characteristic of any structural member is its actual strength, which must be large enough to resist, with some margin to spare, all foreseeable loads that may act on it during the life of the structure, without failure or other distress. It is logical, therefore, to proportion members, i.e., to select concrete dimensions and reinforcement, so that member strengths are adequate to resist forces resulting from certain hypothetical overload stages, significantly above loads expected actually to occur in service. This design concept is known as *strength design*.

For reinforced concrete structures at loads close to and at failure, one or both of the materials, concrete and steel, are invariably in their nonlinear inelastic range. That is, concrete in a structural member reaches its maximum strength and subsequent fracture at stresses and strains far beyond the initial elastic range in which stresses and strains are fairly proportional. Similarly, steel close to and at failure of the member is usually stressed beyond its elastic domain into and even beyond the yield region.

Consequently, the nominal strength of a member must be calculated on the basis of this inelastic behavior of the materials.

A member designed by the strength method must also perform in a satisfactory way under normal service loading. For example, beam deflections must be limited to acceptable values, and the number and width of flexural cracks at service loads must be controlled. Serviceability limit conditions are an important part of the total design, although attention is focused initially on strength.

Historically, members were proportioned so that stresses in the steel and concrete resulting from normal service loads were within specified limits. These limits, known as *allowable stresses*, were only fractions of the failure stresses of the materials. For members proportioned on such a service load basis, the margin of safety was provided by stipulating allowable stresses under service loads that were appropriately small fractions of the compressive concrete strength and the steel yield stress. We now refer to this basis for design as *service load design*. Allowable stresses, in practice, were set at about one-half the concrete compressive strength and one-half the yield stress of the steel.

Because of the difference in realism and reliability, over the past several decades the strength design method has displaced the older service load design method. However, the older method is still used occasionally and is the design basis for many older structures. Throughout this text, strength design is presented almost exclusively.

1.6 Design Codes and Specifications

The design of concrete structures such as those of Figs. 1.1 to 1.12 is generally done within the framework of codes giving specific requirements for materials, structural analysis, member proportioning, etc. The International Building Code (Ref. 1.2) is an example of a consensus code governing structural design and is often adopted by local municipalities. The responsibility of preparing material-specific portions of the codes rests with various professional groups, trade associations, and technical institutes. In contrast with many other industrialized nations, the United States does not have an official, government-sanctioned, national code.

The American Concrete Institute (ACI) has long been a leader in such efforts. As one part of its activity, the American Concrete Institute has published the widely recognized *Building Code Requirements for Structural Concrete* (Ref. 1.10), which serves as a guide in the design and construction of reinforced concrete buildings. The ACI Code has no official status in itself. However, it is generally regarded as an authoritative statement of current good practice in the field of reinforced concrete. As a result, it has been incorporated into the International Building Code and similar codes, which in turn are adopted by law into municipal and regional building codes that do have legal status. Its provisions thereby attain, in effect, legal standing. Most reinforced concrete buildings and related construction in the United States are designed in accordance with the current ACI Code. It has also served as a model document for many other countries. A second ACI publication, *Commentary on Building Code Requirements for Structural Concrete* (Ref. 1.11), provides background material and rationale for the Code provisions. The American Concrete Institute also publishes important journals and standards, as well as recommendations for the analysis and design of special types of concrete structures such as the tanks of Fig. 1.11.

Most highway bridges in the United States are designed according to the requirements of the AASHTO bridge specifications (Ref. 1.3) which not only contain the provisions relating to loads and load distributions mentioned earlier, but also include detailed provisions for the design and construction of concrete bridges. Many of the provisions follow ACI Code provisions closely, although a number of significant differences will be found.

The design of railway bridges is done according to the specifications of the AREMA *Manual of Railway Engineering* (Ref. 1.4). It, too, is patterned after the ACI Code in most respects, but it contains much additional material pertaining to railway structures of all types.

No code or design specification can be construed as a substitute for sound engineering judgment in the design of concrete structures. In structural practice, special circumstances are frequently encountered where code provisions can serve only as a guide, and the engineer must rely upon a firm understanding of the basic principles of structural mechanics applied to reinforced or prestressed concrete, and an intimate knowledge of the nature of the materials.

1.7 Safety Provisions of the ACI Code

The safety provisions of the ACI Code are given in the form of Eqs. (1.3*b*) and (1.3*c*) using strength reduction factors and load factors. These factors are based to some extent on statistical information but to a larger degree on experience, engineering judgment, and compromise. In words, the design strength $\cdot S_n$ of a structure or member must be at least equal to the required strength U calculated from the factored loads, i.e.,

$$\text{Design strength} \geq \text{required strength}$$

or

$$\cdot S_n \geq U \tag{1.4}$$

The nominal strength S_n is computed (usually somewhat conservatively) by accepted methods. The required strength U is calculated by applying appropriate load factors to the respective service loads: dead load D, live load L, wind load W, earthquake load E, earth pressure H, fluid pressure F, impact allowance I, and environmental effects T that may include settlement, creep, shrinkage, and temperature change. Loads are defined in a general sense, to include either loads or the related internal effects such as moments, shears, and thrusts. Thus, in specific terms for a member subjected, say, to moment, shear, and axial load:

$$\cdot M_n \geq M_u \tag{1.5a}$$

$$\cdot V_n \geq V_u \tag{1.5b}$$

$$\cdot P_n \geq P_u \tag{1.5c}$$

where the subscripts n denote the nominal strengths in flexure, shear, and axial load, respectively, and the subscripts u denote the factored load moment, shear, and axial load. In computing the factored load effects on the right, load factors may be applied either to the service loads themselves or to the internal load effects calculated from the service loads.

The load factors specified in the ACI Code, to be applied to calculated dead loads and those live and environmental loads specified in the appropriate codes or standards, are summarized in Table 1.2. These are consistent with the concepts introduced in

TABLE 1.2
Factored load combinations for determining required strength in the ACI Code

Condition	Factored Load or Load Effect
Basic[b]	$U = 1.2D + 1.6L$
Dead plus Fluid[b]	$U = 1.4(D + F)$
Snow, Rain, Temperature, and Wind	$U = 1.2(D + F + T) + 1.6(L + H)$ 1 $0.5(L_r$ or S or $R)$
	$U = 1.2D + 1.6(L_r$ or S or $R) + (1.0L$ or $0.8W)$
	$U = 1.2D + 1.6W + 1.0L + 0.5 \cdot L_r$ or S or R
	$U = 0.9D + 1.6W + 1.6H$
Earthquake	$U = 1.2D + 1.0E + 1.0L + 0.2S$
	$U = 0.9D + 1.0E + 1.6H$

[a] Where the following represent the loads or related internal moments or forces resulting from the listed factors: D = dead load; E = earthquake; F = fluids; H = weight or pressure from soil; L = live load; L_r = roof live load; R = rain; S = snow; T = cumulative effects of temperature, creep, shrinkage, and differential settlement; W = wind.

[b] The ACI Code includes F or H loads in the load combinations. The "Basic" load condition of $1.2D + 1.6L$ reflects the fact that most buildings have neither F nor H loads present and that $1.4D$ rarely governs design.

Section 1.4 and with SEI/ASCE 7, *Minimum Design Loads for Buildings and Other Structures* (Ref. 1.1), and allows design of composite structures using combinations of structural steel and reinforced concrete. For individual loads, lower factors are used for loads known with greater certainty, e.g., dead load, compared with loads of greater variability, e.g., live loads. Further, for load combinations such as dead plus live loads plus wind forces, reductions are applied to one load or the other that reflect the improbability that an excessively large live load coincides with an unusually high windstorm. The factors also reflect, in a general way, uncertainties with which internal load effects are calculated from external loads in systems as complex as highly indeterminate, inelastic reinforced concrete structures which, in addition, consist of variable-section members (because of tension cracking, discontinuous reinforcement, etc.). Finally, the load factors also distinguish between two situations, particularly when horizontal forces are present in addition to gravity, i.e., the situation where the effects of all simultaneous loads are additive, as distinct from that in which various load effects counteract each other. For example, in a retaining wall the soil pressure produces an overturning moment, and the gravity forces produce a counteracting stabilizing moment.

In all cases in Table 1.2, the controlling equation is the one that gives the largest factored load effect U.

The strength reduction factors · in the ACI Code are given different values depending on the state of knowledge, i.e., the accuracy with which various strengths can be calculated. Thus, the value for bending is higher than that for shear or bearing. Also, · values reflect the probable importance, for the survival of the structure, of the particular member and of the probable quality control achievable. For both these reasons, a lower value is used for columns than for beams. Table 1.3 gives the · values specified in the ACI Code.

The joint application of strength reduction factors (Table 1.3) and load factors (Table 1.2) is aimed at producing approximate probabilities of understrength of the order of 1/100 and of overloads of 1/1000. This results in a probability of structural failure of the order of 1/100,000.

TABLE 1.3
Strength reduction factors in the ACI Code

Strength Condition	Strength Reduction Factor ·
Tension-controlled sections	0.90
Compression-controlled sections[a]	
Members with spiral reinforcement	0.70
Other reinforced members	0.65
Shear and torsion	0.75
Bearing on concrete	0.65
Post-tensioned anchorage zones	0.85
Strut-and-tie models[b]	0.75

[a] Chapter 3 contains a discussion of the linear variation of · between tension and compression-controlled sections. Chapter 8 discusses the conditions that allow an increase in · for spirally reinforced columns.
[b] Strut-and-tie models are described in Chapter 10.

In addition to the values given in Table 1.3, ACI Code Appendix B, "Alternative Provisions for Reinforced and Prestressed Concrete Flexural and Compression Members," allows the use of load factors and strength reduction factors from previous editions of the ACI Code. The load factors and strength reduction factors of ACI Code Appendix B are calibrated in conjunction with the detailed requirements of that appendix. Consequently, they may not be interchanged with the provisions of the main body of the Code.

1.8 Fundamental Assumptions for Reinforced Concrete Behavior

The chief task of the structural engineer is the design of structures. *Design* is the determination of the general shape and all specific dimensions of a particular structure so that it will perform the function for which it is created and will safely withstand the influences that will act on it throughout its useful life. These influences are primarily the loads and other forces to which it will be subjected, as well as other detrimental agents, such as temperature fluctuations, foundation settlements, and corrosive influences. *Structural mechanics* is one of the main tools in this process of design. As here understood, it is the body of scientific knowledge that permits one to predict with a good degree of certainty how a structure of given shape and dimensions will behave when acted upon by known forces or other mechanical influences. The chief items of behavior that are of practical interest are (1) the strength of the structure, i.e., that magnitude of loads of a given distribution which will cause the structure to fail, and (2) the deformations, such as deflections and extent of cracking, that the structure will undergo when loaded under service conditions.

The fundamental propositions on which the mechanics of reinforced concrete is based are as follows:

1. The internal forces, such as bending moments, shear forces, and normal and shear stresses, at any section of a member are in equilibrium with the effects of the external loads at that section. This proposition is not an assumption but a fact, because any body or any portion thereof can be at rest only if all forces acting on it are in equilibrium.

2. The strain in an embedded reinforcing bar (unit extension or compression) is the same as that of the surrounding concrete. Expressed differently, it is assumed that perfect bonding exists between concrete and steel at the interface, so that no slip can occur between the two materials. Hence, as the one deforms, so must the other. With modern deformed bars (see Section 2.14), a high degree of mechanical interlocking is provided in addition to the natural surface adhesion, so this assumption is very close to correct.
3. Cross sections that were plane prior to loading continue to be plane in the member under load. Accurate measurements have shown that when a reinforced concrete member is loaded close to failure, this assumption is not absolutely accurate. However, the deviations are usually minor, and the results of theory based on this assumption check well with extensive test information.
4. In view of the fact that the tensile strength of concrete is only a small fraction of its compressive strength (see Section 2.9), the concrete in that part of a member which is in tension is usually cracked. While these cracks, in well-designed members, are generally so narrow as to be hardly visible (they are known as *hairline* cracks), they evidently render the cracked concrete incapable of resisting tension stress. Correspondingly, it is assumed that concrete is not capable of resisting any tension stress whatever. This assumption is evidently a simplification of the actual situation because, in fact, concrete prior to cracking, as well as the concrete located between cracks, does resist tension stresses of small magnitude. Later in discussions of the resistance of reinforced concrete beams to shear, it will become apparent that under certain conditions this particular assumption is dispensed with and advantage is taken of the modest tensile strength that concrete can develop.
5. The theory is based on the actual stress-strain relationships and strength properties of the two constituent materials (see Sections 2.8 and 2.14) or some reasonable equivalent simplifications thereof. The fact that nonelastic behavior is reflected in modern theory, that concrete is assumed to be ineffective in tension, and that the joint action of the two materials is taken into consideration results in analytical methods which are considerably more complex, and also more challenging, than those that are adequate for members made of a single, substantially elastic material.

These five assumptions permit one to predict by calculation the performance of reinforced concrete members only for some simple situations. Actually, the joint action of two materials as dissimilar and complicated as concrete and steel is so complex that it has not yet lent itself to purely analytical treatment. For this reason, methods of design and analysis, while using these assumptions, are very largely based on the results of extensive and continuing experimental research. They are modified and improved as additional test evidence becomes available.

1.9 Behavior of Members Subject to Axial Loads

Many of the fundamentals of the behavior of reinforced concrete, through the full range of loading from zero to ultimate, can be illustrated clearly in the context of members subject to simple axial compression or tension. The basic concepts illustrated here will be recognized in later chapters in the analysis and design of beams, slabs, eccentrically loaded columns, and other members subject to more complex loadings.

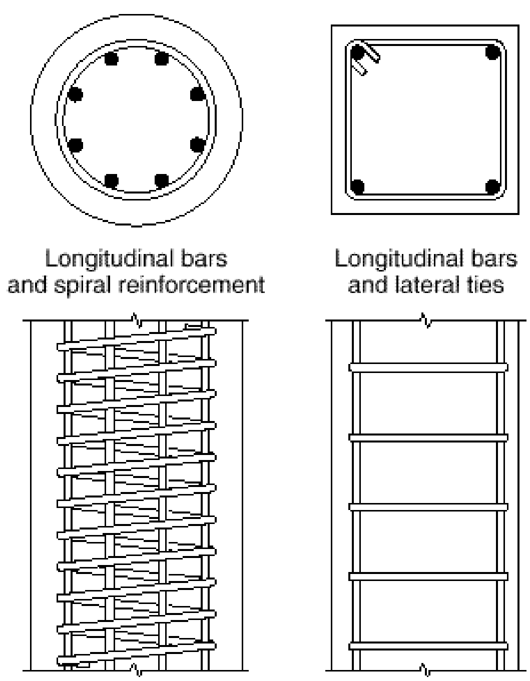
a. Axial Compression

In members that sustain chiefly or exclusively axial compression loads, such as building columns, it is economical to make the concrete carry most of the load. Still, some steel reinforcement is always provided for various reasons. For one, very few members are truly axially loaded; steel is essential for resisting any bending that may exist. For another, if part of the total load is carried by steel with its much greater strength, the cross-sectional dimensions of the member can be reduced—the more so, the larger the amount of reinforcement.

The two chief forms of reinforced concrete columns are shown in Fig. 1.15. In the square column, the four longitudinal bars serve as main reinforcement. They are held in place by transverse small-diameter steel ties that prevent displacement of the main bars during construction operations and counteract any tendency of the compression-loaded bars to buckle out of the concrete by bursting the thin outer cover. On the left is shown a round column with eight main reinforcing bars. These are surrounded by a closely spaced spiral that serves the same purpose as the more widely spaced ties but also acts to confine the concrete within it, thereby increasing its resistance to axial compression. The discussion that follows applies to tied columns.

When axial load is applied, the compression strain is the same over the entire cross section, and in view of the bonding between concrete and steel, is the same in the two materials (see propositions 2 and 3 in Section 1.8). To illustrate the action of such a member as load is applied, Fig. 1.16 shows two typical stress-strain curves, one for a concrete with compressive strength $f'_c = 4000$ psi and the other for a steel with yield stress $f_y = 60{,}000$ psi. The curves for the two materials are drawn on the same graph using different vertical stress scales. Curve b has the shape which would be

FIGURE 1.15
Reinforced concrete columns.



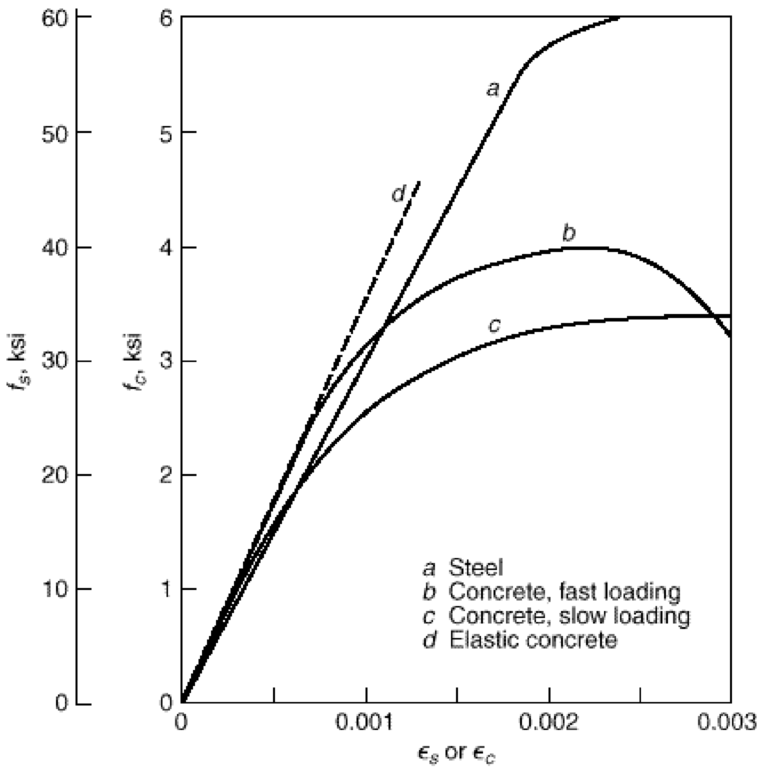


FIGURE 1.16
Concrete and steel stress-strain curves.

obtained in a concrete cylinder test. The rate of loading in most structures is considerably slower than that in a cylinder test, and this affects the shape of the curve. Curve *c*, therefore, is drawn as being characteristic of the performance of concrete under slow loading. Under these conditions, tests have shown that the maximum reliable compressive strength of reinforced concrete is about $0.85 f'_c$, as shown.

Elastic Behavior At low stresses, up to about $f'_c/2$, the concrete is seen to behave nearly elastically, i.e., stresses and strains are quite closely proportional; the straight line *d* represents this range of behavior with little error for both rates of loading. For the given concrete the range extends to a strain of about 0.0005. The steel, on the other hand, is seen to be elastic nearly to its yield point of 60 ksi, or to the much greater strain of about 0.002.

Because the compression strain in the concrete, at any given load, is equal to the compression strain in the steel,

$$\epsilon_c = \frac{f_c}{E_c} = \epsilon_s = \frac{f_s}{E_s}$$

from which the relation between the steel stress f_s and the concrete stress f_c is obtained as

$$f_s = \frac{E_s}{E_c} f_c = n f_c \tag{1.6}$$

where $n = E_s/E_c$ is known as the *modular ratio*.

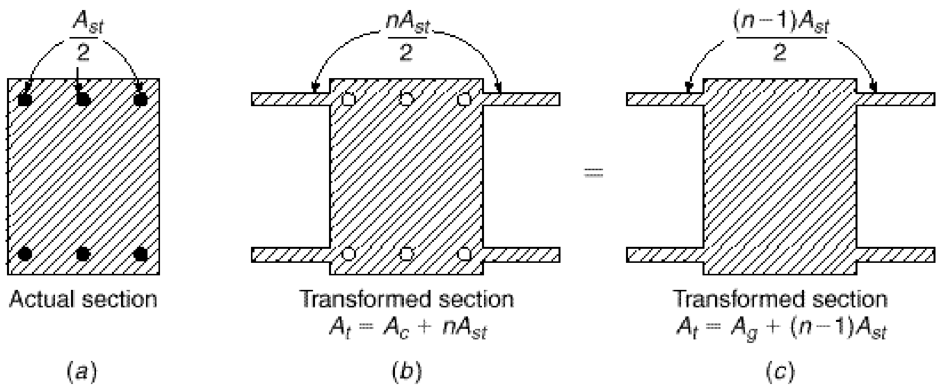


FIGURE 1.17
Transformed section in axial compression.

Let A_c = net area of concrete, i.e., gross area minus area occupied by reinforcing bars
A_g = gross area
A_{st} = total area of reinforcing bars
P = axial load

Then

$$P = f_c A_c + f_s A_{st} = f_c A_c + n f_c A_{st}$$

or

$$P = f_c(A_c + nA_{st}) \tag{1.7}$$

The term $A_c + nA_{st}$ can be interpreted as the area of a fictitious concrete cross section, the so-called *transformed area*, which when subjected to the particular concrete stress f_c results in the same axial load P as the actual section composed of both steel and concrete. This transformed concrete area is seen to consist of the actual concrete area plus n times the area of the reinforcement. It can be visualized as shown in Fig. 1.17. That is, in Fig. 1.17*b* the three bars along each of the two faces are thought of as being removed and replaced, at the same distance from the axis of the section, with added areas of fictitious concrete of total amount nA_{st}. Alternatively, as shown in Fig. 1.17*c*, one can think of the area of the steel bars as replaced with concrete, in which case one has to add to the gross concrete area A_g so obtained only $(n - 1)A_{st}$ to obtain the same total transformed area. Therefore, alternatively,

$$P = f_c[A_g + (n - 1)A_{st}] \tag{1.8}$$

If load and cross-sectional dimensions are known, the concrete stress can be found by solving Eq. (1.7) or (1.8) for f_c, and the steel stress can be calculated from Eq. (1.6). These relations hold in the range in which the concrete behaves nearly elastically, i.e., up to about 50 to 60 percent of f'_c. For reasons of safety and serviceability, concrete stresses in structures under normal conditions are kept within this range. Therefore, these relations permit one to calculate *service load stresses*.

EXAMPLE 1.1

A column made of the materials defined in Fig. 1.16 has a cross section of 16 × 20 in. and is reinforced by six No. 9 (No. 29) bars, disposed as shown in Fig. 1.17. (See Tables A.1 and A.2 of Appendix A for bar diameters and areas and Section 2.14 for a description of bar size designations.) Determine the axial load that will stress the concrete to 1200 psi. The modular ratio n may be assumed equal to 8. (In view of the scatter inherent in E_c, it is customary and satisfactory to round off the value of n to the nearest integer.)

Solution. One finds $A_g = 16 \times 20 = 320$ in^2, and from Appendix A, Table A.2, $A_{st} = 6.00$ in^2 or 1.88 percent of the gross area. The load on the column, from Eq. (1.8), is $P = 1200[320 + (8 - 1)6.00] = 434{,}000$ lb. Of this total load, the concrete is seen to carry $P_c = f_c A_c = f_c(A_g - A_{st}) = 1200(320 - 6) = 377{,}000$ lb, and the steel $P_s = f_s A_{st} = (nf_c)A_{st} = 9600 \times 6 = 57{,}600$ lb, which is 13.3 percent of the total axial load.

Inelastic Range Inspection of Fig. 1.16 shows that the elastic relationships that have been used so far cannot be applied beyond a strain of about 0.0005 for the given concrete. To obtain information on the behavior of the member at larger strains and, correspondingly, at larger loads, it is therefore necessary to make direct use of the information in Fig. 1.16.

EXAMPLE 1.2

One may want to calculate the magnitude of the axial load that will produce a strain or unit shortening $\cdot_c = \cdot_s = 0.0010$ in the column of the preceding example. At this strain the steel is seen to be still elastic, so that the steel stress $f_s = \cdot_s E_s = 0.001 \times 29{,}000{,}000 = 29{,}000$ psi. The concrete is in the inelastic range, so that its stress cannot be directly calculated, but it can be read from the stress-strain curve for the given value of strain.

1. If the member has been loaded at a fast rate, curve *b* holds at the instant when the entire load is applied. The stress for $\cdot = 0.001$ can be read as $f_c = 3200$ psi. Consequently, the total load can be obtained from

$$P = f_c A_c + f_s A_{st} \tag{1.9}$$

which applies in the inelastic as well as in the elastic range. Hence, $P = 3200(320 - 6) + 29{,}000 \times 6 = 1{,}005{,}000 + 174{,}000 = 1{,}179{,}000$ lb. Of this total load, the steel is seen to carry 174,000 lb, or 14.7 percent.

2. For slowly applied or sustained loading, curve *c* represents the behavior of the concrete. Its stress at a strain of 0.001 can be read as $f_c = 2400$ psi. Then $P = 2400 \times 314 + 29{,}000 \times 6 = 754{,}000 + 174{,}000 = 928{,}000$ lb. Of this total load, the steel is seen to carry 18.8 percent.

Comparison of the results for fast and slow loading shows the following. Owing to creep of concrete, a given shortening of the column is produced by a smaller load when slowly applied or sustained over some length of time than when quickly applied. More important, the farther the stress is beyond the proportional limit of the concrete, and the more slowly the load is applied or the longer it is sustained, the smaller the share of the total load carried by the concrete, and the larger the share carried by the steel. In the sample column, the steel was seen to carry 13.3 percent of the load in the elastic range, 14.7 percent for a strain of 0.001 under fast loading, and 18.8 percent at the same strain under slow or sustained loading.

Strength The one quantity of chief interest to the structural designer is *strength*, i.e., the maximum load that the structure or member will carry. Information on stresses, strains, and similar quantities serves chiefly as a tool for determining carrying capacity. The performance of the column discussed so far indicates two things: (1) in the range of large stresses and strains that precede attainment of the maximum load and subsequent failure, elastic relationships cannot be used; (2) the member behaves differently under fast and under slow or sustained loading and shows less resistance to the latter than to the former. In usual construction, many types of loads, such as the weight of the structure and any permanent equipment housed therein, are sustained, and others are applied at slow rates. For this reason, to calculate a reliable magnitude of compressive strength, curve *c* of Fig. 1.16 must be used as far as the concrete is concerned.

The steel reaches its tensile strength (peak of the curve) at strains on the order of 0.08 (see Fig. 2.15). Concrete, on the other hand, fails by crushing at the much smaller strain of about 0.003 and, as seen from Fig. 1.16 (curve c), reaches its maximum stress in the strain range of 0.002 to 0.003. Because the strains in steel and concrete are equal in axial compression, the load at which the steel begins to yield can be calculated from the information in Fig. 1.16.

If the small knee prior to yielding of the steel is disregarded, i.e., if the steel is assumed to be sharp-yielding, the strain at which it yields is

$$\epsilon_y = \frac{f_y}{E_s} \tag{1.10}$$

or

$$\epsilon_y = \frac{60{,}000}{29{,}000{,}000} = 0.00207$$

At this strain, curve c of Fig. 1.16 indicates a stress of 3200 psi in the concrete; therefore, by Eq. (1.9), the load in the member when the steel starts yielding is $P_y = 3200 \times 314 + 60{,}000 \times 6 = 1{,}365{,}000$ lb. At this load the concrete has not yet reached its full strength, which, as mentioned before, can be assumed as $0.85 f'_c = 3400$ psi for slow or sustained loading, and therefore the load on the member can be further increased. During this stage of loading, the steel keeps yielding at constant stress. Finally, the ultimate load† of the member is reached when the concrete crushes while the steel yields, i.e.,

$$P_n = 0.85 f'_c A_c + f_y A_{st} \tag{1.11}$$

Numerous careful tests have shown the reliability of Eq. (1.11) in predicting the ultimate strength of a concentrically loaded reinforced concrete column, provided its slenderness ratio is small so that buckling will not reduce its strength.

For the particular numerical example, $P_n = 3400 \times 314 + 60{,}000 \times 6 = 1{,}068{,}000 + 360{,}000 = 1{,}428{,}000$ lb. At this stage the steel carries 25.2 percent of the load.

Summary In the elastic range, the steel carries a relatively small portion of the total load of an axially compressed member. As member strength is approached, there occurs a redistribution of the relative shares of the load resisted by concrete and steel, the latter taking an increasing amount. The ultimate load, at which the member is on the point of failure, consists of the contribution of the steel when it is stressed to the yield point plus that of the concrete when its stress has attained a value of $0.85 f'_c$, as reflected in Eq. (1.11).

b. Axial Tension

The tension strength of concrete is only a small fraction of its compressive strength. It follows that reinforced concrete is not well suited for use in tension members because the concrete will contribute little, if anything, to their strength. Still, there are situations

† Throughout this book quantities that refer to the strength of members, calculated by accepted analysis methods, are furnished with the subscript n, which stands for "nominal." This notation is in agreement with the ACI Code. It is intended to convey that the actual strength of any member is bound to deviate to some extent from its calculated, nominal value because of inevitable variations of dimensions, materials properties, and other parameters. Design in all cases is based on this nominal strength, which represents the best available estimate of the actual member strength.

in which reinforced concrete is stressed in tension, chiefly in tie rods in structures such as arches. Such members consist of one or more bars embedded in concrete in a symmetrical arrangement similar to compression members (see Figs. 1.15 and 1.17).

When the tension force in the member is small enough for the stress in the concrete to be considerably below its tensile strength, both steel and concrete behave elastically. In this situation, all of the expressions derived for elastic behavior in compression in Section 1.9*a* are identically valid for tension. In particular, Eq. (1.7) becomes

$$P = f_{ct}(A_c + nA_{st}) \tag{1.12}$$

where f_{ct} is the tensile stress in the concrete.

However, when the load is further increased, the concrete reaches its tensile strength at a stress and strain on the order of one-tenth of what it could sustain in compression. At this stage, the concrete cracks across the entire cross section. When this happens, it ceases to resist any part of the applied tension force, since, evidently, no force can be transmitted across the air gap in the crack. At any load larger than that which caused the concrete to crack, the steel is called upon to resist the entire tension force. Correspondingly, at this stage,

$$P = f_s A_{st} \tag{1.13}$$

With further increased load, the tensile stress f_s in the steel reaches the yield point f_y. When this occurs, the tension members cease to exhibit small, elastic deformations but instead stretch a sizable and permanent amount at substantially constant load. This does not impair the strength of the member. Its elongation, however, becomes so large (on the order of 1 percent or more of its length) as to render it useless. Therefore, the maximum useful strength P_{nt} of a tension member is the force that will just cause the steel stress to reach the yield point. That is,

$$P_{nt} = f_y A_{st} \tag{1.14}$$

To provide adequate safety, the force permitted in a tension member under normal service loads should be limited to about $\frac{1}{2} P_{nt}$. Because the concrete has cracked at loads considerably smaller than this, concrete does not contribute to the carrying capacity of the member in service. It does serve, however, as fire and corrosion protection and often improves the appearance of the structure.

There are situations, though, in which reinforced concrete is used in axial tension under conditions in which the occurrence of tension cracks must be prevented. A case in point is a circular tank (see Fig. 1.11). To provide watertightness, the hoop tension caused by the fluid pressure must be prevented from causing the concrete to crack. In this case, Eq. (1.12) can be used to determine a safe value for the axial tension force P by using, for the concrete tension stress f_{ct}, an appropriate fraction of the tensile strength of the concrete, i.e., of the stress that would cause the concrete to crack.

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1.11. *Commentary on Building Code Requirements for Structural Concrete*, ACI 318R-02, American Concrete Institute, Farmington Hills, MI, 2002 (published as a part of Ref. 1.10).

PROBLEMS

1.1. A 16 × 20 in. column is made of the same concrete and reinforced with the same six No. 9 (No. 29) bars as the column in Examples 1.1 and 1.2, except that a steel with yield strength $f_y = 40$ ksi is used. The stress-strain diagram of this reinforcing steel is shown in Fig. 2.15 for $f_y = 40$ ksi. For this column determine (*a*) the axial load that will stress the concrete to 1200 psi; (*b*) the load at which the steel starts yielding; (*c*) the maximum load; (*d*) the share of the total load carried by the reinforcement at these three stages of loading. Compare results with those calculated in the examples for $f_y = 60$ ksi, keeping in mind, in regard to relative economy, that the price per pound for reinforcing steels with 40 and 60 ksi yield points is about the same.

1.2. The area of steel, expressed as a percentage of gross concrete area, for the column of Problem 1.1 is lower than would often be used in practice. Recalculate the comparisons of Problem 1.1 using f_y of 40 ksi and 60 ksi as before, but for a 16 × 20 in. column reinforced with eight No. 11 (No. 36) bars. Compare your results with those of Problem 1.1.

1.3. A square concrete column with dimensions 22 × 22 in. is reinforced with a total of eight No. 10 (No. 32) bars arranged uniformly around the column perimeter. Material strengths are $f_y = 60$ ksi and $f'_c = 4000$ psi, with stress-strain curves as given by curves *a* and *c* of Fig. 1.16. Calculate the percentages of total load carried by the concrete and by the steel as load is gradually increased from 0 to failure, which is assumed to occur when the concrete strain reaches a limit value of 0.0030. Determine the loads at strain increments of 0.0005 up to the failure strain and graph your results, plotting load percentages vs. strain. The modular ratio may be assumed at $n = 8$ for these materials.