11 Design of Reinforcement at Joints

11.1 Introduction

Most reinforced concrete failures occur not because of any inadequacies in analysis of the structure or in design of the members but because of inadequate attention to the detailing of reinforcement. Most often, the problem is at the connections of main structural elements (Ref. 11.1).

There is an increasing tendency in modern structural practice for the engineer to rely upon a detailer, employed by the reinforcing bar fabricator, to provide the joint design. Certainly, in many cases, standard details such as those found in the ACI Detailing Manual (Ref. 11.2) can be followed, but only the design engineer, with the complete results of analysis of the structure at hand, can make this judgment. In many other cases, special requirements for force transfer require that joint details be fully specified on the engineering drawings, including bend configurations and cutoff points for main bars and provision of supplementary reinforcement.

The basic requirement at joints is that all of the forces existing at the ends of the members must be transmitted through the joint to the supporting members. Complex stress states exist at the junction of beams and columns, for example, that must be recognized in designing the reinforcement. Sharp discontinuities occur in the direction of internal forces, and it is essential to place reinforcing bars, properly anchored, to resist the resulting tension. Some frequently used connection details, when tested, have been found to provide as little as 30 percent of the strength required (Refs. 11.1 and 11.3).

In recent years, important research has been directed toward establishing a better basis for joint design (Refs. 11.4 and 11.5). Full-scale tests of beam-column joints have led to improved design methods such as those described in *Recommendations for Design of Beam-Column Joints in Monolithic Reinforced Concrete Structures,* reported by ACI-ASCE Committee 352 (Ref. 11.6). Although they are not a part of the ACI Code, such recommendations provide a basis for the safe design of beam-column joints both for ordinary construction and for buildings subject to seismic forces. Other tests have given valuable insight into the behavior of beam-girder joints, wall junctions, and other joint configurations, thus providing a sound basis for design.

Practicality of the joint design should not be overlooked. Beam reinforcement entering a beam-column joint must clear the vertical column bars, and timely consideration of this fact in selecting member widths and bar size and spacing can avoid costly delays in the field. Similarly, beam steel and girder steel, intersecting at right angles at a typical beam-girder-column joint, cannot be in the same horizontal plane as they enter the joint. Figure 11.1 illustrates the congestion of reinforcing bars at such

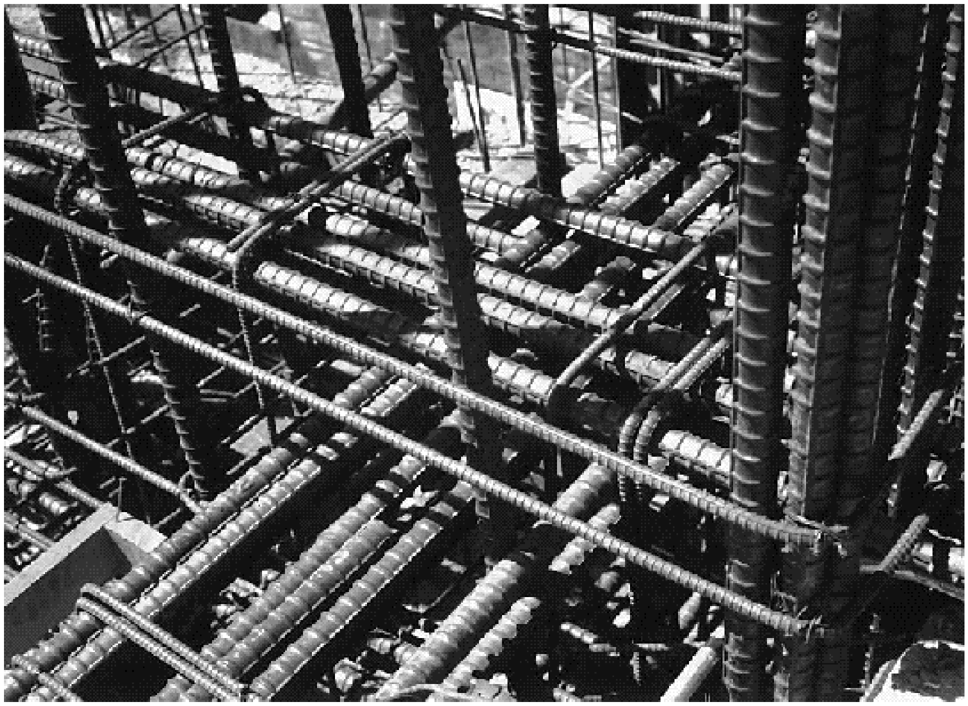


FIGURE 11.1
Steel congestion at beam-girder-column joint.

an intersection. Concrete placement in such a region is difficult at best, but is assisted with the use of plasticizer admixtures.

Most of this chapter treats the design of joint regions for typical continuous-frame monolithic structures that are designed according to the strength requirements of the ACI Code for gravity loads or normal wind loads. Joints connecting members that must sustain strength under reversals of deformation into the inelastic range, as in earthquakes, represent a separate category and are covered in Chapter 20. Brackets and corbels, although they are most often a part of precast buildings rather than monolithic construction, have features in common with monolithic joints, and these will be covered here.

11.2 BEAM-COLUMN JOINTS

A *beam-column joint* is defined as the portion of a column within the depth of the beams that frame into it. Formerly, the design of monolithic joints was limited to providing adequate anchorage for the reinforcement. However, the increasing use of high-strength concrete, resulting in smaller member cross sections, and the use of larger-diameter and higher-strength reinforcing bars now require that more attention be given to joint design and detailing. Although very little guidance is provided by the ACI Code, the ACI-ASCE Committee 352 report *Recommendations for Design of Beam-Column Joints in Monolithic Reinforced Concrete Structures* (Ref. 11.6) provides a basis for the design of joints in both ordinary structures and structures required to resist heavy cyclic loading into the inelastic range.

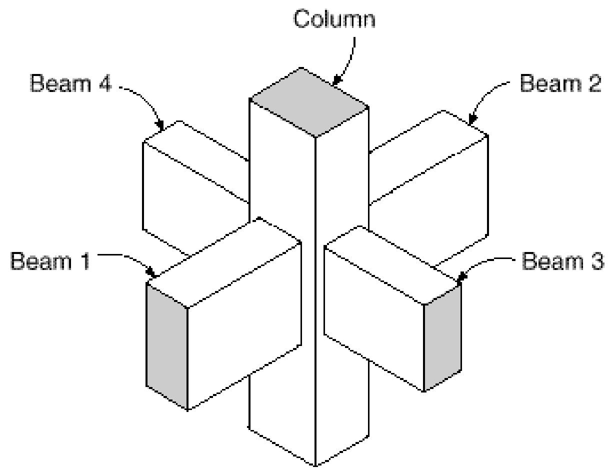


FIGURE 11.2
Typical monolithic interior beam-column joint.

a. Classification of Joints

Reference 11.6 classifies structural joints into two categories. A *Type 1* joint connects members in an ordinary structure designed on the basis of strength, according to the main body of the ACI Code, to resist gravity and normal wind load. A *Type 2* joint connects members designed to have sustained strength under deformation reversals into the inelastic range, such as members in a structure designed for earthquake motions, very high winds, or blast effects. Only Type 1 joints will be considered in this chapter.

Figure 11.2 shows a typical *interior joint* in a monolithic reinforced concrete frame, with beams 1 and 2 framing into opposite faces of the column and beams 3 and 4 framing into the column faces in the perpendicular direction. An *exterior joint* would include beams 1, 2, and 3, or in some cases only beams 1 and 2. A *corner joint* would include only beams 1 and 3, or occasionally only a single beam 1. A joint may have beams framing into it from two perpendicular directions as shown, but for purposes of analysis and design each direction can be considered separately.

b. Joint Loads and Resulting Forces

The joint region must be designed to resist forces that the beams and column transfer to the joint, including axial loads, bending, torsion, and shear. Figure 11.3*a* shows joint loads acting on the free body of a typical joint of a frame subject to gravity loads, with moments M_1 and M_2 acting on opposite faces, in the opposing sense. In general these moments will be unequal, with their difference equilibrated by the sum of the column moments M_3 and M_4. Figure 11.3*b* shows the resulting forces to be transmitted through the joint. Similarly, Fig. 11.4*a* shows the loads on a joint in a structure subjected to sidesway loading. The corresponding joint forces are shown in Fig. 11.4*b*. Only for very heavy lateral loading, such as from seismic forces, would the moments acting on opposite faces of the joint act in the same sense, as shown here, producing very high horizontal shears within the joint.

According to the recommendations by Committee 352, the forces to be considered in designing joint regions are not those determined from the conventional frame analysis; rather, they are calculated based on the *nominal strengths of the members*.

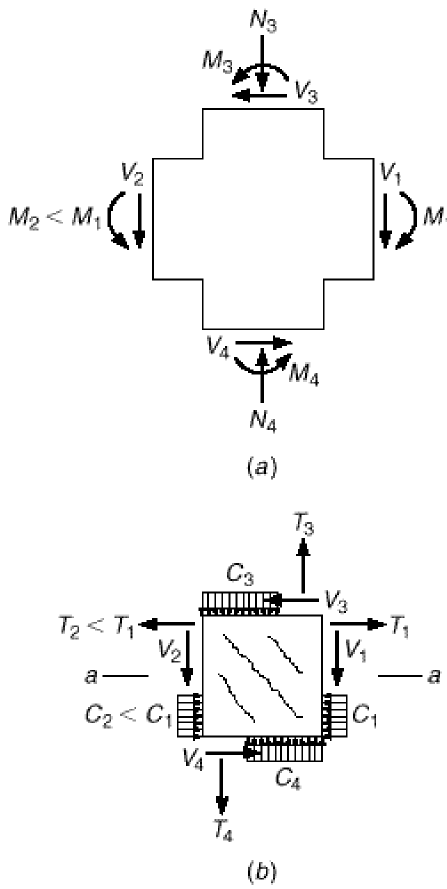


FIGURE 11.3
Joint loads and forces resulting from gravity loads: (*a*) forces and moments on the free body of a joint region; (*b*) resulting internal forces.

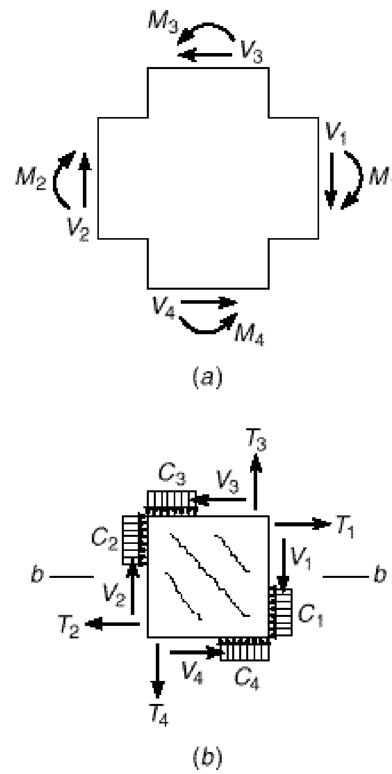


FIGURE 11.4
Joint loads and forces resulting from lateral loads: (*a*) forces and moments on the free body of a joint; (*b*) resulting internal forces.

Where a typical underreinforced beam meets the column face, the tension force from the negative moment reinforcement at the top of the beam is to be taken as $T = A_s f_y$, and the compression force at the face is from equilibrium $C = T$, not the nominal compressive capacity of the concrete. The design moment applied at the joint face is that corresponding to these maximum forces, $M_u = M_n = A_s f_y(d - a{\cdot}2)$, rather than that from the overall analysis of the frame. Note that the inclusion of the usual strength reduction factor · would be unconservative in the present case because it would reduce the forces for which the joint is to be designed; it is therefore not included in this calculation.

With the moment applied to each joint face found in this way, the corresponding column forces for joint design are those forces required to keep the connection in equilibrium. To illustrate, the column shears V_3 and V_4 of Figs. 11.3*a* and 11.4*a* are calculated based on the free body of the column between inflection points, as shown in Fig. 11.5. The inflection points generally can be assumed at column midheight, as shown.

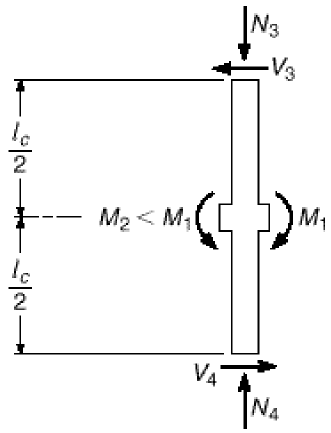


FIGURE 11.5
Free-body diagram of an interior column and joint region.

c. Shear Strength of a Joint

A joint subject to the forces shown in Figs. 11.3*b* or 11.4*b* will develop a pattern of diagonal cracking owing to the diagonal tensile stresses that result from the normal forces and shears, as indicated by those figures. The approach used by Committee 352 is to limit the shear force on a horizontal plane through the joint to a value established by tests. The design basis is

$$V_u \leq \cdot\ V_n \tag{11.1}$$

where V_u is the applied shear force, V_n is the nominal shear strength of the joint, and · is taken equal to 0.75.

The shear force V_u is to be calculated on a horizontal plane at midheight of the joint, such as plane *a-a* of Fig. 11.3*b* or plane *b-b* of Fig. 11.4*b*, by summing horizontal forces acting on the joint above that plane. For example, in Fig. 11.3*b* the joint shear on plane *a-a* is

$$V_u = T_1 - T_2 - V_3$$

and in Fig. 11.4*b*, the joint shear on plane *b-b* is

$$V_u = T_1 + C_2 - V_3$$
$$= T_1 + T_2 - V_3$$

The nominal shear strength V_n is given by the equation

$$V_n = \cdot\ \cdot\ \overline{f_c'}\, b_j h \tag{11.2}$$

where b_j is the effective joint width in inches, h is the thickness in inches of the column in the direction of the load being considered, and · $\overline{f_c'}$ is expressed in psi units. The value of f_c' used in Eq. (11.2) is not to be taken greater than 6000 psi, even though the actual strength may be larger, because of the lack of research information on connections using high-strength concrete. As discussed in Chapter 20, ACI Code 21.5 follows similar procedures for the design of joints in moment resistant frames, the only difference being that lower values for the coefficient · are recommended.

The coefficient · in Eq. (11.2) depends on the confinement of the joint provided by the beams framing into it, as follows:

	Gravity frames	Moment resisting frames
Interior joint	· = 24	· = 20
Exterior joint	· = 20	· = 15
Corner joint	· = 15	· = 12

The definitions of interior, exterior, and corner joints were discussed in Section 11.2a and shown in Fig. 11.2. However, there are restrictions to be applied for purposes of determining · as follows:

1. An *interior joint* has beams framing into all four sides of the joint. However, to be classified as an interior joint, the beams should cover at least $\frac{3}{4}$ the width of the column, and the total depth of the shallowest beam should not be less than $\frac{3}{4}$ the total depth of the deepest beam. Interior joints that do not satisfy this requirement should be classified as *exterior joints.*
2. An *exterior joint* has at least two beams framing into opposite sides of the joint. However, to be classified as an exterior joint, the widths of the beams on the two opposite faces of the joint should cover at least $\frac{3}{4}$ the width of the column, and the depths of these two beams should be not less than $\frac{3}{4}$ the total depth of the deepest beam framing into the joint. Joints that do not satisfy this requirement should be classified as *corner joints.*

For joints with beams framing in from two perpendicular directions, as for a typical interior joint, the horizontal shear should be checked independently in each direction. Although such a joint is designed to resist shear in two directions, only one classification is made for the joint in this case (i.e., only one value of · is selected based on the joint classification, and that value is used to compute V_n when checking the design shear capacity in each direction).

According to Committee 352 recommendations, the effective joint width b_j to be used in Eq. (11.2) depends on the transverse width of the beams that frame into the column as well as the transverse width of the column. With regard to the beam width b_b, if there is a single beam framing into the column in the load direction, then b_b is the width of that beam. If there are two beams in the direction of shear, one framing into each column face, then b_b is the average of the two beam widths. In reference to Fig. 11.6*a*, when the beam width is less than the column width, the effective joint width is the average of the beam width and column width, but it should not exceed the beam width plus one-half the column depth h on each side of the beam. That is,

$$b_j = \frac{b_b + b_c}{2} \qquad \text{and} \qquad b_j \le b_b + h \tag{11.3}$$

If the beam frames flush with one face of the column, as is common for exterior joints, the same criteria result in an effective joint width of

$$b_j = \frac{b_b + b_c}{2} \qquad \text{and} \qquad b_j \le b_b + \frac{h}{2} \tag{11.4}$$

as shown in Fig. 11.6*b*. If the beam width b_b *exceeds* the column width (permitted for Type 1 joints only), the effective joint width b_j is equal to the column width b_c, as shown in Fig. 11.6*c*.

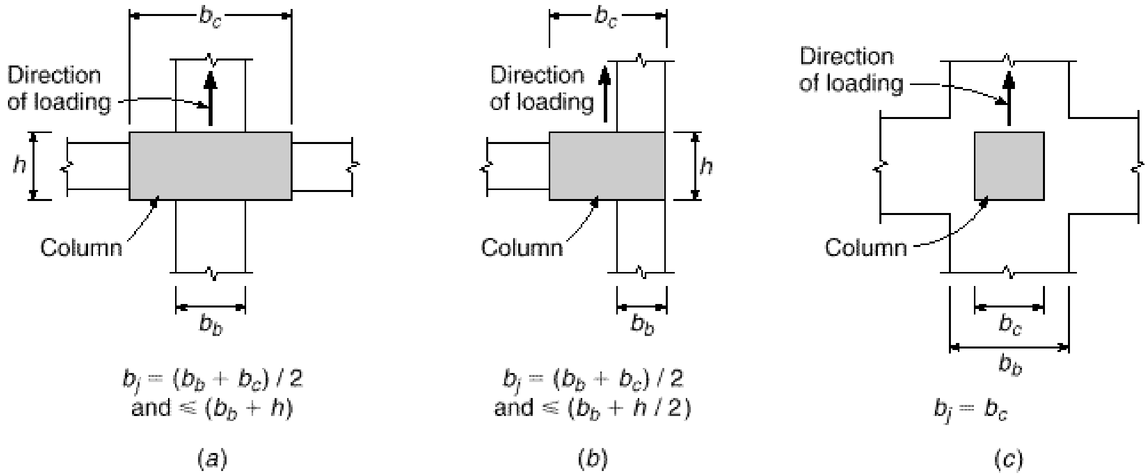


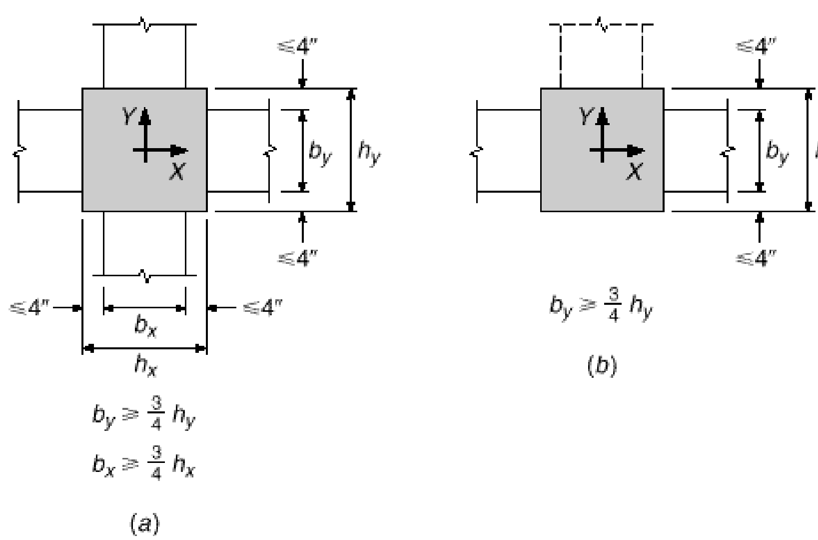
FIGURE 11.6
Determination of effective joint width *b*: (*a*) interior joint; (*b*) exterior or corner joint; (*c*) beam wider than column.

d. Confinement and Transverse Joint Reinforcement

The successful performance of a beam-column joint depends strongly on the lateral confinement of the joint. Confinement has two benefits: (a) the core concrete is strengthened and its strain capacity improved, and (b) the vertical column bars are prevented from buckling outward. Confinement can be provided either by the beams that frame into the joint or by special column ties provided within the joint region.

Confinement by beams is illustrated in Fig. 11.7. According to Committee 352 recommendations, if beams frame into four sides of the joint, as in Fig. 11.7*a*, adequate confinement is provided if each beam width is at least $\frac{3}{4}$ the width of the intersected column face and if no more than 4 in. of column face is exposed on either side of the beam. Where beams frame into only two sides of the joint, as in Fig. 11.7*b*, adequate

FIGURE 11.7
Confinement of joint concrete by beams: (*a*) confinement in *X* and *Y* directions; (*b*) confinement in *X* direction only.



confinement can be assumed *in the direction of the beams* if the beam widths are at least $\frac{3}{4}$ the column width and no more than 4 in. of concrete is exposed on either side of the beams. In the other direction, transverse reinforcement must be provided for confinement. The presence of a third beam, but not a fourth, in the perpendicular direction does not modify the requirement for transverse reinforcement.

If adequate confinement is not provided by beams according to these criteria, then transverse reinforcement must be provided. If confinement steel is needed, it must meet all the usual requirements for column ties (see Section 8.2). In addition, there must be at least two layers of ties between the top and bottom flexural steel in the beams at the joint, and the vertical center-to-center spacing of these ties must not exceed 12 in. If the beam-column joint is part of the primary system for resisting non-seismic lateral loads, this maximum spacing is reduced to 6 in. For joints that are not confined by beams on four sides, ACI Code 11.11 requires that the ties satisfy Eq. (4.13).

e. Anchorage and Development of Beam Reinforcement

For interior joints, normally the flexural reinforcement in a beam entering one face of the joint is continued through the joint to become the flexural steel for the beam entering the opposite face. Therefore, for loadings associated with Type 1 joints, pullout is unlikely, and no special recommendations are made. However, for exterior or corner joints, where one or more of the beams do not continue beyond the joint, a problem of bar anchorage exists. The critical section for development of the yield strength of the beam steel is at the face of the column. Column dimensions seldom permit development of the steel entering the joint by straight embedment alone, and hooks are usually needed for the negative beam reinforcement. Ninety degree hooks are used, with the hook extending toward and beyond the middepth of the joint. If the bottom bars entering the joint need to develop their strength $A_s f_y$ at the face of the joint, as they do if the beam is a part of a primary lateral load-resisting system, they should have 90° hooks also, in this case turned upward to extend toward the middepth of the joint. Requirements for development of hooked bars given in Chapter 5 are applicable in both cases, including modification factors for concrete cover and for enclosure with ties or stirrups.

EXAMPLE 11.1

Design of exterior Type 1 joint. The exterior joint shown in Fig. 11.8 is a part of a continuous, monolithic, reinforced concrete frame designed to resist gravity loads only. Member section dimensions $b \times h$ and reinforcements are as shown. The frame story height is 12 ft. Material strengths are $f'_c = 4000$ psi and $f_y = 60{,}000$ psi. Design the joint, following the recommendations of the Committee 352 report.

Solution. First the joint geometry must be carefully laid out, to be sure that beam bars and column bars do not interfere with one another and that placement and vibration of the concrete are practical. In this case, bar layout is simplified by making the column 4 in. wider than the beams. Column steel is placed with the usual 1.5 in. of concrete outside of the No. 4 (No. 13) ties. Beam top and bottom bars are placed just inside the outer column bars. The slight offset of the center top beam bars to avoid the center column bars is of no concern. Top bars of the spandrel beams are placed just under the top normal beam bars, except for the outer spandrel bar, which is above the hook shown in Fig. 11.8*b*. Bottom bars enter the joint at different levels without interference.

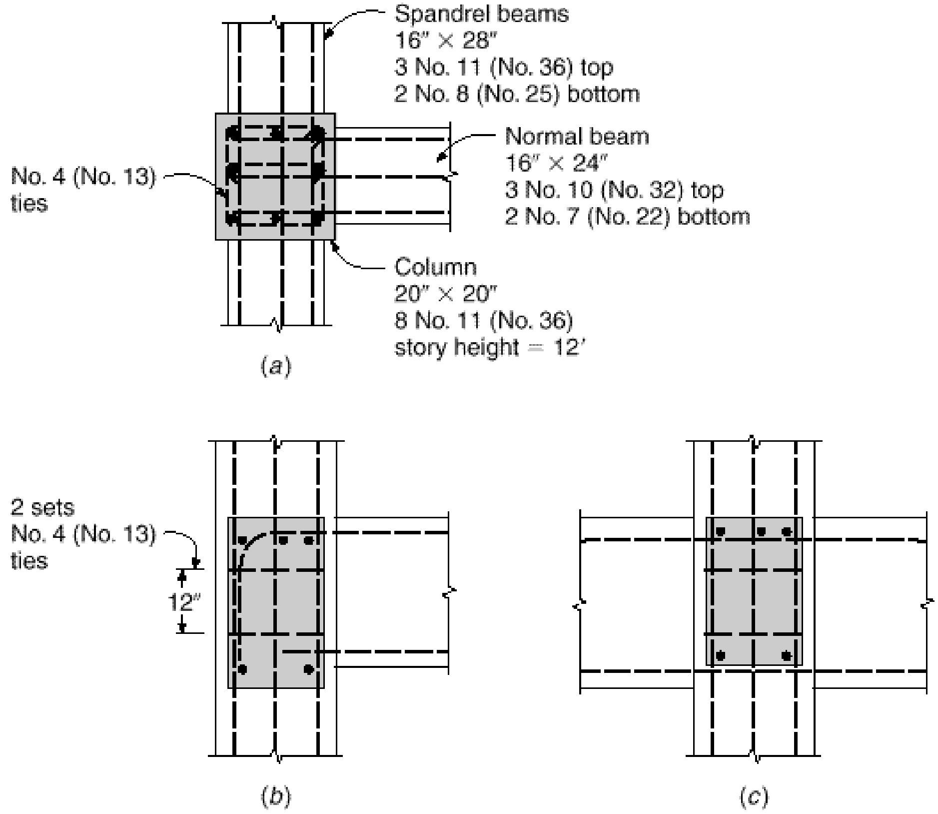


FIGURE 11.8
Exterior beam-column joint for Example 11.1: (*a*) plan view; (*b*) cross section through spandrel beam; (*c*) cross section through normal beam. Note that beam stirrups and column ties outside of the joint are not shown.

No anchorage problems exist for the spandrel beam top reinforcement, which is continuous through the joint. However, the normal beam top steel must be provided with hooks to develop its yield strength at the face of the column. Referring to Table 5.3, the basic development length for No. 10 (No. 32) hooked bars is

$$l_{dh} = \frac{0.02\beta\lambda f_y}{\sqrt{f'_c}} d_b = \frac{0.02 \times 1 \times 1 \times 60{,}000}{\sqrt{4000}} 1.27 = 24.1 \text{ in.}$$

Being inside the column bars, the beam top bars have side cover of $1.5 + 0.5 + 1.4 = 3.4$ in. This exceeds 2.5 in., so a modification factor of 0.7 is applicable, and the required hook development length is

$$l_{dh} = 24.1 \times 0.7 = 16.9 \text{ in.}$$

If the hooked bars are carried down just inside the column ties, the actual embedded length is $20.0 - 1.5 - 0.5 = 18.0$ in., exceeding 16.9 in., so development is ensured. None of the beams are a part of the primary, lateral load–resisting system of the frame, so the bottom bars simply can be carried 6 in. into the face of the joint and stopped.

Next the shear strength of the joint must be checked. In the direction of the spandrel beams, moments applied to the joint will be about the same and acting in the opposite sense, so very little joint shear is expected in that direction. However, the normal beam will subject the joint to horizontal shears. In reference to Fig. 11.9*a*, which shows a free-body sketch of the top half of the joint, the maximum force from the beam top steel is

$$A_s f_y = 3.81 \times 60 = 229 \text{ kips}$$

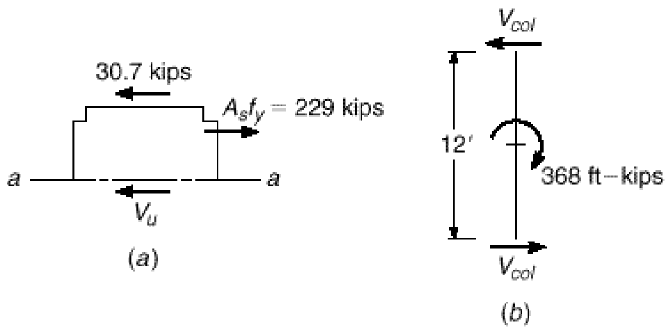


FIGURE 11.9
Basis of column shear for Example 11.1: (*a*) horizontal forces on joint free-body sketch; (*b*) free-body sketch of column between inflection points.

The joint moment is calculated based on this tensile force. The normal beam effective depth is $d = 24.0 - 1.5 - 0.5 - 1.27 \cdot 2 = 21.4$ in. and with stress block depth $a = A_s f_y \cdot 0.85 f'_c b_w = 229 \cdot (0.85 \times 4 \times 16) = 4.21$ in., the design moment is

$$M_u = M_n = A_s f_y \cdot d - \frac{a}{2} \cdot = \frac{229}{12} \cdot 21.4 - \frac{4.21}{2} \cdot = 368 \text{ ft-kips}$$

Column shears corresponding to this joint moment are found based on the free body of the column between assumed midheight inflection points, as shown in Fig. 11.9*b*: $V_{col} = 368 \cdot 12 = 30.7$ kips. Then summing horizontal forces on the joint above the middepth plane *a-a*, the joint shear in the direction of the normal beam is

$$V_u = 229 - 30.7 = 198 \text{ kips}$$

For purposes of calculating the joint shear strength, the joint can be classified as exterior, because the 16 in. width of the spandrel beams exceeds $\frac{3}{4}$ the column width of 15 in., and the spandrels are the deepest beams framing into the joint. Thus, · = 20. The effective joint width is

$$b_j = \frac{b_b + b_c}{2} = \frac{16 + 20}{2} = 18 \text{ in.}$$

but not to exceed $b_b + h = 16 + 20 = 36$ in., which does not control. Then the nominal and design shear strengths of the joint are respectively

$$V_n = \cdot \cdot \overline{f'_c}\, b_j h = 20 \cdot \overline{4000} \times 18 \times \frac{20}{1000} = 455 \text{ kips}$$

$$\cdot V_n = 0.75 \times 455 = 341 \text{ kips}$$

The applied shear $V_u = 198$ kips does not exceed the design strength, so shear is satisfactory.

Confinement is provided in the direction of the spandrel beams by the beams themselves because the spandrel width of 16 in. exceeds $\frac{3}{4}$ the column width and no more than 4 in. of column face is exposed on either side. However, in the direction of the normal beam, confinement must be provided by column ties within the joint. Two sets of No. 4 (No. 13) ties will be provided, as shown in Fig. 11.8*a* and *b*. The clear distance between column bars is 5.89 in. here, less than 6 in., so strictly speaking the single-leg cross tie is not required. However, it will improve the joint confinement, guard against outward buckling of the central No. 11 (No. 36) column bar, and add little to the cost of construction, so it will be specified as shown in Fig. 11.8*a*. The ties satisfy Eq. (4.13) several times over. Note that a 90° hook at one end, rather than the 135° bend shown, would meet ACI Code tie anchorage requirements and would facilitate steel fabrication.

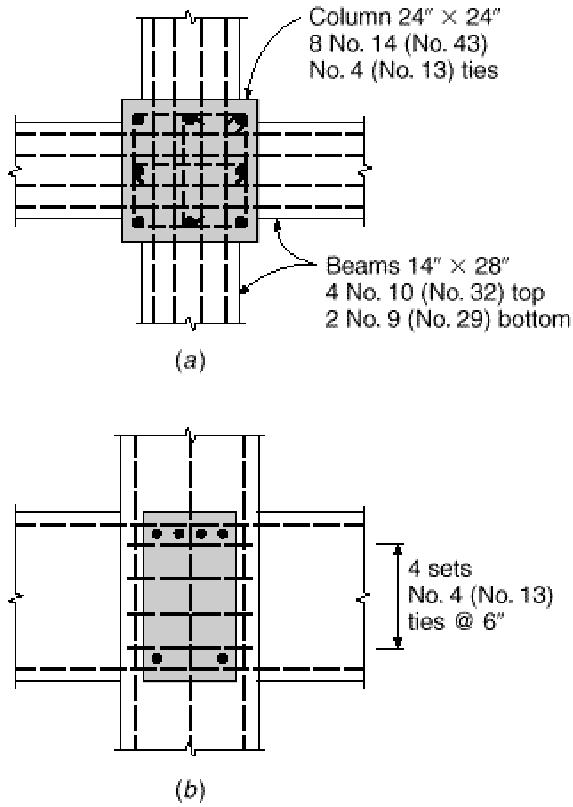


FIGURE 11.10
Interior beam-column joint for Example 11.2: (*a*) plan view; (*b*) section through beam.

EXAMPLE 11.2

Design of interior Type 1 joint. Figure 11.10 shows a proposed interior joint of a reinforced concrete building, with beam and column dimensions and reinforcement as indicated. The building frame is to carry gravity loads and normal wind loads. Design and detail the joint reinforcement.

SOLUTION. Because the joint is to be a part of the primary, lateral load–resisting system, beam bottom bars as well as top bars are carried straight through the joint for anchorage. In such cases, it is usually convenient to lap splice the bottom steel near the point of inflection of the beams.

In Fig. 11.10*a* and *b*, top and bottom beam bars entering the joint in one direction must pass, respectively, under and over the corresponding bars in the perpendicular direction. It will be assumed that this has been recognized by adjusting the effective depths in designing the beams. Because the column is 10 in. wider than the beams, the outer beam bars can be passed inside the corner column bars without interference. Four bars are used for the beam top steel in order to avoid interference with the center column bar.

Even the combination of normal wind loading with gravity loads should not produce large unbalanced moment on opposite faces of this interior column, and it can be safely assumed that joint shear will not be critical. However, confinement of the joint region by the beams is considered inadequate because (a) the beam width of 14 in. is less than $\frac{3}{4}$ the column width of 24 in., and (b) the exposed column face outside the beam is $(24 - 14)/2 = 5$ in., which exceeds the 4 in. limit. Consequently, transverse column ties must be added within the joint for confinement. For the 24 in. square column, the spacing between the vertical bars exceeds

6 in., so it is necessary, according to the ACI Code, to provide ties to support the intermediate bars as well as the corner bars. Three ties are used per set, as shown in Fig. 11.10*a*. Since the joint is a part of the lateral load–resisting system, the maximum vertical spacing of these tie sets is 6 in. Four sets within the joint, as indicated in Fig. 11.10*b*, are adequate to satisfy this requirement.

f. Wide-Beam Joints

In multistory buildings, to reduce the construction depth of each floor and to reduce the overall building height, wide shallow beams are sometimes used. Joint design in cases where the beams are wider than the column introduces some important concepts not addressed in the Committee 352 report, although most of the report's provisions can be applied. It is important to equilibrate all of the forces applied to the joint. The tension from the top bars in the usual case, with beam width no greater than the column, will be equilibrated by the horizontal component of a diagonal compression strut within the joint. The diagonal compression at the ends of the strut, in turn, is equilibrated by the beam compression and the thrust from the column. (See Section 11.3 for a more complete description of the strut-and-tie model.) If the outer bars of the normal beam pass *outside* of the column, as they might in wide-beam designs, the diagonal strut also will be outside of the column, with no equilibrating vertical compression at its upper and lower ends. The outer parts of the beam would tend to shear off, resulting in premature failure.

Two possibilities exist to overcome this problem. The first requires that all of the beam top steel be placed within the width of the column, and preferably inside the outer column bars. Second, if the normal beam bars are carried outside the joint, vertical stirrups can be provided through the joint region to carry the vertical component of thrust from the compression strut.

In extreme but not unusual cases, very wide beams are used, several times wider than the column, with beam depth only about 2 times the slab depth. In such cases, a safe basis for joint design is to treat the wide beam as a slab and follow the recommendations for slab-column connections contained in Chapter 13.

EXAMPLE 11.3 **Design of exterior Type 1 joint with wide beams.** Figure 11.11 shows a typical exterior joint in the floor of a wide-beam structure, designed to resist gravity loads. Here the beams in each direction are 8 in. wider than the corresponding column dimension. Check the proposed joint geometry and shear strength, and design the transverse joint reinforcement. Material strengths are $f_c' = 4000$ psi and $f_y = 60{,}000$ psi. Story height is 12 ft.

Solution. For the present case, all normal beam top steel is passed inside the core of the joint, terminating in 90° hooks at the outside of the column. Top steel in the spandrel beams is continuous through the joint but is also carried inside the joint core. Bottom beam bars, in each case, can be spread across the width of the beam, and they are carried only 6 in. into the joint for the normal beam because the joint is not a part of the primary, lateral load–resisting system. The bottom spandrel beam bars are continued to provide structural integrity (ACI Code 7.13). Beam stirrups outside of the joint, not shown in Fig. 11.11, would be carried outside of the outer bottom bars and bent up. They would require small-diameter horizontal bars inside the hooks for proper anchorage at the upper ends of their vertical legs.

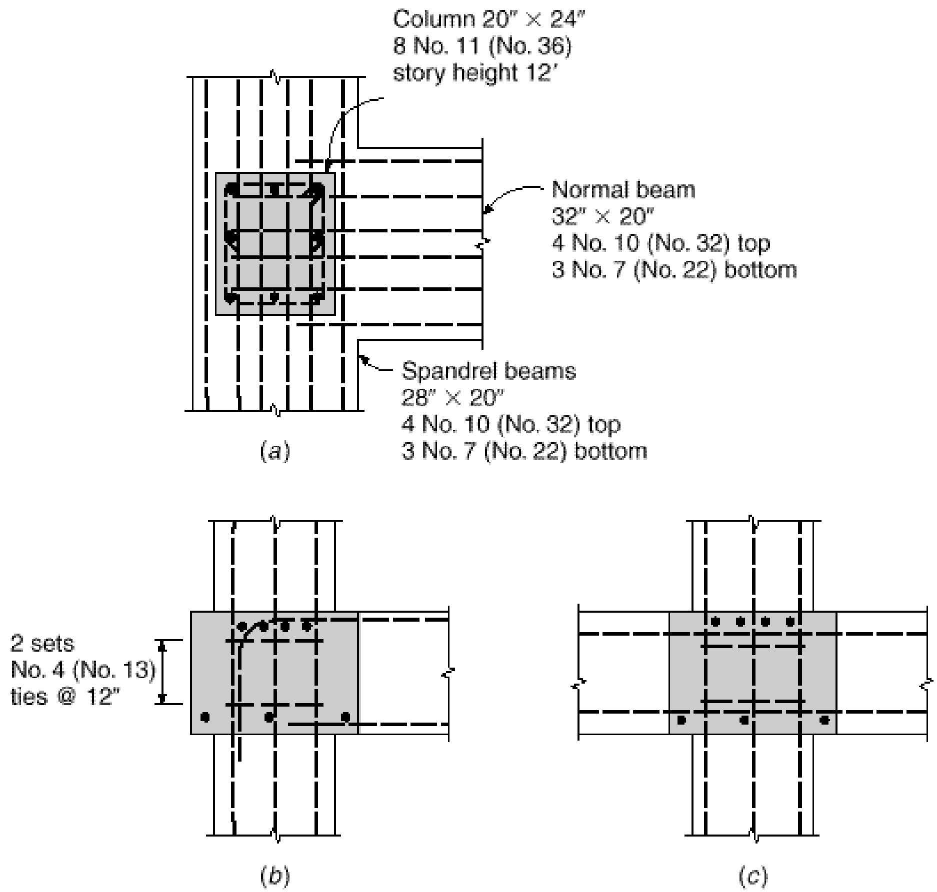


FIGURE 11.11
Exterior beam-column joint for Example 11.3: (*a*) plan view; (*b*) section through spandrel beam; (*c*) section through normal beam.

Checking the required development length of the No. 10 (No. 32) top bars of the normal beam gives

$$l_{dh} = \left(\frac{0.02\beta\lambda f_y}{\sqrt{f'_c}}\right) d_b = \left(\frac{0.02 \times 1 \times 1 \times 60{,}000}{\sqrt{4000}}\right) 1.27 = 24.1 \text{ in.}$$

With lateral cover well in excess of 2.5 in., a modification factor of 0.7 is applicable, and the necessary hook development length is

$$l_{dh} = 24.1 \times 0.7 = 16.9 \text{ in.}$$

If the hooks are carried down in the plane of the outer column bars, the available embedment is $20.0 - 1.5 - 0.5 = 18.0$ in., exceeding the minimum required embedment.

Moments from the spandrels on either side of the joint will be about equal, so no joint shear problem exists in that direction. In the direction of the normal beam, shear must be checked. The tensile force applied by the top bars is $A_s f_y = 5.08 \times 60 = 305$ kips. The depth of the beam compressive stress block is $a = A_s f_y / 0.85 f'_c b_w = 305/(0.85 \times 4 \times 32) = 2.80$ in., and the corresponding moment is

$$M_u = M_n = A_s f_y \left(d - \frac{a}{2}\right) = \frac{305}{12}\left(17.6 - \frac{2.80}{2}\right) = 412 \text{ ft-kips}$$

Column shears are based on a free body corresponding to that of Fig. 11.9*b*, and are equal to $V_{col} = 412/12 = 34.3$ kips. Thus, the joint shear at middepth is $V_u = 305 - 34.3 = 270$ kips.

The spandrel beams provide full-width joint confinement in their direction, and the joint can be classed as exterior, so $\gamma = 20$. In the perpendicular direction, when the beam width exceeds the column width, the joint width b_j is to be taken equal to the column width (24 in. in the present case). The nominal and design shear strengths are respectively

$$V_n = \gamma \sqrt{f_c'}\, b_j h = 20\sqrt{4000} \times 24 \times 20/1000 = 607 \text{ kips}$$

$$\phi V_n = 0.75 \times 607 = 455 \text{ kips}$$

Because the design strength is well above the applied shear of 270 kips, the shear requirement is met.

Transverse confinement steel must be provided in the direction of the normal beam, between the top and bottom bars of the normal beam, with spacing not to exceed 12 in. Two sets of No. 4 (No. 13) column ties will be used, as shown in Fig. 11.11. In addition to the hoop around the outside bars, a single-leg cross tie is required for the middle column bars because the clear distance between column bars exceeds 6 in. The ties satisfy Eq. (4.13).

11.3 Strut-and-Tie Model for Joint Behavior

Although the Committee 352 report (Ref. 11.6) is an important contribution to the safe design of joints of certain standard configurations, the recommendations are based mainly on test results. Consequently, they must be restricted to joints whose geometry closely matches that of the tested joints. This leads to many seemingly arbitrary geometric limitations, and little guidance is provided for the design of joints that may not meet these limitations. An illustration of this is the wide-beam joint discussed in Section 11.2f. Such joints are mentioned only very briefly in the report.

Good physical models are available for many aspects of reinforced concrete behavior—for example, for predicting the flexural strength of a beam or the strength of an eccentrically loaded column—but no clear physical model is evident in the Committee 352 recommendations for the behavior of a joint. For this reason, among others, increasing attention is being given to the strut-and-tie models, described in Chapter 10, as a basis for the design of D-regions in joints.

The essential features of a strut-and-tie model of joint behavior may be understood with reference to Fig. 11.12, which shows a joint of a frame subject to lateral loading, with clockwise moments from the beams equilibrated by counterclockwise moments from the columns. The line of action of the horizontal forces C_1 and T_2 intersects that of the vertical forces C_3 and T_4 at a *nodal zone*, where the resultant force is equilibrated by a diagonal *compression strut* within the joint. At the lower end of the strut, the diagonal compression equilibrates the resultant of the horizontal forces T_1 and C_2 and the vertical forces T_3 and C_4. The tension bars must be well anchored by extension into and through the joint, or in the case of discontinuous bars (such as the top beam steel in an exterior joint) by hooks. The concrete within the nodal zone is subjected to a biaxial or, in many cases, a triaxial state of stress.

With this simple model, the flow of forces in a joint is easily visualized, satisfaction of the requirements of equilibrium is confirmed, and the need for proper anchorage of bars is emphasized. In a complete strut-and-tie model analysis, through proper attention to deformations within the joint, serviceability is ensured through control of cracking.

According to the strut-and-tie model, the main function of the column ties required within the joint region by conventional design procedures, in addition to preventing

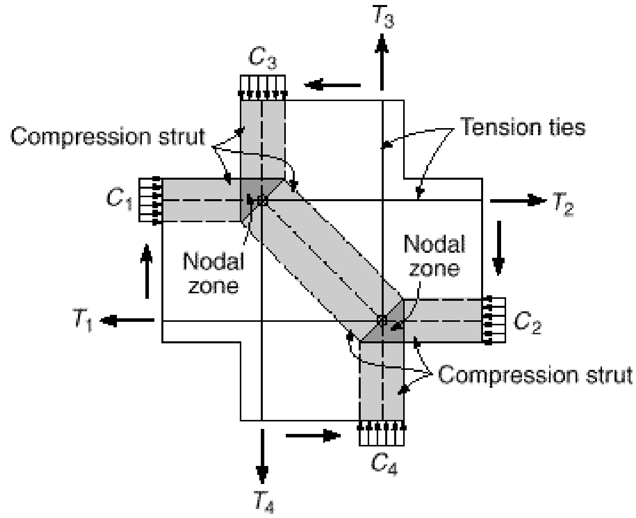


FIGURE 11.12
Strut-and-tie model for behavior of a beam-column joint.

outward buckling of the vertical column bars, is to confine the concrete in the compression strut, thereby improving both its strength and ductility, and to control the cracking that may occur owing to diagonal tension perpendicular to the axis of the compression strut. The main load is carried by the uniaxially loaded struts and ties.

The strut-and-tie model not only provides valuable insights into the behavior of ordinary beam-column joints but also represents an important tool for the design of joints that fall outside of the limited range of those considered in Ref. 11.6. In the sections of this chapter that follow, a number of types of joints will be considered that occur commonly in reinforced concrete structures, for which the strut-and-tie models provide essential aid in developing proper bar details.

11.4 BEAM-TO-GIRDER JOINTS

Commonly in concrete construction, secondary floor beams are supported by primary girders, as shown in Fig. 11.13*a* and *b*. It is often assumed that the reaction from the floor beam is more or less uniformly distributed through the depth of the interface between beam and girder. This incorrect assumption is perhaps encouraged by the ACI Code "$V_c + V_s$" approach to shear design, which makes use of a nominal average shear stress in the concrete, $v_c = V_c \cdot b_w d$, suggesting a uniform distribution of shear stress through the beam web.

The actual behavior of a diagonally cracked beam, as indicated by tests, is quite different, and the flow of forces can be represented in somewhat simplified form by the truss model of the beam shown in Fig. 11.13*c* (Ref. 11.7). The main reaction is delivered from beam to girder by a diagonal compression strut *mn*, which applies its thrust near the bottom of the carrying girder. Failure to provide for this thrust may result in splitting off the concrete at the bottom of the girder followed by collapse of the beam. A graphic example of lack of support for diagonal compression at the junction of a beam and its supporting girder is shown in Fig. 11.14.

Proper detailing of steel in the region of such a joint requires the use of well-anchored "hanger" stirrups in the girder, as shown in Fig. 11.13*a* and *b* to provide for the downward thrust of the compression strut at the end of the beam (Refs. 11.8 and

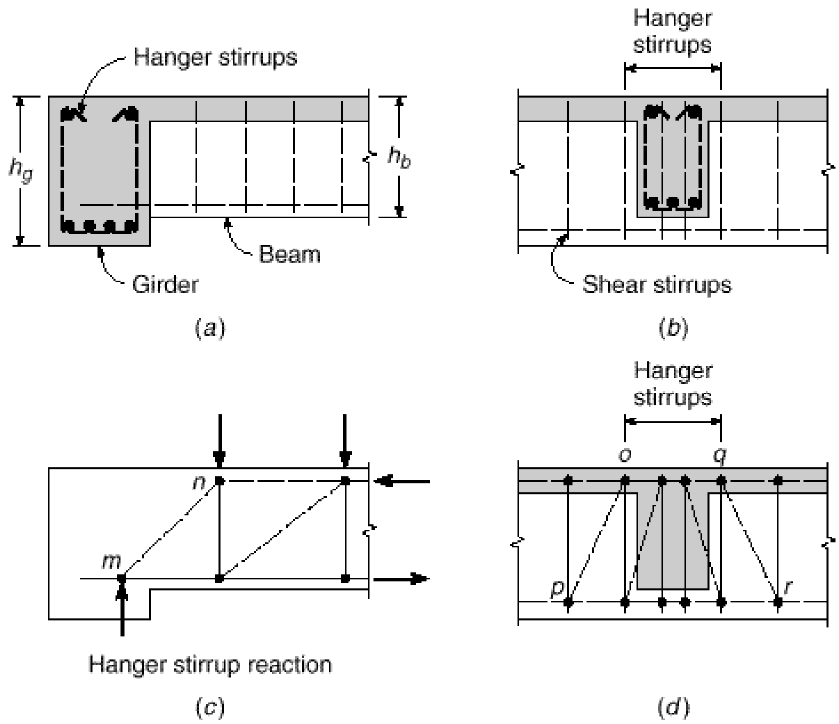


FIGURE 11.13
Main girder supporting secondary beam: (*a*) cross section through girder showing hanger stirrups; (*b*) cross section through beam; (*c*) truss model showing transfer of beam load to girder at load near ultimate; (*d*) truss model showing transfer of load into the girder.

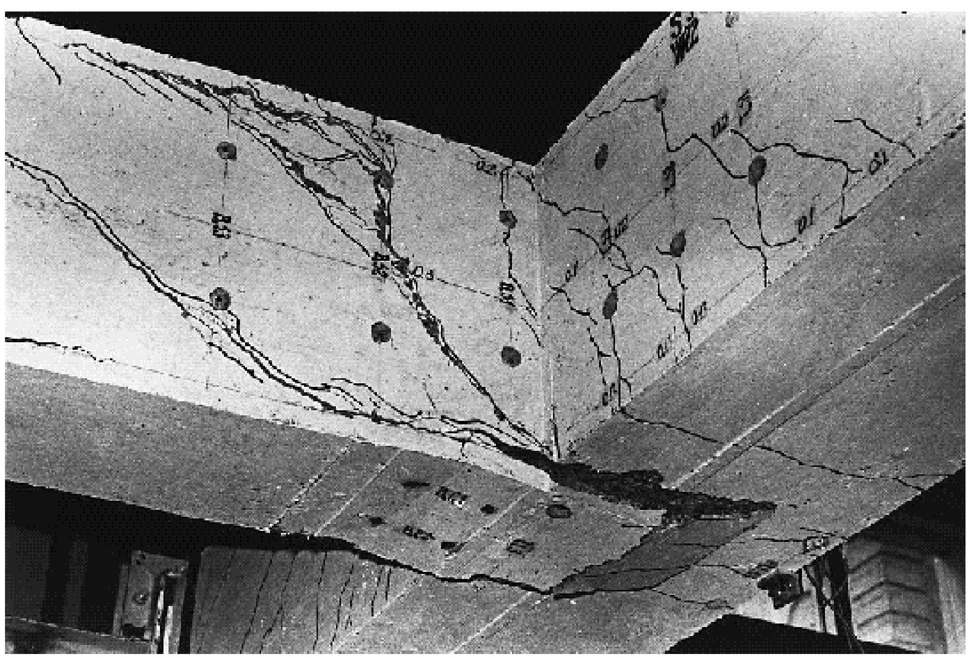


FIGURE 11.14
Failure due to lack of support for diagonal compression in beam-girder joint. (*Courtesy of M. P. Collins, University of Toronto.*)

11.9). These stirrups serve as tension ties to transmit the reaction of the beam to the compression zone of the girder, where it can be equilibrated by diagonal compression struts in the girder. The hanger stirrups, which are required *in addition* to the normal girder stirrups required for shear, can be designed based on equilibrating part or all of the reaction from the beam, with the hanger stirrups assumed to be stressed to their yield stress f_y at the factored load stage.

The strut-and-tie model allows visualization of the transfer of the beam load along the girder as seen in Fig. 11.13*d*. The compression struts *op* and *qr* complete the shear transfer into the girder. The orientation of these compression struts depends on the location of the beam relative to the girder end.

If the beam and girder are the same depth, the hangers should take the full reaction. However, if the beam depth is much less than that of the girder, hangers may prove unnecessary. It is suggested in Ref. 11.10 that hanger stirrups be placed to resist a downward force of V_s^*, where

$$V_s^* = \frac{h_b}{h_g} V \tag{11.5}$$

Here h_b is the depth of the beam, h_g is the depth of the carrying girder, as indicated by Fig. 11.13, and V is the end reaction received from the beam.

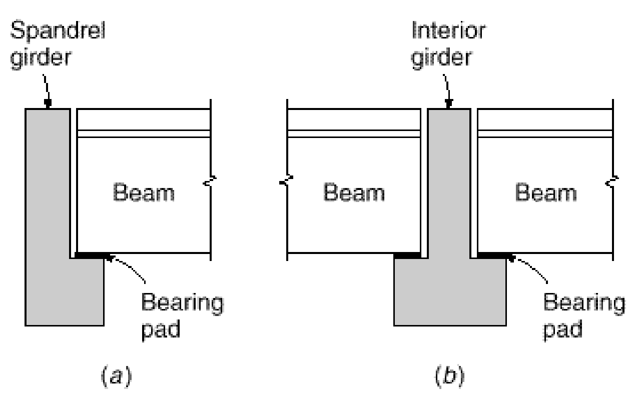
Hangers will also be unnecessary if the factored beam shear is less than · V_c (as is usually the case for one-way joists, for example), because in such a case diagonal cracks would not form in the supported member. The predictions of the truss model would not be valid, and the reaction would be more nearly uniform through the depth.

The hanger stirrups should pass around the main flexural reinforcement of the girder, as shown in Fig. 11.13. If the beam and girder have the same depth, the main flexural bars in the girder should pass below those entering the connection from the beam to provide the best possible reaction platform for the diagonal compression strut.

11.5 Ledge Girders

Frequently in precast concrete construction, an L or inverted T girder is used to provide a seat, or ledge, to support precast beams framing into the carrying girder from the perpendicular direction. Typical ledge girder cross sections are shown in Fig. 11.15. The end reaction of the beams introduces a heavy concentrated load near the bottom of such girders, requiring special reinforcement in the projecting ledge and in the girder web.

FIGURE 11.15
Ledge girders carrying precast T beams: (*a*) L girder providing exterior support for T beam; (*b*) inverted T girder carrying two T beam reactions.



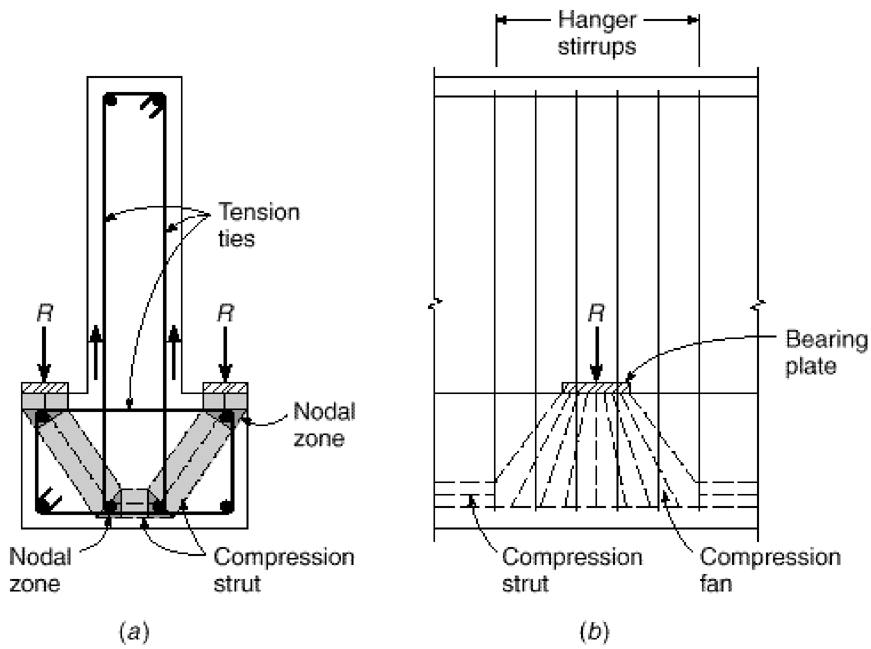
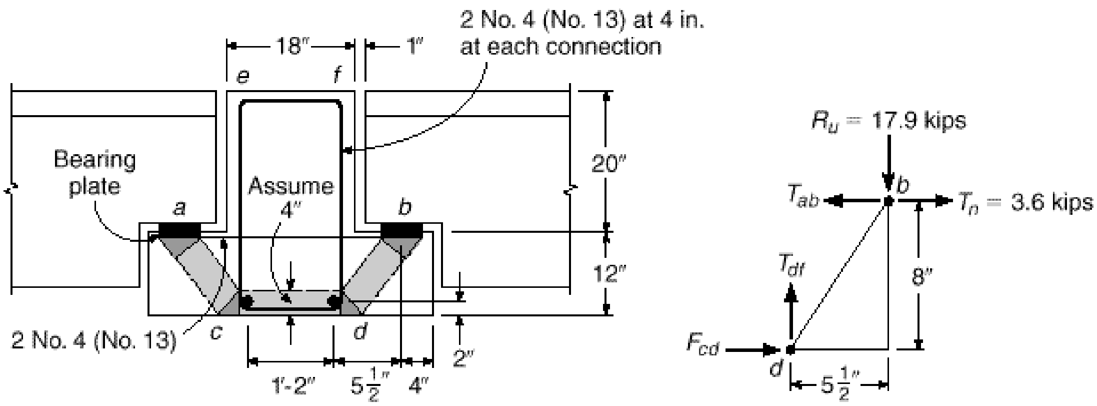


FIGURE 11.16
Strut-and-tie model for behavior of inverted T ledge girder: (*a*) girder cross section; (*b*) side elevation.

The design of such reinforcement is facilitated through use of a strut-and-tie model, as illustrated in Fig. 11.16. The downward reaction of the supported beam creates a compression fan in the ledge that distributes the reaction along a length greater than that of the bearing plate, as shown in Fig. 11.16*b*. The horizontal components of the fan are equilibrated by a compression strut along the lower flange of the girder.

In the cross section view of Fig. 11.16*a*, the downward thrust under the bearing plate is equilibrated by a diagonal compression strut, with the outward thrust at the top of the strut causing tension in the upper horizontal leg of closed hoop stirrups in the lower part of the girder. In many cases, a short structural steel angle is used just under the bearing plate, and the main tie at the top of the ledge is welded to the angle to ensure positive anchorage. At the bottom of the diagonal strut, the horizontal component of thrust is equilibrated by the opposing thrust from the other side, and the vertical component causes tension in stirrups that extend to the top of the girder. These stirrups are used in addition to those required for girder shear. Proper anchorage at the nodes is ensured by passing longitudinal bars inside the bends of both sets of stirrups.

EXAMPLE 11.4 **Inverted T beam connection design.** The inverted T beam shown in Fig. 11.17*a* supports 40 ft long, 12 ft wide double T beams. The width of the double T stem is 4.75 in. and the bearing plate is 6 in. long. The dead load of the double T is 71 psf, including self-weight, and the beam carries a live load of 40 psf. A horizontal force is taken equal to 20 percent of the vertical reaction. Design the reinforcement in the inverted T at the double T bearing point. Material properties are $f'_c = 6000$ psi and $f_y = 60{,}000$ psi.



(*a*) Strut-and-tie model (*b*) Truss detail

FIGURE 11.17
Strut-and-tie model for Example 11.4.

Solution. The factored loads on the beam stem for a 6-ft tributary width are

$$q_u = 1.2 \times 71 + 1.6 \times 40 = 149 \text{ psf}$$

$$R_u = 0.149 \text{ psf} \times 6 \text{ ft} \times 40 \text{ ft} \cdot 2 = 17.9 \text{ kips}$$

and

$$T_u = 0.2 \times 17.9 = 3.6 \text{ kips}$$

The bearing area under the double T leg is 6 in. by 4.75 in. = 28.5 in^2, giving a nodal bearing stress of

$$f_n = \frac{17.9}{28.5} = 0.63 \text{ ksi}$$

which is well below the nominal capacity of the nodes and bottle-shaped or rectangular struts. The low stress is used to demonstrate an alternative solution methodology. By using the low stress, the node and strut capacities are adequate by inspection; however, the size of strut *cd* must be confirmed. Solving for the geometry and forces in Fig. 11.17*b*, T_{ab} = 15.9 kips, T_{df} = 17.9 kips, and strut *cd* carries F_{cd} = 12.3 kips. The thickness of the strut is assumed as 4.75 in., the same as the bearing plate. Therefore, the width of strut *cd* is

$$w_{cd} = \frac{12.3}{0.63 \times 4.75} = 4.11 \text{ in.}$$

This is slightly more than the 4 in. assumed. A minor modification to the bearing stress would make this acceptable; therefore, the design continues with the selected geometry. The required area of steel for tie *ab* is

$$A_s = \frac{T_{ab}}{\cdot f_y} = \frac{15.9}{0.75 \times 60} = 0.35 \text{ in}^2$$

which is satisfied by using two No. 4 (No. 13) bars welded to each bearing plate. For tie *df*,

$$A_s = \frac{T_{df}}{\cdot f_y} = \frac{17.9}{0.75 \times 60} = 0.40 \text{ in}^2$$

which is, also, met using two No. 4 (No. 13) closed stirrups at 4 in. on center at each load point.

11.6 Corners and T Joints

In many common types of reinforced concrete structures, moments and other forces must be transmitted around corners. Some examples, shown in Fig. 11.18, include gable frames, retaining walls, liquid storage tanks, and large box culverts. Reinforcement detailing at the corners is rarely obvious. A comprehensive experimental study of such joints by Nilsson and Losberg (Ref. 11.3) showed that many commonly used joint details will transmit only a small fraction of their assumed strength. Ideally, the joint should resist a moment at least as large as the calculated failure moment of the members framing into it (i.e., the *joint efficiency* should be at least 100 percent). Tests have shown that, for common reinforcing details, joint efficiency may be as low as 30 percent.

Corner joints may be subjected to opening moments, causing flexural tension on the inside of the joint, or closing moments, causing tension on the outside. Generally, the first case is the more difficult to detail properly.

Consider, for example, a corner joint subjected to opening moments, such as an exterior corner of the liquid storage tank shown in the plan view in Fig. 11.18*d*. Figure 11.19*a* shows the system of forces acting on such a corner. The reinforcing bar pattern shown is *not* recommended. Formation of crack 1, radiating inward from the corner, is perhaps obvious. Crack 2, which may lead to splitting off the outside corner, may not be so obvious. However, the resultant of the two compressive forces C, having a magnitude $C\sqrt{2}$, is equilibrated by the resultant tension $T\sqrt{2}$. These two forces, one applied near the outer corner and one near the inner corner, require high tensile stress between the two, leading to formation of crack 2 as shown. The same conclusion is reached considering a small concrete element A in the corner. It is subjected to the shearing forces shown as a result of the forces C and T from the entering members. The resultant of these shearing stresses is 45° principal tension across the corner, confirming formation of crack 2.

One may, at first, be tempted to add an L-shaped bar around the outside of the corner in an attempt to confine the outer concrete. Such a bar would serve no purpose, however, because the bar would be in compression and may actually assist in pushing

FIGURE 11.18
Structures with corners subject to opening or closing moments: (*a*) gable frame; (*b*) earth-retaining wall; (*c*) liquid storage tank; (*d*) plan view of multicell liquid storage tank; (*e*) large box culvert.

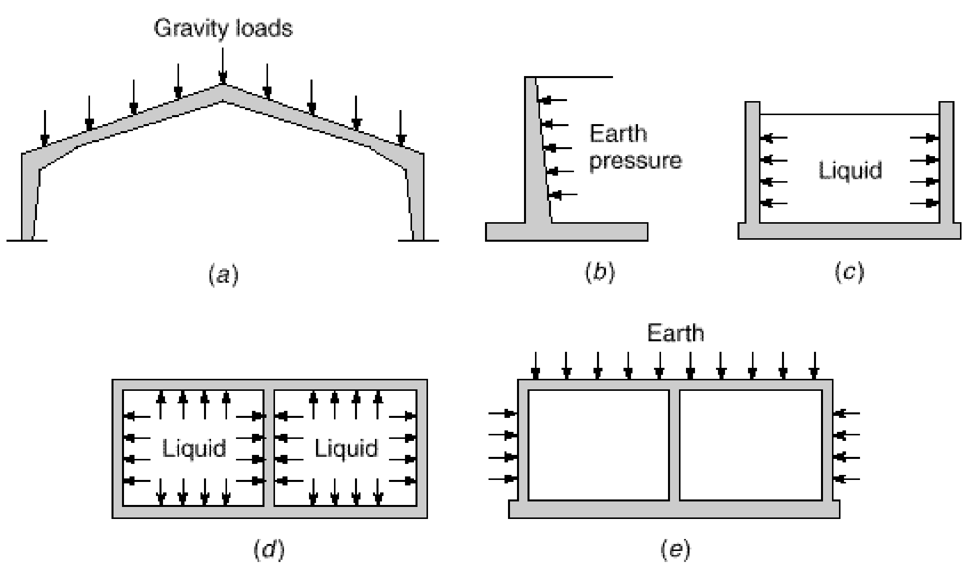
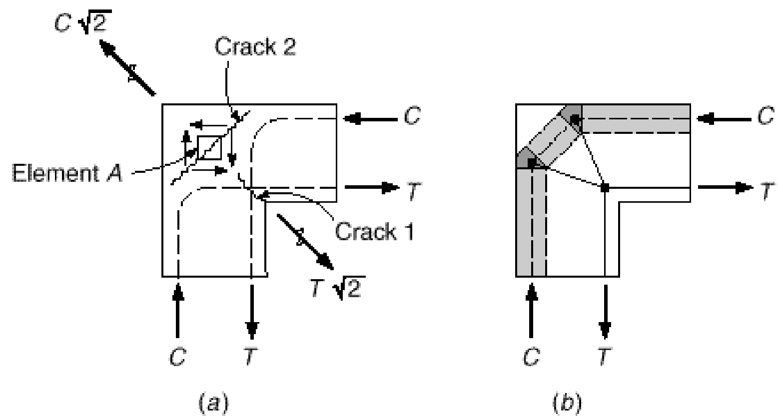


FIGURE 11.19
Corner joint subject to opening moments: (*a*) cracking in an improperly designed joint; (*b*) strut-and-tie model of joint behavior.



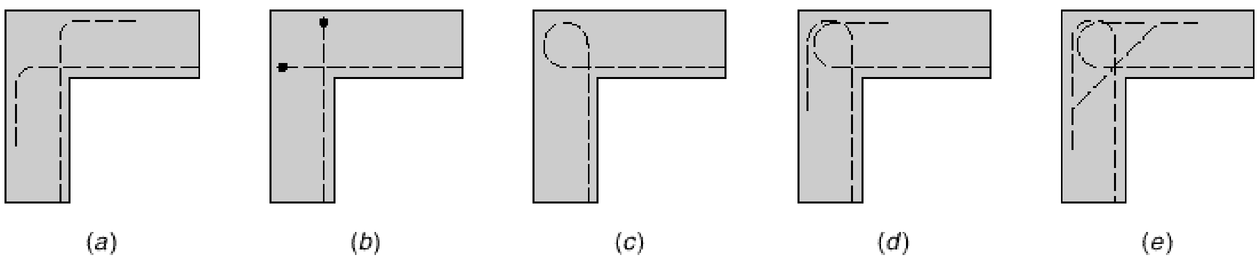


FIGURE 11.20
Efficiencies of corner joints subject to opening moments for various reinforcing details: (*a*) 32 percent; (*b*) 68 percent; (*c*) 77 percent; (*d*) 87 percent; (*e*) 115 percent. (*After Ref. 11.3.*)

the corner off. The strut-and-tie model of Fig. 11.19*b* provides valuable insight into the needed reinforcement, indicating that, in addition to well-anchored tensile bars to transmit the forces *T* into the joint, some form of radial reinforcement is required to permit the compressive forces *C* to "turn the corner."

Test results for a large number of joints with alternative bar details are reported in Ref. 11.3. Comparative efficiencies for some specific details, relating the maximum moment transmitted by the corner joint to the flexural capacity of the entering members, are summarized in Fig. 11.20. In all cases, the reinforcement ratio of the entering members is 0.75 percent. Figure 11.20*a* is a simple detail, probably often used, but it provides joint efficiency of only 32 percent. The details in Fig. 11.20*b*, reinforced with bent bars in the form of hairpins with the plane of the hooks parallel to the inside face of the joint, provides efficiency of 68 percent. In Fig. 11.20*c*, the main reinforcement is simply looped and continued out the other leg of the joint, resulting in an efficiency of 77 percent. The somewhat similar detail shown in Fig. 11.20*d*, in which the bars entering the joint are terminated with separate loops, gives an efficiency of 87 percent. The best performance results from the detail shown in Fig. 11.20*e*—the same as in Fig. 11.20*d* except for the addition of a diagonal bar. This improves joint efficiency to 115 percent, so that the joint is actually stronger than the design strength of the members framing into it. It was determined experimentally that the area of the diagonal bar should be about one-half that of the main reinforcement.

The joints between the vertical wall and horizontal base slab of retaining walls (see Fig. 11.18*b*) are also subjected to opening moments. Tests of such joints confirm

FIGURE 11.21
Comparative efficiencies of T joints subject to bending moment: (*a*) 24 to 40 percent depending on reinforcement ratio; (*b*) 82 to 110 percent depending on reinforcement ratio. (*After Ref. 11.3.*)

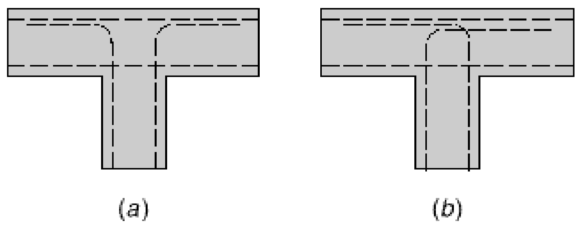
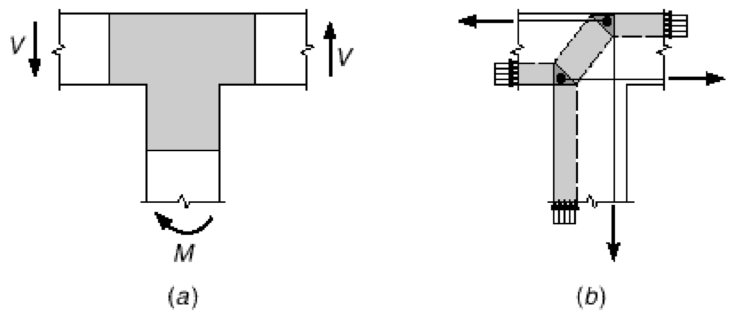


FIGURE 11.22
T joint behavior subjected to moment: (*a*) bending moment and resulting shear forces; (*b*) strut-and-tie model.



the benefit of placing a diagonal bar similar to Fig. 11.20*e*. Retaining wall bar details will be discussed further in Chapter 17.

T joints also may be subjected to bending moments, such as if only one cell of the multiple-cell liquid storage tank of Fig. 11.18*d* were filled. Tests of such joints, reported in Ref. 11.3, again indicate the importance of proper detailing. The reinforcing bar arrangement shown in Fig. 11.21*a*, which is sometimes seen, permits a joint efficiency of only 24 to 40 percent, but the simple rearrangement shown in Fig. 11.21*b* improves the efficiency to between 82 and 110 percent. In both cases, efficiency depends upon the main reinforcement ratio in the entering members, with highest efficiency corresponding to the lowest tensile reinforcement ratio.

A strut-and-tie model for the T joint confirms the research results presented above. Figure 11.22*a* shows that a clockwise moment applied to the stem of the T is resisted by shear forces at the inflection points of the T-top. The strut-and-tie model in Fig. 11.22*b* clearly shows that the stem reinforcement must hook to the left for the joint to be effective, just as shown in Fig. 11.21*b*.

Joints subjected to closing moments, with main reinforcement passing around the corner close to the outside face, cause few detailing problems because the main tension steel from the entering members can be carried around the outside of the corner. There is, however, a risk of splitting the concrete in the plane of the bend, or concrete crushing inside the bend. The efficiency of such joints can be improved by increasing the bend radius of the bar.

11.7 BRACKETS AND CORBELS

Brackets such as shown in Fig. 11.23*a* are widely used in precast construction for supporting precast beams at the columns. When they project from a wall, rather than from a column, they are properly called corbels, although the two terms are often used interchangeably. Brackets are designed mainly to provide for the vertical reaction V_u at the end of the supported beam, but unless special precautions are taken to avoid horizon-

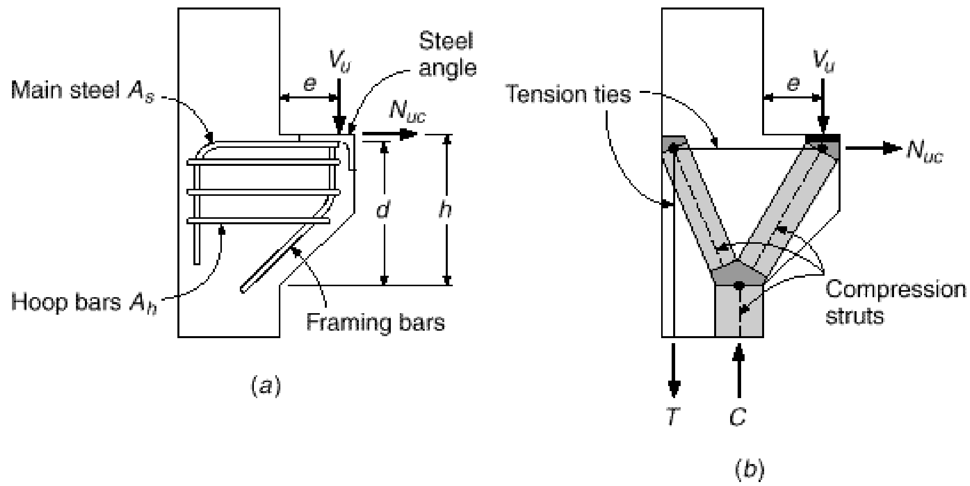


FIGURE 11.23
Typical reinforced concrete bracket: (*a*) loads and reinforcement; (*b*) strut-and-tie model for internal forces.

tal forces caused by restrained shrinkage, creep (in the case of prestressed beams), or temperature change, they must also resist a horizontal force N_{uc}.

Steel bearing plates or angles are generally used on the top surface of the brackets, as shown, to provide a uniform contact surface and to distribute the reaction. A corresponding steel bearing plate or angle is usually provided at the lower corner of the supported member. If the two plates are welded together, horizontal forces clearly must be allowed for in the design. Even with Teflon or elastomeric bearing pads, frictional forces will develop due to volumetric change.

The structural performance of a bracket can be visualized easily by means of the strut-and-tie model shown in Fig. 11.23*b*. The downward thrust of the load V_u is equilibrated by the vertical component of the reaction from the diagonal compression strut that carries the load down into the column. The outward thrust at the top of the strut is balanced by the tension in the horizontal tie bars across the top of the bracket; these also take the tension, if any, imparted by the horizontal force N_{uc}. At the left end of the horizontal tie, the tension is equilibrated by the horizontal component of thrust from the second compression strut shown. The vertical component of this thrust requires the tensile force shown acting downward at the left side of the supporting column.

The steel required, according to the strut-and-tie model, is shown in Fig. 11.23*a*. The main bars A_s must be carefully anchored because they need to develop their full yield strength f_y directly under the load V_u, and for this reason they are usually welded to the underside of the bearing angle and a 90° hook is provided for anchorage at the left side. Closed hoop bars with area A_h confine the concrete in the two compression struts and resist a tendency for splitting in a direction parallel to the thrust. The framing bars shown are usually of about the same diameter as the stirrups and serve mainly to improve the stirrup anchorage at the outer face of the bracket.

The bracket may also be considered as a very short cantilevered beam, with flexural tension at the column face resisted by the top bars A_s. Either concept will result in about the same area of main reinforcement.

A second possible mode of failure is by direct shear along a plane more or less flush with the vertical face of the main part of the column. Shear-friction reinforcement crossing such a crack (see Section 4.9) would include the area A_s previously placed in the top tie and the area A_h from the hoops below it. Other failure modes include flexural tension failure, with yielding of the top bars followed by crushing of the concrete at the bottom of the bracket; crushing of the concrete under the bearing angle (particularly if end rotation of the supported beam causes the force V_u to be applied too close to the outer corner of the bracket); and direct tension failure, if the horizontal force N_{uc} is larger than anticipated.

The provisions of ACI Code 11.9 for the design of brackets and corbels have been developed mainly based on tests (Refs. 11.9, 11.11, and 11.12) and relate to the flexural model of bracket behavior. They apply to brackets and corbels with a shear span ratio e/d of 1.0 or less (see Fig. 11.23*a*). The distance d is measured at the column face, and the depth at the outside edge of the bearing area must not be less than $0.5d$. The usual design basis is employed, i.e., $M_u \le \phi M_n$ and $V_u \le \phi V_n$, and for brackets and corbels (for which shear dominates the design), ϕ is to be taken equal to 0.75 for all strength calculations, including flexure and direct tension as well as shear.

The section at the face of the supporting column must simultaneously resist the shear V_u, the moment $M_u = V_u e + N_{uc}(h - d)$, and the horizontal tension N_{uc}. Unless special precautions are taken, a horizontal tension not less than 20 percent of the vertical reaction must be assumed to act. This tensile force is to be regarded as live load, and a load factor of 1.6 should be applied.

An amount of steel A_f to resist the moment M_u can be found by the usual methods for flexural design. Thus,

$$A_f = \frac{M_u}{\phi f_y (d - a/2)} \tag{11.6}$$

where $a = A_f f_y / 0.85 f'_c b$. An additional area of steel A_n must be provided to resist the tensile component of force:

$$A_n = \frac{N_{uc}}{\phi f_y} \tag{11.7}$$

The total area required *for flexure and direct tension* at the top of the bracket is thus

$$A_s \ge A_f + A_n \tag{11.8}$$

Design for shear is based on the shear-friction method of Section 4.9, and the total shear-friction reinforcement A_{vf} is found by

$$A_{vf} = \frac{V_u}{\phi \mu f_y} \tag{11.9}$$

where the friction factor μ for monolithic construction is 1.40 for normal-weight concrete, 1.19 for sand-lightweight concrete, and 1.05 for all-lightweight concrete. The value of $V_n = V_u/\phi$ must not exceed the smaller of $0.2 f'_c b_w d$ or $800 b_w d$ at the support face for normal-weight concrete or the smaller of $(0.2 - 0.07e/d) f'_c bd$ or $(800 - 280e/d)bd$ for lightweight concrete. Then, according to ACI Code 11.9, the total area required *for shear plus direct tension* at the top of the bracket is

$$A_s \ge \frac{2}{3} A_{vf} + A_n \tag{11.10}$$

with the remaining part of A_{vf} placed in form of closed hoops having area A_h in the lower part of the bracket, as shown in Fig. 11.23*a*.

Thus, the total steel area A_s required at the top of the bracket is equal to the larger of the values given by Eq. (11.8) or (11.10). An additional restriction, that A_s must not be less than $0.04(f'_c/f_y)bd$, is intended to avoid the possibility of sudden failure upon formation of a flexural tensile crack at the top of the bracket.

According to the ACI Code, closed hoop stirrups having area A_h (see Fig. 11.23*a*) not less than $0.5(A_s - A_n)$ must be provided and be uniformly distributed within two-thirds of the effective depth adjacent to and parallel to A_s. This requirement is more clearly stated as follows:

$$A_h \geq 0.5A_f \qquad \text{and} \qquad \geq \frac{1}{3}A_{vf} \tag{11.11}$$

EXAMPLE 11.5

Design of column bracket. A column bracket having the general features shown in Fig. 11.24 is to be designed to carry the end reaction from a long-span precast girder. Vertical reactions from service dead and live loads are 25 kips and 51 kips, respectively, applied at $e = 5.5$ in. from the column face. A steel bearing plate will be provided for the girder, which will rest directly on a $5 \times 3 \times \frac{3}{8}$ in. steel angle anchored at the outer corner of the bracket. Bracket reinforcement will include main steel A_s welded to the underside of the steel angle, closed hoop stirrups having total area A_h distributed appropriately through the bracket depth, and framing bars in a vertical plane near the outer face. Select appropriate concrete dimensions, and design and detail all reinforcement. Material strengths are $f'_c = 5000$ psi and $f_y = 60{,}000$ psi.

Solution. The vertical factored load to be carried is

$$V_u = 1.2 \times 25 + 1.6 \times 51 = 112 \text{ kips}$$

In the absence of a roller or low-friction support pad, a horizontal tensile force of

$$N_{uc} = 0.20 \times 112 = 22.4 \text{ kips}$$

will be included. According to the shear friction provisions of the ACI Code, the nominal shear strength V_n must not exceed $0.2f'_cbd$ or $800bd$. With $f'_c = 5000$ psi, the second limit controls. Then, with $V_u = \cdot\ V_n$ and with the column width $b = 12$ in.,

$$112 = 0.75 \times 0.800 \times 12d$$

from which $d = 15.56$ in. Estimating 1 in. from the center of the main steel to the top surface of the bracket, a total depth $h = 17$ in. will be selected, with d approximately equal to 16 in., the exact value depending on the bar diameter chosen for A_s. If a 45° slope is used, as indicated in Fig. 11.24, the bracket depth at the outside of the bearing area will be 8 in. This is not less than $0.5d = 8.0$ in., as required. For the bracket geometry selected, $e \cdot d = 5.5 \cdot 16 = 0.34$. This does not exceed the 1.0 limit imposed by the ACI Code.

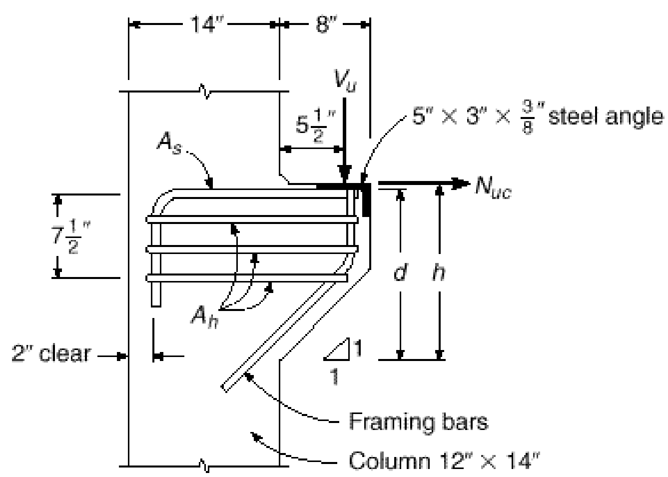


FIGURE 11.24
Column bracket design example.

The total shear friction steel is found from Eq. (11.9):

$$A_{vf} = \frac{V_u}{\cdot \ \cdot \ f_y} = \frac{112}{0.75 \times 1.4 \times 60} = 1.78 \text{ in}^2$$

The bending moment to be resisted is

$$M_u = V_u e + N_{uc} \cdot h - d \cdot$$
$$= 112 \times 5.5 + 22.4 \times 1 = 638 \text{ kips}$$

The depth of the flexural compression stress block will be estimated to be 2 in., so, from Eq. (11.6),

$$A_f = \frac{M_u}{\cdot f_y \cdot d - a \cdot 2 \cdot} = \frac{638}{0.75 \times 60 \cdot 16 - 1.0 \cdot} = 0.95 \text{ in}^2$$

Checking the stress block depth gives

$$a = \frac{A_f f_y}{0.85 f_c b} = \frac{0.95 \times 60}{0.85 \times 5 \times 12} = 1.12 \text{ in}$$

so the revised steel area is

$$A_f = \frac{638}{0.75 \times 60 \cdot 16 - 0.56 \cdot} = 0.92 \text{ in}^2$$

The tensile force of 22.4 kips requires an additional steel area, from Eq. (11.7), of

$$A_n = \frac{N_{uc}}{\cdot f_y} = \frac{22.4}{0.75 \times 60} = 0.50 \text{ in}^2$$

Thus, from Eqs. (11.8) and (11.10), respectively, the total steel area at the top of the bracket must not be less than

$$A_s \geq A_f + A_n = 0.92 + 0.50 = 1.42 \text{ in}^2$$

nor less than

$$A_s \geq \frac{2}{3} A_{vf} + A_n = \frac{2}{3} \times 1.78 + 0.50 = 1.69 \text{ in}^2$$

The second requirement controls here. The minimum steel requirement of

$$A_{s,min} = 0.04 \frac{f_c}{f_y} bd = 0.04 \times \frac{5}{60} \times 12 \times 16 = 0.64 \text{ in}^2$$

is seen not to control. A total of three No. 7 (No. 22) bars, providing $A_s = 1.80$ in^2, will be used.

Closed hoop steel having a total area A_h not less than $0.5(A_s - A_n)$ must be provided. Thus,

$$A_h \geq 0.5 A_f = 0.5 \times 0.92 = 0.46 \text{ in}^2$$

and

$$A_h \geq 0.5 \times \frac{2}{3} A_{vf} = \frac{1}{3} \times 1.78 = 0.60 \text{ in}^2$$

The second requirement controls. Three No. 3 (No. 10) closed hoops will be provided, giving total area $A_h = 0.66$ in^2. These must be placed within $\frac{2}{3}$ of the effective depth of the main steel. A spacing of 2.5 in. will be satisfactory, as indicated in Fig. 11.24. A pair of No. 3 (No. 10) framing bars will be added at the inside corner of the hoops to improve anchorage, as shown.

Anchorage of the No. 7 (No. 22) bars will be provided at the right end by welding to the underside of the steel angle and at the left end by a standard 90° bend (see Fig. 5.10). The basic development length for hooked bars (Table 5.3) is

$$l_{dh} = \frac{0.02 f_y}{\sqrt{f_c'}} d_b = \frac{0.02 \times 1 \times 1 \times 60{,}000}{\sqrt{5000}} 0.875 = 14.8 \text{ in}$$

Two modification factors apply here. The first is 0.7, provided at least 2 in. cover is maintained at the end of the hook, and the second is (required A_s)/(provided A_s) = 1.69/1.80 = 0.94. Thus, the required development length past the face of the column is

$$l_{dh} = 14.8 \times 0.7 \times 0.94 = 9.74 \text{ in.}$$

This requirement is easily met. The hook extension will be $12d_b = 12 \times 0.875 = 10.5$ in. For the hoop bars, a standard 1'35° hook, as shown in Fig. 5.9*b*, will be used.

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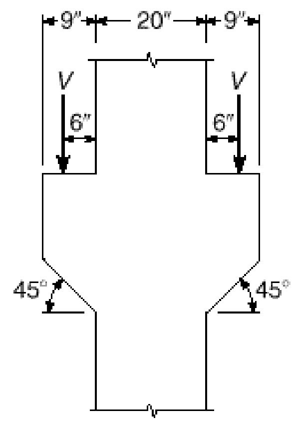
Problems

11.1. An interior Type 1 joint, which is to be considered a part of the primary lateral load–resisting system, is to be designed. The 16 in. square column, with main steel consisting of four No. 11 (No. 36) bars, is intersected by two 12 × 18 in. beams in the *X* direction, reinforced with three No. 10 (No. 32) top bars and three No. 8 (No. 25) bottom bars. In the *Y* direction, there are two 12 × 22 in. girders, reinforced with three No. 11 (No. 36) top bars and three No. 9 (No. 29) bottom bars. Concrete cover is 2.5 in. to the center of the bars, except for the top steel in the girders, which is carried just under the top steel of the beams. Design and detail the joint, using $f_c' = 4000$ psi and $f_y = 60{,}000$ psi. Specify placement of all bars and cutoff points.

11.2. A typical exterior joint of the building of Problem 11.1 is identical to the interior joint except that the 12 × 18 in. beam occurs on one side of the column only; the girders frame into two opposite faces, as before. All reinforcement is the same as for the joint of Problem 11.1. Design and detail the joint, specifying bar placement, cutoff points, and details such as bar hook dimensions.

11.3. The precast columns of a proposed parking garage will incorporate symmetrical brackets to carry the end reactions of short girders that, in turn, carry long-span precast, prestressed double T floor units. The girder reactions will be applied 6 in. from the column face, as shown in Fig. P11.3, and a total width of bracket of 9 in. must be provided for proper bearing. Column width in the perpendicular direction is 20 in. Service load reactions applied at the top face of the brackets are 45 kips dead load and 36 kips live load. Select all unspecified concrete dimensions and design and detail the reinforcement. A corner angle is suggested at the outer top edge of the bracket. Column material strengths are $f_c' = 6000$ psi and $f_y = 60{,}000$ psi.

FIGURE P11.3



11.4. The stem of a 60 ft long, 8 ft wide simply supported single T beam rests on the ledge of the inverted T beam shown in Fig. P11.4. The T beam has a bearing area 6 in. thick and 4 in. parallel to the axis of the T. The applied service load is 85 psf dead load, including self-weight, and 50 psf live load. Design the connection detail under the stem using $f_c' = 5000$ psi and $f_y = 60{,}000$ psi.

FIGURE P11.4