# 10 Strut-and-Tie Models

## 10.1 Introduction

Reinforced concrete beam theory is based on equilibrium, compatibility, and the constitutive behavior of the materials, steel and concrete. Of particular importance is the assumption that strain varies linearly through the depth of a member and that, as a result, plane sections remain plane. This assumption is validated by St. Venant's principle, which stipulates that strains induced by discontinuities in load or in member cross section vary in an approximately linear fashion at distances greater than or equal to the greatest cross-sectional dimension $h$ from the point of load application. St. Venant's principle underlies the development of beam theory as presented in Chapters 1 and 3.

St. Venant's principle, however, does not apply at points closer than the distance $h$ to discontinuities in applied load or geometry. This leads to the identification of so-called *discontinuity regions* within reinforced concrete members near concentrated loads, openings, or changes in cross section. Because of their geometry, the full volume of deep beams and column brackets qualify as discontinuity regions. Thus, reinforced concrete structures may be divided into regions where beam theory is valid, often referred to as *B-regions*, and regions where discontinuities affect member behavior, known as *D-regions*. A number of D-regions are illustrated in Fig. 10.1.

At low stresses, when the concrete is elastic and uncracked, the stresses within D-regions may be computed using finite element analysis and elasticity theory. When concrete cracks, the strain field is disrupted, causing a redistribution of the internal forces. Once this happens, it is possible to represent the internal forces within discontinuity regions using a statically determinate truss, referred to as a *strut-and-tie model*. This allows a complex design problem to be greatly simplified—producing a safe solution that satisfies statics. As shown in Fig. 10.2, strut-and-tie models consist of concrete compression *struts*, steel tension *ties*, and joints that are referred to as *nodal zones* (for consistency of presentation, struts are represented by dashed lines and ties are represented by solid lines).

## 10.2 Development of Strut-and-Tie Models

Strut-and-tie models evolved in the early 1980s in Europe (Refs. 10.1 to 10.4). Their use is permitted by ACI Code 8.3.4 and defined in Appendix A of the Code (Ref. 10.5). As defined, strut-and-tie models divide members into D-regions and B-regions. A D-region is that portion of a member that is within a distance equal to the member height

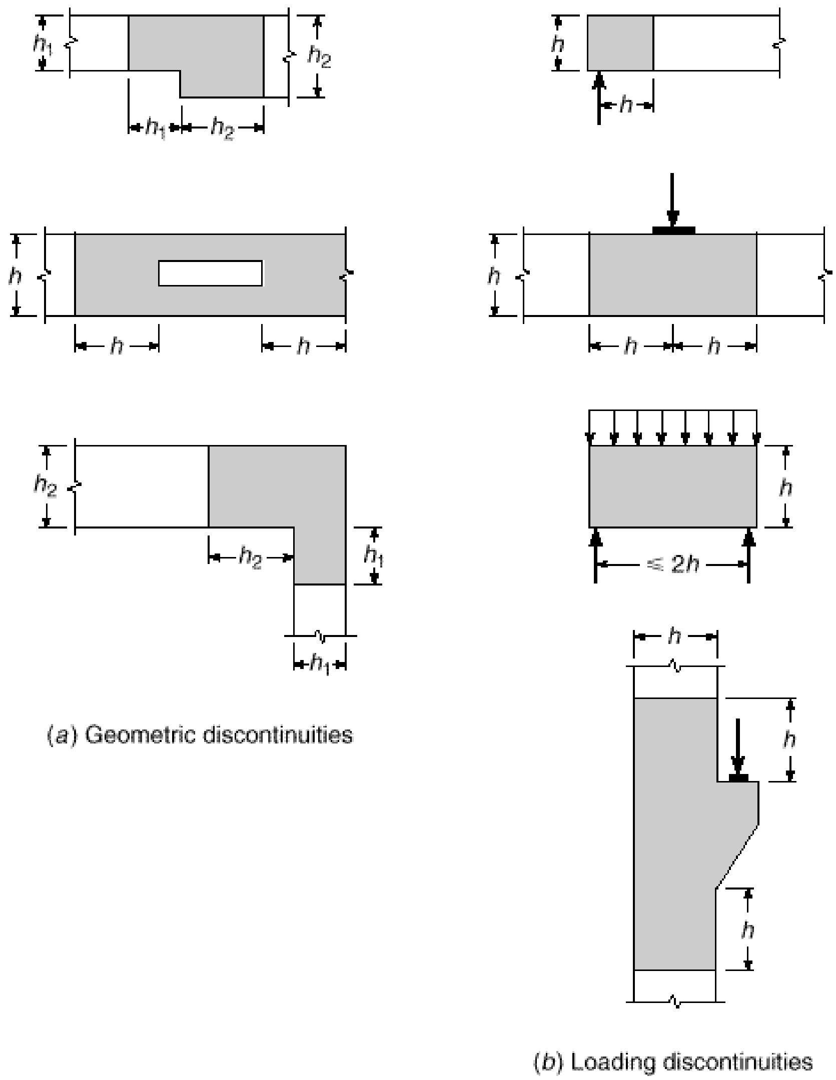



**FIGURE 10.1**
Geometric and load discontinuities for D-regions.

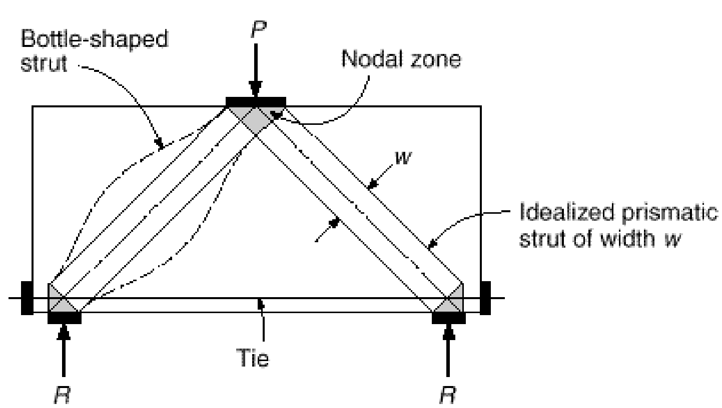



**FIGURE 10.2**
Strut-and-tie model.

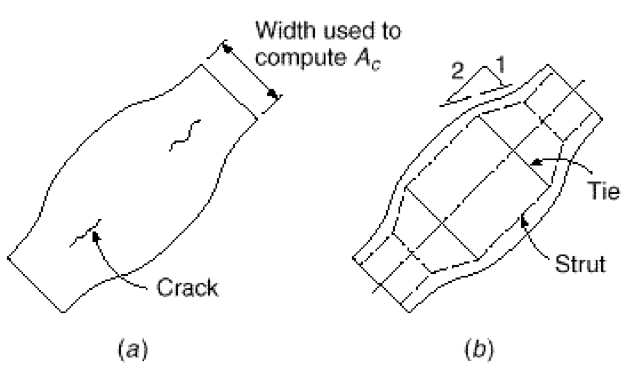
$h$[†] from a force or geometric discontinuity, as shown in Fig. 10.1. B-regions are, in general, any portions of a member outside of D-regions. The assumption is that within B-regions stress varies linearly through the member cross section and plane sections remain plane.

Strut-and-tie models are applied within D-regions. Models consist of struts and ties connected at nodal zones that are capable of transferring loads to the supports or adjacent B-regions. The cross-sectional dimensions of the struts and ties are designated as thickness and width. Thickness $b$ is perpendicular to the plane of the truss model and width $w$ is measured in the plane of the model, as shown in Fig. 10.2.

## a. Struts

A strut is an internal compression member. It may consist of a single element, parallel elements, or a fan-shaped compression field. Along its length, a strut may be rectangular or *bottle-shaped*, in which case the compression field spreads laterally between nodal zones, as shown in Fig. 10.3. For design purposes, a strut is typically idealized as a prismatic member between two nodes. The dimensions of the cross section of the strut are established by the contact area between the strut and the nodal zone. Bottle-shaped struts are wider at the center than at the ends and form where the compression field is free to spread laterally. As the compression zone spreads along the length of bottle-shaped struts, tensile stresses perpendicular to the axis of the strut may result in longitudinal cracking. For simplicity in design, bottle-shaped struts are idealized as having linearly tapered ends and uniform center sections. The linear taper is taken at a slope of 1:2 to the axis of the compression force, as shown in Fig. 10.3*b*. The capacity of a strut is a function of the effective concrete compressive strength, which is affected by lateral stresses within the struts. Because of longitudinal splitting, bottle-shaped struts are weaker than rectangular struts, even though they possess a larger cross section at midlength. Transverse reinforcement is designed to control longitudinal splitting and proportioned using a strut-and-tie model that forms within the strut element, as shown in Fig. 10.3*b*.

**FIGURE 10.3**
Bottle-shaped strut.




[†] The ACI Code defines a D-region based on the member height $h$ or effective depth $d$. No guidance is provided when to use $h$ or $d$. The member height $h$ is used in this text because it is conservative, always defining a larger D-region than that defined by the effective depth $d$.

## b. Ties

A tie is a tension member within a strut-and-tie model. Ties consist of reinforcement (prestressed or nonprestressed) plus a portion of the concrete that is concentric with and surrounds the axis of the tie. The surrounding concrete defines the tie area and the region available to anchor the tie. For design purposes, it is assumed that the concrete within the tie does not carry any tensile force. Even though the tensile capacity of the concrete is not used in design, it assists in reducing tie deformation at service load.

## c. Nodal Zones

Nodes are points within strut-and-tie models where the axes of struts, ties, and concentrated loads intersect. A nodal zone is the volume of concrete around a node where force transfer occurs. A nodal zone may be treated as a single region or may be subdivided into two smaller zones to equilibrate forces. For example, the nodal zone shown in Fig. 10.4*a* is subdivided, as shown in Fig. 10.4*b*, where two reactions $R_1$ and $R_2$ equilibrate the vertical components of strut forces $C_1$ and $C_2$.

For equilibrium, at least three forces must act on a node. Nodes are classified by the sign of these forces (Fig. 10.5). Thus, a *C-C-C* node resists three compressive forces and a *C-C-T* node resists two compressive forces and one tensile force. Both

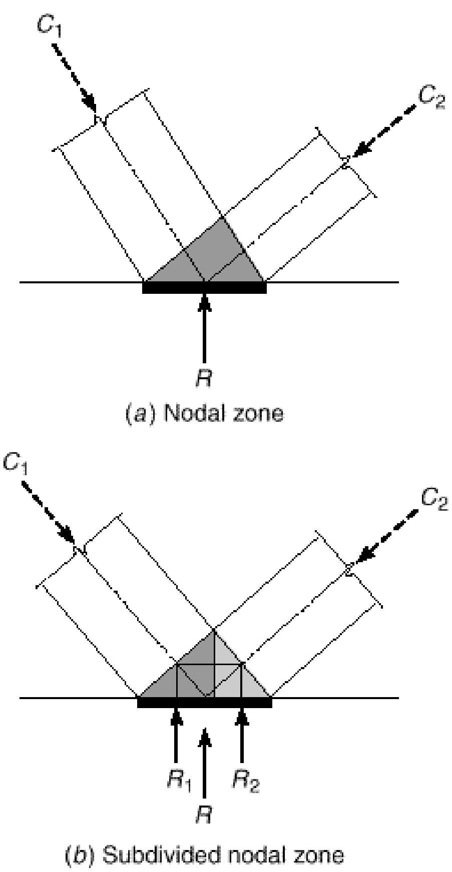
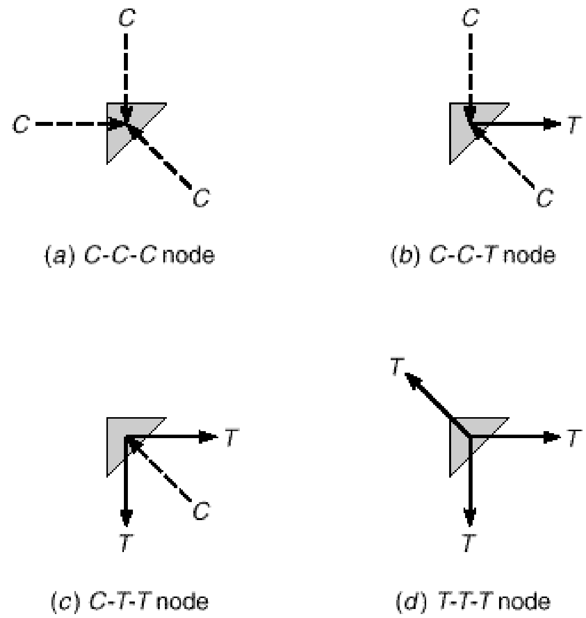



**FIGURE 10.4**
Subdivision of nodal zones.

**FIGURE 10.5**
Classification of nodes.




tensile and compressive forces place nodes in compression because tensile forces are treated as if they pass through the node and apply a compressive force on the far side, or anchorage face. Thus, within the plane of a strut-and-tie model truss, nodal zones are considered to be in a state of *hydrostatic* compression, as shown in Fig. 10.6*a*. Nodal zone dimensions $w_{n1}$, $w_{n2}$, and $w_{n3}$ are proportional to the applied compressive forces. The dimension of one side of a nodal zone is often determined based on the contact area of the load, for example by a bearing plate, column base, or beam support. The dimensions of the remaining sides are established to maintain a constant level of stress $p$ within the node. By selecting nodal zone dimensions that are proportional to the applied loads, the stresses on the faces of the nodal zone are equal.[†]

The length of a hydrostatic zone is often not adequate to allow for anchorage of tie reinforcement. For this reason, an *extended nodal zone*, defined by the intersection of the nodal zone and the associated strut (shown in light shading in Fig. 10.6*b* and *c*), is used. An extended nodal zone may be regarded as that portion of the overlap region between struts and ties that is not already counted as part of a primary node. It increases the length within which the tensile force from the tie can be transferred to the concrete and, thus, defines the available anchorage length for ties. Ties may be developed outside of the nodal and extended nodal zones if needed, as shown to the left of the node in Fig. 10.6*c*.

## 10.3 STRUT-AND-TIE DESIGN METHODOLOGY

Strut-and-tie models are used in several ways during the design process. At the *conceptual* design level, sketching a strut-and-tie model provides insight into structural behavior and detailing requirements. Examples of conceptual design can be seen in the development of connection details in Chapter 11. Strut-and-tie models may be used to

[†] The state of stress within a nodal zone is not truly hydrostatic since out-of-plane stresses are not considered.

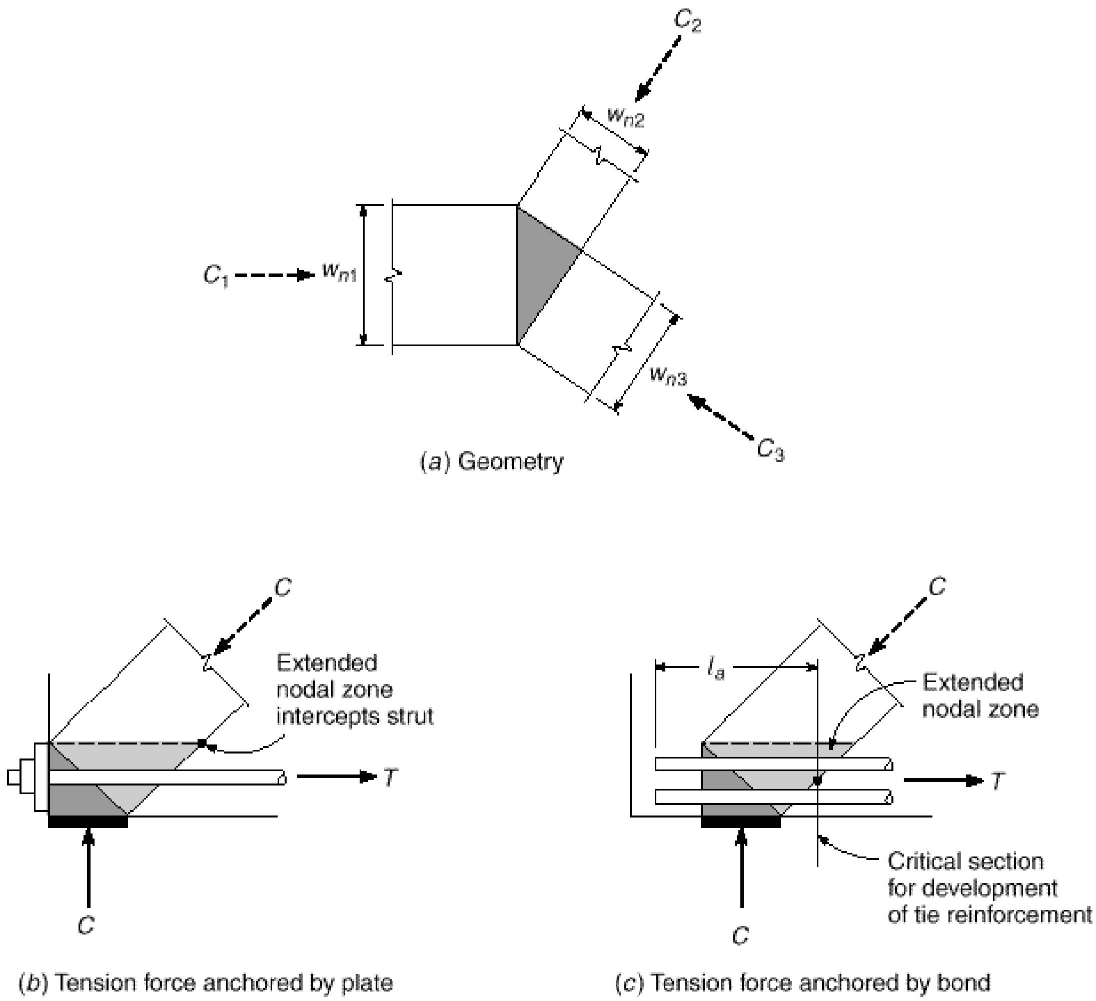



**FIGURE 10.6**
Nodal zones and extended nodal zones.

*validate* design details, such as for special reinforcement configurations. Finally, strut-and-tie models may form the basis for *detailed* design of a member.

Application of a detailed strut-and-tie model involves completion of the following steps.

1. Define and isolate the D-regions.
2. Compute the force resultants on each D-region boundary.
3. Select a truss model to transfer the forces across a D-region.
4. Select dimensions for strut-and-tie nodal zones.
5. Verify the capacity of the strut both at midlength and at the nodal interface.
6. Design the ties and the tie anchorage.
7. Prepare design details and check minimum reinforcement requirements.

As will be described shortly, the design process requires interaction between these steps.

According to ACI Code A.2.6, design using a strut-and-tie model requires that

$$\phi F_n \geq F_u \tag{10.1}$$

where $F_u$ = factored force acting in strut, tie, bearing area, or nodal zone
$F_n$ = nominal capacity of strut, tie, or nodal zone
$\phi$ = strength-reduction factor

In addition to strength criteria, service level performance must be considered in design because strut-and-tie models, which are based on strength, do not necessarily satisfy serviceability requirements. To this end, the spacing of reinforcement within ties should be checked using Eq. (6.3). ACI Code 11.8.3 limits the nominal shear strength of deep beams to $10\phi\sqrt{f'_c}b_w d$. This limit should be checked prior to beginning a detailed design, as described in Section 10.4d.

## a. D-region

A D-region extends on both sides of a discontinuity a distance equal to the member height $h$. At geometric discontinuities, a D-region may have different dimensions on either side of the discontinuity, as shown in Fig. 10.1.

## b. Force Resultants on D-region Boundaries

Once the D-region is defined, the next step involves determining the magnitude, location, and direction of the resultant forces acting on the D-region boundaries. These forces serve as input for the strut-and-tie model and assist in establishing the geometry of the truss model. When one face of a D-region is loaded with a uniform or linearly varying stress field, or when a face is loaded by bending of a concrete section, it may be necessary to subdivide the boundary into segments corresponding to struts or ties and then to compute the resultant force on each segment, as shown in Fig. 10.7. For example, in Fig. 10.7*a*, the distributed load along the top of the deep beam is represented by four concentrated loads, and the stresses at the beam-column interface are represented by concentrated reactions. In Fig. 10.7*b*, the moments at the faces of the beam-column joint are represented by couples consisting of tensile and compressive forces acting at the interfaces between the members and the joint.

## c. The Truss Model

The truss representing the strut-and-tie model must fit within the envelope defined by the D-region. The selection of struts and ties is made at the discretion of the designer and, therefore, multiple solutions are possible. The layout of a truss model is constrained by the geometric requirement that struts must intersect only at nodal zones. Ties may cross struts. An effective model will represent a minimum energy distribution through the D-region (Refs. 10.1 and 10.4), i.e., within the model, forces should follow the stiffest load path. Because struts are typically much stiffer than ties, a model with a minimum number of tension ties is generally preferred. Alternative truss models

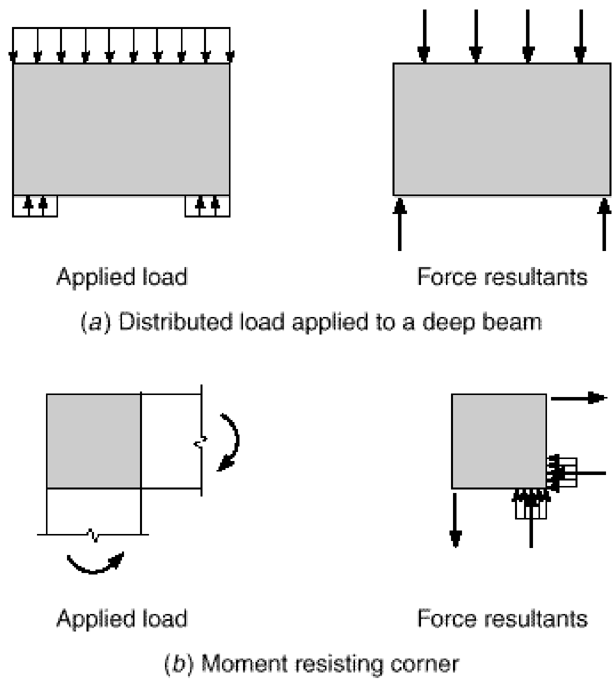



**FIGURE 10.7**
Resolution of forces in a D-region.

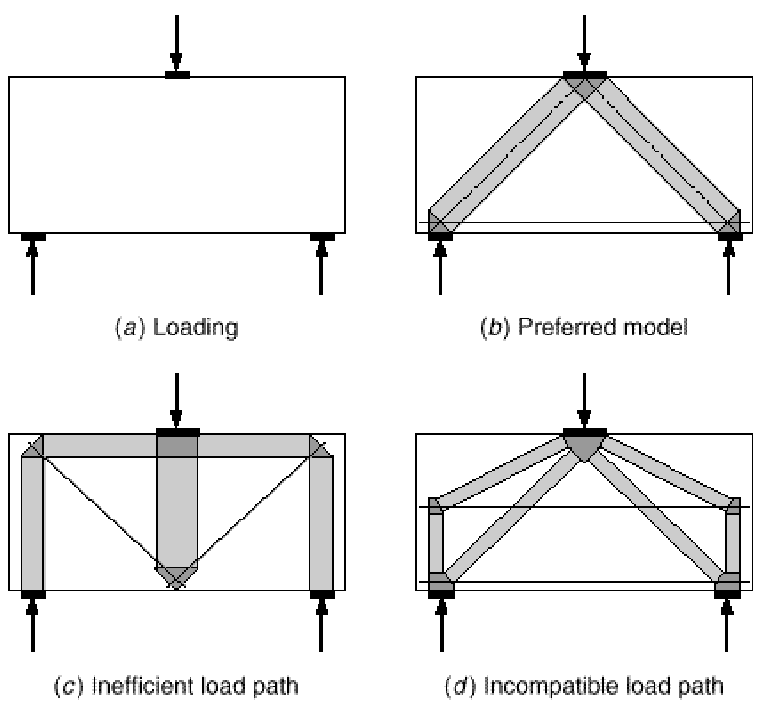



**FIGURE 10.8**
Alternatives for a deep beam truss model.

for a deep beam are compared in Fig. 10.8. Figure 10.8*a* shows a deep beam subjected to a concentrated load at midspan. Figure 10.8*b* shows the preferred strut-and-tie model for this beam and loading condition. In this case, struts carry the load directly to nodal regions at the supports, which are, in turn, connected by a single tension tie. The model in Fig. 10.8*c* shows an ineffective load path, with a single strut carrying the load to a node at the bottom of the beam that is supported by two diagonal tension ties, which are, in turn, supported by vertical struts over the supports. In this instance, the number of transfer points and tension ties is greater, as is the flexibility of the truss, indicating a solution that is much less effective than that shown in Fig. 10.8*b*. Lastly, Fig. 10.8*d* illustrates a model with multiple struts and ties. This particular layout is not only unduly complex, but includes an upper tension tie that will be effective only after extensive yielding and possible failure of the lower tension tie.

Theoretically, there may be a unique minimum energy solution for a strut-and-tie model. Practically, any model that satisfies equilibrium and pays attention to structural stiffness will prove satisfactory. Using the rationale just discussed allows the designer to select a logical model that effectively mobilizes ties and minimizes the potential for excessive cracking. Finite element analyses and solutions based on the theory of elasticity for the full structure can provide an indication of where maximum stresses occur. A truss model that provides struts in regions of high compression and ties in regions of high tension based on these analyses will, in general, provide an efficient load path.

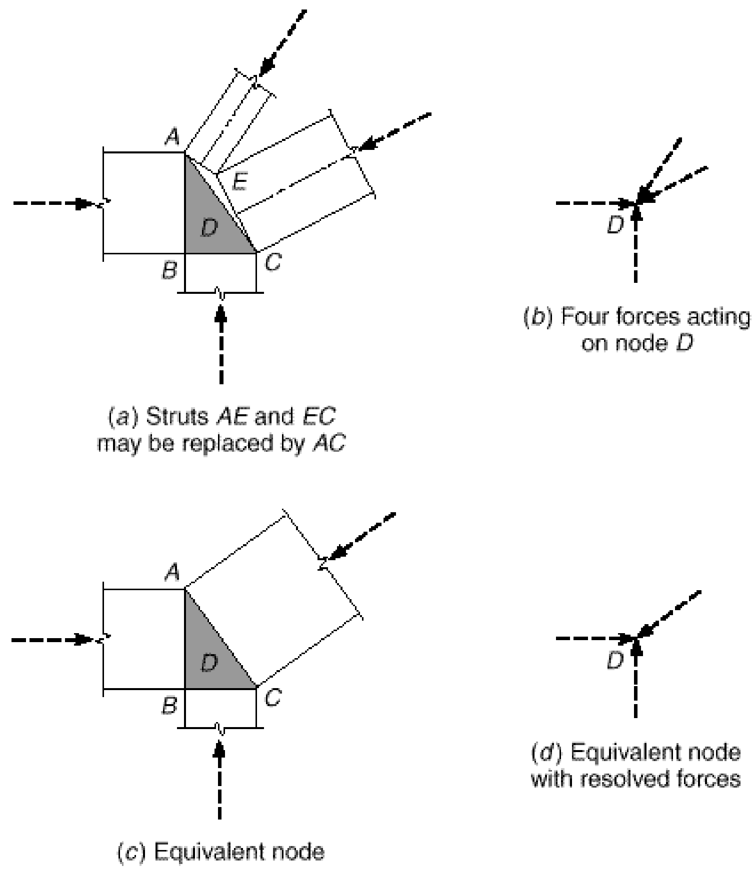
## d. Selecting Dimensions for Struts and Nodal Zones

The struts, ties, and nodal zones within the truss that represents a strut-and-tie model have finite widths that must be considered when selecting the dimensions of the truss. The width of each truss member depends on the magnitude of the forces and the dimensions of the adjoining elements. An external element, such as a bearing plate or column, can serve to define a nodal zone. If the bearing area is too small, a high hydrostatic pressure results, and the corresponding width of the struts will not be sufficient to carry the applied load. The solution in this case is to increase the size of the bearing surface and, thus, reduce the contact pressures. Some designers intentionally select struts and nodes that are large enough to keep the compressive stresses low; in this case, only the tension ties require detailed design. To minimize cracking and to reduce complications that may result from incompatibility in the deformations due to struts shortening and ties elongating in nearly the same plane, the angle between struts and ties at a node should be greater than 25°.

The design of nodal zones is based on the assumption that the principal stresses within the intersecting struts and ties are parallel to the axes of these truss members. The widths of the struts and ties are, in general, proportional to the magnitude of the force in the elements. If two or more struts converge on the same face, such as shown in Fig. 10.9*a* and *b*, it is generally necessary to resolve the forces into a single force and to orient the face of the nodal zone so that it is perpendicular to the combined force, as shown in Fig. 10.9*c* and *d*. Some geometric arrangements preclude establishing a purely hydrostatic node. In these cases, the width of the strut is determined by the geometry of the bearing plate or tension tie, as shown in Fig. 10.10*a*.

The thickness of the strut, tie, and nodal zone is typically equal to the thickness of the member. If the thickness of the bearing area is less than the thickness of the member, it may be necessary to reduce the strut and tie thickness or to add reinforcement perpendicular to the principal plane of the member to add confinement and prevent splitting. In this instance, a strut-and-tie model may be used to determine the requirements for transverse reinforcement in a manner that is similar to that used to reinforce bottle-shaped struts.

**FIGURE 10.9**
Resolution of forces in nodal zones.




## e. Capacity of Struts

Strut capacity is based on both the strength of the strut itself and the strength of the nodal zone. If a strut does not have sufficient capacity, the design must be revised by providing compression reinforcement or by increasing the size of the nodal zone. This may, in turn, affect the size of the bearing plate or column.

## f. Design of Ties and Anchorage

To control cracking in a D-region, ties are designed so that the stress in the reinforcement is below yield at service loads. The geometry of the tie must be selected so that the reinforcement fits within the tie dimensions.

Anchorage for ties is provided within the nodal and extended nodal zones plus regions on the far side of the node that may be available based on the geometry of the member. Figure 10.10*a* illustrates an extended nodal zone and the length available for anchorage of ties $l_a$. In this case, the tie is extended to the left of the nodal zone to allow for full development of the reinforcement. The shape of the extended nodal zone is a function of the strut angle · and the width of the tie $w_t$. Figure 10.10*a* illustrates the geometry and dimensions of a *C-C-T* node with a tension tie that contains multiple layers of reinforcement. Figure 10.10*b* shows a *C-T-T* nodal zone. If insufficient

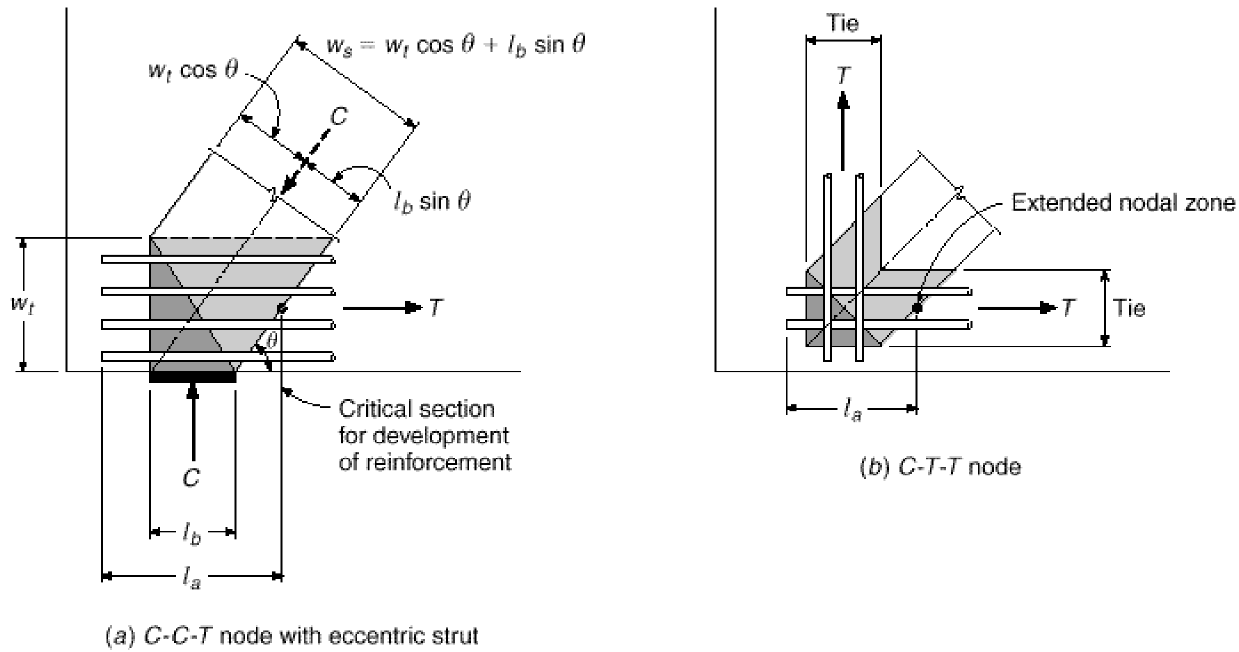



**FIGURE 10.10**
Extended nodal zone definition.

length is available to anchor the reinforcement within the nodal and extended nodal zones, the reinforcement must extend beyond the node or a hook or mechanical anchor must be used to fully develop the reinforcement.

### g. Design Details and Minimum Reinforcement Requirements

A complete design includes verification that (1) tie reinforcement can be placed in the section, (2) nodal zones are confined by compressive forces or tension ties, and (3) minimum reinforcement requirements are satisfied. Reinforcement within ties must meet the ACI Code requirements for bar spacing (see Section 3.6c) and fit within the overall width and thickness of the tie. Tie details should be reviewed to ensure that ties are adequately developed on the far side of nodes by tension development length, hooks, or mechanical anchorage. Shear reinforcement requirements are satisfied by ensuring that the factored shear is less than the ACI Code maximum, as described in Chapter 4, longitudinal cracking of bottle-shaped struts is controlled, or the minimum reinforcement requirements described in Section 10.4d are met.

## 10.4 ACI Provisions for Strut-and-Tie Models

ACI Code Appendix A provides guidance for sizing struts, nodes, and ties. It addresses the performance of highly stressed compression zones that may be adjacent to or crossed by cracks in a member, the effect of stresses in nodal zones, and the requirements for bond and anchorage of ties. The effective compressive strength of concrete $0.85f'_c$ is modified by a factor · to account for the effects of cracks (caused by spreading compressive resultants) and confining reinforcement in struts and the anchorage of ties in nodal zones.

The balance of this section describes the steps needed to calculate the capacity of struts, verify nodal zones, and design ties and tie anchorage. A strength-reduction factor $\phi = 0.75$ is used for struts, ties, nodal zones, and bearing areas.

## a. Strength of Struts

The strength of a strut is limited based on the strength of the concrete in the strut and the strength of the nodal zones at the ends of the strut. The nominal compressive strength of a strut $F_{ns}$ is given as

$$F_{ns} = f_{cu} A_c \tag{10.2}$$

where $f_{cu}$ is the effective compressive strength of the concrete in a strut or nodal zone and $A_c$ is the cross-sectional area at one end of the strut, which is equal to the product of the strut thickness and the strut width. The effective strength of concrete in a strut is

$$f_{cu} = 0.85 \beta_s f'_c \tag{10.3}$$

where $\beta_s$ is a factor that accounts for the effects of cracking and confining reinforcement within the strut, with values ranging from 1.0 for a strut with a uniform cross-sectional area over its length to 0.4 for struts in tension members or the tension flanges of members (Table 10.1). Intermediate values include 0.75 for struts with a width at midsection that is larger than the width at the nodes (bottle-shaped struts) that are crossed by transverse reinforcement to resist the transverse tensile force resulting from the compressive force spreading in the strut and $0.6\lambda$ for bottle-shaped struts without the required transverse reinforcement, where $\lambda$ is the correction factor related to the unit weight of concrete: 1.0 for normal-weight concrete, 0.85 for sand-lightweight concrete, and 0.75 for all-lightweight concrete. $\beta_s = 0.60$ for all other cases, as when parallel diagonal cracks divide the web struts or when diagonal cracks are likely to turn and cross a strut, as shown in Fig. 10.11.

Compression steel may be added to increase the strength of a strut, so that

$$F_{ns} = f_{cu} A_c + A'_s f'_s \tag{10.4}$$

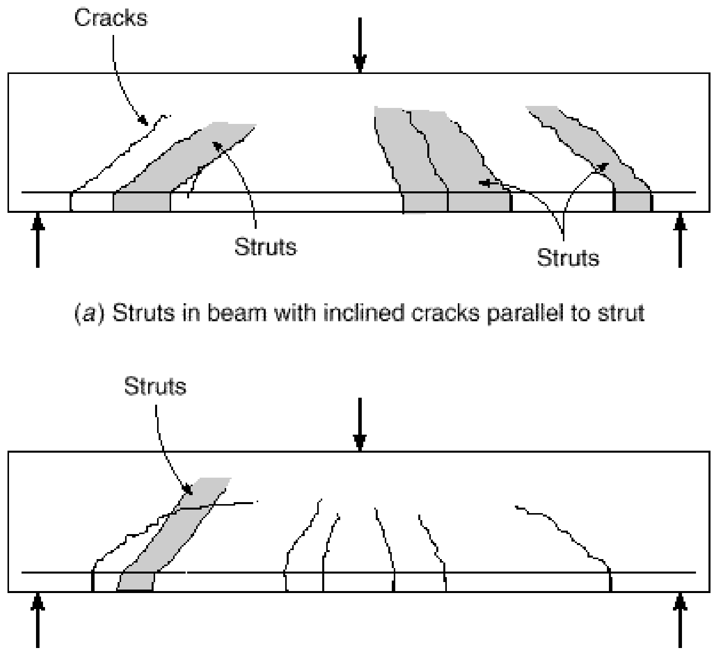
where $f'_s$ is based on the strain in the concrete at peak stress. For Grades 40 and 60 reinforcement, $f'_s = f_y$.

**TABLE 10.1**
**$\beta_s$ values for strut strength**

| Condition | $\beta_s$ |
|---|---|
| Strut with uniform cross section over its entire length | 1.0 |
| Strut with the width at midsection larger than the width at the nodes (bottle-shaped strut) and with reinforcement satisfying transverse requirements | 0.75 |
| Strut with the width at midsection larger than the width at the nodes (bottle-shaped strut) and reinforcement not satisfying transverse requirements | $0.60\lambda$ * |
| Struts in tension members or in the tension flange of members | 0.40 |
| All other cases, Fig. 10.11 | 0.60 |

* $\lambda$ equals 1.0 for normal-weight concrete, 0.85 for sand-lightweight concrete, and 0.75 for all-lightweight concrete.

**FIGURE 10.11**
Beam cracking conditions for $\beta_s = 0.6$.




(*a*) Struts in beam with inclined cracks parallel to strut

(*b*) Struts crossed by skew cracks

To design transverse reinforcement for bottle-shaped struts, ACI Code A.3.3 permits the assumption that the compressive force in the struts spreads at a slope of two longitudinal to one transverse along the axis of the strut, as shown in Fig. 10.3*b*. For $f'_c \leq 6000$ psi, the ACI Code considers the transverse reinforcement requirement to be satisfied if the strut is crossed by layers of reinforcement that satisfy

$$\sum \frac{A_{si}}{bs_i} \sin \alpha_i \geq 0.003 \tag{10.5}$$

where $A_{si}$ is the total area of reinforcement at spacing $s_i$ in a layer of reinforcement with bars at an angle $\alpha_i$ to the axis of the strut, and $b$ is the thickness of the strut. The reinforcement may be perpendicular to the strut axis or may be placed in an orthogonal grid pattern. The subscript $i$ denotes the layer of reinforcement. The values $s_i$ and $\alpha_i$ are shown in Fig. 10.12.

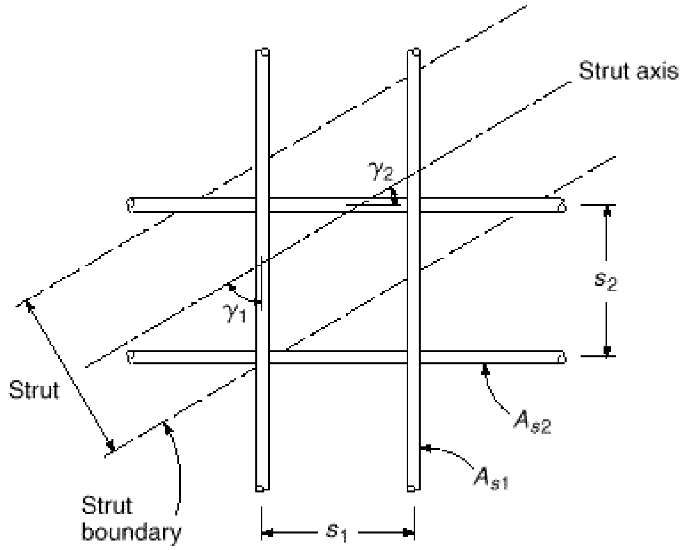
The ACI Code provides no clear guidance to indicate when a strut should be considered as rectangular or bottle-shaped. Some researchers suggest that horizontal struts be represented as rectangular and inclined struts represented as bottle-shaped (Ref. 10.6). Others simply assume a bottle-shaped strut will develop, using the lower values of $\beta_s$ for design (Ref. 10.7). Examples in this text use rectangular horizontal struts and bottle-shaped inclined struts.

## b. Strength of Nodal Zones

The nominal compressive strength of a nodal zone is

$$F_{nn} = f_{cu} A_n \tag{10.6}$$

**FIGURE 10.12**
Details of reinforcement crossing a strut.




where $f_{cu}$ is the effective strength of the concrete in the nodal zone and $A_n$ is (1) the area of the face of the nodal zone taken perpendicular to the line of action of the force from the strut or tie or (2) the area of a section through the nodal zone taken perpendicular to the line of action of the resultant force on the section. The latter condition occurs when multiple struts intersect a node, as shown in Fig. 10.9.

The effective concrete strength in a nodal zone is

$$f_{cu} = 0.85\cdot_n f'_c \tag{10.7}$$

where $f'_c$ is the compressive strength of the concrete in the nodal zone, and $\cdot_n$ is a factor that reflects the degree of disruption in nodal zones due to the incompatibility of tensile strains in ties with compressive strains in struts. $\cdot_n = 1.0$ for *C-C-C* nodes, 0.80 for *C-C-T* nodes, and 0.60 for *C-T-T* or *T-T-T* nodes. Values of $\cdot_n$ are summarized in Table 10.2.

## c. Strength of Ties

The nominal strength of ties $F_{nt}$ is the sum of the strengths of the reinforcing steel and prestressing steel within the tie.

$$F_{nt} = A_{st}f_y + A_{ps}(f_{pe} + \Delta f_p) \tag{10.8}$$

where $A_{st}$ = area of reinforcing steel
$f_y$ = yield strength of reinforcing steel
$A_{ps}$ = area of prestressing steel, if any
$f_{pe}$ = effective stress in prestressing steel
$\Delta f_p$ = increase in prestressing steel stress due to factored load

The sum $f_{pe} + \Delta f_p$ must be less than or equal to the yield stress of the prestressing reinforcement $f_{py}$. $A_{ps}$ is zero for nonprestressed members. The value of $\Delta f_p$ may be found by analysis, or, in lieu of formal analysis, ACI Code A.4.1 allows a value 60,000 psi to be used for bonded tendons and 10,000 psi to be used for unbonded tendons.

**TABLE 10.2**
**· values for node strength**

| Nodal Zone Condition | Classification | · |
|---|---|---|
| Bounded by struts or bearing area | *C-C-C* | 1.0 |
| Anchoring one tie | *C-C-T* | 0.80 |
| Anchoring two or more ties | *C-T-T* or *T-T-T* | 0.60 |

The effective width of a tie $w_t$ depends on the distribution of the tie reinforcement. If the reinforcement in a tie is placed in a single layer, the effective width of a tie may be taken as the diameter of the largest bars in the tie plus twice the cover to the surface of the bars. Alternatively, the width of a tie may be taken as the width of the anchor plates. The practical upper limit for tie width $w_{t,max}$ is equal to the width corresponding to the width of a hydrostatic nodal zone given as

$$w_{t,max} = \frac{F_{nt}}{bf_{cu}} \tag{10.9}$$

where $f_{cu}$ is the effective nodal zone compressive stress given in Eq. (10.6) and $b$ is the thickness of the strut.

Ties must be anchored before they leave the extended nodal zone at a point defined by the centroid of the bars in the tie and the extension of the outlines of either the strut or the bearing area, as shown in Fig. 10.10. If the combined lengths of the nodal zone and extended nodal zone are inadequate to provide for development of the reinforcement, additional anchorage may be obtained by extending the reinforcement beyond the nodal zone, using 90° hooks, or by using a mechanical anchor. If the tie is anchored with a 90° hook, the hooks should be confined by reinforcement extending into the beam from supporting members to avoid splitting of the concrete within the anchorage region.

## d. ACI Shear Requirements for Deep Beams

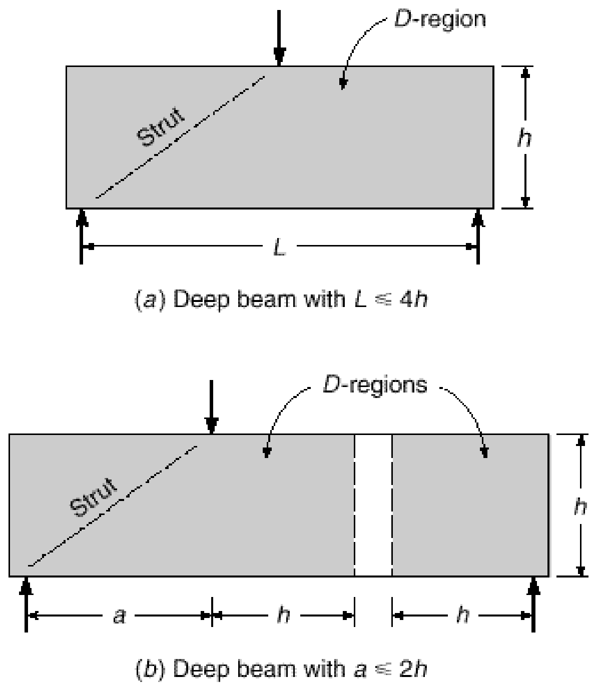
Beams with clear spans less than or equal to 4 times the total member depth or with concentrated loads placed within twice the member depth of a support are classified as deep beams, according to ACI Code 11.7 and 11.8.[†] Examples of deep beams are shown in Fig. 10.13. ACI Code 11.8.2 allows such members to be designed either by using a nonlinear analysis or by applying the strut-and-tie method of ACI Code Appendix A. While solutions based on nonlinear strain distributions are available (Ref. 10.8), the strut-and-tie approach allows a rational design solution.

ACI Code 11.8.3 specifies that the nominal shear in a deep beam may not exceed $10\sqrt{f'_c}b_w d$, where $b_w$ is the width of the web and $d$ is the effective depth. ACI Code 10.8.4 and 10.8.5 provide minimum steel requirements for horizontal and vertical reinforcement within a deep beam. The minimum reinforcement perpendicular to a span is

$$A_v \geq 0.0025 b_w s \tag{10.10}$$

[†] The ACI Code does not specify the magnitude of the concentrated load at a beam end needed to invoke the deep beam provisions of Section 11.8. A level of professional judgment is required if a low-magnitude concentrated load is placed at the end of a beam. Deep beam design may not be required in this situation.

**FIGURE 10.13**
Deep beam D-regions.




where $s$ is the spacing of the reinforcement. The minimum reinforcement parallel to a span is

$$A_{vh} \geq 0.0015 b_w s_2 \tag{10.11}$$

where $s_2$ is the spacing of the reinforcement perpendicular to the longitudinal reinforcement. Spacings $s$ and $s_2$ may not exceed $d/5$ or 12 in. ACI Code 11.8.6 allows Eq. (10.5) to be used in lieu of Eqs. (10.10) and (10.11). For strut-and-tie models, $b_w$ equals the thickness of the element $b$.

## 10.5 APPLICATIONS

While there are a number of possible applications for a strut-and-tie model, ACI Code 11.8 and 11.9 specifically allow deep beam and column bracket design to be completed with this method. The following examples examine the details of deep beams and dapped beam end design by the strut-and-tie method. Additional examples of strut-and-tie modeling may be found in Chapter 11 and in Refs. 10.9 and 10.10.

### a. Deep Beams

Deep beams represent one of the principal applications of strut-and-tie models, since the alternative under ACI Code 11.8 is a nonlinear analysis. Two examples of deep beam design are presented next, one that includes the application of concentrated loads at the upper surface of a transfer girder and a second that addresses design for distributed as well as concentrated loads.

**EXAMPLE 10.1** **Deep beam.** A transfer girder is to carry two 24 in. square columns, each with factored loads of 1200 kips located at the third points of its 36 ft span, as shown in Fig. 10.14*a*. The beam has a thickness of 2 ft and a total height of 12 ft. Design the beam for the given loads, ignoring the self-weight, using $f'_c = 5000$ psi and $f_y = 60{,}000$ psi.

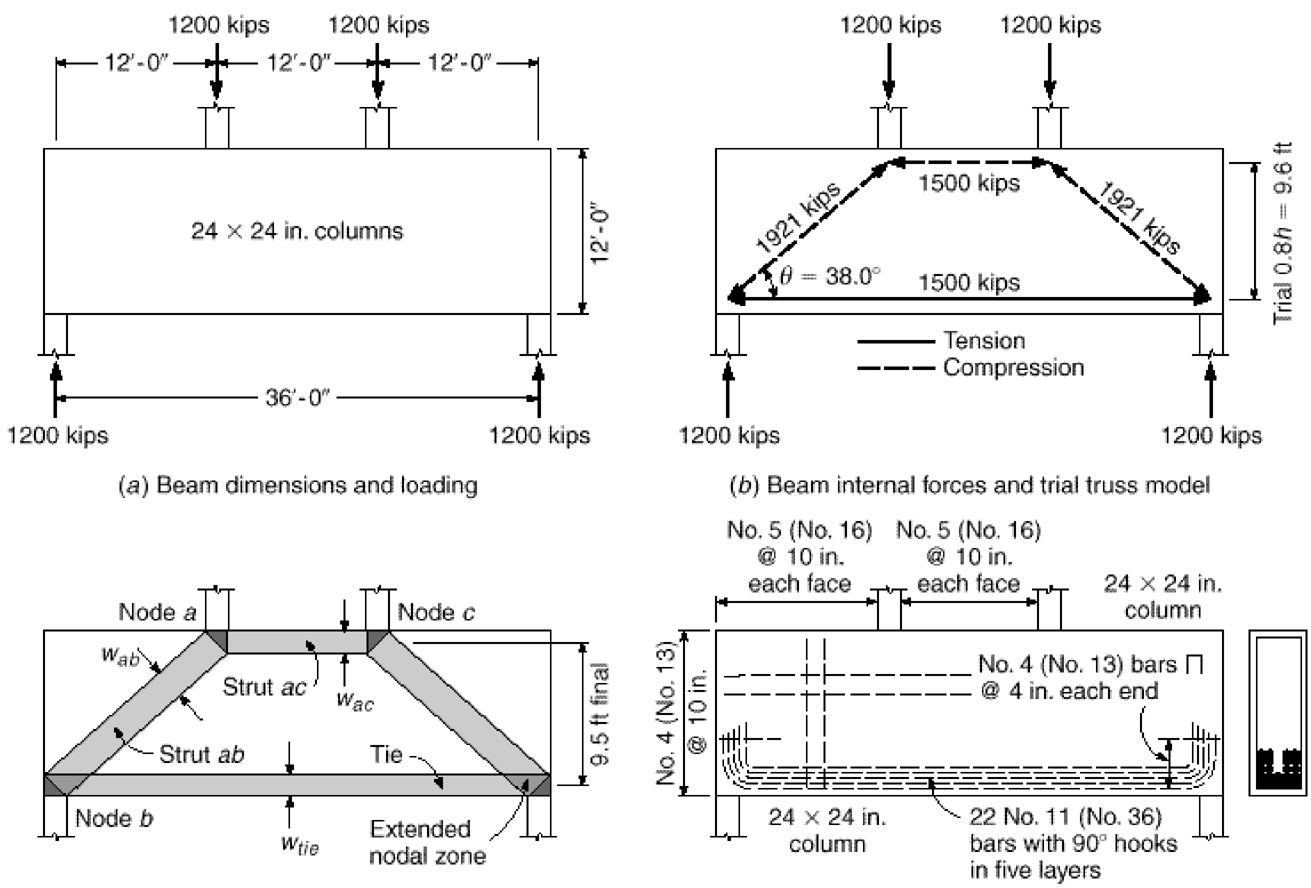
**Solution.** The span-to-depth ratio for the beam is 3, thereby qualifying it as a deep beam. A strut-and-tie solution will be used.

**Definition of D-region**

All of the supports and loads are within $h$ of each other or the supports, so the entire structure may be characterized as a D-region. The thickness of the struts and ties is equal to the thickness of the beam $b = 24$ in. Assuming an effective depth $d = 0.9h = 0.9 \times 12 = 10.8$ ft in the middle third of the beam, the maximum design shear capacity of the beam is $\phi V_n = \phi 10\sqrt{f'_c}b_w d = 0.75 \times 10\sqrt{5000} \times 24 \times 10.8 \times 12/1000 = 1650$ kips. This is greater than $V_u = 1200$ kips. Thus, the design may continue.

**Force resultants on D-region boundaries**

The 1200 kip column loads on the upper face of the beam are equilibrated by 1200 kip reactions at the supports, as shown in Fig. 10.14*b*. Based on an assumed center-to-center distance




(*a*) Beam dimensions and loading

(*b*) Beam internal forces and trial truss model

(*c*) Schematic showing widths of struts, ties, and nodes in final truss model

(*d*) Reinforcement details

**FIGURE 10.14**
Deep beam design for Example 10.1.

between the horizontal strut and the tie of 0.8$h$, the trial diagonal struts form at an angle · = 38.66° and carry a load of 1921 kips. A horizontal 1500 kip compression strut runs between the two column loads and a 1500 kip tension tie runs between the bottom nodes.

**The truss model**
Based on the beam geometry and loading, a single truss is sufficient to carry the column loads, as shown in Fig. 10.14$c$. The truss has a trapezoidal shape. This is an acceptable solution since the nodes are not true pins and instability within the plane of the truss is not a concern in a strut-and-tie model. The truss geometry is established by the assumed intersection of the struts and ties and used to determine · .

**Selecting dimensions for strut and nodal zones**
The nodal stress $p$ is determined by the average stress under the columns. Thus, $p$ = 1200 kips· (24 in. × 24 in.) = 2.08 ksi. The width of strut $ac$, found using $p$, is

$$w_{ac} = F_{ac} \cdot (b \times p) = 1500 \cdot (24 \times 2.08) = 30.0 \text{ in.}$$

Similarly, $w_{ab}$ = 38.5 in. and $w_{tie}$ = 30.0 in. The center-to-center distance between the horizontal strut $ac$ and the tie is 12 − 30· 12 = 9.5 ft, or 0.79$h$. The angle · between the diagonal strut $ab$ and the tie is thus 38.3°. Using an angle of 38.0° gives a revised force in strut $ab$ of 1949 kips. Similarly, the revised force in strut $ac$ and the tie is 1536 kips, while the widths are revised to $w_{ab}$ = 39.0 in., and $w_{ac}$ and $w_{tie}$ = 30.7 in. (Note: After iteration, the actual angle becomes 38.2°. The value of 38.0° is conservative and is, thus, retained.)

**Capacity of struts**
The horizontal strut $ac$ will be assumed to have a uniform cross section, while the diagonal struts will be considered as bottle-shaped because of the greater width available. Strut capacity is given in Eqs. (10.2) and (10.3), which, when combined, give $\cdot F_{ns} = \cdot \cdot_s 0.85 f'_c w_i b$, where $w_i$ is the width of the strut and $\cdot_s$ is 1.0 for a rectangular strut. For strut $ac$,

$$\cdot F_{ns} = 0.75 \times 1.0 \times 0.85 \times 5000 \times 30.7 \times 24 \cdot 1000 = 2349 \text{ kips} > 1536 \text{ kips}$$

Therefore, strut $ac$ is adequate. Similarly, for strut $ab$,

$$\cdot F_{ns} = 0.75 \times 0.75 \times 0.85 \times 5000 \times 39.0 \times 24 \cdot 1000 = 2238 \text{ kips} > 1949 \text{ kips}$$

From Eqs. (10.6) and (10.7), the capacity of the nodal zone is $\cdot F_{nn} = \cdot \cdot_n 0.85 f'_c w_i b$. At $a$, a $C$-$C$-$C$ node, $\cdot_n$ = 1.0, and at $b$, a $C$-$C$-$T$ node, $\cdot_n$ = 0.80. Thus, the capacity of strut $ab$ is established at node $b$ and

$$\cdot F_{nn} = 0.75 \times 0.80 \times 0.85 \times 5000 \times 39.0 \times 24 \cdot 1000 = 2387 \text{ kips} \gtrsim 1949 \text{ kips}$$

Similarly, the nodal end capacity of strut $ac$ is 2349 kips with $\cdot_n$ = 1.0. Thus, the capacity at the end of the struts and at the nodes exceeds the factored loads, and thus, the struts are adequate.

**Design ties and anchorage**
The tie design consists of three steps, selection of the area of steel, design of the anchorage, and validation that the tie fits within the available tie width. The steel area is computed as $A_s = F_{tu} \cdot \cdot f_y$ = 1536 (0.75 × 60) = 34.1 in$^2$. This is satisfied by using 22 No. 11 (No. 36) bars, having a total area of $A_s$ = 34.3 in$^2$. Placing the bars in two layers of five bars and three layers of four bars, while allowing for 2.5 in. clear cover to the bottom of the beam and $4\frac{1}{2}$ clear spacing between layers, results in a total tie width of 5 × 1.41 + 4 × 4.5 + 2 × 2.5 = 30.0 in., matching the tie dimension.

The anchorage length $l_d$ for No. 11 (No. 36) bars (from Table A.10 in Appendix A) is 42$d_b$ = 59.2 in. The length of the nodal zone and extended nodal zone is 24 + 0.5 × 30.7 cot 38.0° = 43.6 in., which is less than $l_d$. The beam geometry does not allow the tie reinforcement to extend linearly beyond the node; therefore, 90° hooks or mechanical anchors are required on the No. 11 (No. 36) bars. Placement details are covered in the next section. Allowing 1.5 in. cover on the sides, No. 5 (No. 16) transverse and horizontal reinforcement, and 2$d_b$ spacing between No. 11 (No. 36) bars, five No. 11 (No. 36) bars require a total thickness of

$b_{reqd} = 2 \times 1.5 + 4 \times 0.625 + 4$ spaces @ $2 \times 1.41 + 5$ bars $\times 1.41 = 23.8$ in., which fits within the 24 in. beam thickness.

**Design details and minimum reinforcement requirements**

ACI Code 11.8.6 requires that shear reinforcement in deep beams satisfy (a) both Eqs. (10.10) and (10.11) or (b) Eq. (10.5). Using Eq. (10.10), the minimum required vertical steel is $A_v \geq 0.0025ts = 0.0025 \times 24 \times 12 = 0.72$ in$^2$/ft. This is satisfied by No. 5 (No. 16) bars at 10 in. placed on each face, giving a total area of reinforcement equal to 0.74 in$^2$/ft. Similarly, using Eq. (10.11), the horizontal reinforcement is $A_{vh} \geq 0.0015ts_1 = 0.0015 \times 24 \times 12 = 0.43$ in$^2$/ft, which is satisfied using No. 4 (No. 13) bars at 10 in. placed on each face, giving 0.48 in$^2$/ft.

Equation (10.5) produces similar steel requirements. Using the reinforcement selected using Eqs. (10.10) and (10.11), two No. 5 (No. 16) bars ($\cdot$ = 38.0°) give $A_v = 0.62$ in$^2$ and two No. 4 (No. 13) bars ($\cdot$ = 52.0°) give $A_{vh} = 0.40$ in$^2$. Equation (10.5) becomes

$$\cdot \; \frac{A_{si}}{bs_i} \sin \cdot_i = \frac{0.62 \sin 49.1° + 0.40 \sin 38.0°}{24 \times 10} = 0.00306 \; \cdot \; 0.003 \text{ req'd}$$

This ensures that sufficient reinforcement is present to control longitudinal splitting in the bottle-shaped struts, as well as satisfying minimum reinforcement requirements.

The large number of No. 11 (No. 36) bars will require either the use of mechanical anchorage or staggering the location of the hooks. Mechanical anchors require less space and would be preferable for concrete placement, but are often more expensive than standard hooks. Figure 10.14*d* shows staggered hooks for the final design. In addition, horizontal U-shaped No. 4 (No. 13) bars are placed at 4 in. ($3d_b = 3 \times 1.41$ in. $= 4.23$ in.) across the end of the beam to confine the No. 11 (No. 36) hooks. The final beam details are given in Fig. 10.14*d*.

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**EXAMPLE 10.2**

**Deep beam with distributed loads.** In addition to the concentrated loads, the transfer girder from Example 10.1 carries a distributed factored load of 3.96 kips/ft applied along its top edge, as shown in Fig. 10.15*a*. Design for the given loads, plus the self-weight, using $f'_c = 5000$ psi and $f_y = 60{,}000$ psi.

**Solution.** The factored self-weight of the beam is 1.2 (12 ft $\times$ 2 ft $\times$ 0.15 kips/ft$^3$) = 4.32 kips/ft. Thus, the total factored distributed load is 4.32 + 3.96 = 8.28 kips/ft, resulting in a total factored load of 8.28 kips/ft $\times$ 37.7 ft = 312 kips, approximately 13 percent of the column loads. The solution follows Example 10.1 and accounts for the distributed loads. For this example, the self-weight of the beam is combined with the superimposed dead load. A more conservative solution could place the self-weight at the bottom of the beam and correspondingly increase the vertical tension tie requirements to transfer the load to the top flange. The top placement is used in this case because the self-weight is a small percentage of the total load and the concentrated forces are moved slightly toward the center of the beam for a conservative placement.

**Definition of D-region**

The entire beam is a D-region, as shown in Fig. 10.15*a*. The maximum factored shear in the beam is $V_u = 1200 + 312 \cdot 2 = 1360$ kips $< \cdot \, 10 \cdot \sqrt{f'_c}\, b_w d = 1650$ kips, the maximum design shear using $d = 10.8$ ft. Thus, the design can continue.

**Force resultants on D-region boundaries**

The 1200 kip column loads are the same as in Example 10.1; however, the lower column reactions are equal to 1360 kips. Maintaining the same lower column size gives a stress at the beam-column interface of $p = 1360 \cdot (24 \times 24) = 2.36$ ksi.

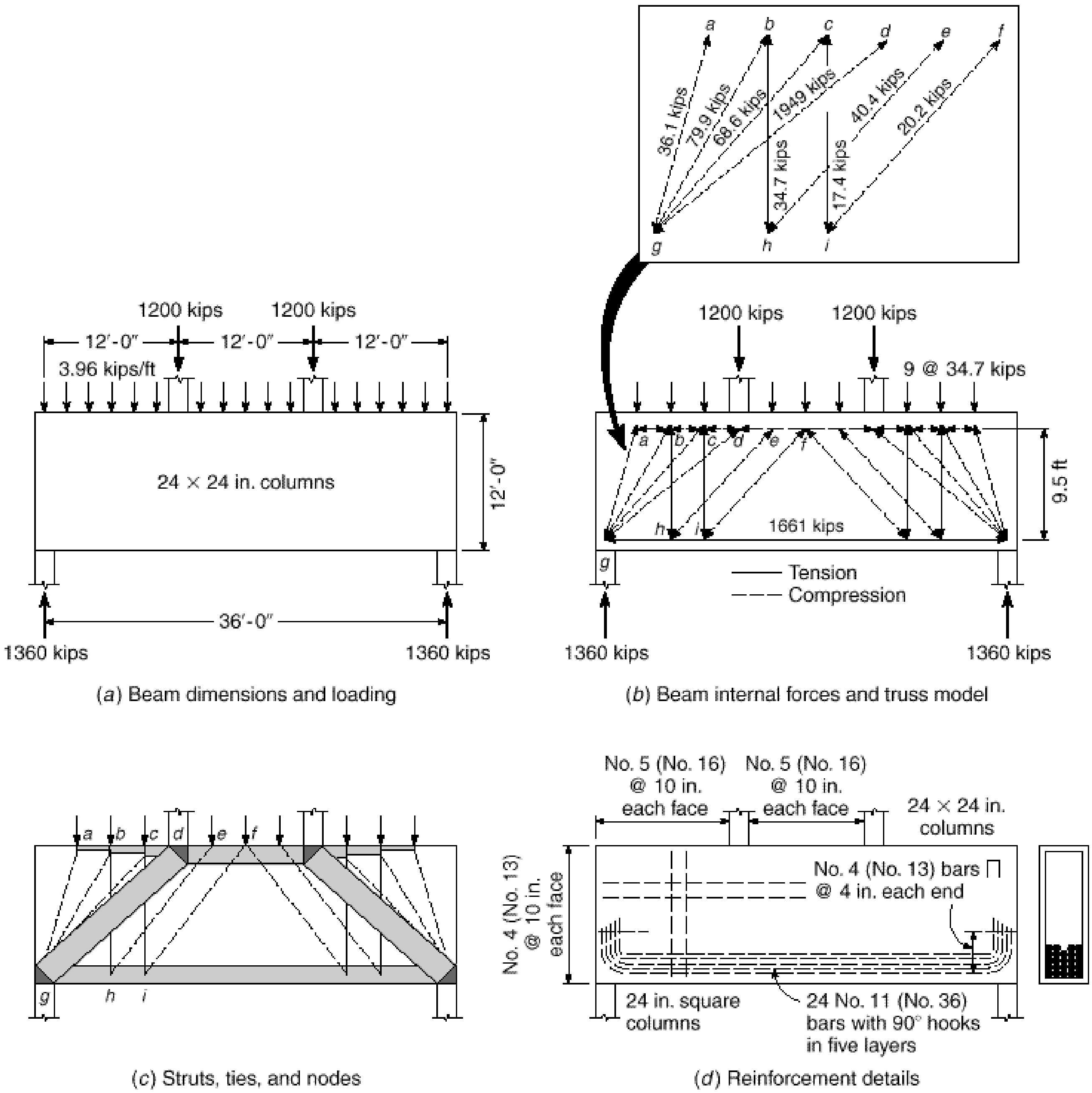


**FIGURE 10.15**
Deep beam with distributed loads for Example 10.2.

The stress on the column node cannot exceed the effective concrete strength. For a *C-C-T* node,

$$p \leq \cdot f_{cu} = \cdot \cdot {}_n 0.85 f'_c = 0.75 \times 0.80 \times 0.85 \times 5000 \cdot 1000 = 2.55 \text{ ksi}$$

Therefore, the bottom column of 24 × 24 in., giving $p = 1360 \cdot (24 \times 24) = 2.36 \text{ ksi} < 2.55 \text{ ksi}$, is adequate. As in Example 10.1, the center-to-center distance between the horizontal strut

**TABLE 10.3**
**Diagonal strut properties and forces for Example 10.2**

| Strut | Vertical Load, kips | Slope, degrees | Axial Load, kips | , in. | | | Strut End Capacity, kips | Horizontal Force, kips |
|---|---|---|---|---|---|---|---|---|
| *dg* | 1200 | 38.0 | 1949 | 34.4 | 0.75 | 0.8 | 1974 | 1536 |
| *ag* | 34.7 | 74.1 | 36.1 | 0.64 | 0.75 | 0.8 | 36.7 | 9.9 |
| *bg* | 69.4 | 60.3 | 79.9 | 1.41 | 0.75 | 0.8 | 81.0 | 39.7 |
| *cg* | 52.1 | 49.4 | 68.6 | 1.21 | 0.75 | 0.8 | 64.9 | 44.6 |
| *eh* | 34.7 | 59.3 | 40.4 | 0.71 | 0.75 | 0.6 | 40.4 | 20.6 |
| *fi* | 17.4 | 59.3 | 20.2 | 0.36 | 0.75 | 0.6 | 20.2 | 10.3 |
| Total tie force | | | | | | | | 1661 |

and the horizontal tie at midspan is taken 9.5 ft to compute the slope of the strut *dg* as = 38.0°. The vertical dimension for struts *ag*, *bg*, and *cg* is assumed to be 10.5 ft because they are anchored closer to the top edge of the beam.

The total distributed load of 312 kips is represented by nine 34.7 kip concentrated loads placed at 3 ft centers, as shown in Fig. 10.15*b*. Distributed loads can be grouped at the discretion of the designer. It would be equally satisfactory to group them into 12 loads, placed one per foot, or combine some load with the column loads. The loads are not combined with the column loads in this example to illustrate design for distributed loads. Using the geometric layout of the loads, strut and tie forces are computed and summarized in Fig. 10.15*b* and Table 10.3.

**The truss model**

In addition to the struts and ties needed to carry the column loads, struts and ties to carry the distributed loads are now included in the truss. The distributed loads between the columns are carried by struts to the bottom chord; tension ties then transfer the vertical component of the load to the top chord, while the horizontal component is transferred to the bottom tie. The geometry of the struts is selected to allow tension ties to be placed vertically. The loads at nodes *a*, *b*, and *c* between the column and the support create a fan of compression struts to node *g*, as shown in Fig. 10.15*b* and *c*.

**Selecting dimensions for strut and nodal zones**

The forces in the "fan" struts are based on the geometry of the struts. The widths of the struts *ag*, *bg*, and *cg* are computed based on the contact area at node *g*, because the capacity of a *C-C-T* node (at the lower end) is lower than that of a *C-C-C* node (at the upper end). The stress on the column at node *g* is $p = 2.36$ ksi. To maintain constant stress in the node, strut *ag* has a width $w_{ag} = F_u \cdot pb = 36.1 \cdot (2.36 \times 24) = 0.64$ in. The dimensions of struts *eh* and *fi*, with no concentrated loads acting directly on either end, are governed by the nodal capacity, since the nodes *h* and *i* are *C-T-T* nodes, with $_n = 0.60$ (Table 10.2). For example, for strut *eh*, $p = f_{cu} = 0.75 \times 0.85 \times 0.60 \times 5000 \cdot 1000 = 1.91$ ksi and width $w_{eh} = F_u \cdot pb = 40.4 \cdot (1.91 \times 24) = 0.88$ in. In this case, the design capacity will exactly equal the factored load. The remaining strut widths, geometries, and loads are summarized in Table 10.3.

**Capacity of struts**

By inspection, the stresses in the fan struts *ag*, *bg*, and *cg* will be critical as bottle-shaped struts. Using $_n = 0.75$ for a *C-C-T* node and $= 0.75$, strut *ag* has a design capacity $F_{ns,ag} = {}_s 0.85 f'_c w_{ab} t = 0.75 \times 0.75 \times 0.85 \times 5000 \times 0.64 \times 24 \cdot 1000 = 36.7$ kips, which is greater than the applied load of 36.1 kips. The strut capacities are summarized in Table 10.3. In all cases, the design strength $F_{ns}$ exceeds the applied forces and the struts are adequate.

**Design ties and anchorage**

Tie design is similar to that in Example 10.1, except that the additional horizontal thrust from the distributed loads increases the force to 1661 kips. The required area of steel for the tie is $A_s = F_{tu} \cdot f_y = 1661 \cdot (0.75 \times 60) = 36.9$ in$^2$ or 24 No. 11 (No. 36) bars. As in Example

10.1, 90° hooks will be required to anchor the tie. The steel will be placed in four layers of five bars and one layer of four bars. Example 10.1 validated that the reinforcement will fit in the available space.

The vertical tie *bh* carries 34.7 kips. The required area of steel for this tie is $A_s = F_{tu} \cdot \cdot f_y = 34.7 \cdot (0.75 \times 60) = 0.77$ in$^2$. Distributed steel is selected for vertical ties because the placement of the struts was arbitrary due to the assumptions made in modeling the distributed load. Thus, the 0.77 in$^2$ is distributed over 3 feet, the center-to-center spacing used for the distributed load. The minimum reinforcement in Example 10.1, No. 5 (No. 16) bars at 10 in. on each face, provides the required steel.

**Design details and minimum reinforcement requirements**

The minimum reinforcement requirements from Example 10.1 remain unchanged. The final details are shown in Fig. 10.15*d*.

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A comparison of Examples 10.1 and 10.2 demonstrates the sensitivity of the design to the applied loading. The addition of the distributed load resulted in an increase in the horizontal tie reinforcement, although the vertical reinforcement, which in the case of Example 10.2 serves as the vertical tie steel, remains unchanged.

## b. Dapped beam ends

Precast and prestressed concrete beams often have *dapped* or notched ends, such as shown in Fig. 10.16, to reduce the floor-to-floor height of buildings. The recess allows structural overlap between the main beams and the floor beams. While the dapped end is advantageous in controlling building floor-to-floor height, it creates two structural problems. First, the shear at the end of the beam must be carried by a much smaller section, and second, the mechanism of load transfer through the notched zone is difficult to represent using conventional design techniques. As a result, dapped-end beams lend themselves to design using strut-and-tie models.

**EXAMPLE 10.3**

**Design of a dapped beam end.** A 24 in. deep precast concrete T beam has a 10 in. thick web that carries factored end reactions of 67 kips in the vertical direction and 13 kips in the horizontal direction, as shown in Fig. 10.16*a*. The beam end is notched 10 in. vertically and 8 in. along the beam axis. The load is transferred to the support through a 4 × 10 in. bearing plate. Design the end reinforcement using $f'_c = 5000$ psi and $f_y = 60{,}000$ psi.

SOLUTION. The combination of the concentrated load and the geometric discontinuity suggest the use of a strut-and-tie solution.

**Definition of D-region**

The D-region for this beam is approximately one structural depth in from the end of the notch. The bearing plate will have longitudinal reinforcement welded to it to allow for load transfer. Therefore, the effective depth at the notch is taken as 13.0 in. The maximum allowable shear capacity is $V_u = \cdot V_n = \cdot 10 \cdot \overline{f'_c} b_w d = 0.75 \times 10 \cdot \overline{5000} \times 10 \times 13.0 \cdot 1000 =$ 68.9 kips. This exceeds the 67 kip applied load, so the section is adequate to proceed with the design.

**Force resultants on D-region boundaries and the truss model**

Three possible truss layouts are considered, as shown in Fig. 10.16*b*. Option 1 includes a vertical strut and a diagonal tension tie to the lower chord of the beam. The presence of several tension ties in this model suggests that this is not a minimum energy solution. Option 2 includes a diagonal compressive strut and a vertical hanger to transfer the load to the bottom chord of the beam. The compression strut below the dapped end transfers the tensile

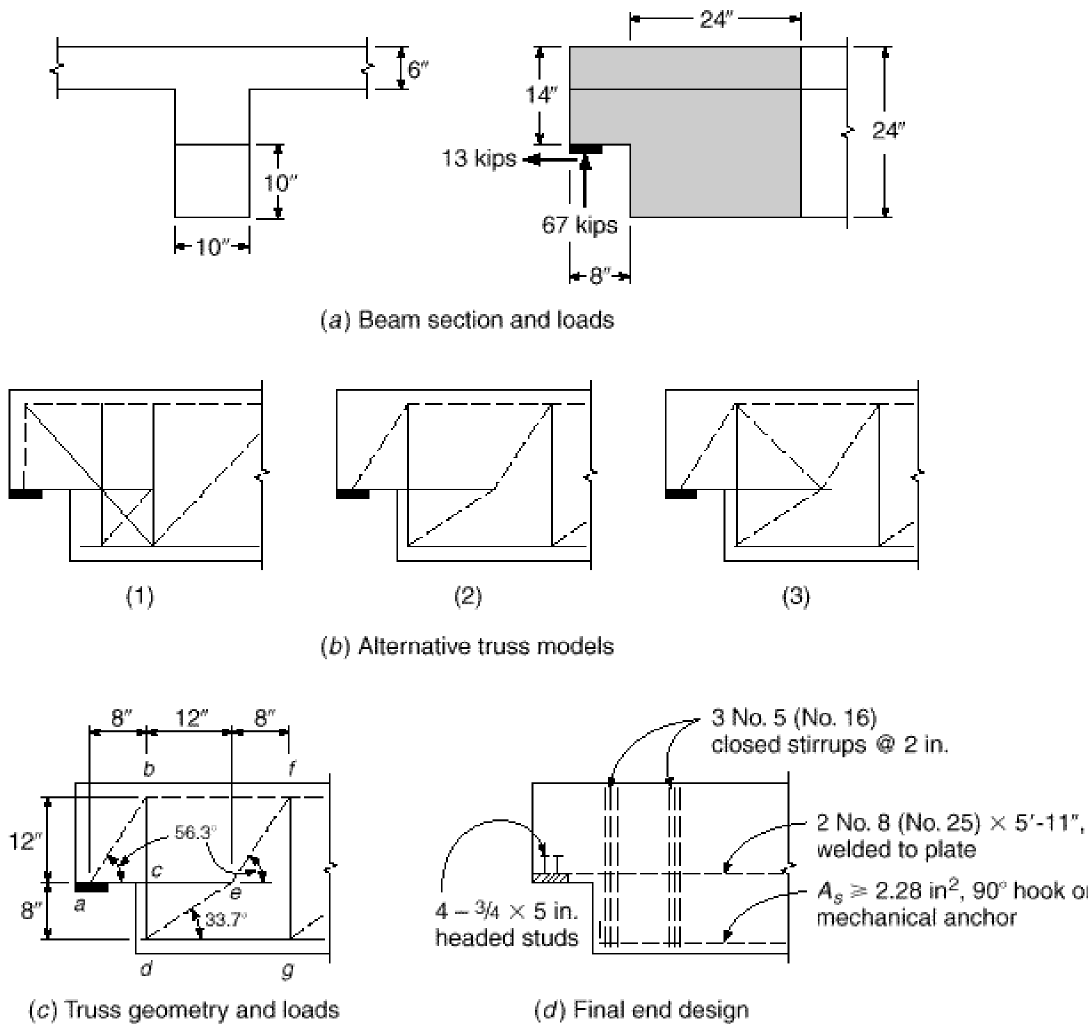


**FIGURE 10.16**
Dapped beam end design for Example 10.3.

reaction to the main longitudinal steel. Option 3 includes an internal triangular truss to balance the reaction within the interior of the beam. Balancing nodal forces at an indeterminate interior four-member joint poses difficulties in joint detailing, but may be desirable if concentrated loads are applied to the top flange. Option 2 is selected as the design choice based on its simplicity and limited number of tension ties. The dimensions of the truss are shown in Fig. 10.16*c*, and the truss forces are summarized in Table 10.4.

**Selecting dimensions for strut and nodal zones**
The nodal zone stress is established at the bearing plate. The stress is $p = V_u \cdot A_b = 67 \cdot (4 \times 10) = 1.68$ ksi. The calculations for strut *ab* follow, and the remaining strut and tie widths and capacities are given in Table 10.4. $F_{u,ab} = V_u \cdot \sin \cdot = 67 \cdot \sin 56.3° = 80.5$ kips. The strut width is $w_{ab} = F_{u,ab} \cdot (p \times 10) = 4.79$ in.

**Capacity of struts**
The strut design capacity is based on the strength of a bottle-shaped strut ($\cdot_s = 0.75$). For strut *ab*, $\cdot F_{ns,ab} = \cdot \cdot_n 0.85 f'_c w_{ab} b = 0.75 \times 0.75 \times 0.85 \times 5.0 \times 4.79 \times 10 = 114.5$ kips. This exceeds the applied load of 80.5 kips, so the strut is adequate. The remaining strut

## TABLE 10.4
## Strut and tie properties and forces for Example 10.3

| Member Type | Member | | | Force, kips | , in. | Capacity, kips | Required, in² | Provided, in² | Reinforcement |
|---|---|---|---|---|---|---|---|---|---|
| **Strut** | *ab* | 0.75 | 0.8 | 80.5 | 4.79 | 114.6 | | | |
| | *bf* | 1.0 | 0.8 | 44.7 | 2.66 | 67.8 | | | |
| | *de* | 0.75 | 0.8 | 120.8 | 7.19 | 171.9 | | | |
| | *ef* | 0.75 | 0.8 | 80.5 | 4.79 | 114.6 | | | |
| **Tie** | *ae* | | | 57.7 | 3.43 | | 1.28 | 1.58 | 2 No. 8 (No. 25) |
| | *bd** | | | 67.0 | 3.99 | | 1.49 | 1.86 | 3 No. 5 (No. 16) |
| | *dg*† | | | 100.5 | 5.98 | | 2.28 | 2.37 | 3 No. 8 (No. 25) |
| | *fg** | | | 67.0 | 3.99 | | 1.49 | 1.86 | 3 No. 5 (No. 16) |

* For ties *bd* and *fg*, use three No. 5 (No. 16) stirrups with two legs each, $A_s$ = 3 stirrups × 2 legs × 0.31 in² = 1.86 in².

† Tie *dg* is an extension of the main longitudinal reinforcement and must have an area ≥ 2.28 in² and a 90° hook or mechanical anchor at node *d*. If the main reinforcement is insufficient, auxiliary reinforcement may be added.

design capacities are summarized in Table 10.4. All exceed the applied forces, as would be expected with the low nodal stress used in this design.

**Design ties and anchorage**

For tie *bd*, $A_s = F_{u,bd} \cdot f_y$ = 67 (0.75 × 60) = 1.49 in². Three No. 5 (No. 16) stirrups provide 1.86 in² of steel. The maximum width for tie *bd* is $w_{bd} = F_{bd}\, pb$ = 67.0 (1.68 × 10) = 3.99 in. Three No. 5 (No. 16) stirrups may be placed within a total width of 2.7 in. and, thus, fit within the maximum tie width for *bd*. Tie *ae* carries both the horizontal component of strut *ab* and the 13 kip horizontal reaction. Therefore, $F_{u,af}$ = 67 × 8 12 + 13.0 = 57.7 kips, requiring an area of steel equal to 1.28 in², which is provided by two No. 8 (No. 25) bars. The anchorage length of the ties exceeds the available nodal dimensions. Therefore, tie *ae* is welded to the plate at node *a* and has a full development length to the right of node *f*. Ties *bd* and *fg* are detailed as closed stirrups. The area of steel and the selected bar sizes for the ties are tabulated in Table 10.4. Stirrups for tie *bd* are grouped together and should be added to any normal shear reinforcement from the B-region of the beam.

**Design details and minimum reinforcement requirements**

Strut *ab* transfers a horizontal thrust to node *a*. Welding the reinforcement for tie *ae* to the plate anchors the tie, but it is not sufficient to ensure that the horizontal component of the strut force is transferred to the tie. Two solutions are possible. First the plate at node *a* may be replaced with a steel angle. A 3.5 in. tall leg is needed to confine the nodal zone width. Alternatively, a more common practice in the precast industry, headed studs are welded to the plate and the connection is designed by the shear friction principles described in Section 4.9. Headed studs have a yield stress of 50,000 psi and a coefficient of friction between concrete and steel of 0.7. Thus, the area of studs required to resist the horizontal components of strut *ab* is $A_{vf} = V_{uf} \cdot f_y$ = 67(8 12) (0.75 × 50 × 0.7) = 1.70 in². Four 3/4 in. diameter × 5 in. long headed studs will be used to provide 1.76 in². The 5 in. length places the head of the stud outside of the nodal zone width.

Tie *dg* is an extension of the main longitudinal reinforcement. An area of Grade 60 steel ≥ 2.28 in² is needed to provide the force in the tie, which should also be checked against the reinforcement requirements for moment in the B-region. A 90° hook or mechanical anchor is required at node *d* to provide full development of the force in tie *dg*. If the beam is prestressed, the prestressing steel and the accompanying compression in the concrete provide an equivalent anchorage.

Minimum reinforcement in the dapped end is $A_{v,min} = 0.0025 b_w s$ = 0.0025 × 10 × 12 = 0.30 in²/ft. This is satisfied by No. 4 (No. 13) bars at 12 in. Since the steel in tie *bd* exceeds this, no further reinforcement is needed. The final connection is detailed in Fig. 10.16*d*.

The examples in this section illustrate both the methodology of strut-and-tie design and the importance of understanding the detailing requirements needed to transfer forces at nodes. Failure to appreciate the need to provide anchorage for the tie in Example 10.1 or to supply thrust resistance for the struts in Example 10.3 can lead to failure. In the examples, the contact area was used to establish the hydrostatic nodal pressure. As discussed, an equally acceptable solution would have been to select the maximum stress for one of the struts. The remaining strut and tie widths would then be adjusted accordingly.

## References

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## Problems

**10.1.** A deep beam with the dimensions and material properties given in Example 10.1 carries a single 24 × 24 in. column with a factored load of 1600 kips located 12 ft from the left end. Design the beam using a strut-and-tie solution that includes the self-weight of the beam. In your solution, include (*a*) a sketch of the load path and truss layout, (*b*) the sizes and geometry of the struts, ties, and nodal zones, and (*c*) a complete sketch of the final design.

**10.2.** Redesign the column bracket shown in Example 11.5 using the strut-and-tie method. Your strut-and-tie model may be based on Fig. 11.23. Material properties remain the same as in Example 11.5.

**10.3.** A 36 in. deep single T beam with a dapped end has a web thickness of 6 in. The factored end reactions are 82 kips in the vertical direction and 18 kips in the horizontal direction. The horizontal force places the beam in tension. The beam end is notched 12 in. high by 10 in. along the beam axis. Design the end connection using a bearing plate that is 3 in. wide with a thickness equal to that of the web. Adjust the bearing plate size if necessary. Specified material strengths are $f'_c = 6000$ psi and $f_y = 60{,}000$ psi.

**10.4.** A transfer girder has an overall depth of 11 ft and spans 22 ft between column supports. In addition to its own weight, it will pick up a uniformly distributed factored load of 3.8 kips/ft from the floor above and will carry a 14 × 14 in. column delivering a concentrated factored load of 1000 kips from floors above at midspan. The girder width must be equal to or less than 16 in. Design the beam for the given loads. Find the girder width and the area and geometry of tie steel, and specify the placement details. Material strengths are $f'_c = 5000$ psi and $f_y = 60{,}000$ psi.