13 Analysis and Design of Slabs

13.1 Types of Slabs

In reinforced concrete construction, slabs are used to provide flat, useful surfaces. A reinforced concrete slab is a broad, flat plate, usually horizontal, with top and bottom surfaces parallel or nearly so. It may be supported by reinforced concrete beams (and is usually cast monolithically with such beams), by masonry or reinforced concrete walls, by structural steel members, directly by columns, or continuously by the ground.

Slabs may be supported on two opposite sides only, as shown in Fig. 13.1*a*, in which case the structural action of the slab is essentially *one-way*, the loads being carried by the slab in the direction perpendicular to the supporting beams. There may be beams on all four sides, as shown in Fig. 13.1*b*, so that *two-way* slab action is obtained. Intermediate beams, as shown in Fig. 13.1*c*, may be provided. If the ratio of length to width of one slab panel is larger than about 2, most of the load is carried in the short direction to the supporting beams and one-way action is obtained in effect, even though supports are provided on all sides.

Concrete slabs may in some cases be carried directly by columns, as shown in Fig. 13.1*d*, without the use of beams or girders. Such slabs are described as *flat plates* and are commonly used where spans are not large and loads not particularly heavy. *Flat slab* construction, shown in Fig. 13.1*e*, is also beamless but incorporates a thickened slab region in the vicinity of the column and often employs flared column tops. Both are devices to reduce stresses due to shear and negative bending around the columns. They are referred to as *drop panels* and *column capitals*, respectively. Closely related to the flat plate slab is the two-way joist, also known as a *grid* or *waffle slab*, shown in Fig. 13.1*f*. To reduce the dead load of solid-slab construction, voids are formed in a rectilinear pattern through use of metal or fiberglass form inserts. A two-way ribbed construction results. Usually inserts are omitted near the columns, so a solid slab is formed to resist moments and shears better in these areas.

In addition to the column-supported types of construction shown in Fig. 13.1, many slabs are supported continuously on the ground, as for highways, airport runways, and warehouse floors. In such cases, a well-compacted layer of crushed stone or gravel is usually provided to ensure uniform support and to allow for proper subgrade drainage.

Reinforcing steel for slabs is primarily parallel to the slab surfaces. Straight bar reinforcement is generally used, although in continuous slabs bottom bars are sometimes bent up to serve as negative reinforcement over the supports. Welded wire reinforcement is commonly employed for slabs on the ground. Bar or rod mats are available for the

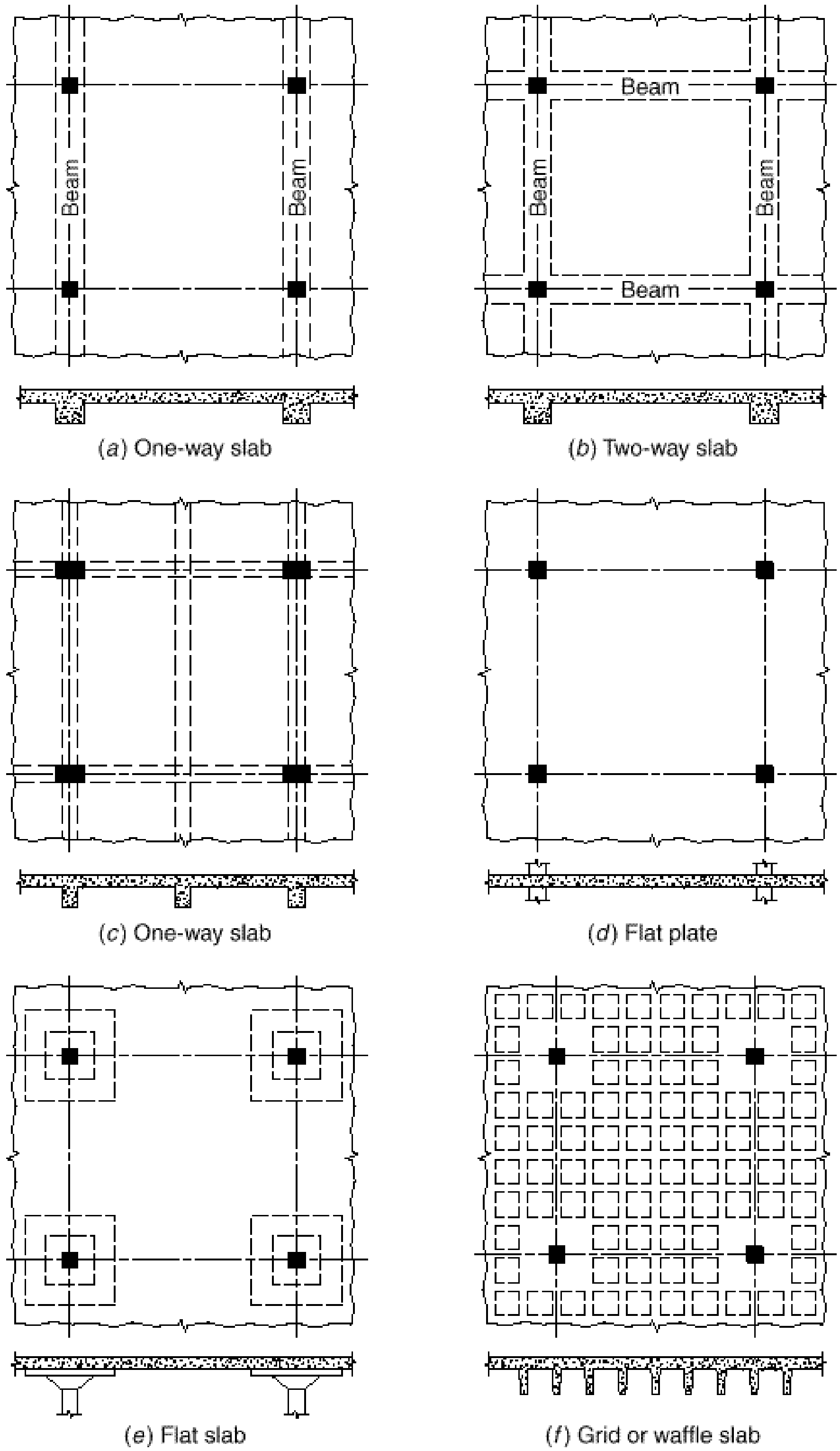


FIGURE 13.1
Types of structural slabs.

heavier reinforcement sometimes needed in highway slabs and airport runways. Slabs may also be prestressed using high tensile strength strands.

Reinforced concrete slabs of the types shown in Fig. 13.1 are usually designed for loads assumed to be uniformly distributed over one entire slab panel, bounded by supporting beams or column centerlines. Minor concentrated loads can be accommodated through two-way action of the reinforcement (two-way flexural steel for two-way slab systems or one-way flexural steel plus lateral distribution steel for one-way systems). Heavy concentrated loads generally require supporting beams.

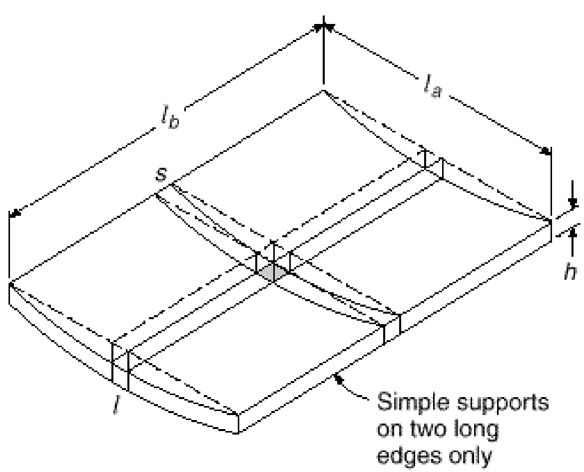
One-way and two-way edge-supported slabs, such as shown in Fig. 13.1*a, b*, and *c*, will be discussed in Sections 13.2 to 13.4. Two-way beamless systems, such as shown in Fig. 13.1*d, e*, and *f*, as well as two-way edge-supported slabs (Fig. 13.1*b*), will be treated in Sections 13.5 to 13.13. Special methods based on limit analysis at the overload state, applicable to all types of slabs, will be presented in Chapters 14 and 15.

13.2 Design of One-Way Slabs

The structural action of a one-way slab may be visualized in terms of the deformed shape of the loaded surface. Figure 13.2 shows a rectangular slab, simply supported along its two opposite long edges and free of any support along the two opposite short edges. If a uniformly distributed load is applied to the surface, the deflected shape will be as shown by the solid lines. Curvatures, and consequently bending moments, are the same in all strips s spanning in the short direction between supported edges, whereas there is no curvature, hence no bending moment, in the long strips l parallel to the supported edges. The surface is approximately cylindrical.

For purposes of analysis and design, a unit strip of such a slab cut out at right angles to the supporting beams, as shown in Fig. 13.3, may be considered as a rectangular beam of unit width, with a depth h equal to the thickness of the slab and a span l_a equal to the distance between supported edges. This strip can then be analyzed by the methods that were used for rectangular beams, the bending moment being computed for the strip of unit width. The load per unit area on the slab becomes the load per unit length on the slab strip. Since all of the load on the slab must be transmitted to the two supporting beams, it follows that all of the reinforcement should be placed

FIGURE 13.2
Deflected shape of uniformly loaded one-way slab.



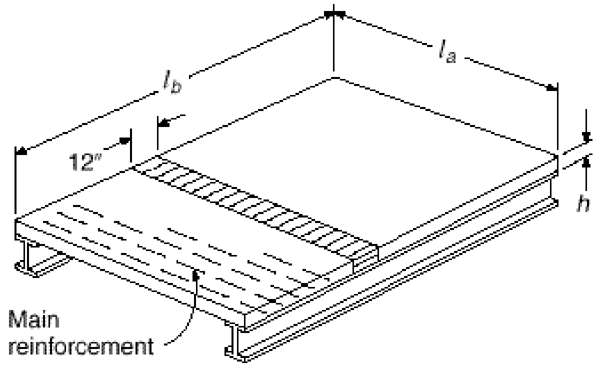


FIGURE 13.3
Unit strip basis for flexural design.

at right angles to these beams, with the exception of any bars that may be placed in the other direction to control shrinkage and temperature cracking. A one-way slab, thus, consists of a set of rectangular beams side by side.

This simplified analysis, which assumes Poisson's ratio to be zero, is slightly conservative. Actually, flexural compression in the concrete in the direction of l_a will result in lateral expansion in the direction of l_b unless the compressed concrete is restrained. In a one-way slab, this lateral expansion is resisted by adjacent slab strips, which tend to expand also. The result is a slight strengthening and stiffening in the span direction, but this effect is small and can be disregarded.

The reinforcement ratio for a slab can be determined by dividing the area of one bar by the area of concrete between two successive bars, the latter area being the product of the depth to the center of the bars and the distance between them, center to center. The reinforcement ratio can also be determined by dividing the average area of steel per foot of width by the effective area of concrete in a 1 ft strip. The average area of steel per foot of width is equal to the area of one bar times the average number of bars in a 1 ft strip (12 divided by the spacing in inches), and the effective area of concrete in a 1 ft (or 12 in.) strip is equal to 12 times the effective depth *d*.

To illustrate the latter method of obtaining the reinforcement ratio · , assume a 5 in. slab with an effective depth of 4 in., with No. 4 (No. 13) bars spaced $4\frac{1}{2}$ in. center to center. The average number of bars in a 12 in. strip of slab is 12·4.5 = $2\frac{2}{3}$ bars, and the average steel area in a 12 in. strip is $2\frac{2}{3} \times 0.20 = 0.533$ in^2. Hence · = 0.533·(12 × 4) = 0.0111. By the other method,

$$\cdot = \frac{0.20}{4.5 \times 4} = 0.0111$$

The spacing of bars that is necessary to furnish a given area of steel per foot of width is obtained by dividing the number of bars required to furnish this area into 12. For example, to furnish an average area of 0.46 in^2·ft, with No. 4 (No. 13) bars, requires $0.46 \div 0.20 = 2.3$ bars per foot; the bars must be spaced not more than12·2.3 = 5.2 in. center to center. The determination of slab steel areas for various combinations of bars and spacings is facilitated by Table A.3 of Appendix A.

Factored moments and shears in one-way slabs can be found either by elastic analysis or through the use of the same coefficients as used for beams (see Chapter 12). If the slab rests freely on its supports, the span length may be taken equal to the clear span plus the depth of the slab but need not exceed the distance between centers of supports, according to ACI Code 8.7.1. In general, center-to-center distances should be used in continuous slab analysis, but a reduction is allowed in negative moments to

TABLE 13.1
Minimum thickness h of nonprestressed one-way slabs

Simply supported	$l/20$
One end continuous	$l/24$
Both ends continuous	$l/28$
Cantilever	$l/10$

account for support width as discussed in Chapter 12. For slabs with clear spans not more than 10 ft that are built integrally with their supports, ACI Code 8.7.4 permits analysis as a continuous slab on knife-edge supports with spans equal to the clear spans and the width of the beams otherwise neglected. If moment and shear coefficients are used, computations should be based on clear spans.

One-way slabs are normally designed with tensile reinforcement ratios well below the maximum permissible value of ρ_{max}. Typical reinforcement ratios range from about 0.004 to 0.008. This is partially for reasons of economy, because the saving in steel associated with increasing the effective depth more than compensates for the cost of the additional concrete, and partially because very thin slabs with high reinforcement ratios would be likely to permit large deflections. Thus, flexural design may start with selecting a relatively low reinforcement ratio, say about $0.25\rho_{max}$, setting $M_u = \phi M_n$ in Eq. (3.38), and solving for the required effective depth d, given that $b = 12$ in. for the unit strip. Alternatively, Table A.5 or Graph A.1 of Appendix A may be used. Table A.9 is also useful. The required steel area per 12 in. strip, $A_s = \rho bd$, is then easily found.

ACI Code 9.5.2 specifies the minimum thickness in Table 13.1 for nonprestressed slabs of normal-weight concrete ($w_c = 145$ pcf) using Grade 60 reinforcement, provided that the slab is not supporting or attached to construction that is likely to be damaged by large deflections. Lesser thicknesses may be used if calculation of deflections indicates no adverse effects. For concretes having unit weight w_c in the range from 90 to 120 pcf, the tabulated values should be multiplied by $(1.65 - 0.005w_c)$, but not less than 1.09. For reinforcement having a yield stress f_y other than 60,000 psi, the tabulated values should be multiplied by $(0.4 + f_y/100{,}000)$. Slab deflections may be calculated, if required, by the same methods as for beams (see Section 6.7).

Shear will seldom control the design of one-way slabs, particularly if low tensile reinforcement ratios are used. It will be found that the shear capacity of the concrete, ϕV_c, will almost without exception be well above the required shear strength V_u at factored loads.

The total slab thickness h is usually rounded to the next higher $\frac{1}{4}$ in. for slabs up to 6 in. thickness, and to the next higher $\frac{1}{2}$ in. for thicker slabs. Best economy is often achieved when the slab thickness is selected to match nominal lumber dimensions. The concrete protection below the reinforcement should follow the requirements of ACI Code 7.7.1, calling for $\frac{3}{4}$ in. below the bottom of the steel (see Fig. 3.12*b*). In a typical slab, 1 in. below the center of the steel may be assumed. The lateral spacing of the bars, except those used only to control shrinkage and temperature cracks (see Section 13.3), should not exceed 3 times the thickness h or 18 in., whichever is less, according to ACI Code 7.6.5. Generally, bar size should be selected so that the actual spacing is not less than about 1.5 times the slab thickness, to avoid excessive cost for bar fabrication and handling. Also, to reduce cost, straight bars are usually used for slab reinforcement, cut off where permitted as described for beams in Section 5.9.

13.3 TEMPERATURE AND SHRINKAGE REINFORCEMENT

Concrete shrinks as it dries out, as was pointed out in Section 2.11. It is advisable to minimize such shrinkage by using concretes with the smallest possible amounts of water and cement compatible with other requirements, such as strength and workability, and by thorough moist-curing of sufficient duration. However, no matter what precautions are taken, a certain amount of shrinkage is usually unavoidable. If a slab of moderate dimensions rests freely on its supports, it can contract to accommodate the shortening of its length produced by shrinkage. Usually, however, slabs and other members are joined rigidly to other parts of the structure and cannot contract freely. This results in tension stresses known as *shrinkage stresses*. A decrease in temperature relative to that at which the slab was cast, particularly in outdoor structures such as bridges, may have an effect similar to shrinkage. That is, the slab tends to contract and if restrained from doing so becomes subject to tensile stresses.

Since concrete is weak in tension, these temperature and shrinkage stresses are likely to result in cracking. Cracks of this nature are not detrimental, provided their size is limited to what are known as *hairline cracks*. This can be achieved by placing reinforcement in the slab to counteract contraction and distribute the cracks uniformly. As the concrete tends to shrink, such reinforcement resists the contraction and consequently becomes subject to compression. The total shrinkage in a slab so reinforced is less than that in one without reinforcement; in addition, whatever cracks do occur will be of smaller width and more evenly distributed by virtue of the reinforcement.

In one-way slabs, the reinforcement provided for resisting the bending moments has the desired effect of reducing shrinkage and distributing cracks. However, as contraction takes place equally in all directions, it is necessary to provide special reinforcement for shrinkage and temperature contraction in the direction perpendicular to the main reinforcement. This added steel is known as *temperature* or *shrinkage reinforcement*, or *distribution steel*.

Reinforcement for shrinkage and temperature stresses normal to the principal reinforcement should be provided in a structural slab in which the principal reinforcement extends in one direction only. ACI Code 7.12.2 specifies the minimum ratios of reinforcement area to *gross concrete area* (i.e., based on the total depth of the slab) shown in Table 13.2, but in no case may such reinforcing bars be placed farther apart than 5 times the slab thickness or more than 18 in. In no case is the reinforcement ratio to be less than 0.0014.

The steel required by the ACI Code for shrinkage and temperature crack control also represents the minimum permissible reinforcement in the span direction of one-way slabs; the usual minimums for flexural steel do not apply.

TABLE 13.2
Minimum ratios of temperature and shrinkage reinforcement in slabs based on gross concrete area

Slabs where Grade 40 or 50 deformed bars are used	0.0020
Slabs where Grade 60 deformed bars or welded wire fabric (smooth or deformed) are used	0.0018
Slabs where reinforcement with yield strength exceeding 60,000 psi measured at yield strain of 0.35 percent is used	$\frac{0.0018 \times 60{,}000}{f_y}$

EXAMPLE 13.1 **One-way slab design.** A reinforced concrete slab is built integrally with its supports and consists of two equal spans, each with a clear span of 15 ft. The service live load is 100 psf, and 4000 psi concrete is specified for use with steel with a yield stress equal to 60,000 psi. Design the slab, following the provisions of the ACI Code.

Solution. The thickness of the slab is first estimated, based on the minimum thickness from Table 13.1: $l/28 = 15 \times 12/28 = 6.43$ in. A trial thickness of 6.50 in. will be used, for which the weight is $150 \times 6.50/12 = 81$ psf. The specified live load and computed dead load are multiplied by the ACI load factors:

$$\begin{aligned} \text{Dead load} &= 81 \times 1.2 = 97 \text{ psf} \\ \text{Live load} &= 100 \times 1.6 = \underline{160 \text{ psf}} \\ \text{Total} &= 257 \text{ psf} \end{aligned}$$

For this case, factored moments at critical sections may be found using the ACI moment coefficients (see Table 12.1):

$$\begin{aligned} &\text{At interior support:} && -M = \tfrac{1}{9} \times 0.257 \times 15^2 = 6.43 \text{ ft-kips} \\ &\text{At midspan:} && +M = \tfrac{1}{14} \times 0.257 \times 15^2 = 4.13 \text{ ft-kips} \\ &\text{At exterior support:} && -M = \tfrac{1}{24} \times 0.257 \times 15^2 = 2.41 \text{ ft-kips} \end{aligned}$$

The maximum reinforcement ratio permitted by the ACI Code is, according to Eq. (3.30*b*):

$$\rho_{max} = 0.85^2 \frac{4}{60} \frac{0.003}{0.003 + 0.004} = 0.021$$

If that maximum value of ρ were actually used, the minimum required effective depth, controlled by negative moment at the interior support, would be found from Eq. (3.38) to be

$$\begin{aligned} d^2 &= \frac{M_u}{\phi \rho f_y b (1 - 0.59 \rho f_y / f_c')} \\ &= \frac{6.43 \times 12}{0.90 \times 0.021 \times 60 \times 12 (1 - 0.59 \times 0.021 \times 60/4)} = 6.96 \text{ in}^2 \\ d &= 2.64 \text{ in.}^{\dagger} \end{aligned}$$

This is less than the effective depth of $6.50 - 1.00 = 5.50$ in. resulting from application of Code restrictions, and the latter figure will be adopted. At the interior support, if the stress-block depth $a = 1.00$ in., the area of steel required per foot of width in the top of the slab is [Eq. (3.37)]

$$A_s = \frac{M_u}{\phi f_y (d - a/2)} = \frac{6.43 \times 12}{0.90 \times 60 \times 5.00} = 0.29 \text{ in}^2$$

Checking the assumed depth a by Eq. (3.32), one gets

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{0.29 \times 60}{0.85 \times 4 \times 12} = 0.43 \text{ in.}$$

A second trial will be made with $a = 0.43$ in. Then

$$A_s = \frac{6.43 \times 12}{0.90 \times 60 \times 5.29} = 0.27 \text{ in}^2$$

for which $a = 0.43 \times 0.27/0.29 = 0.40$ in. No further revision is necessary. At other critical-moment sections, it will be satisfactory to use the same lever arm to determine steel areas, and

† The depth is more easily found using Graph A.1 of Appendix A. For $\rho = \rho_{max}$, $M_u/\phi bd^2 = 1026$, from which $d = 2.64$ in. Table A.5*a* may also be used.

At midspan: $$A_s = \frac{4.13 \times 12}{0.90 \times 60 \times 5.29} = 0.17 \text{ in}^2$$

At exterior support: $$A_s = \frac{2.41 \times 12}{0.90 \times 60 \times 5.29} = 0.10 \text{ in}^2$$

The minimum reinforcement is that required for control of shrinkage and temperature cracking. This is

$$A_s = 0.0018 \times 12 \times 6.50 = 0.14 \text{ in}^2$$

per 12 in. strip. This requires a small increase in the amount of steel used at the exterior support.

The factored shear force at a distance d from the face of the interior support is

$$V_u = 1.15 \times \frac{257 \times 15}{2} - 257 \times \frac{5.50}{12} = 2100 \text{ lb}$$

By Eq. (4.12*b*), the nominal shear strength of the concrete slab is

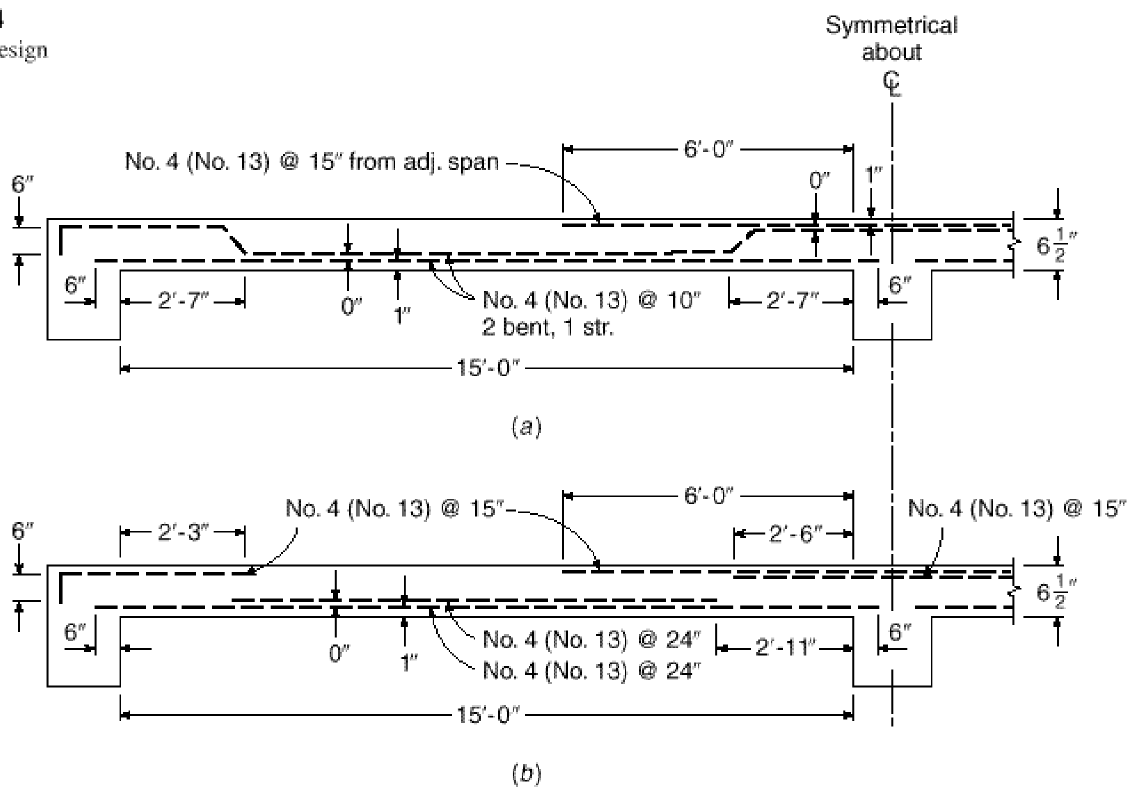
$$V_n = V_c = 2 \cdot \sqrt{f_c'}\, bd = 2 \cdot \sqrt{4000} \times 12 \times 5.50 = 8350 \text{ lb}$$

Thus, the design strength of the concrete slab, $\phi V_c = 0.75 \times 8350 = 6260$ lb, is well above the required strength in shear of $V_u = 2100$.

The required tensile steel areas may be provided in a variety of ways, but whatever the selection, due consideration must be given to the actual placing of the steel during construction. The arrangement should be such that the steel can be placed rapidly with the minimum of labor costs even though some excess steel is necessary to achieve this end.

Two possible arrangements are shown in Fig. 13.4. In Fig. 13.4*a*, bent bars are used, while in Fig. 13.4*b* all bars are straight.

FIGURE 13.4
One-way slab design example.



In the arrangement of Fig. 13.4*a*, No. 4 (No. 13) bars at 10 in. furnish 0.24 in^2 of steel at midspan, slightly more than required. If two-thirds of these bars are bent upward for negative reinforcement over the interior support, the average spacing of such bent bars at the interior support will be $(10 + 20)/2 = 15$ in. Since an identical pattern of bars is bent upward from the other side of the support, the effective spacing of the No. 4 (No. 13) bars over the interior support is $7\frac{1}{2}$ in. This pattern satisfies the required steel area of 0.27 in^2 per foot width of slab over the support. The bars bent at the interior support will also be bent upward for negative reinforcement at the exterior support, providing reinforcement equivalent to No. 4 (No. 13) bars at 15 in., or 0.16 in^2 of steel.

Note that it is not necessary to achieve uniform spacing of reinforcement in slabs, and that the steel provided can be calculated safely on the basis of average spacing, as in the example. Care should be taken to satisfy requirements for both minimum and maximum spacing of principal reinforcement, however.

The locations of bend and cutoff points shown in Fig. 13.4*a* were obtained using Graph A.3 of Appendix A, as explained in Section 5.9 and Table A.10 (see also Fig. 5.14).

The arrangement shown in Fig. 13.4*b* uses only straight bars. Although it is satisfactory according to the ACI Code (since the shear stress does not exceed two-thirds of that permitted), cutting off the shorter positive and negative bars as shown leads to an undesirable condition at the ends of those bars, where there will be concentrations of stress in the concrete. The design would be improved if the negative bars were cut off 3 ft from the face of the interior support rather than 2 ft 6 in. as shown, and if the positive steel were cut off at 2 ft 2 in. rather than at 2 ft 11 in. This would result in an overlap of approximately 2*d* of the cut positive and negative bars. Figure 5.15*a* suggests a somewhat simpler arrangement that would also prove satisfactory.

The required area of steel to be placed normal to the main reinforcement for purposes of temperature and shrinkage crack control is 0.14 in^2. This will be provided by No. 4 (No. 13) bars at 16 in. spacing, placed directly on top of the main reinforcement in the positive-moment region and below the main steel in the negative-moment zone.

13.4 BEHAVIOR OF TWO-WAY EDGE-SUPPORTED SLABS

The slabs discussed in Sections 13.2 and 13.3 deform under load into an approximately cylindrical surface. The main structural action is one-way in such cases, in the direction normal to supports on two opposite edges of a rectangular panel. In many cases, however, rectangular slabs are of such proportions and are supported in such a way that two-way action results. When loaded, such slabs bend into a dished surface rather than a cylindrical one. This means that at any point the slab is curved in both principal directions, and since bending moments are proportional to curvatures, moments also exist in both directions. To resist these moments, the slab must be reinforced in both directions, by at least two layers of bars perpendicular, respectively, to two pairs of edges. The slab must be designed to take a proportionate share of the load in each direction.

Types of reinforced concrete construction that are characterized by two-way action include slabs supported by walls or beams on all sides (Fig. 13.1*b*), flat plates (Fig. 13.1*d*), flat slabs (Fig. 13.1*e*), and waffle slabs (Fig. 13.1*f*).

The simplest type of two-way slab action is that represented by Fig. 13.1*b*, where the slab, or slab panel, is supported along its four edges by relatively deep, stiff, monolithic concrete beams or by walls or steel girders. If the concrete edge beams are shallow or are omitted altogether, as they are for flat plates and flat slabs, deformation of the floor system along the column lines significantly alters the distribution of

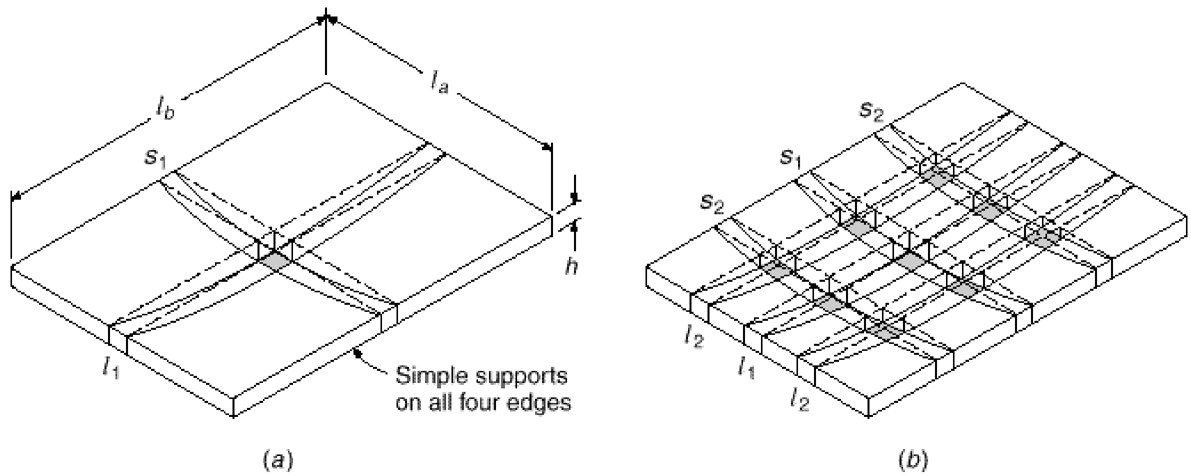


FIGURE 13.5
Two-way slab on simple edge supports: (*a*) bending of center strips of slab; (*b*) grid model of slab.

moments in the slab panel itself (Ref. 13.1). Two-way systems of this type are considered separately, beginning in Section 13.5. The present discussion pertains to the former type, in which edge supports are stiff enough to be considered unyielding.

Such a slab is shown in Fig. 13.5*a*. To visualize its flexural performance, it is convenient to think of it as consisting of two sets of parallel strips, in each of the two directions, intersecting each other. Evidently, part of the load is carried by one set and transmitted to one pair of edge supports, and the remainder by the other.

Figure 13.5*a* shows the two center strips of a rectangular plate with short span l_a and long span l_b. If the uniform load is w per square foot of slab, each of the two strips acts approximately like a simple beam, uniformly loaded by its share of w. Because these imaginary strips actually are part of the same monolithic slab, their deflections at the intersection point must be the same. Equating the center deflections of the short and long strips gives

$$\frac{5w_a l_a^4}{384EI} = \frac{5w_b l_b^4}{384EI} \qquad (a)$$

where w_a is the share of the load w carried in the short direction and w_b is the share of the load w carried in the long direction. Consequently,

$$\frac{w_a}{w_b} = \frac{l_b^4}{l_a^4} \qquad (b)$$

One sees that the larger share of the load is carried in the short direction, the ratio of the two portions of the total load being inversely proportional to the fourth power of the ratio of the spans.

This result is approximate because the actual behavior of a slab is more complex than that of the two intersecting strips. An understanding of the behavior of the slab itself can be gained from Fig. 13.5*b*, which shows a slab model consisting of two sets of three strips each. It is seen that the two central strips s_1 and l_1 bend in a manner similar to that shown in Fig. 13.5*a*. The outer strips s_2 and l_2, however, are not only bent but also twisted. Consider, for instance, one of the intersections of s_2 with l_2. It is seen that at the intersection the exterior edge of strip l_2 is at a higher elevation than the interior edge, while at the nearby end of strip l_2 both edges are at the same elevation; the strip is twisted. This twisting results in torsional stresses and torsional moments that are seen to be most pronounced near the corners. Consequently, the total load on the slab is carried not only by the bending moments in two directions but also by the twisting moments. For this reason, bending moments in elastic slabs are smaller than would be computed for sets of unconnected strips loaded by w_a and w_b. For instance, for a simply supported square slab, $w_a = w_b = w/2$. If only bending were present, the maximum moment in each strip would be

$$\frac{(w/2)l^2}{8} = 0.0625wl^2 \tag{c}$$

The exact theory of bending of elastic plates shows that, actually, the maximum moment in such a square slab is only $0.048wl^2$, so that in this case the twisting moments relieve the bending moments by about 25 percent.

The largest moment occurs where the curvature is sharpest. Figure 13.5*b* shows this to be the case at midspan of the short strip s_1. Suppose the load is increased until this location is overstressed, so that the steel at the middle of strip s_1 is yielding. If the strip were an isolated beam, it would now fail. Considering the slab as a whole, however, one sees that no immediate failure will occur. The neighboring strips (those parallel as well as those perpendicular to s_1), being actually monolithic with it, will take over any additional load that strip s_1 can no longer carry until they, in turn, start yielding. This inelastic redistribution will continue until in a rather large area in the central portion of the slab all the steel in both directions is yielding. Only then will the entire slab fail. From this reasoning, which is confirmed by tests, it follows that slabs need not be designed for the absolute maximum moment in each of the two directions (such as $0.048wl^2$ in the example given in the previous paragraph), but only for a smaller average moment in each of the two directions in the central portion of the slab. For instance, one of the several analytical methods in general use permits a square slab to be designed for a moment of $0.036wl^2$. By comparison with the actual elastic maximum moment $0.048wl^2$, it is seen that, owing to inelastic redistribution, a moment reduction of 25 percent is provided.

The largest moment in the slab occurs at midspan of the short strip s_1 of Fig. 13.5*b*. It is evident that the curvature, and hence the moment, in the short strip s_2 is less than at the corresponding location of strip s_1. Consequently, a variation of short-span moment occurs in the long direction of the span. This variation is shown qualitatively in Fig. 13.6. The short-span moment diagram in Fig. 13.6*a* is valid only along the center strip at 1-1. Elsewhere, the maximum-moment value is less, as shown. Other moment ordinates are reduced proportionately. Similarly, the long-span moment diagram in Fig. 13.6 applies only at the longitudinal centerline of the slab; elsewhere, ordinates are reduced according to the variation shown. These variations in maximum moment across the width and length of a rectangular slab are accounted for in an approximate way in most practical design methods by designing for a reduced moment in the outer quarters of the slab span in each direction.

It should be noted that only slabs with side ratios less than about 2 need be treated as two-way slabs. From Eq. (*b*) above, it is seen that, for a slab of this proportion, the share of the load carried in the long direction is only on the order of one-sixteenth of that in the short direction. Such a slab acts almost as if it were spanning in the short direction only. Consequently, rectangular slab panels with an aspect ratio of 2 or more may be reinforced for one-way action, with the main steel perpendicular to the long edges.

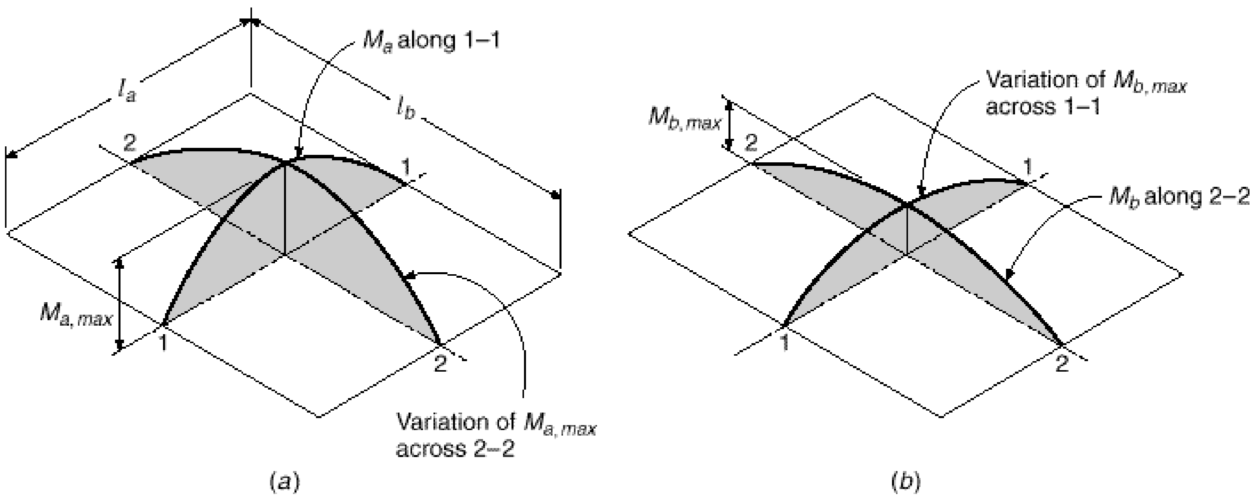


FIGURE 13.6
Moments and moment variations in a uniformly loaded slab with simple supports on four sides.

Consistent with the assumptions of the analysis of two-way edge-supported slabs, the main flexural reinforcement is placed in an orthogonal pattern, with reinforcing bars parallel and perpendicular to the supported edges. As the positive steel is placed in two layers, the effective depth d for the upper layer is smaller than that for the lower layer by one bar diameter. Because the moments in the long direction are the smaller ones, it is economical to place the steel in that direction on top of the bars in the short direction. The stacking problem does not exist for negative reinforcement perpendicular to the supporting edge beams except at the corners, where moments are small.

Either straight bars, cut off where they are no longer required, or bent bars may be used for two-way slabs, but economy of bar fabrication and placement will generally favor all straight bars. The precise locations of inflection points (or lines of inflection) are not easily determined, because they depend upon the side ratio, the ratio of live to dead load, and continuity conditions at the edges. The standard cutoff and bend points for beams, summarized in Fig. 5.16, may be used for edge-supported slabs as well.

According to ACI Code 13.3.1, the minimum reinforcement in each direction for two-way slabs is that required for shrinkage and temperature crack control, as given in Table 13.2. For two-way systems, the spacing of flexural reinforcement at critical sections must not exceed 2 times the slab thickness h.

The twisting moments discussed earlier are usually of consequence only at exterior corners of a two-way slab system, where they tend to crack the slab at the bottom along the panel diagonal, and at the top perpendicular to the panel diagonal. Special reinforcement should be provided at exterior corners in both the bottom and top of the slab, for a distance in each direction from the corner equal to one-fifth the longer span of the corner panel, as shown in Fig. 13.7. The reinforcement at the top of the slab should be parallel to the diagonal from the corner, while that at the bottom should be perpendicular to the diagonal. Alternatively, either layer of steel may be placed in two bands parallel to the sides of the slab. The positive and negative reinforcement, in any case, should be of a size and spacing equivalent to that required for the maximum positive moment (per foot of width) in the panel, according to ACI Code 13.3.6.

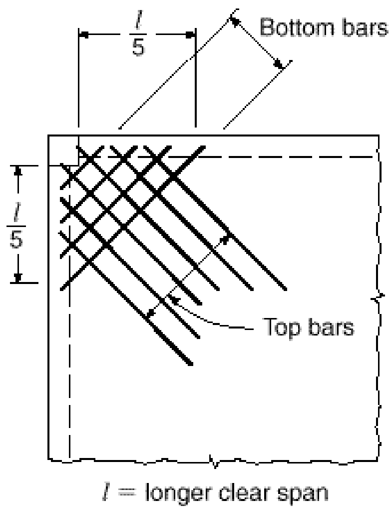


FIGURE 13.7
Special reinforcement at exterior corners of a beam-supported two-way slab.

13.5 Two-Way Column-Supported Slabs

When two-way slabs are supported by relatively shallow, flexible beams (Fig. 13.1*b*), or if column-line beams are omitted altogether, as for flat plates (Fig. 13.1*d*), flat slabs (Fig. 13.1*e*), or two-way joist systems (Fig. 13.1*f*), a number of new considerations are introduced. Figure 13.8*a* shows a portion of a floor system in which a rectangular slab panel is supported by relatively shallow beams on four sides. The beams, in turn, are carried by columns at the intersection of their centerlines. If a surface load w is applied, that load is shared between imaginary slab strips l_a in the short direction and l_b in the long direction, as described in Section 13.4. The portion of the load that is carried by the long strips l_b is delivered to the beams $B1$ spanning in the short direction of the panel. The portion carried by the beams $B1$ plus that carried directly in the short direction by the slab strips l_a, sums up to 100 percent of the load applied to the panel. Similarly, the short-direction slab strips l_a deliver a part of the load to long-direction beams $B2$. That load, plus the load carried directly in the long direction by the slab, includes 100 percent of the applied load. It is clearly a requirement of statics that, for column-supported construction, *100 percent of the applied load must be carried in each direction*, jointly by the slab and its supporting beams (Ref. 13.2).

A similar situation is obtained in the flat plate floor shown in Fig. 13.8*b*. In this case beams are omitted. However, broad strips of the slab centered on the column lines in each direction serve the same function as the beams of Fig. 13.8*a*; for this case, also, the full load must be carried in each direction. The presence of drop panels or column capitals (Fig. 13.1*e*) in the double-hatched zone near the columns does not modify this requirement of statics.

Figure 13.9*a* shows a flat plate floor supported by columns at *A, B, C,* and *D*. Figure 13.9*b* shows the moment diagram for the direction of span l_1. In this direction, the slab may be considered as a broad, flat beam of width l_2. Accordingly, the load per foot of span is wl_2. In any span of a continuous beam, the sum of the midspan positive moment and the average of the negative moments at adjacent supports is equal to the

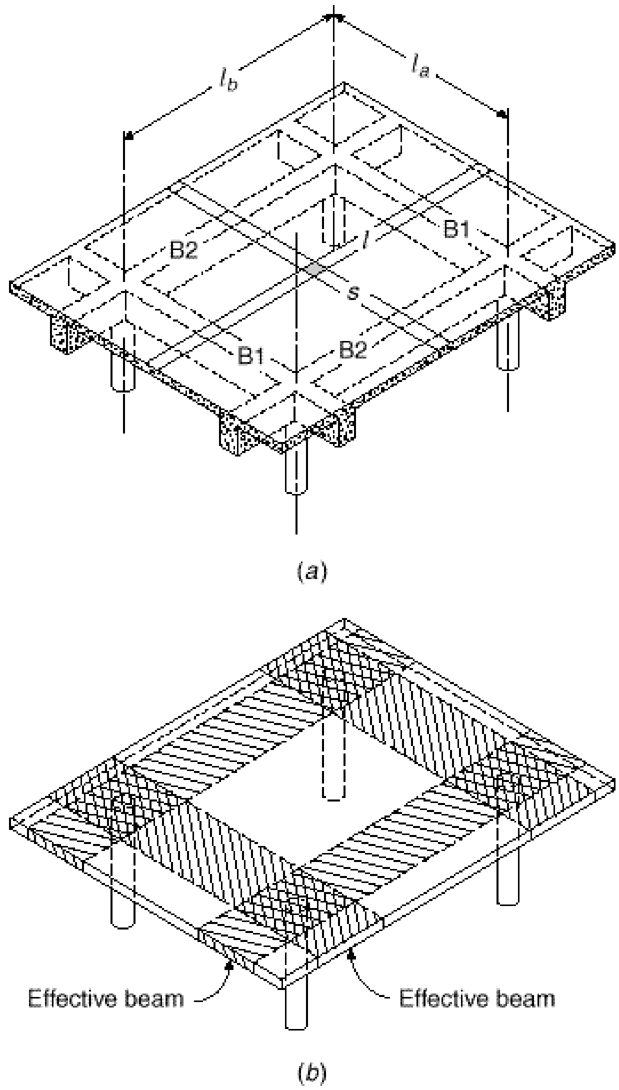


FIGURE 13.8
Column-supported two-way slabs: (*a*) two-way slab with beams; (*b*) two-way slab without beams.

midspan positive moment of a corresponding simply supported beam. In terms of the slab, this requirement of statics may be written

$$\frac{1}{2}(M_{ab} + M_{cd}) + M_{ef} = \frac{1}{8} w l_2 l_1^2 \qquad (a)$$

A similar requirement exists in the perpendicular direction, leading to the relation

$$\frac{1}{2}(M_{ac} + M_{bd}) + M_{gh} = \frac{1}{8} w l_1 l_2^2 \qquad (b)$$

These results disclose nothing about the relative magnitudes of the support moments and span moments. The proportion of the total static moment that exists at each critical section can be found from an elastic analysis that considers the relative span lengths in adjacent panels, the loading pattern, and the relative stiffness of the

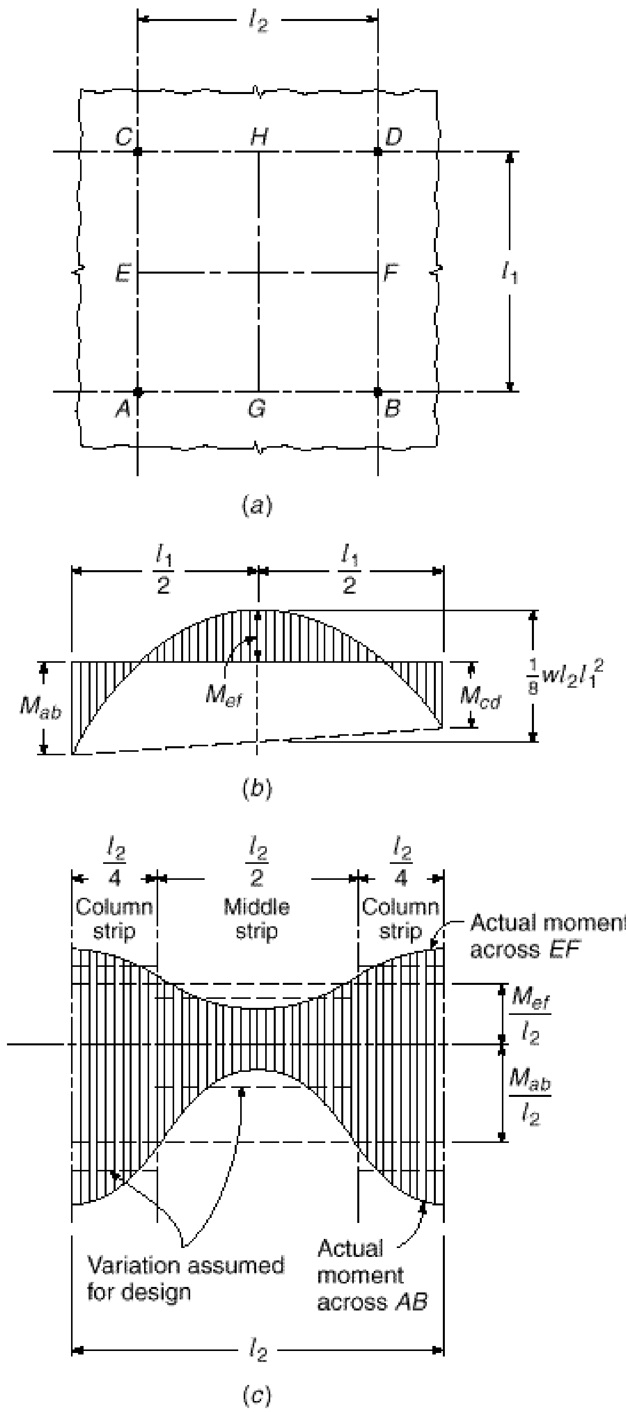


FIGURE 13.9
Moment variation in column-supported two-way slabs: (*a*) critical moment sections; (*b*) moment variation along a span; (*c*) moment variation across the width of critical sections.

supporting beams, if any, and that of the columns. Alternatively, empirical methods that have been found to be reliable under restricted conditions may be adopted.

The moments across the width of critical sections such as *AB* or *EF* are not constant but vary as shown qualitatively in Fig. 13.9*c*. The exact variation depends on the presence or absence of beams on the column lines, the existence of drop panels and

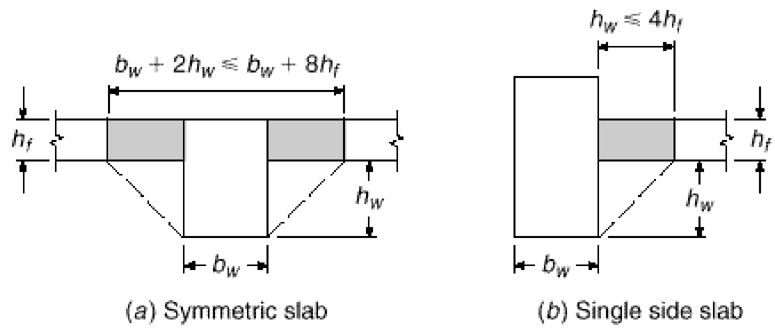


FIGURE 13.10
Portion of slab to be included with beam.

column capitals, as well as on the intensity of the load. For design purposes, it is convenient to divide each panel as shown in Fig. 13.9*c* into column strips, having a width of one-fourth the panel width, on each side of the column centerlines, and middle strips in the one-half panel width between two column strips. Moments may be considered constant within the bounds of a middle strip or column strip, as shown, unless beams are present on the column lines. In the latter case, while the beam must have the same curvature as the adjacent slab strip, the beam moment will be larger in proportion to its greater stiffness, producing a discontinuity in the moment-variation curve at the lateral face of the beam. Since the total moment must be the same as before, according to statics, the slab moments must be correspondingly less.

Chapter 13 of the ACI Code deals in a unified way with all such two-way systems. Its provisions apply to slabs supported by beams and to flat slabs and flat plates, as well as to two-way joist slabs. While permitting design "by any procedure satisfying the conditions of equilibrium and geometrical compatibility," specific reference is made to two alternative approaches: a semiempirical *direct design method* and an approximate elastic analysis known as the *equivalent frame method.*

In either case, a typical panel is divided, for purposes of design, into *column strips* and *middle strips*. A column strip is defined as a strip of slab having a width on each side of the column centerline equal to one-fourth the smaller of the panel dimensions l_1 and l_2. Such a strip includes column-line beams, if present. A middle strip is a design strip bounded by two column strips. In all cases, l_1 is defined as the span in the direction of the moment analysis and l_2 as the span in the lateral direction measured center to center of the support. In the case of monolithic construction, beams are defined to include that part of the slab on each side of the beam extending a distance equal to the projection of the beam above or below the slab h_w (whichever is greater) but not greater than 4 times the slab thickness (see Fig. 13.10).

13.6 Direct Design Method for Column-Supported Slabs

Moments in two-way slabs can be found using the semiempirical direct design method, subject to the following restrictions:

1. There must be a minimum of three continuous spans in each direction.
2. The panels must be rectangular, with the ratio of the longer to the shorter spans within a panel not greater than 2.
3. The successive span lengths in each direction must not differ by more than one-third of the longer span.

4. Columns may be offset a maximum of 10 percent of the span in the direction of the offset from either axis between centerlines of successive columns.
5. Loads must be due to gravity only and the live load must not exceed 2 times the dead load.
6. If beams are used on the column lines, the relative stiffness of the beams in the two perpendicular directions, given by the ratio $\cdot\ _1 l_2^2 \cdot\ _2 l_1^2$, must be between 0.2 and 5.0 (see below for definitions).

a. Total Static Moment at Factored Loads

For purposes of calculating the total static moment M_o in a panel, the clear span l_n in the direction of moments is used. The clear span is defined to extend from face to face of the columns, capitals, brackets, or walls but is not to be less than $0.65l_1$. The total factored moment in a span, for a strip bounded laterally by the centerline of the panel on each side of the centerline of supports, is

$$M_o = \frac{w_u l_2 l_n^2}{8} \tag{13.1}$$

b. Assignment of Moments to Critical Sections

For interior spans, the total static moment is apportioned between the critical positive and negative bending sections according to the following ratios:

$$\text{Negative factored moment: Neg } M_u = 0.65M_o \tag{13.2}$$

$$\text{Positive factored moment: Pos } M_u = 0.35M_o \tag{13.3}$$

as illustrated by Fig. 13.11. The critical section for negative bending is taken at the face of rectangular supports, or at the face of an equivalent square support having the same cross-sectional area as a round support.

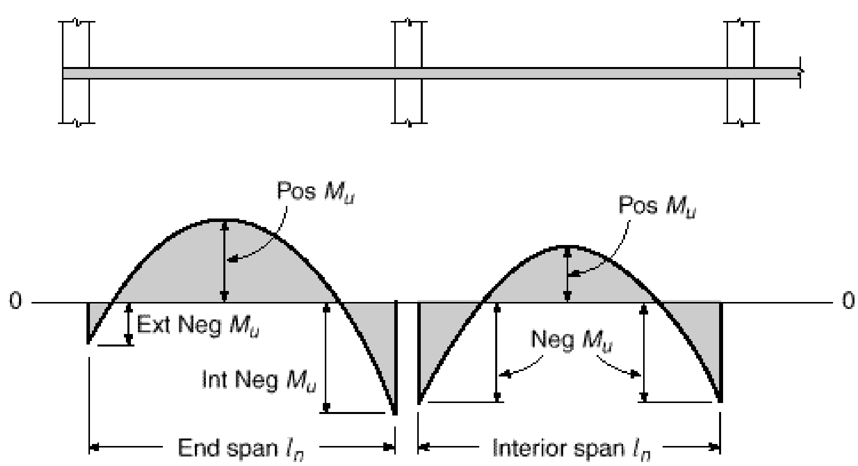


FIGURE 13.11
Distribution of total static moment M_o to critical sections for positive and negative bending.

TABLE 13.3
Distribution factors applied to static moment for positive and negative moments in end span

	( )	( )	( )	( )	( )
	Exterior Edge Unrestrained	Slab with Beams between All Supports	Slab without Beams between Interior Supports		Exterior Edge Fully Restrained
			Without Edge Beam	With Edge Beam	
Interior negative moment	0.75	0.70	0.70	0.70	0.65
Positive moment	0.63	0.57	0.52	0.50	0.35
Exterior negative moment	0	0.16	0.26	0.30	0.65

In the case of end spans, the apportionment of the total static moment among the three critical moment sections (interior negative, positive, and exterior negative, as illustrated by Fig. 13.11) depends upon the flexural restraint provided for the slab by the exterior column or the exterior wall, as the case may be, and depends also upon the presence or absence of beams on the column lines. ACI Code 13.6.3 specifies five alternative sets of moment distribution coefficients for end spans, as shown in Table 13.3 and illustrated in Fig. 13.12.

In case (*a*), the exterior edge has no moment restraint, such as would be the condition with a masonry wall, which provides vertical support but no rotational restraint. Case (*b*) represents a two-way slab with beams on all sides of the panels. Case (*c*) is a flat plate, with no beams at all, while case (*d*) is a flat plate in which a beam is provided along the exterior edge. Finally, case (*e*) represents a fully restrained edge, such as that obtained if the slab is monolithic with a very stiff reinforced concrete wall. The appropriate coefficients for each case are given in Table 13.3 and are based on three-dimensional elastic analysis modified to some extent in the light of tests and practical experience (Refs. 13.3 to 13.10).

At interior supports, negative moments may differ for spans framing into the common support. In such a case, the slab should be designed to resist the larger of the two moments, unless a special analysis based on relative stiffnesses is made to distribute the unbalanced moment (see Chapter 12). Edge beams if they are used, or the edge of the slab if they are not, must be designed to resist in torsion their share of the exterior negative moment indicated by Table 13.3 (see Chapter 7).

c. Lateral Distribution of Moments

Having distributed the moment M_o to the positive and negative-moment sections as just described, the designer still must distribute these design moments across the width of the critical sections. For design purposes, as discussed in Section 13.5, it is convenient to consider the moments constant within the bounds of a middle strip or column strip unless there is a beam present on the column line. In the latter case, because

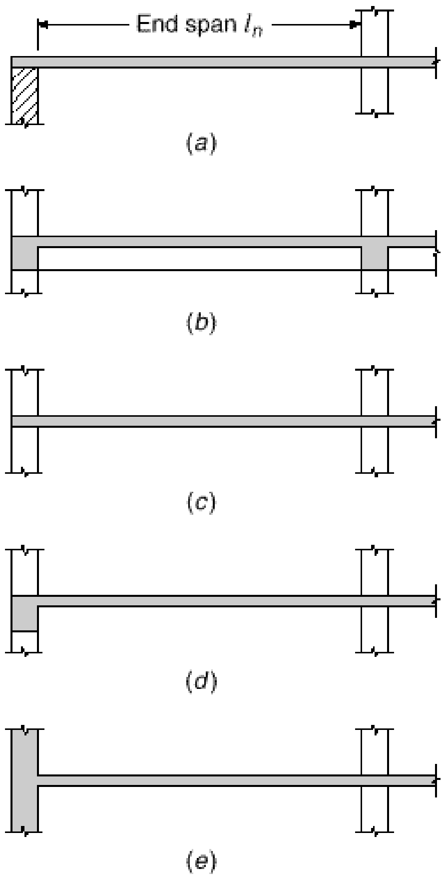


FIGURE 13.12
Conditions of edge restraint considered in distributing total static moment M_o to critical sections in an end span: (*a*) exterior edge unrestrained, e.g., supported by a masonry wall; (*b*) slab with beams between all supports; (*c*) slab without beams, i.e., flat plate; (*d*) slab without beams between interior supports but with edge beam; (*e*) exterior edge fully restrained, e.g., by monolithic concrete wall.

of its greater stiffness, the beam will tend to take a larger share of the column-strip moment than the adjacent slab. The distribution of total negative or positive moment between slab middle strips, slab column strips, and beams depends upon the ratio $l_2 \cdot l_1$, the relative stiffness of the beam and the slab, and the degree of torsional restraint provided by the edge beam.

A convenient parameter defining the relative stiffness of the beam and slab spanning in either direction is

$$\cdot = \frac{E_{cb} I_b}{E_{cs} I_s} \tag{13.4}$$

in which E_{cb} and E_{cs} are the moduli of elasticity of the beam and slab concrete (usually the same) and I_b and I_s are the moments of inertia of the effective beam and the slab. Subscripted parameters $\cdot_1$ and $\cdot_2$ are used to identify · computed for the directions of l_1 and l_2, respectively.

The flexural stiffnesses of the beam and slab may be based on the gross concrete section, neglecting reinforcement and possible cracking, and variations due to column capitals and drop panels may be neglected. For the beam, if present, I_b is based on the effective cross section defined as in Fig. 13.10. For the slab, I_s is taken equal to $bh^3 \cdot 12$, where b in this case is the width between panel centerlines on each side of the beam.

The relative restraint provided by the torsional resistance of the effective transverse edge beam is reflected by the parameter β_t, defined as

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s} \tag{13.5}$$

where I_s, as before, is calculated for the slab spanning in direction l_1 and having width bounded by panel centerlines in the l_2 direction. The constant C pertains to the torsional rigidity of the effective transverse beam, which is defined according to ACI Code 13.7.5 as the largest of the following:

1. A portion of the slab having a width equal to that of the column, bracket, or capital in the direction in which moments are taken.
2. The portion of the slab specified in 1 plus that part of any transverse beam above and below the slab.
3. The transverse beam defined as in Fig. 13.10.

The constant C is calculated by dividing the section into its component rectangles, each having smaller dimension x and larger dimension y, and summing the contributions of all the parts by means of the equation

$$C = \sum \left(1 - 0.63\frac{x}{y}\right)\frac{x^3y}{3} \tag{13.6}$$

The subdivision can be done in such a way as to maximize C.

With these parameters defined, ACI Code 13.6.4 distributes the negative and positive moments between column strips and middle strips, assigning to the column strips the percentages of positive and negative moments shown in Table 13.4. Linear interpolations are to be made between the values shown.

Implementation of these provisions is facilitated by the interpolation charts of Graph A.4 of Appendix A. Interior negative and positive-moment percentages can be

TABLE 13.4
Column-strip moment, percent of total moment at critical section

		l_2/l_1		
		0.5	**1.0**	**2.0**
Interior negative moment				
$\alpha_1 l_2/l_1 = 0$		75	75	75
$\alpha_1 l_2/l_1 \geq 1.0$		90	75	45
Exterior negative moment				
$\alpha_1 l_2/l_1 = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	75	75	75
$\alpha_1 l_2/l_1 \geq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	90	75	45
Positive moment				
$\alpha_1 l_2/l_1 = 0$		60	60	60
$\alpha_1 l_2/l_1 \geq 1.0$		90	75	45

read directly from the charts for known values of $l_2 \cdot l_1$ and $\cdot\, {}_1 l_2 \cdot l_1$. For exterior negative moment, the parameter $\cdot\, {}_t$ requires an additional interpolation, facilitated by the auxiliary diagram on the right side of the charts. To illustrate its use for $l_2 \cdot l_1 = 1.55$ and $\cdot\, {}_1 l_2 \cdot l_1 = 0.6$, the dotted line indicates moment percentages of 100 for $\cdot\, {}_t = 0$ and 65 for $\cdot\, {}_t = 2.5$. Projecting to the right as indicated by the arrow to find the appropriate vertical scale of 2.5 divisions for an intermediate value of $\cdot\, {}_t$, say 1.0, then upward and finally to the left, one reads the corresponding percentage of 86 on the main chart.

The column-line beam spanning in the direction l_1 is to be proportioned to resist 85 percent of the column-strip moment if $\cdot\, {}_1 l_2 \cdot l_1$ is equal to or greater than 1.0. For values between one and zero, the proportion to be resisted by the beam may be obtained by linear interpolation. Concentrated or linear loads applied directly to such a beam should be accounted for separately.

The portion of the moment not resisted by the column strip is proportionately assigned to the adjacent half-middle strips. Each middle strip is designed to resist the sum of the moments assigned to its two half-middle strips. A middle strip adjacent and parallel to a wall is designed for twice the moment assigned to the half-middle strip corresponding to the first row of interior supports.

d. Shear in Slab Systems with Beams

Special attention must be given to providing the proper resistance to shear, as well as to moment, when designing by the direct method. According to ACI Code 13.6.8, beams with $\cdot\, {}_1 l_2 \cdot l_1 \geq 1.0$ must be proportioned to resist the shear caused by loads on a tributary area defined as shown in Fig. 13.13. For values of $\cdot\, {}_1 l_2 \cdot l_1$ between one and

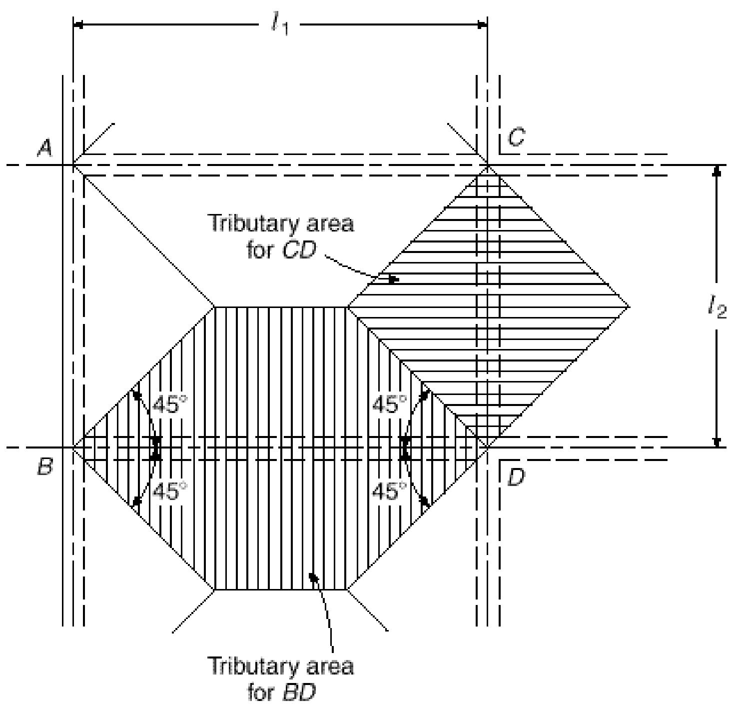


FIGURE 13.13
Tributary areas for shear calculation.

zero, the proportion of load carried by beam shear is found by linear interpolation. The remaining fraction of the load on the shaded area is assumed to be transmitted directly by the slab to the columns at the four corners of the panel, and the shear stress in the slab computed accordingly (see Section 13.10).

e. Design of Columns

Columns in two-way construction must be designed to resist the moments found from analysis of the slab-beam system. The column supporting an edge beam must provide a resisting moment equal to the moment applied from the edge of the slab (see Table 13.4). At interior locations, slab negative moments are found, assuming that dead and full live loads act. For the column design, a more severe loading results from partial removal of the live load. Accordingly, ACI Code 13.6.9 requires that interior columns resist a moment

$$M = 0.07[(w_d + 0.5w_l)l_2 l_n^2 - w_d' l_2' (l_n')^2] \tag{13.7}$$

In Eq. (13.7), the primed quantities refer to the shorter of the two adjacent spans (assumed to carry dead load only), and the unprimed quantities refer to the longer span (assumed to carry dead load and half live load). In all cases, the moment is distributed to the upper and lower columns in proportion to their relative flexural stiffness.

13.7 Flexural Reinforcement for Column-Supported Slabs

Consistent with the assumptions made in analysis, flexural reinforcement in two-way slab systems is placed in an orthogonal grid, with bars parallel to the sides of the panels. Bar diameters and spacings may be found as described in Section 13.2. Straight bars are generally used throughout, although in some cases positive-moment steel is bent up where no longer needed, to provide for part or all of the negative requirement. To provide for local concentrated loads, as well as to ensure that tensile cracks are narrow and well distributed, a maximum bar spacing at critical sections of 2 times the total slab thickness is specified by ACI Code 13.3.2 for two-way slabs. At least the minimum steel required for temperature and shrinkage crack control (see Section 13.3) must be provided. For protection of the steel against damage from fire or corrosion, at least $\frac{3}{4}$ in. concrete cover must be maintained.

Because of the stacking that results when bars are placed in perpendicular layers, the inner steel will have an effective depth 1 bar diameter less than the outer steel. For flat plates and flat slabs, the stacking problem relates to middle-strip positive steel and column-strip negative bars. In two-way slabs with beams on the column lines, stacking occurs for the middle-strip positive steel, and in the column strips is important mainly for the column-line beams, because slab moments are usually very small in the region where column strips intersect.

In the discussion of the stacking problem for two-way slabs supported by walls or stiff edge beams, in Section 13.4 it was pointed out that, because curvatures and moments in the short direction are greater than in the long direction of a rectangular panel, short-direction bars are normally placed closer to the top or bottom surface of the slab, with the larger effective depth d, and long-direction bars are placed inside these, with the smaller d. For two-way beamless flat plates, or slabs with relatively flexible edge beams, things are not so simple.

Consider a rectangular interior panel of a flat plate floor. If the slab column strips provided unyielding supports for the middle strips spanning in the perpendicular direction, the short-direction middle-strip curvatures and moments would be the larger. In fact, the column strips deflect downward under load, and this softening of the effective support greatly reduces curvatures and moments in the supported middle strip.

For the entire panel, including both middle strips and column strips in each direction, the moments in the long direction will be larger than those in the short direction, as is easily confirmed by calculating the static moment M_o in each direction for a rectangular panel. Noting that the apportioning of M_o first to negative and positive-moment sections, and then laterally to column and middle strips, is done by applying exactly the same ratios in each direction to the corresponding section, it is clear that the middle-strip positive moments (for example) are larger in the long direction than the short direction, exactly the opposite of the situation for the slab with stiff edge beams. In the column strips, positive and negative moments are larger in the long than in the short direction. On this basis, the designer is led to place the long-direction negative and positive bars, in both middle and column strips, closer to the top or bottom surface of the slab, respectively, with the larger effective depth.

If column-line beams are added, and if their stiffness is progressively increased for comparative purposes, it will be found that the short-direction slab moments gradually become dominant, although the long-direction beams carry larger moments than the short-direction beams. This will be clear from a careful study of Table 13.4.

The situation is further complicated by the influence of the ratio of short to long side dimensions of a panel, and by the influence of varying conditions of edge restraint (e.g., corner vs. typical exterior vs. interior panel). The best guide in specifying steel placement order in areas where stacking occurs is the relative magnitudes of design moments obtained from analysis for a particular case, with maximum d provided for the bars resisting the largest moment. No firm rules can be given. For square slab panels, many designers calculate the required steel area based on the average effective depth, thus obtaining the same bar size and spacing in each direction. This is slightly conservative for the outer layer, and slightly unconservative for the inner steel. Redistribution of loads and moments before failure would provide for the resulting differences in capacities in the two directions.

Reinforcement cutoff points could be calculated from moment envelopes if available; however, when the direct design method is used, moment envelopes and lines of inflection are not found explicitly. In such a case (and often when the equivalent frame method of Section 13.9 is used as well), standard bar cutoff points from Fig. 13.14 are used, as recommended in the ACI Code.

ACI Code 13.3.8.5 requires that all bottom bars within the column strip in each direction be continuous or spliced with Class A splices (see Section 5.11a) or mechanical or welded splices located as shown in Fig. 13.14. At least two of the column strip bars in each direction must pass within the column core and must be anchored at exterior supports. The continuous column strip bottom steel is intended to provide some residual ability to carry load to adjacent supports by catenary action if a single support should be damaged or destroyed. The two continuous bars through the column can be considered to be "integrity steel" and are provided to give the slab some residual capacity following a single punching shear failure.

The need for special reinforcement at the exterior corners of two-way beam-supported slabs was described in Section 13.4, and typical corner reinforcement is shown in Fig. 13.7. According to ACI Code 13.3.6, such reinforcement is required for slabs with beams between supporting columns if the value of · given by Eq. (13.3) is greater than 1.0.

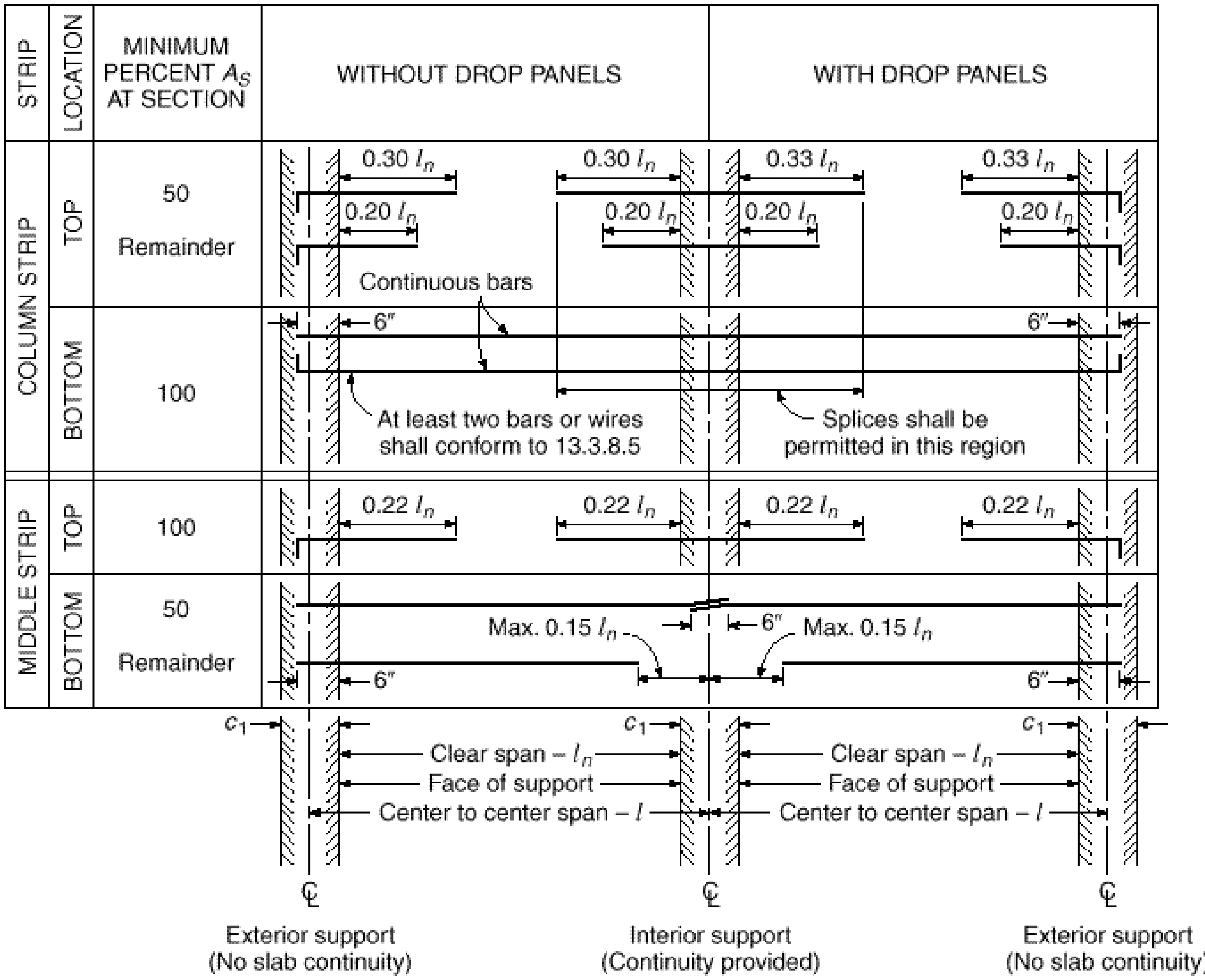


FIGURE 13.14
Minimum length of slab reinforcement in a slab without beams.

13.8 Depth Limitations of the ACI Code

To ensure that slab deflections in service will not be troublesome, the best approach is to compute deflections for the total load or load component of interest and to compare the computed deflections with limiting values. Methods have been developed that are both simple and acceptably accurate for predicting deflections of two-way slabs. A method for calculating the deflection of two-way column-supported slabs will be found in Section 13.13.

Alternatively, deflection control can be achieved indirectly by adhering to more or less arbitrary limitations on minimum slab thickness, limitations developed from review of test data and study of the observed deflections of actual structures. As a result of efforts to improve the accuracy and generality of the limiting equations, they have become increasingly complex.

ACI Code 9.5.3 establishes minimum thicknesses for two-way construction designed according to the methods of ACI Code Chapter 13, i.e., for slabs designed by

TABLE 13.5
Minimum thickness of slabs without interior beams

Yield Stress ·, psi	Without Drop Panels			With Drop Panels		
	Exterior Panels		Interior Panels	Exterior Panels		Interior Panels
	Without Edge Beams	With Edge Beams		Without Edge Beams	With Edge Beams	
40,000	$l_n \cdot 33$	$l_n \cdot 36$	$l_n \cdot 36$	$l_n \cdot 36$	$l_n \cdot 40$	$l_n \cdot 40$
60,000	$l_n \cdot 30$	$l_n \cdot 33$	$l_n \cdot 33$	$l_n \cdot 33$	$l_n \cdot 36$	$l_n \cdot 36$
75,000	$l_n \cdot 28$	$l_n \cdot 31$	$l_n \cdot 31$	$l_n \cdot 31$	$l_n \cdot 34$	$l_n \cdot 34$

[a] Slabs with beams along exterior edges. The value of · for the edge beam shall not be less than 0.8.

either the equivalent frame method or the direct design method. Simplified criteria are included pertaining to slabs without interior beams (flat plates and flat slabs with or without edge beams), while more complicated limit equations are to be applied to slabs with beams spanning between the supports on all sides. In both cases, minimum thicknesses less than the specified value may be used if calculated deflections are within Code-specified limits, as quoted in Table 6.2.

a. Slabs without Interior Beams

The minimum thickness of two-way slabs without interior beams, according to ACI Code 9.5.3.2, must not be less than provided by Table 13.5. Edge beams, often provided even for two-way slabs otherwise without beams to improve moment and shear transfer at the exterior supports, permit a reduction in minimum thickness of about 10 percent in exterior panels. In all cases, the minimum thickness of slabs without interior beams must not be less than the following:

For slabs without drop panels	5 in.
For slabs with drop panels	4 in.

b. Slabs with Beams on All Sides

The parameter used to define the relative stiffness of the beam and slab spanning in either direction is · , calculated from Eq. (13.4) of Section 13.6c, above. Then $\cdot_m$ is defined as the average value of · for all beams on the edges of a given panel. According to ACI Code 9.5.3.3, for $\cdot_m$ equal to or less than 0.2, the minimum thicknesses of Table 13.5 shall apply.

For $\cdot_m$ greater than 0.2 but not greater than 2.0, the slab thickness must not be less than

$$h = \frac{l_n \cdot 0.8 + f_y \cdot 200{,}000 \cdot}{36 + 5 \cdot \cdot\cdot_m - 0.2 \cdot} \qquad (13.8a)$$

and not less than 5.0 in.

For α_m greater than 2.0, the thickness must not be less than

$$h = \frac{l_n(0.8 + f_y/200{,}000)}{36 + 9\beta} \qquad (13.8b)$$

and not less than 3.5 in.,

where l_n = clear span in long direction, in.
α_m = average value of α for all beams on edges of a panel [see Eq. (13.4)]
β = ratio of clear span in long direction to clear span in short direction

At discontinuous edges, an edge beam must be provided with a stiffness ratio α not less than 0.8; otherwise the minimum thickness provided by Eq. (13.8*a*) or (13.8*b*) must be increased by at least 10 percent in the panel with the discontinuous edge.

In all cases, slab thickness less than the stated minimum may be used if it can be shown by computation that deflections will not exceed the limit values of Table 6.2.

Equations (13.8*a*) and (13.8*b*) can be restated in the general form

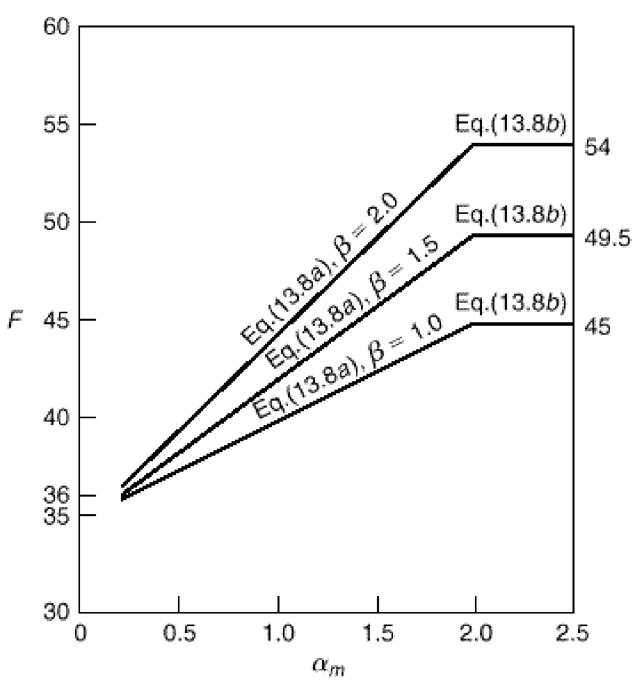
$$h = \frac{l_n(0.8 + f_y/200{,}000)}{F} \qquad (13.8c)$$

where F is the value of the denominator in each case. Figure 13.15 shows the value of F as a function of α_m, for comparative purposes, for three panel aspect ratios β:

1. Square panel, with $\beta = 1.0$
2. Rectangular panel, with $\beta = 1.5$
3. Rectangular panel, with $\beta = 2.0$, the upper limit of applicability of Eqs. (13.8*a*) and (13.8*b*)

Note that, for α_m less than 0.2, column-line beams have little effect, and minimum thickness is given by Table 13.5. For stiff, relatively deep edge beams, with α_m of 2 or greater, Eq. (13.8*b*) governs. Equation (13.8*a*) provides a transition for slabs with shallow column-line beams having α_m in the range from 0.2 to 2.0.

FIGURE 13.15
Parameter F governing minimum thickness of two-way slabs; minimum thickness $h = l_n(0.8 + f_y/200{,}000)/F$.



EXAMPLE 13.2 **Design of two-way slab with edge beams.**[†] A two-way reinforced concrete building floor system is composed of slab panels measuring 20 × 25 ft in plan, supported by shallow column-line beams cast monolithically with the slab, as shown in Fig. 13.16. Using concrete with $f'_c = 4000$ psi and steel with $f_y = 60{,}000$ psi, design a typical exterior panel to carry a service live load of 144 psf in addition to the self-weight of the floor.

Solution. The floor system satisfies all limitations stated in Section 13.6, and the ACI direct design method will be used. For illustrative purposes, only a typical exterior panel, as shown in Fig. 13.16, will be designed. The depth limitations of Section 13.8 will be used as a guide to the desirable slab thickness. To use Eqs. (13.8*a*) and (13.8*b*), a trial value of $h = 7$ will be introduced, and beam dimensions 14 × 20 in. will be assumed, as shown in Fig. 13.16. The effective flange projection beyond the face of the beam webs is the lesser of $4h_f$ or h_w, and in the present case is 13 in. The moment of inertia of the T beams will be estimated as multiples of that of the rectangular portion as follows:

For the edge beams: $I = \frac{1}{12} \times 14 \times 20^3 \times 1.5 = 14{,}000 \text{ in}^4$

For the interior beams: $I = \frac{1}{12} \times 14 \times 20^3 \times 2 = 18{,}700 \text{ in}^4$

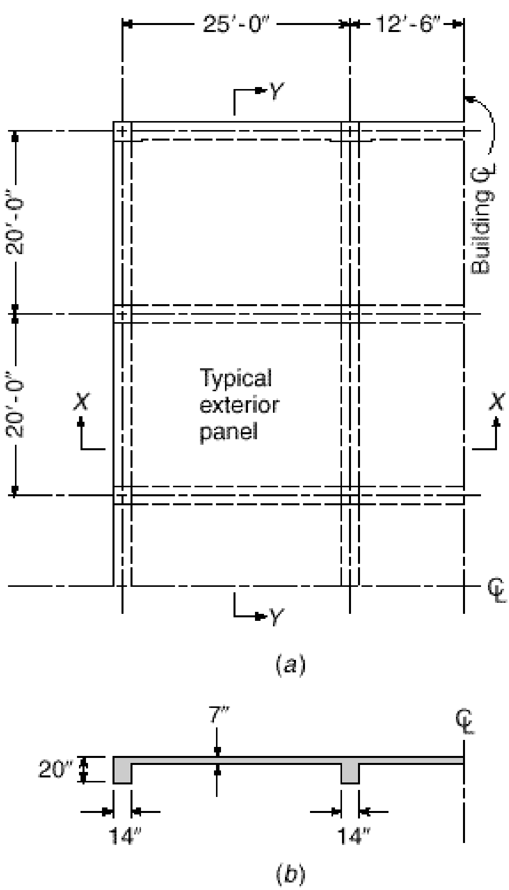


FIGURE 13.16
Two-way slab floor with beams on column lines: (*a*) partial floor plan; (*b*) section *X-X* (section *Y-Y* similar).

[†] The design of a two-way slab without beams, i.e., a flat plate floor system, which may also be done by the direct design method if the restrictions of Section 13.6 are met, will be illustrated by an example in Section 13.7.

For the slab strips:

For the 13.1 ft edge width: $I = \frac{1}{12} \times 13.1 \times 12 \times 7^3 = 4500 \text{ in}^4$
For the 20 ft width: $I = \frac{1}{12} \times 20 \times 12 \times 7^3 = 6900 \text{ in}^4$
For the 25 ft width: $I = \frac{1}{12} \times 25 \times 12 \times 7^3 = 8600 \text{ in}^4$

Thus, for the edge beam · = 14,000·4500 = 3.1, for the two 25 ft long beams · = 18,700·6900 = 2.7, and for the 20 ft long beam · = 18,700·8600 = 2.2, producing an average value $\cdot_m$ = 2.7. The ratio of long to short clear spans is · = 23.8·18.8 = 1.27. Then the minimum thickness is not to be less than that given by Eq. (13.8*b*):

$$h = \frac{286 \cdot 0.8 + 60 \cdot 200 \cdot}{36 + 9 \times 1.27} = 6.63 \text{ in.}$$

The 3.5 in. limitation of Section 13.8 clearly does not control in this case, and the 7 in. depth tentatively adopted will provide the basis of further calculation.

For a 7 in. slab, the dead load is $\frac{7}{12} \times 150 = 88$ psf. Applying the usual load factors to obtain design load gives

$$w = 1.2 \times 88 + 1.6 \times 144 = 336 \text{ psf}$$

For the *short-span direction*, for the slab-beam strip centered on the interior column line, the total static design moment is

$$M_o = \frac{1}{8} \times 0.336 \times 25 \times 18.8^2 = 371 \text{ ft-kips}$$

This is distributed as follows:

Negative design moment = 371 × 0.65 = 241 ft-kips
Positive design moment = 371 × 0.35 = 130 ft-kips

The column strip has a width of 2 × 20·4 = 10 ft. With $l_2 \cdot l_1$ = 25·20 = 1.25 and $\cdot_1 l_2 \cdot l_1$ = 2.2 × 25·20 = 2.75, Graph A.4 of Appendix A indicates that 68 percent of the negative moment, or 163 ft-kips, is taken by the column strip, of which 85 percent, or 139 ft-kips, is taken by the beam and 24 ft-kips by the slab. The remaining 78 ft-kips is allotted to the slab middle strip. Graph A.4 also indicates that 68 percent of the positive moment, or 88 ft-kips, is taken by the column strip, of which 85 percent, or 75 ft-kips, is assigned to the beam and 13 ft-kips to the slab. The remaining 42 ft-kips is taken by the slab middle strip.

A similar analysis is performed for the slab-beam strip at the edge of the building, based on a total static design moment of

$$M_o = \frac{1}{8} \times 0.336 \times 13.1 \times 18.8^2 = 194 \text{ ft-kips}$$

of which 65 percent is assigned to the negative and 35 percent to the positive bending sections as before. In this case, $\cdot_1 l_2 \cdot l_1$ = 3.1 × 25·20 = 3.9, The distribution factor for column-strip moment, from Graph A.4, is 68 percent for positive and negative moments as before, and again 85 percent of the column-strip moments is assigned to the beams.

In summary, the short-direction moments, in ft-kips, are as follows:

	Beam Moment	Column-Strip Slab Moment	Middle-Strip Slab Moment
Interior slab-beam strip—20 ft span			
Negative	139	24	78
Positive	75	13	42
Exterior slab-beam strip—20 ft span			
Negative	73	13	40
Positive	39	7	22

The total static design moment in the *long direction* of the exterior panel is

$$M_o = \frac{1}{8} \times 0.336 \times 20 \times 23.8^2 = 476 \text{ ft-kips}$$

This will be apportioned to the negative and positive moment sections according to Table 13.3, and distributed laterally across the width of critical moment sections with the aid of Graph A.4. The moment ratios to be applied to obtain exterior negative, positive, and interior negative moments are, respectively, 0.16, 0.57, and 0.70. The torsional constant for the edge beam is found from Eq. (13.6) for a 14 × 20 in. rectangular shape with a 7 × 13 in. projecting flange:

$$C = \left(1 - 0.63 \times \frac{14}{20}\right)\frac{14^3 \times 20}{3} + \left(1 - 0.63 \times \frac{7}{13}\right)\frac{7^3 \times 13}{3} = 11{,}210$$

With $l_2/l_1 = 0.80$, $\alpha_1 l_2/l_1 = 2.7 \times 20/25 = 2.2$, and from Eq. (13.5), $\beta_t = 11{,}210/(2 \times 6900) = 0.81$, Graph A.4 indicates that the column strip will take 93 percent of the exterior negative moment, 81 percent of the positive moment, and 81 percent of the interior negative moment. As before, the column-line beam will account for 85 percent of the column-strip moment. The results of applying these moment ratios are as follows:

	Beam Moment	Column-Strip Slab Moment	Middle-Strip Slab Moment
Exterior negative—25 ft span	60	11	5
Positive—25 ft span	187	33	51
Interior negative—25 ft span	229	40	63

It is convenient to tabulate the design of the slab reinforcement, as shown in Table 13.6. In the 25 ft direction, the two half-column strips may be combined for purposes of calculation into one strip of 106 in. width. In the 20 ft direction, the exterior half-column strip and the interior half-column strip will normally differ and are treated separately. Factored moments from the previous distributions are summarized in column 3 of the table.

The short-direction positive steel will be placed first, followed by the long-direction positive bars. If $\frac{3}{4}$ in. clear distance below the steel is allowed and use of No. 4 (No. 13) bars is anticipated, the effective depth in the short direction will be 6 in., while that in the long direction will be 5.5 in. A similar situation occurs for the top steel.

After calculating the design moment per foot strip of slab (column 6), find the minimum effective slab depth required for flexure. For the material strengths to be used, the maximum permitted reinforcement ratio is $\rho_{max} = 0.0206$. For this ratio,

$$d^2 = \frac{M_u}{\phi \rho f_y b (1 - 0.59 \rho f_y / f_c')}$$

$$= \frac{M_u}{0.90 \times 0.0206 \times 60{,}000 \times 12 (1 - 0.59 \times 0.0206 \times 60/4)} = \frac{M_u}{10{,}920}$$

Hence $d = \sqrt{M_u/10{,}920}$. Thus, the following minimum effective depths are needed:

In 25 ft direction: $d = \sqrt{6.30 \times \dfrac{12{,}000}{10{,}920}} = 2.63$ in.

In 20 ft direction: $d = \sqrt{5.20 \times \dfrac{12{,}000}{10{,}920}} = 2.39$ in.

both well below the depth dictated by deflection requirements. An underreinforced slab results. The required reinforcement ratios (column 7) are conveniently found from Table A.5 with $R = M_u/\phi bd^2$ or from Table A.9. Note that a minimum steel area equal to 0.0018 times the gross concrete area must be provided for control of temperature and shrinkage cracking. For a 12 in. slab strip, the corresponding area is $0.0018 \times 7 \times 12 = 0.151$ in^2. Expressed in terms of minimum reinforcement ratio for actual effective depths, this gives

TABLE 13.6
Design of slab reinforcement

(1)	(2) Location	(3) ft-kips	(4) in.	(5) in.	(6) × 12/ , ft-kips/ft	(7)	(8) in^2	(9) Number of No. 4 (No. 13) Bars
25 ft span Two half-column strips	Exterior negative	11	106	5.5	1.25	0.0023[a]	1.34	7
	Positive	33	106	5.5	3.74	0.0023	1.34	7
	Interior negative	40	106	5.5	4.53	0.0029	1.69	9
Middle strip	Exterior negative	5	120	5.5	0.50	0.0023[a]	1.52	9[b]
	Positive	51	120	5.5	5.10	0.0033	2.18	11
	Interior negative	63	120	5.5	6.30	0.0041	2.71	14
20 ft span Exterior half-column strip	Negative	13	53	6	2.94	0.0021[a]	0.67	4
	Positive	7	53	6	1.58	0.0021[a]	0.67	4
Middle strip	Negative	78	180	6	5.20	0.0028	3.03	16
	Positive	42	180	6	2.80	0.0021[a]	2.27	13[b]
Interior half-column strip	Negative	12	53	6	2.71	0.0021[a]	0.67	4
	Positive	6.5	53	6	1.47	0.0021[a]	0.67	4

[a] Reinforcement ratio controlled by shrinkage and temperature requirements.
[b] Number of bars controlled by maximum spacing requirements.

In 25 ft direction: $$_{min} = \frac{0.151}{5.5 \times 12} = 0.0023$$

In 20 ft direction: $$_{min} = \frac{0.151}{6 \times 12} = 0.0021$$

This requirement controls at the locations indicated in Table 13.6.

The total steel area in each band is easily found from the reinforcement ratio and is given in column 8. Finally, with the aid of Table A.2, the required number of bars is obtained. Note that in two locations, the number of bars used is dictated by the maximum spacing requirement of 2 × 7 = 14 in.

The shear capacity of the slab is checked on the basis of the tributary areas shown in Fig. 13.13. At a distance d from the face of the long beam,

$$V_u = 0.336\left(10 - \frac{14}{2 \times 12} - \frac{6}{12}\right) = 3.00 \text{ kips}$$

The design shear strength of the slab is

$$\phi V_c = 0.75 \times 2\sqrt{4000} \times 12 \times \frac{6}{1000}$$
$$= 6.83 \text{ kips}$$

well above the shear applied at factored loads.

Each beam must be designed for its share of the total static moment, as found in the above calculations, as well as the moment due to its own weight; this moment may be distributed to positive and negative bending sections, using the same ratios used for the static moments due to slab loads. Beam shear design should be based on the loads from the tributary areas shown in Fig. 13.13. Since no new concepts would be introduced, the design of the beams will not be presented here.

Since 0.85 × 93 = 79 percent of the exterior negative moment in the long direction is carried directly to the column by the column-line beam in this example, torsional stresses in the spandrel beam are very low and may be disregarded. In other circumstances, the spandrel beams would be designed for torsion following the methods of Chapter 7.

13.9 Equivalent Frame Method

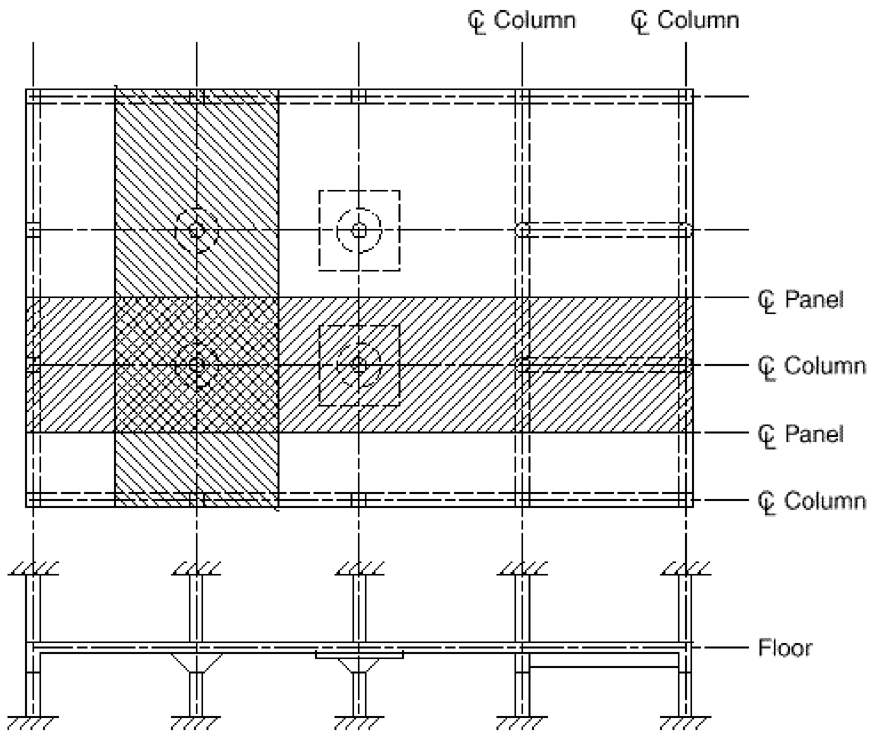
a. Basis of Analysis

The direct design method for two-way slabs described in Section 13.6 is useful if each of the six restrictions on geometry and load is satisfied by the proposed structure. Otherwise, a more general method is needed. One such method, proposed by Peabody in 1948 (Ref. 13.11), was incorporated in subsequent editions of the ACI Code as *design by elastic analysis*. The method was greatly expanded and refined based on research in the 1960s (Refs. 13.12 and 13.13), and it appears in Chapter 13 of the current ACI Code as the *equivalent frame method*.

It will be evident that the equivalent frame method was derived with the assumption that the analysis would be done using the moment distribution method (see Chapter 12). If analysis is done by computer using a standard frame analysis program, special modeling devices are necessary. This point will be discussed further in Section 13.9e.

By the equivalent frame method, the structure is divided, for analysis, into continuous frames centered on the column lines and extending both longitudinally and transversely, as shown by the shaded strips in Fig. 13.17. Each frame is composed of a row of columns and a broad continuous beam. The beam, or slab beam, includes the portion of the slab bounded by panel centerlines on either side of the columns, together with column-line beams or drop panels, if used. For vertical loading, each floor with its columns may be analyzed separately, with the columns assumed to be fixed at the floors above and below. In calculating bending moment at a support, it is convenient and sufficiently accurate to assume that the continuous frame is completely fixed at the support two panels removed from the given support, provided the frame continues past that point.

FIGURE 13.17
Building idealization for equivalent frame analysis.



b. Moment of Inertia of Slab Beam

Moments of inertia used for analysis may be based on the concrete cross section, neglecting reinforcement, but variations in cross section along the member axis should be accounted for.

For the beam strips, the first change from the midspan moment of inertia normally occurs at the edge of drop panels, if they are used. The next occurs at the edge of the column or column capital. While the stiffness of the slab strip could be considered infinite within the bounds of the column or capital, at locations close to the panel centerlines (at each edge of the slab strip), the stiffness is much less. According to ACI Code 13.7.3, from the center of the column to the face of the column or capital, the moment of inertia of the slab is taken equal to the value at the face of the column or capital, divided by the quantity $(1 - c_2/l_2)^2$, where c_2 and l_2 are the size of the column or capital and the panel span, respectively, both measured transverse to the direction in which moments are being determined.

Accounting for these changes in moments of inertia results in a member, for analysis, in which the moment of inertia varies in a stepwise manner. The stiffness factors, carryover factors, and uniform-load fixed-end moment factors needed for moment distribution analysis (see Chapter 12) are given in Table A.13*a* of Appendix A for a slab without drop panels and in Table A.13*b* for a slab with drop panels with a depth equal to 1.25 times the slab depth and a length equal to one-third the span length.

c. The Equivalent Column

In the equivalent frame method of analysis, the columns are considered to be attached to the continuous slab beam by torsional members that are transverse to the direction of the span for which moments are being found; the torsional member extends to the panel centerlines bounding each side of the slab beam under study. Torsional deformation of these transverse supporting members reduces the effective flexural stiffness provided by the actual column at the support. This effect is accounted for in the analysis by use of what is termed an *equivalent column* having stiffness less than that of the actual column.

The action of a column and the transverse torsional member is easily explained with reference to Fig. 13.18, which shows, for illustration, the column and transverse beam at the exterior support of a continuous slab-beam strip. From Fig. 13.18, it is clear that the rotational restraint provided at the end of the slab spanning in the direction l_1 is influenced not only by the flexural stiffness of the column but also by the torsional stiffness of the edge beam AC. With distributed torque m_t applied by the slab and resisting torque M_t provided by the column, the edge-beam sections at A and C will rotate to a greater degree than the section at B, owing to torsional deformation of the edge beam. To allow for this effect, the actual column and beam are replaced by an equivalent column, so defined that the total flexibility (inverse of stiffness) of the equivalent column is the sum of the flexibilities of the actual column and beam. Thus,

$$\frac{1}{K_{ec}} = \frac{1}{K_c} + \frac{1}{K_t} \tag{13.9}$$

where K_{ec} = flexural stiffness of equivalent column
K_c = flexural stiffness of actual column
K_t = torsional stiffness of edge beam

all expressed in terms of moment per unit rotation. In computing K_c, the moment of inertia of the actual column is assumed to be infinite from the top of the slab to the bottom of the slab beam, and I_g is based on the gross concrete section elsewhere along the length. Stiffness factors for such a case are given in Table A.13*c*.

The effective cross section of the transverse torsional member, which may or may not include a beam web projecting below the slab, as shown in Fig. 13.18, is the

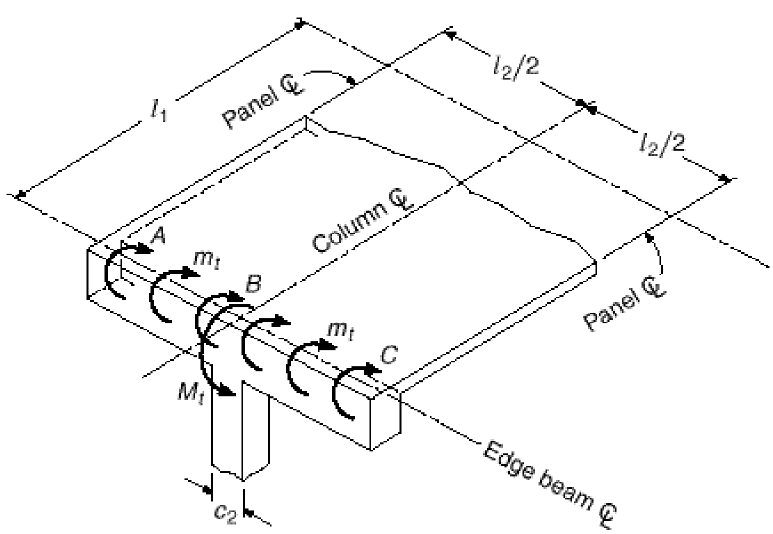


FIGURE 13.18
Torsion at a transverse supporting member illustrating the basis of the equivalent column.

same as defined earlier in Section 13.6c. The torsional constant C is calculated by Eq. (13.5) based on the effective cross section so determined. The torsional stiffness K_t can then be calculated by the expression

$$K_t = \sum \frac{9E_{cs}C}{l_2\left(1 - \frac{c_2}{l_2}\right)^3} \qquad (13.10)$$

where E_{cs} = modulus of elasticity of slab concrete
c_2 = size of rectangular column, capital, or bracket in direction l_2
C = cross-sectional constant [see Eq. (13.6)]

The summation applies to the typical case in which there are slab beams (with or without edge beams) on both sides of the column. The length l_2 is measured center-to-center of the supports and, thus, may have different values in each of the summation terms in Eq. (13.10), if the transverse spans are unequal.

If a panel contains a beam parallel to the direction in which moments are being determined, the value of K_t obtained from Eq. (13.10) leads to values of K_{ec} that are too low. Accordingly, it is recommended that in such cases the value of K_t found by Eq. (13.10) be multiplied by the ratio of the moment of inertia of the slab with such a beam to the moment of inertia of the slab without it.

The concept of the equivalent column, illustrated with respect to an exterior column, is employed at *all* supporting columns for each continuous slab beam, according to the equivalent frame method.

d. Moment Analysis

With the effective stiffness of the slab-beam strip and the supports found as described, the analysis of the equivalent frame can proceed by moment distribution (see Chapter 12).

In keeping with the requirements of statics (see Section 13.5), equivalent beam strips in each direction must each carry 100 percent of the load. If the live load does not exceed three-quarters of the dead load, maximum moment may be assumed to occur at all critical sections when the full factored live load (plus factored dead load) is on the entire slab, according to ACI Code 13.7.6. Otherwise pattern loadings must be used to maximize positive and negative moments. Maximum positive moment is calculated with three-quarters factored live load on the panel and on alternate panels, while maximum negative moment at a support is calculated with three-quarters factored live load on the adjacent panels only. Use of three-quarters live load rather than the full value recognizes that maximum positive and negative moments cannot occur simultaneously (since they are found from different loadings) and that redistribution of moments to less highly stressed sections will take place before failure of the structure occurs. Factored moments must not be taken less than those corresponding to full factored live load on all panels, however.

Negative moments obtained from that analysis apply at the centerlines of supports. Since the support is not a knife-edge but a rather broad band of slab spanning in the transverse direction, some reduction in the negative design moment is proper (see also Section 12.5a). At interior supports, the critical section for negative bending, in both column and middle strips, may be taken at the face of the supporting column or capital, but in no case at a distance greater than $0.175l_1$ from the center of the column, according to ACI Code 13.7.7. To avoid excessive reduction of negative moment at the exterior supports (where the distance to the point of inflection is small) for the case where columns are provided with capitals, the critical section for negative bending in the direction perpendicular to an edge should be taken at a distance from the face of support not greater than one-half the projection of the capital beyond the face of the support.

With positive and negative design moments obtained as just described, it still remains to distribute these moments across the widths of the critical sections. For design purposes, the total strip width is divided into column strip and adjacent half-middle strips, defined previously, and moments are assumed constant within the bounds of each. The distribution of moments to column and middle strips is done using the same percentages given in connection with the direct design method. These are summarized in Table 13.4 and by the interpolation charts of Graph A.4 of Appendix A.

The distribution of moments and shears to column-line beams, if present, is in accordance with the procedures of the direct design method also. Restriction 6 of Section 13.6, pertaining to the relative stiffness of column-line beams in the two directions, applies here also if these distribution ratios are used.

EXAMPLE 13.3 **Design of flat plate floor by equivalent frame method.** An office building is planned using a flat plate floor system with the column layout as shown in Fig. 13.19. No beams, drop panels, or column capitals are permitted. Specified live load is 100 psf and dead load will include the weight of the slab plus an allowance of 20 psf for finish floor plus suspended loads. The columns will be 18 in. square, and the floor-to-floor height of the structure will be 12 ft. Design the interior panel *C*, using material strengths $f_y = 60{,}000$ psi and $f'_c =$ 4000 psi. Straight-bar reinforcement will be used.

Solution. Minimum thickness *h* for a flat plate, according to the ACI Code, may be found from Table 13.5.† For the present example, the minimum *h* for the exterior panel is

$$h = \frac{20.5 \times 12}{30} = 8.20 \text{ in.}$$

This will be rounded upward for practical reasons, with calculations based on a trial thickness of 8.5 in. for all panels. Thus the dead load of the slab is $150 \times 8.5 \cdot 12 = 106$ psf, to which the superimposed dead load of 20 psf must be added. The factored design loads are

$$1.2w_d = 1.2 \cdot 106 + 20 \cdot = 151 \text{ psf}$$

$$1.6w_l = 1.6 \times 100 = 160 \text{ psf}$$

The structure is identical in each direction, permitting the design for one direction to be used for both (an average effective depth to the tensile steel will be used in the calculations). While the restrictions of Section 13.6 are met and the direct design method of analysis is permissible, the equivalent frame method will be adopted to demonstrate its features. Moments will be found by the method of moment distribution.

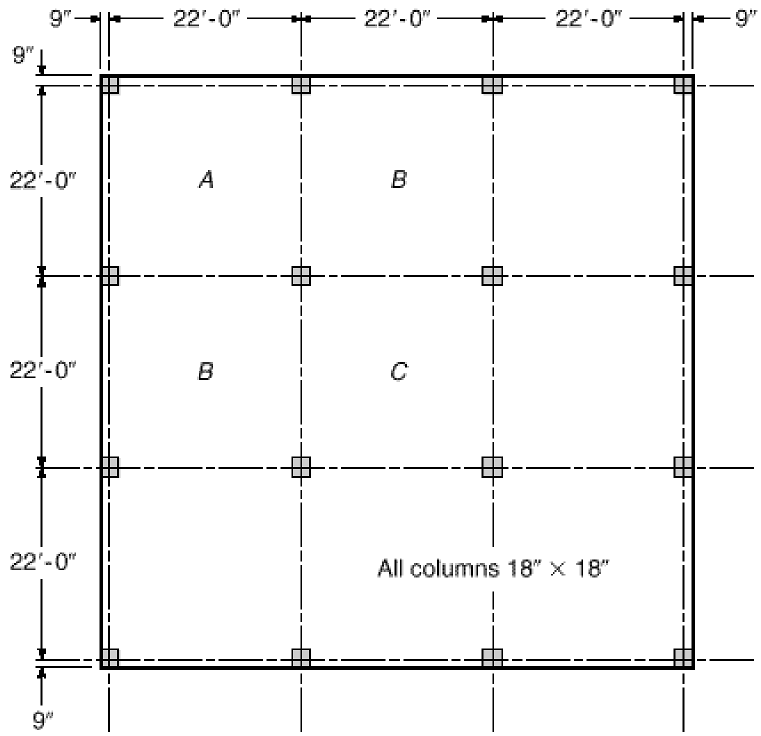
For flat plate structures, it is usually acceptable to calculate stiffnesses as if all members were prismatic, neglecting the increase in stiffness within the joint region, as it generally has negligible effect on design moments and shears. Then, for the slab spans,

$$K_s = \frac{4E_c I_c}{l}$$

$$= \frac{4E_c \cdot 264 \times 8.5^3 \cdot}{12 \times 264} = 205E_c$$

† In many flat plate floors, the minimum slab thickness is controlled by requirements for shear transfer at the supporting columns, and *h* is determined either to avoid supplementary shear reinforcement or to limit the excess shear to a reasonable margin above that which can be carried by the concrete. Design for shear in flat plates and flat slabs will be treated in Section 13.10.

FIGURE 13.19
Two-way flat plate floor.



and the column stiffnesses are

$$K_c = \frac{4E_c \cdot 18 \times 18^3 \cdot}{12 \times 144} = 243E_c$$

Calculation of the equivalent column stiffness requires consideration of the torsional deformation of the transverse strip of slab that functions as the supporting beam. Applying the criteria of the ACI Code establishes that the effective torsional member has width 18 in. and depth 8.5 in. For this section, the torsional constant C from Eq. (13.5) is

$$C = \cdot\ 1 - 0.63 \times \frac{8.5}{18} \cdot\ 8.5^3 \times \frac{18}{3} = 2590 \text{ in}^4$$

and the torsional stiffness, from Eq. (13.10), is

$$K_t = \frac{9E_c \times 2590}{264 \cdot 1 - 1.5 \cdot 22 \cdot^3} = 109E_c$$

From Eq. (13.9), accounting for two columns and two torsional members at each joint,

$$\frac{1}{K_{ec}} = \frac{1}{2 \times 243E_c} + \frac{1}{2 \times 109E_c}$$

from which $K_{ec} = 151E_c$. Distribution factors at each joint are then calculated in the usual way.

For the present example, the ratio of service live load to dead load is $100 \cdot 126 = 0.79$, and because this exceeds 0.75, according to ACI Code 13.7.6 maximum positive and negative moments must be found based on pattern loadings, with full factored dead load in place and three-quarters factored live load positioned to cause the maximum effect. In addition, the design moments must not be less than those produced by full factored live and dead loads on all panels. Thus three load cases must be considered: (*a*) full factored dead and live

TABLE 13.7
Moments in flat plate floor, ft-kips

Panel	B		C		B	
Joint	**1**	**2**	**2**	**3**	**3**	**4**
(*a*) 311 psf all panels						
Fixed-end moments	+276	−276	+276	−276	+276	−276
Final moments	+125	−323	+295	−295	+323	−125
Span moment in *C*			119			
(*b*) 151 psf panels *B* and 271 psf panel *C*						
Fixed-end moments	+134	−134	+240	−240	+134	−134
Final moments	+50	−200	+224	−224	+200	−50
Span moment in *C*			137			
(*c*) 271 psf panels *B* (left) and *C* and 151 psf panel *B* (right)						
Fixed-end moments	+240	−240	+240	−240	+134	−134
Final moments	+107	−290	+274	−207	+191	−52
Span moment in *C*			120			

load, 311 psf, on all panels; (*b*) factored dead load of 151 psf on all spans plus three-quarters factored live load, 120 psf, on panel *C*; and (*c*) full factored dead load on all spans and three-quarters live load on first and second spans. Fixed-end moments and final moments obtained from moment distribution are summarized in Table 13.7. The results indicate that load case (*a*) controls the slab design in the support region, while load case (*b*) controls at the midspan of panel *C*. Moment diagrams for the two controlling cases are shown in Fig. 13.20*a*. According to the ACI Code the critical section at interior supports may be taken at the face of supports, but not greater than $0.175l_1$ from the column centerline. The former criterion controls here, and the negative design moment is calculated by subtracting the area under the shear diagram between the centerline and face of support, for load case (*a*), from the negative moment at the support centerline. The shear diagram for load case (*a*) is given in Fig. 13.20*b*, with the adjusted design moments shown in Fig. 13.20*a*.

Because the effective depth for all panels will be the same, and because the negative steel for panel *C* will continue through the support region to become the negative steel for panels *B*, the larger negative moment found for the panels *B* will control. Accordingly, the design negative moment is 262 ft-kips and the design positive moment is 137 ft-kips.†

Moments will be distributed laterally across the slab width according to Table 13.4, which indicates that 75 percent of the negative moment will be assigned to the column strip and 60 percent of the positive moment assigned to the column strip. The design of the slab reinforcement is summarized in Table 13.8.

Other important aspects of the design of flat plates include design for punching shear at the columns, which may require supplementary shear reinforcement, and transfer of unbalanced moments to the columns, which may require additional flexural bars in the negative bending region of the column strips or adjustment of spacing of negative steel. These considerations are of special importance at exterior columns and corner columns, such as shown in Fig. 13.19. Shear and moment transfer at the columns will be discussed in Sections 13.10 and 13.11, respectively.

† When slab systems that meet the restrictions of the direct design method are designed by the equivalent frame method, according to ACI Code 13.7.7 the resulting design moments may be reduced proportionately so that the sum of the positive and average negative moments in a span are no greater than M_o calculated for the direct design method according to Eq. (13.1). There is no theoretical basis for this. The reduction is less than 5 percent in the present example, and it will not be included in the design calculations.

FIGURE 13.20
Design moments and shears for flat plate floor interior panel *C*: (*a*) moments; (*b*) shears.

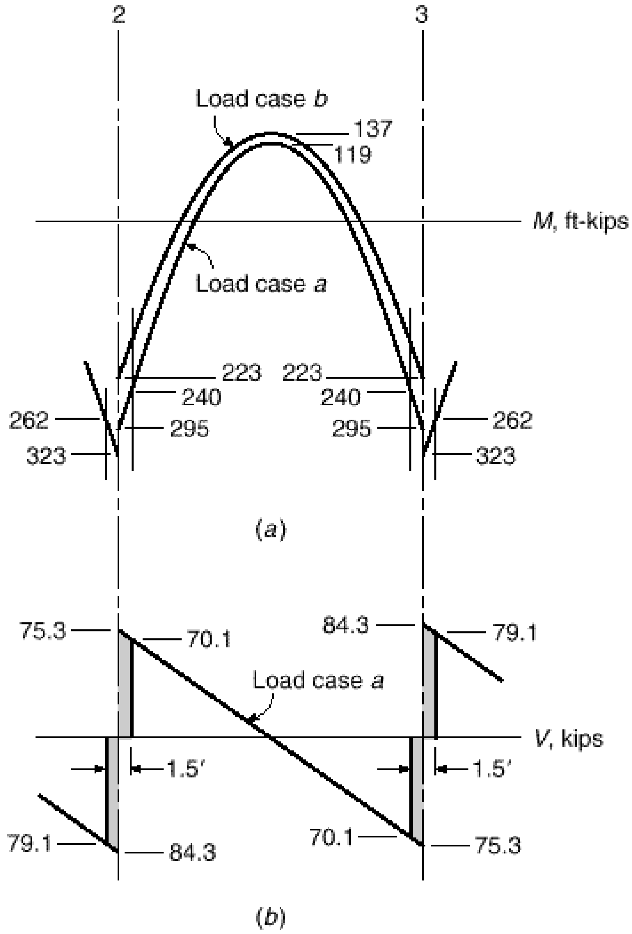


TABLE 13.8
Design of flat plate reinforcement

(1)	(2) Location	(3) ft-kips	(4) in.	(5) in.	(6) × 12 ft-kips/ft	(7)	(8) in^2	(9) Number and Size of Bars
Column strip	Negative	196	132	7	17.82	0.0075	6.65	16 No. 6 (No. 19)
	Positive	82	132	7	7.45	0.0029	2.68	9 No. 5 (No. 16)
Two half-middle strips	Negative	66	132	7	6.00	0.0023	2.13	8 No. 5 (No. 16)[a]
	Positive	55	132	7	5.00	0.0020	1.85	8 No. 5 (No. 16)[a]

[a] Number of bars controlled by maximum spacing requirement.

e. Equivalent Frame Analysis by Computer

It is clear that the equivalent frame method, as described in the ACI Code and the ACI Code Commentary, is oriented toward analysis using the method of moment distribution. Presently, most offices make use of computers, and frame analysis is done using general-purpose programs based on the direct stiffness method. Plane frame analysis programs can be used for slab analysis based on the concepts of the equivalent frame method, but the frame must be specially modeled. Variable moments of inertia along the axis of slab-beams and columns require nodal points (continuous joints) between sections where I is to be considered constant (i.e., in the slab at the junction of slab and drop panel, drop panel and capital, and in the columns at the bottom of the capitals). In addition, it is necessary to compute K_{ec} for each column, and then to compute the equivalent value of the moment of inertia for the column.

Alternately, a three-dimensional frame analysis may be used, in which the torsional properties of the transverse supporting beams may be included directly. A third option is to make use of specially written computer programs, the most widely used being "ADOSS-Analysis and Design of Reinforced Concrete Slab Systems," developed by the Portland Cement Association (Skokie, Illinois).

13.10 SHEAR DESIGN IN FLAT PLATES AND FLAT SLABS

When two-way slabs are supported directly by columns, as in flat slabs and flat plates, or when slabs carry concentrated loads, as in footings, shear near the columns is of critical importance. Tests of flat plate structures indicate that, in most practical cases, the capacity is governed by shear (Ref. 13.14).

a. Slabs without Special Shear Reinforcement

Two kinds of shear may be critical in the design of flat slabs, flat plates, or footings. The first is the familiar beam-type shear leading to diagonal tension failure. Applicable particularly to long narrow slabs or footings, this analysis considers the slab to act as a wide beam, spanning between supports provided by the perpendicular column strips. A potential diagonal crack extends in a plane across the entire width l_2 of the slab. The critical section is taken a distance d from the face of the column or capital. As for beams, the design shear strength · V_c must be at least equal to the required strength V_u at factored loads. The nominal shear strength V_c should be calculated by either Eq. (4.12*a*) or Eq. (4.12*b*), with $b_w = l_2$ in this case.

Alternatively, failure may occur by *punching shear*, with the potential diagonal crack following the surface of a truncated cone or pyramid around the column, capital, or drop panel, as shown in Fig. 13.21*a*. The failure surface extends from the bottom of the slab, at the support, diagonally upward to the top surface. The angle of inclination with the horizontal, · (see Fig. 13.21*b*), depends upon the nature and amount of reinforcement in the slab. It may range between about 20 and 45°. The critical section for shear is taken perpendicular to the plane of the slab and a distance d·2 from the periphery of the support, as shown. The shear force V_u to be resisted can be calculated as the total factored load on the area bounded by panel centerlines around the column less the load applied within the area defined by the critical shear perimeter, unless significant moments must be transferred from the slab to the column (see Section 13.11).

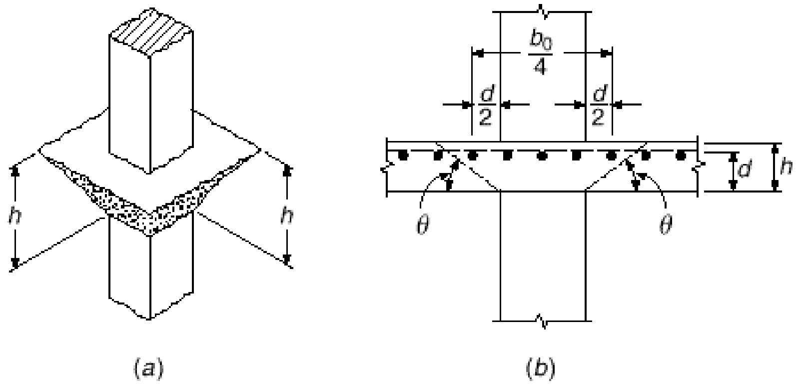


FIGURE 13.21
Failure surface defined by punching shear.

At such a section, in addition to the shearing stresses and horizontal compressive stresses due to negative bending moment, vertical or somewhat inclined compressive stress is present, owing to the reaction of the column. The simultaneous presence of vertical and horizontal compression increases the shear strength of the concrete. For slabs supported by columns having a ratio of long to short sides not greater than 2, tests indicate that the nominal shear strength may be taken equal to

$$V_c = 4\sqrt{f'_c}\, b_o d \tag{13.11a}$$

according to ACI Code 11.12.2, where b_o = the perimeter along the critical section.

However, for slabs supported by very rectangular columns, the shear strength predicted by Eq. (13.11*a*) has been found to be unconservative. According to tests reported in Ref. 13.15, the value of V_c approaches $2\sqrt{f'_c}\, b_o d$ as β_c, the ratio of long to short sides of the column, becomes very large. Reflecting this test data, ACI Code 11.12.2 states further that V_c in punching shear shall not be taken greater than

$$V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f'_c}\, b_o d \tag{13.11b}$$

The variation of the shear strength coefficient, as governed by Eqs. (13.11*a*) and (13.11*b*) is shown in Fig. 13.22 as a function of β_c.

Further tests, reported in Ref. 13.16, have shown that the shear strength V_c decreases as the ratio of critical perimeter to slab depth b_o/d increases. Accordingly, ACI Code 11.12.2 states that V_c in punching shear must not be taken greater than

$$V_c = \left(\frac{\alpha_s d}{b_o} + 2\right)\sqrt{f'_c}\, b_o d \tag{13.11c}$$

where α_s is 40 for interior columns, 30 for edge columns, and 20 for corner columns, i.e., columns having critical sections with 4, 3, or 2 sides, respectively.

Thus, according to the ACI Code, the punching shear strength of slabs and footings is to be taken as the smallest of the values of V_c given by Eqs. (13.11*a*), (13.11*b*), and (13.11*c*). The design strength is taken as ϕV_c as usual, where $\phi = 0.75$ for shear. The basic requirement is then $V_u \le \phi V_c$.

For columns with nonrectangular cross sections the ACI Code indicates that the perimeter b_o must be of minimum length, but need not approach closer than $d/2$ to the perimeter of the reaction area. The manner of defining the critical perimeter b_o and the ratio β_c for such irregular support configurations is illustrated in Fig. 13.23.

FIGURE 13.22
Shear strength coefficient for flat plates as a function of ratio β_c of long side to short side of support.

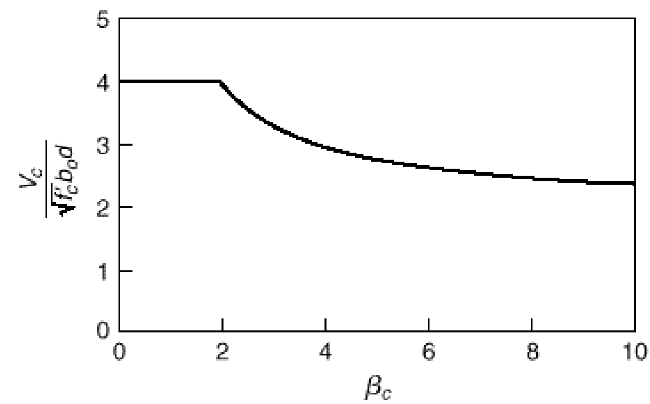
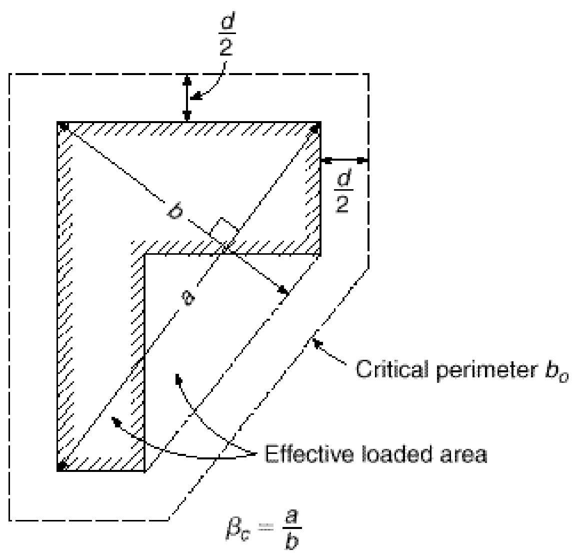


FIGURE 13.23
Punching shear for columns of irregular shape.

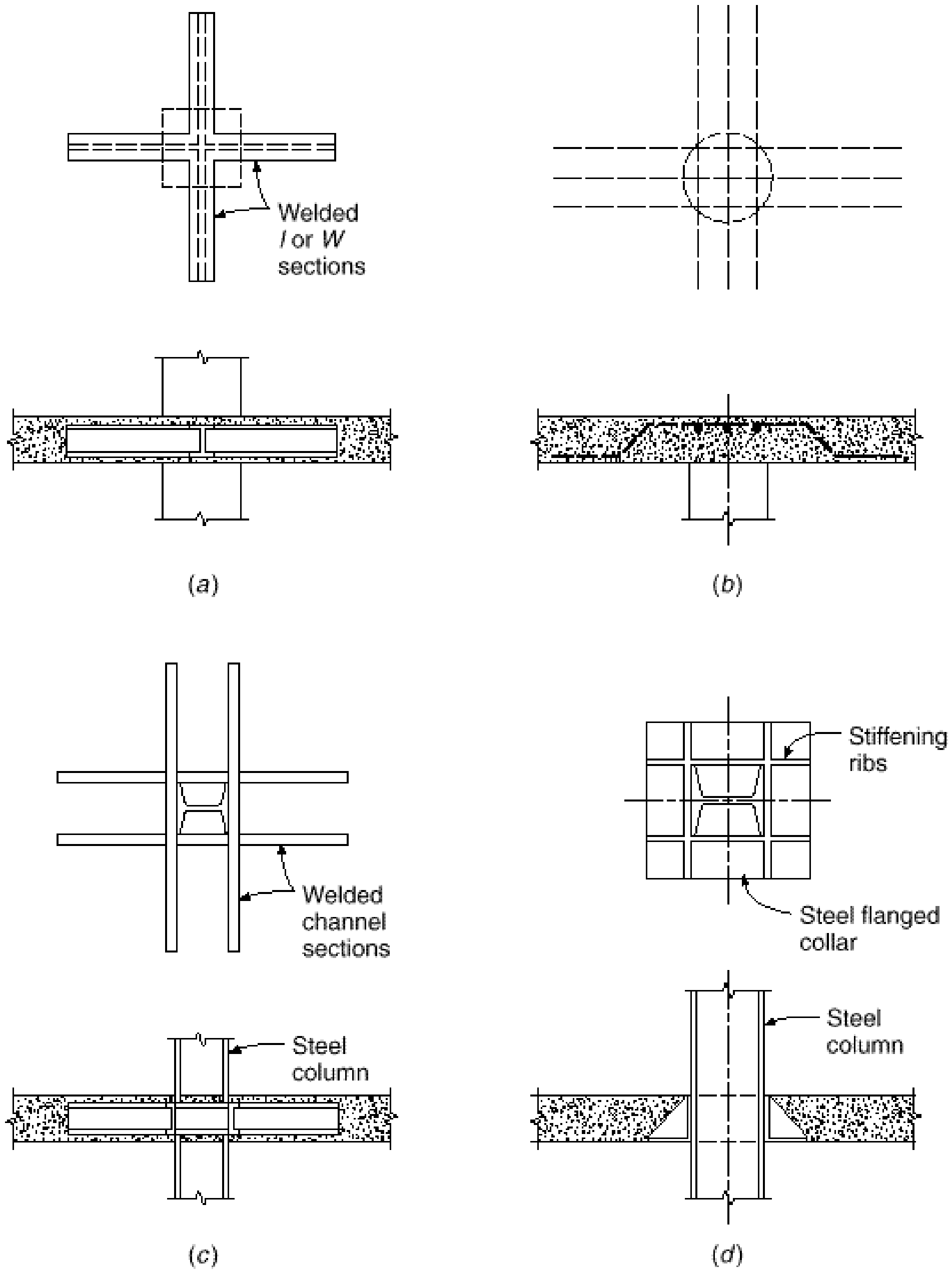


b. Types of Shear Reinforcement

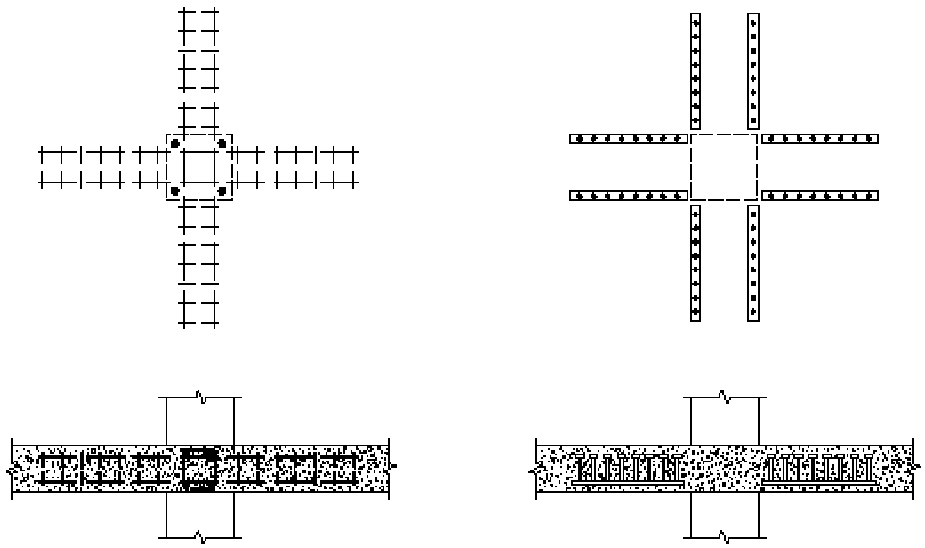
Special shear reinforcement is often used at the supports for flat plates, and sometimes for flat slabs as well. It may take several forms. A few common types are shown in Fig. 13.24.

The *shearheads* shown in Fig. 13.24*a* and *c* consist of standard structural steel shapes embedded in the slab and projecting beyond the column. They serve to increase the effective perimeter b_o of the critical section for shear. In addition, they may contribute to the negative bending resistance of the slab. The reinforcement shown in Fig. 13.24*a* is particularly suited for use with concrete columns. It consists of short lengths of I or wide-flange beams, cut and welded at the crossing point so that the arms are continuous through the column. Normal negative slab reinforcement passes over the top of the structural steel, while bottom bars are stopped short of the shearhead. Column bars pass vertically at the corners of the column. The effectiveness of this type of shearhead has been documented by tests by Corley and Hawkins Ref. 13.17. The channel frame in Fig. 13.24*c* is very similar in its action, but is adapted for

FIGURE 13.24
Shear reinforcement for flat plates (*continued on next page*).



use with steel columns. The bent-bar arrangement in Fig. 13.24*b* is suited for use with concrete columns. The bars are usually bent at 45° across the potential diagonal tension crack, and extend along the bottom of the slab a distance sufficient to develop their strength by bond. The flanged collar in Fig. 13.24*d* is designed mainly for use with lift-slab construction (see Chapter 18). It consists of a flat bottom plate with vertical stiffening ribs. It may incorporate sockets for lifting rods, and usually is used in conjunction with shear pads welded directly to the column surfaces below the collar to transfer the vertical reaction.



(e) (f)

FIGURE 13.24
(*continued*)

Another type of shear reinforcement is illustrated in Fig. 13.24*e*, where vertical stirrups have been used in conjunction with supplementary horizontal bars radiating outward in two perpendicular directions from the support, to form what are termed *integral beams* contained entirely within the slab thickness. These beams act in the same general way as the shearheads shown in Fig. 13.24*a* and *c*. Adequate anchorage of the stirrups is difficult in slabs thinner than about 10 in. ACI Code 11.12.3 requires the slab effective depth d to be at least 6 in., but not less than 16 times the diameter of the shear reinforcement. In all cases, closed hoop stirrups should be used, with a large-diameter horizontal bar at each bend point, and the stirrups must be terminated with a standard hook (Ref. 13.18).

A more recent development is the shear stud reinforcement shown in Fig. 13.24*f*. This consists of large-head studs welded to steel strips. The strips are supported on wire chairs during construction to maintain the required concrete cover to the bottom of the slab below the strip, and the usual cover is maintained over the top of the head. Because of the positive anchorage provided by the stud head and the steel strip, these devices are more effective, according to tests, than either the bent bar or integral beam reinforcement (Refs. 13.19 and 13.20). In addition, they can be placed more easily, with less interference with other reinforcement, than other types of shear steel.

c. Design of Bent Bar Reinforcement

If shear reinforcement in the form of bars is used (Fig. 13.24*b*), the limit value of nominal shear strength V_n, calculated at the critical section $d/2$ from the support face, may be increased to $6\sqrt{f'_c}\,b_o d$ according to ACI Code 11.12.3. The shear resistance of the concrete, V_c, is reduced to $2\sqrt{f'_c}\,b_o d$, and reinforcement must provide for the excess shear above ϕV_c. The total bar area A_v crossing the critical section at slope angle α is easily obtained by equating the vertical component of the steel force to the excess shear force to be accommodated:

$$\phi A_v f_y \sin\alpha = V_u - \phi V_c$$

Where inclined shear reinforcement is all bent at the same distance from a support, $V_s = A_v f_y \sin\alpha$ is not to exceed $3\sqrt{f'_c}\, b_o d$, according to ACI Code 11.5.6. The required area of reinforcement for shear is found by transposing the preceding equation:

$$A_v = \frac{V_u - \phi V_c}{\phi f_y \sin\alpha} \tag{13.12}$$

Successive sections at increasing distances from the support must be investigated and reinforcement provided where V_u exceeds ϕV_c as given by Eq. (13.11).[†] Only the center three-quarters of the inclined portion of the bent bars can be considered effective in resisting shear, and full development length must be provided past the location of peak stress in the steel, which is assumed to be at slab middepth $d/2$.

EXAMPLE 13.4

Design of bar reinforcement for punching shear. A flat plate floor has thickness $h = 7\frac{1}{2}$ in. and is supported by 18 in. square columns spaced 20 ft on centers each way. The floor will carry a total factored load of 300 psf. Check the adequacy of the slab in resisting punching shear at a typical interior column, and provide shear reinforcement, if needed, using bent bars similar to Fig. 13.24*b*. An average effective depth $d = 6$ in. may be used. Material strengths are $f_y = 60{,}000$ psi and $f'_c = 4000$ psi.

Solution. The first critical section for punching shear is a distance $d/2 = 3$ in. from the column face, providing a shear perimeter $b_o = 24 \times 4 = 96$ in. Based on the tributary area of loaded floor, the factored shear is

$$V_u = 300(20^2 - 2^2) = 118{,}800 \text{ lb}$$

and, if no shear reinforcement is used, the design strength of the slab, controlled by Eq. (13.11*a*), is

$$\phi V_c = 0.75 \times 4\sqrt{4000} \times 96 \times 6 = 109{,}300 \text{ lb}$$

confirming that shear reinforcement is required. Bars bent at 45° will be used in two directions, as shown in Fig. 13.25. When shear strength is provided by a combination of reinforcement and concrete, the concrete contribution is reduced to

$$\phi V_c = 0.75 \times 2\sqrt{4000} \times 96 \times 6 = 54{,}600 \text{ lb}$$

and so the shear V_s to be resisted by the reinforcement is

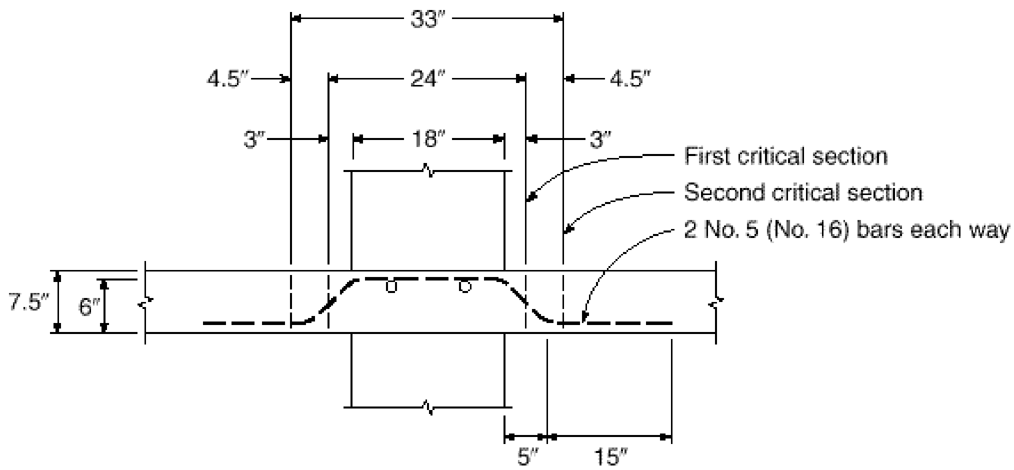
$$V_s = \frac{V_u - \phi V_c}{\phi} = \frac{118{,}800 - 54{,}600}{0.75} = 85{,}600 \text{ lb}$$

This is below the maximum permissible value of $3\sqrt{4000} \times 96 \times 6 = 109{,}300$ lb. The required bar area is then found from Eq. (13.12) to be

$$A_v = \frac{85{,}600}{60{,}000 \times 0.707} = 2.02 \text{ in}^2$$

[†] ACI Code 11.12.3 and ACI Commentary R11.12.3 are ambiguous regarding the value of V_c to be used for flat plate slabs beyond the region where shear reinforcement is required. In general, for slabs where shear reinforcement is not required, V_c is calculated from Eqs. (13.11*a* to 13.11*c*), with V_c in most cases equal to $4\sqrt{f'_c}b_o d$. When shear reinforcement is provided, the limiting shear may be increased to a maximum of $6\sqrt{f'_c}b_o d$; however, the shear reinforcement must be designed to carry all shear in excess of ϕV_c with $V_c = 2\sqrt{f'_c}b_o d$. This seems to imply that the reduction in V_c to one-half its normal value applies only where there is a sharing of the force between concrete and steel reinforcement and that, in the region where shear reinforcement is *not* required, the full concrete contribution of $4\sqrt{f'_c}\, b_o d$ can be used. Examples 13.4 and 13.6, which follow, have been prepared on that basis. The alternative interpretation is that, if shear reinforcement is required at the column, then the concrete contribution is reduced to $2\sqrt{f'_c}b_o d$ throughout the slab. This more conservative interpretation could be adopted in many cases without significant cost increase, because of the rapid increase in V_c with increasing distance from the column resulting from the increase in concrete shear perimeter b_o, as well as the reduction in net shear force V_u.

FIGURE 13.25
Bar reinforcement for punching shear in flat plate slab.



A total of four bars will be used (two in each direction), and with eight legs crossing the critical section, the necessary area per bar is $2.02/8 = 0.25$ in^2. No. 5 (No. 16) bars will be used as shown in Fig. 13.25. The upper limit of $V_n = 6\sqrt{f'_c}\,b_o d$ is automatically satisfied in this case, given the more stringent limit on V_s.

With bars bent at 45° and effective through the center three-fourths of the inclined length, the next critical section is approximately $\frac{3}{4}$ times the effective depth, or 4.5 in., past the first, as shown, giving a shear perimeter of $33 \times 4 = 132$ in. The factored shear at that critical section is

$$V_u = 300(20^2 - 2.75^2) = 117{,}700 \text{ lb}$$

and the design capacity of the concrete is

$$\phi V_c = 0.75 \times 4\sqrt{4000} \times 132 \times 6 = 150{,}300 \text{ lb}$$

confirming that no additional bent bars are needed. The No. 5 (No. 16) bars will be extended along the bottom of the slab the full development length of 15 in., as shown in Fig. 13.25.

d. Design of Shearhead Reinforcement

If embedded structural steel shapes are used, as shown in Fig. 13.24*a* and *c*, the limiting value of V_n may be increased to $7\sqrt{f'_c}\,b_o d$. Such a shearhead, provided it is sufficiently stiff and strong, has the effect of moving the critical section out away from the column, as shown in Fig. 13.26. According to ACI Code 11.12.4, this critical section crosses each arm of the shearhead at a distance equal to three-quarters of the projection beyond the face of the support, and is defined so that the perimeter is a minimum. It need not approach closer than $d/2$ to the face of the support.

Moving the critical section out in this way provides the double benefit of increasing the effective perimeter b_o and decreasing the total shear force for which the slab must be designed. The nominal shear V_n at the new critical section must not be taken greater than $4\sqrt{f'_c}\,b_o d$, according to ACI Code 11.12.4.

Tests reported in Ref. 13.17 indicate that throughout most of the length of a shearhead arm the shear is constant, and, further, that the part of the total shear carried

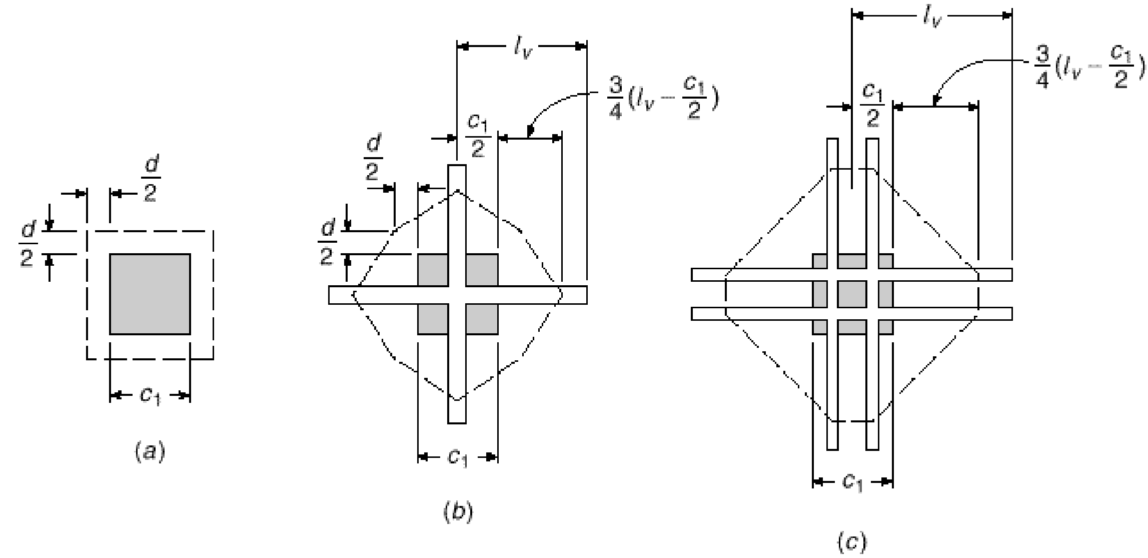


FIGURE 13.26
Critical sections for shear for flat plates: (*a*) no shearhead; (*b*) small shearhead; (*c*) large shearhead.

by the shearhead arm is proportional to $\cdot\ _v$, its relative flexural stiffness, compared with that of the surrounding concrete section:

$$\cdot\ _v = \frac{E_s I_s}{E_c I_c} \tag{13.13}$$

The concrete section is taken with an effective width of $c_2 + d$, where c_2 is the width of the support measured perpendicular to the arm direction. Properties are calculated for the cracked, transformed section, including the shearhead. The observation that shear is essentially constant, at least up to the diagonal cracking load, implies that the reaction is concentrated largely at the end of the arm. Thus, if the total shear at the support is V and if the shearhead has $\cdot$ identical arms (generally $\cdot = 4$ for shearheads at interior columns), the constant shear force in each arm is equal to $\cdot\ _v V \cdot \cdot$.

If the load is increased past that which causes diagonal cracking immediately around the column, tests indicate that the increased shear above the cracking shear V_c is carried mostly by the steel shearhead, and that the shear force in the projecting arm within a distance from the column face equal to h_v, the depth of the arm, assumes a nearly constant value greater than $\cdot\ _v V \cdot \cdot$. This increased value is very nearly equal to the total shear per arm $V_u \cdot \cdot \cdot$ minus the shear carried by the partially cracked concrete. The latter term is equal to $(V_c \cdot \cdot)(1 - \cdot\ _v)$; hence, the idealized shear diagram shown in Fig. 13.27*b* is obtained.

The moment diagram of Fig. 13.27*c* is obtained by integration of the shear diagram. If V_c is equal to $V_n \cdot 2 = V_u \cdot 2 \cdot$, as tests indicate for shearheads of common proportions, it is easily confirmed that the plastic moment M_p at the face of the support, for which the shearhead arm must be proportioned, is

$$M_p = \frac{V_u}{2 \cdot \cdot} \cdot h_v + \cdot\ _v \cdot l_v - \frac{c_1}{2} \cdot \cdot \tag{13.14}$$

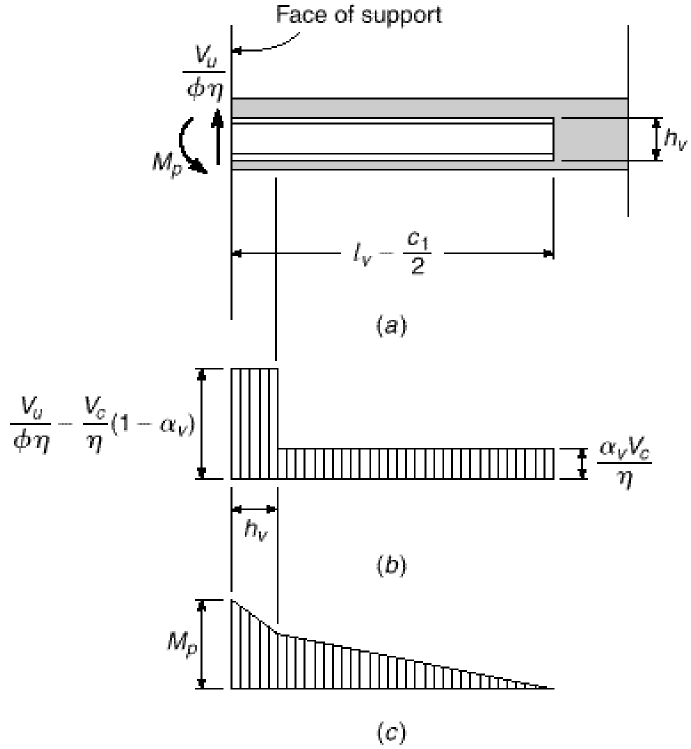


FIGURE 13.27
Stress resultants in shearhead arm: (*a*) shearhead arm; (*b*) shear; (*c*) moment.

in which · = 0.90, the capacity-reduction factor for tension-controlled members.

According to ACI Code 11.12.4, the value of · $_v$ must be at least equal to 0.15; more flexible shearheads have proved ineffective. The compression flange must not be more than 0.3d from the bottom surface of the slab, and the steel shapes used must not be deeper than 70 times the web thickness.

For flexural design of the slab, moments found at the support centerline by the equivalent frame method are reduced to moments at the support face, assumed to be the critical section for moment. By the direct design method, support-face moments are calculated directly through the use of the clear-span distance. If shearheads are used, they have the effect of reducing the design moment in the column strips still further by increasing the effective support width. This reduction is proportional to the share of the load carried by the shearhead and to its size, and can be estimated conservatively (see Fig. 13.27*b* and *c*) by the expression

$$M_v = \frac{\cdot\ \cdot\ {}_v V_u}{2\cdot} \cdot\ l_v - \frac{c_1}{2}\cdot \tag{13.15}$$

where · = 0.90. According to ACI Code 11.12.4, the reduction may not be greater than 30 percent of the total design moment for the slab column strip, or greater than the change in column-strip moment over the distance l_v, or greater than M_p given by Eq. (13.14).

Limited test information pertaining to shearheads at a slab edge indicates that behavior may be substantially different due to torsional and other effects. If shearheads are to be used at an edge or corner column, special attention must be given to anchorage of the embedded steel within the column. The use of edge beams or a cantilevered slab edge may be preferred.

EXAMPLE 13.5

Design of shearhead reinforcement. A flat plate slab $7\frac{1}{2}$ in. thick is supported by 10 in. square columns and is reinforced for negative bending with No. 5 (No. 16) bars 5 in. on centers in each direction, with an average effective depth d of 6 in. The concrete strength f_c' is 3000 psi. The slab must transfer a factored shear V_u of 107,000 lb to the column. What special slab reinforcement is required, if any, at the column to transfer the factored shear?

Solution. The nominal shear strength at the critical section $d/2$ from the face of the column is found from Eq. (13.11*a*) to be

$$V_c = 4\sqrt{3000} \times 64 \times 6 = 84.1 \text{ kips}$$

and $\phi V_c = 0.75 \times 84.1 = 63.1$ kips. This is less than $V_u = 107$ kips, indicating that shear reinforcement is necessary. A shearhead similar to Fig. 13.24*a* will be used, fabricated from I-beam sections with $f_y = 50$ ksi. Maintaining $\frac{3}{4}$ in. clearance below such steel, bar clearance at the top of the slab permits use of an I beam with a $4\frac{5}{8}$ in. depth; a nominal 4 in. section will be used. With such reinforcement, the upper limit of shear V_n on the critical section is $7\sqrt{3000}\,(64 \times 6) = 147$ kips, and $\phi V_n = 0.75 \times 147 = 110$ kips, above the value of V_u to be resisted. The required perimeter b_o can be found by setting $V_u = \phi V_c$, where V_c is given by Eq. (13.11*a*):

$$b_o = \frac{V_u}{4\phi\sqrt{f_c'}\,d} = \frac{107{,}000}{4 \times 0.75\sqrt{3000} \times 6} = 109 \text{ in.}$$

(Note that the actual shear force to be transferred at the critical section is slightly less than 107 kips because a part of the floor load is within the effective perimeter b_o; however, the difference is small except for very large shearheads.) The required projecting length l_v of the shearhead arm is found from geometry, with b_o expressed in terms of l_v;

$$b_o = 4\left[\sqrt{2}\left(\frac{c_1}{2} + \frac{3}{4}\left(l_v - \frac{c_1}{2}\right)\right)\right] = 109 \text{ in.}$$

from which $l_v = 24.0$ in. To determine the required plastic section modulus for the shear arm, it is necessary to assume a trial value of the relative stiffness α_v. After selecting 0.25 for trial, the required moment capacity is found from Eq. (13.14):

$$M_p = \frac{107{,}000}{8 \times 0.90}\left[4 + 0.25\left(24.0 - 5\right)\right] = 130{,}000 \text{ in-lb}$$

A standard I beam S4 × 7.7, with yield stress of 50 ksi, provides 176,000 in-lb resistance and will tentatively be adopted. The E_sI_s value provided by the beam is 174×10^6 in^2-lb. The effective cross section of the slab strip is shown in Fig. 13.28. Taking moments of the composite cracked section about the bottom surface to locate the neutral axis gives

$$y = \frac{8.90 \times 6 + 19.9 \times 2.75 + 8y^2}{8.90 + 19.9 + 16y}$$

from which $y = 2.29$ in. The moment of inertia of the composite section is

$$I_c = \tfrac{1}{3} \times 16 \times 2.29^3 + 8.90 \times 3.71^2 + 6 \times 9 + 19.9 \times 0.46^2 = 244 \text{ in}^4$$

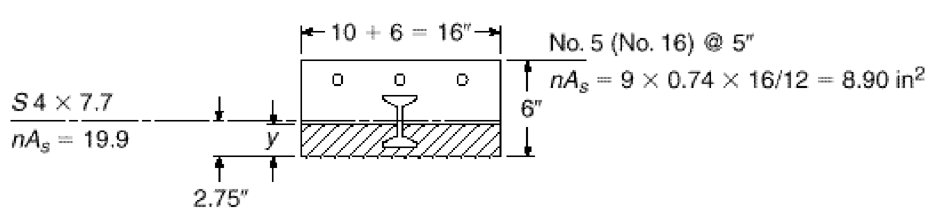


FIGURE 13.28
Effective section of slab.

the flexural stiffness of the effective composite slab strip is

$$E_c I_c = 3.1 \times 10^6 \times 244 = 756 \times 10^6 \text{ in}^2\text{-lb}$$

and, from Eq. (13.13),

$$\alpha_v = \frac{174}{756} = 0.23$$

This is greater than the specified minimum of 0.15 and close to the 0.25 value assumed earlier. The revised value of M_p is

$$M_p = \frac{107{,}000}{8 \times 0.90}\left[4 + 0.23(24.0 - 5)\right] = 124{,}400 \text{ in-lb}$$

The 4 in. beam is adequate. The calculated length l_v of 24.0 in. will be retained. The reduction in column-strip moment in the slab may be based on this actual length. From Eq. (13.15),

$$M_v = \frac{0.90 \times 0.23 \times 107{,}000}{8}(24 - 5) = 52{,}600 \text{ in-lb}$$

This value is less than M_p, as required by specification, and must also be less than 30 percent of the design negative moment in the column strip and less than the change in the column-strip moment in the distance l_v.

e. Design of Integral Beams with Vertical Stirrups

Steel shearheads of the type described in Section 13.10d have not been widely used, primarily because of their cost, but also because of difficulty in placing the slab flexural reinforcement to pass the structural steel sections and because of interference with the column steel. The bent bar shear reinforcement cages of Section 13.10c are less expensive, but also lead to troublesome congestion of reinforcement in the column-slab joint region. Shear reinforcement using vertical stirrups in *integral beams,* as shown in Fig. 13.24*e,* avoids much of this difficulty.

The first critical section for shear design in the slab is taken at $d/2$ from the column face, as usual, and the stirrups, if needed, are extended outward from the column in four directions for the typical interior case (three or two directions for exterior or corner columns, respectively), until the concrete alone can carry the shear, with $V_c = 4\sqrt{f'_c}\,b_o d$ at the second critical section.† Within the region adjacent to the column, where shear resistance is provided by a combination of concrete and steel, the nominal shear strength V_n must not exceed $6\sqrt{f'_c}\,b_o d$, according to ACI Code 11.12.3. In this region, the concrete contribution is reduced to $V_c = 2\sqrt{f'_c}\,b_o d$. The second critical section crosses each integral beam at a distance $d/2$ measured outward from the last stirrup and is located so that its perimeter b_o is a minimum (i.e., for the typical case, defined by 45 degree lines between the integral beams). The required spacing of the vertical stirrups s is found using Eq. (4.14*a*), but must not exceed $d/2$, with the first line of stirrups not more than $d/2$ from the column face. The spacing of the stirrup legs

† Neither the ACI Code nor the ACI Commentary makes clear whether Eqs. (13.11*b*) and (13.11*c*) are to be applied at successive critical sections past the first, immediately adjacent to the column. The research on which these equations were based considered only the first critical section at the column. Except in extreme cases, the aspect ratio of the column, in Eq. (13.11*b*), seems less relevant with increasing distance from the column; however, the b_o/d ratio, in Eq. (13.11*c*), may be influential, and that equation might conservatively be applied.

(measured parallel to the face of the column) in the first line of shear reinforcement must not exceed $2d$.

The problem of anchorage of the shear reinforcement in shallow flat plates is critical, and closed hoop stirrups, terminating in standard hooks, always should be provided with interior corner bars to improve pullout resistance.

EXAMPLE 13.6

Design of an integral beam with vertical stirrups. The flat plate slab with 7.5 in. total thickness and 6 in. effective depth shown in Fig. 13.29 is carried by 12 in. square columns 15 ft on centers in each direction. A factored load of 120 kips must be transmitted from the slab to a typical interior column. Concrete and steel strengths used are, respectively, f'_c = 4000 psi and f_y = 60,000 psi. Determine if shear reinforcement is required for the slab, and if so, design integral beams with stirrups to carry the excess shear.

Solution. The design shear strength of the concrete alone at the critical section $d/2$ from the face of the column, by the controlling Eq. (13.11a), is

$$\phi V_c = 0.75 \times 4\sqrt{4000} \times 72 \times 6 = 82.0 \text{ kips}$$

This is less than V_u = 120 kips, indicating that shear reinforcement is required. The effective depth d = 6 in. just satisfies the minimum allowed to use stirrup reinforcement, as described in Section 13.10b. In this case, the maximum design strength allowed by the ACI Code is

$$\phi V_n = 0.75 \times 6\sqrt{4000} \times 72 \times 6 = 122.9 \text{ kips}$$

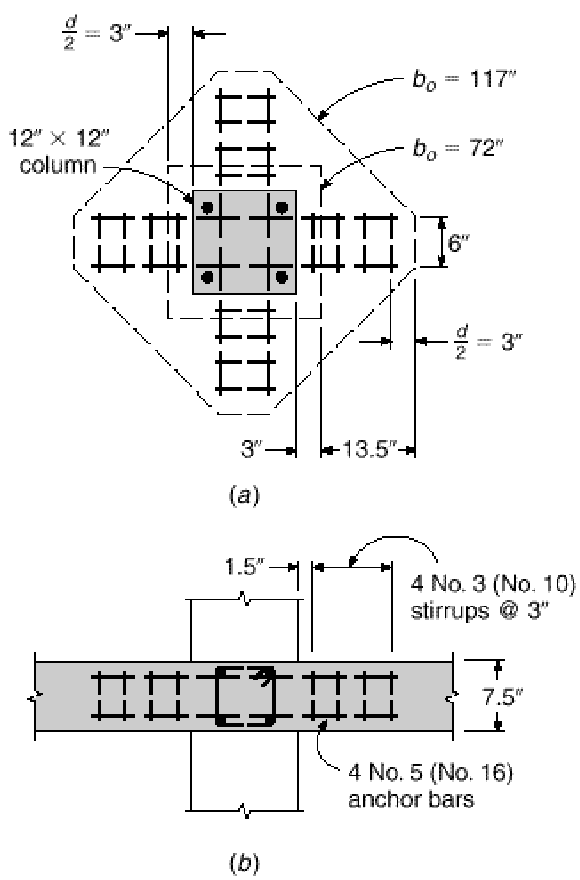


FIGURE 13.29
Vertical stirrup shear reinforcement for slab in Example 13.6.

satisfactorily above the actual V_u. When shear is resisted by combined action of concrete and bar reinforcement, the concrete contribution is reduced to

$$\phi V_c = 0.75 \times 2\sqrt{4000} \times 72 \times 6 = 41.0 \text{ kips}$$

No. 3 (No. 10) vertical closed hoop stirrups will be used since d must be $\geq$ 16 times the stirrup diameter ($d/16 = \frac{3}{8}$ in.) and arranged along four integral beams as shown in Fig. 13.29. Thus, the A_v provided is $4 \times 2 \times 0.11 = 0.88$ in^2 at the first critical section, a distance $d/2$ from the column face, and the required spacing can be found from Eq. (4.14a):

$$s = \frac{\phi A_v f_y d}{V_u - \phi V_c} = \frac{0.75 \times 0.88 \times 60 \times 6}{120 - 41.0} = 3.01 \text{ in.}$$

However, the maximum spacing of $d/2 = 3$ in. controls here, and No. 3 (No. 10) stirrups at a constant spacing of 3 in. will be used. In other cases, stirrup spacing might be increased with distance from the column, as excess shear is less, although this would complicate placement of the reinforcement and generally save little steel.

The required perimeter of the second critical section, at which the concrete alone can carry the shear, is found from the controlling Eq. (13.11a) as follows:

$$\phi V_c = 0.75 \times 4\sqrt{4000} \times b_o \times 6 = 120{,}000 \text{ lb}$$

from which the minimum perimeter $b_o = 105.4$ in. It is easily confirmed that this requires a minimum projection of the critical section past the face of the column of 11.39 in. Four stirrups at a constant 3 in. spacing will be sufficient, the first placed at $s/2 = 1.5$ in. $\leq d/2 =$ 3 in. from the column face, as indicated in Fig. 13.29. This provides a perimeter b_o at the second critical section of $(16.5\sqrt{2} + 6) \times 4 = 117$ in., exceeding the requirement.

Four longitudinal No. 5 (No. 16) bars will be provided inside the corners of each closed hoop stirrup, as shown, to provide for proper anchorage of the shear reinforcement.

f. Design of Shear Stud Reinforcement

Slab shear reinforcement consisting of integral beams with stirrups, as described in Section 13.10e, is probably the most widely used type at present. However, the cage that is formed by the stirrups and longitudinal anchor bars may be difficult to install. Also, the slab-column joint region is somewhat congested, with top and bottom slab steel running in two perpendicular directions, with vertical bars in the column, and with the stirrups. Congestion can become critical when the slab has openings, which are frequently required, at or near the column faces.

Shear stud reinforcing strips, as shown in Fig. 13.24f and in Fig. 13.30a and b, are widely used in Germany, Switzerland, and Canada (Refs. 13.19 and 13.20). They are mentioned briefly in ACI Code Commentary R11.12.3, although no specific design provisions are included. Use in the United States is increasing.

These devices are composed of vertical bars with anchor heads at their top, welded to a steel strip at the bottom. Multiple strips are arranged in two perpendicular directions for square and rectangular columns or usually in radial directions for circular columns. They are secured in position in the forms before the top and bottom flexural steel is in place. The steel strip rests on bar chairs to maintain the needed concrete cover below the steel and is held in position by nails through holes in the strip.

For design purposes, an individual stud is considered to be the equivalent of one vertical leg of one stirrup. Design can then proceed following the general procedures

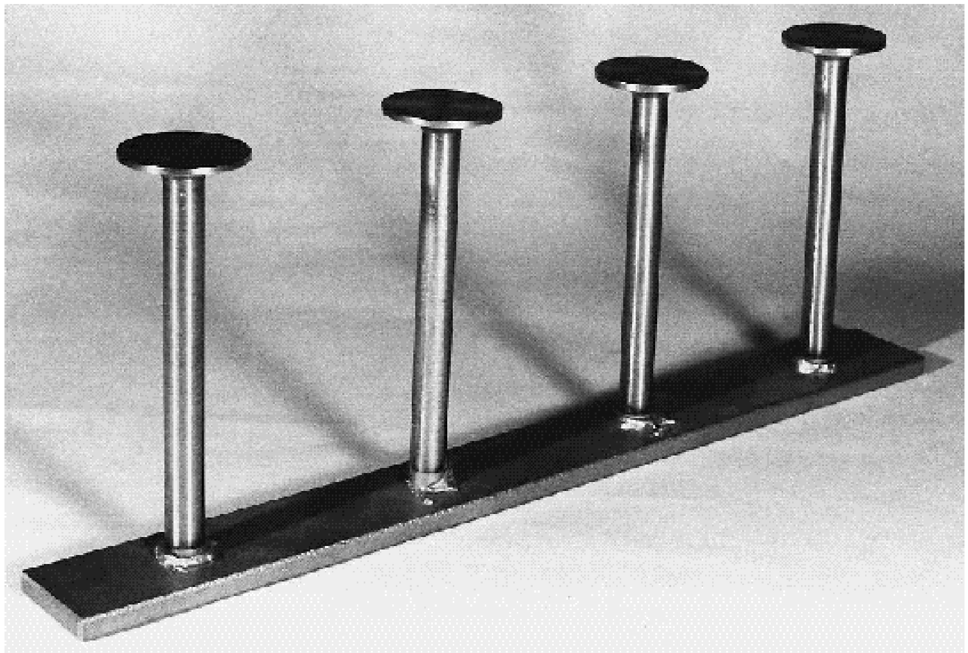


FIGURE 13.30a
Shear stud reinforcement for concrete slabs: shear stud assembly. (*Courtesy of Amin Ghali and Walter H. Dilger.*)

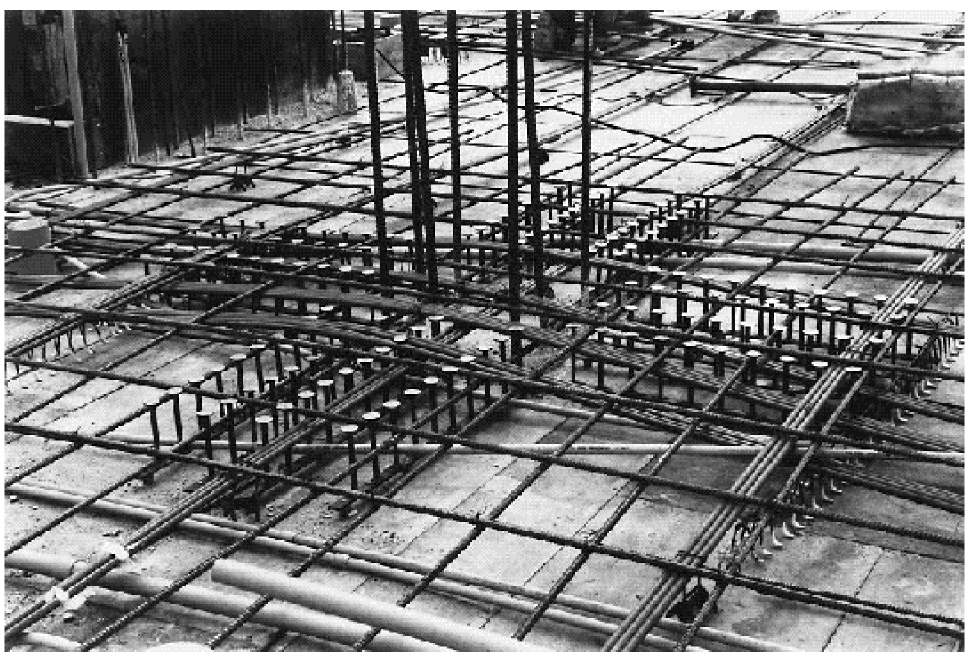


FIGURE 13.30b
Shear stud reinforcements for concrete slabs: shear reinforcement installed in forms for prestressed concrete slab. (*Courtesy of Amin Ghali and Walter H. Dilger.*)

illustrated in Section 13.10e for stirrup shear reinforcement. However, based on extensive testing (Refs. 13.21 and 13.22), some modifications have been proposed. Ghali (Refs. 13.19 and 13.20) recommends the following:

1. The upper limit for the nominal shear stress at $d/2$ from the column face is increased to $8\sqrt{f'_c}\,b_o d$.

2. The allowable stud spacing is increased to between $2d/3$ and $3d/4$, depending on the maximum nominal shear stress at factored loads.
3. Within the shear-reinforced zone, the contribution of the concrete is increased to $3\sqrt{f'_c}\,b_o d$.

In addition to the above, Ghali has recommended the following details:

(a) Top anchors are in the form of circular or square plates, the areas of which are at least 10 times the area of the stem.
(b) When the top anchor plates and the bottom strips are of uniform thickness, the thickness should be greater than or equal to one-half the stud diameter.
(c) If the top anchor plate is tapered, the thickness at the connection with the stem should be greater than or equal to $\frac{2}{3}$ the stud diameter.
(d) The width of the bottom strip should be greater than or equal to 2.5 times the stud diameter.
(e) Bottom anchor strips should be aligned with the column faces of square or rectangular columns.
(f) In the direction parallel to the column face, the distance between anchor strips should not exceed twice the effective depth of the slab.
(g) The minimum concrete cover above and below the stud strips should be as normally specified for slab bars, and the cover should not exceed the minimum plus $\frac{1}{2}$ the bar diameter of the flexural reinforcement.

Further recommendations are found in Refs. 13.23 and 13.24 pertaining to the use of shear stud reinforcement at exterior and corner columns, where special problems always exist because of lack of symmetry, reduced perimeter of the critical section, and relatively large unbalanced moments.

13.11 Transfer of Moments at Columns

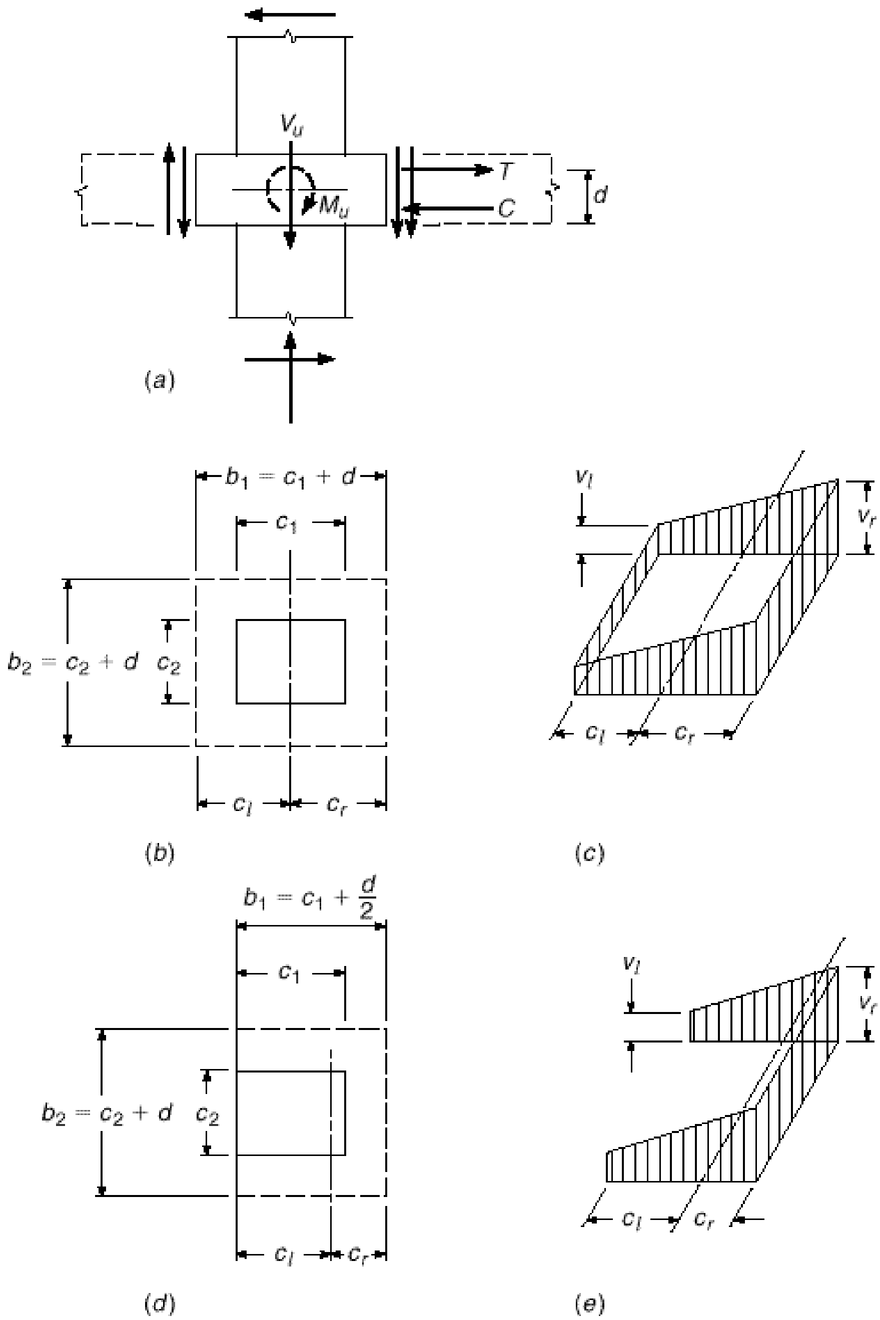
The analysis for punching shear in flat plates and flat slabs presented in Section 13.10 assumed that the shear force V_u was resisted by shearing stresses uniformly distributed around the perimeter b_o of the critical section, a distance $d/2$ from the face of the supporting column. The nominal shear strength V_c was given by Eqs. (13.11*a*), (13.11*b*), and (13.11*c*).

If significant moments are to be transferred from the slab to the columns, as would result from unbalanced gravity loads on either side of a column or from horizontal loading due to wind or seismic effects, the shear stress on the critical section is no longer uniformly distributed.

The situation can be modeled as shown in Fig. 13.31*a*. Here V_u represents the total vertical reaction to be transferred to the column, and M_u represents the unbalanced moment to be transferred, both at factored loads. The vertical force V_u causes shear stress distributed more or less uniformly around the perimeter of the critical section as assumed earlier, represented by the inner pair of vertical arrows, acting downward. The unbalanced moment M_u causes additional loading on the joint, represented by the outer pair of vertical arrows, which add to the shear stresses otherwise present on the right side, in the sketch, and subtract on the left side.

Tests indicate that for square columns about 60 percent of the unbalanced moment is transferred by flexure (forces T and C in Fig. 13.31*a*) and about 40 percent by shear stresses on the faces of the critical section (Ref. 13.25). For rectangular

FIGURE 13.31
Transfer of moment from slab to column: (*a*) forces resulting from vertical load and unbalanced moment; (*b*) critical section for an interior column; (*c*) shear stress distribution for an interior column; (*d*) critical section for an edge column; (*e*) shear stress distribution for an edge column.



columns, it is reasonable to suppose that the portion transferred by flexure increases as the width of the critical section that resists the moment increases, i.e., as $c_2 + d$ becomes larger relative to $c_1 + d$ in Fig. 13.31*b*. According to ACI Code 13.5.3, the moment considered to be transferred by flexure is

$$M_{ub} = \cdot_f M_u \tag{13.16a}$$

where

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1/b_2}} \tag{13.16b}$$

and b_1 = the width of the critical section for shear measured in the direction of the span for which the moments are determined

b_2 = the width of critical section for shear measured in the direction perpendicular to b_1

The value of γ_f may be modified if certain conditions are met: For unbalanced moments about an axis parallel to the edge of exterior supports, γ_f may be increased to 1.0, provided that the factored shear V_u at the edge support does not exceed $0.75\phi V_c$ or at a corner support does not exceed $0.5\phi V_c$. For unbalanced moments at interior supports and about an axis perpendicular to the edge at exterior supports, γ_f may be increased by up to 25 percent, provided that $V_u \leq 0.4\phi V_c$. In all of these cases, the reinforcement ratio ρ within 1.5h on either side of the column or column capital may not exceed $0.375\rho_b$.

The moment assumed to be transferred by shear, by ACI Code 11.12.6 is

$$M_{ub} = (1 - \gamma_f)M_u = \gamma_v M_u \tag{13.16c}$$

It is seen that for a square column Eqs. (13.16*a*), (13.16*b*), and (13.16*c*) indicate that 60 percent of the unbalanced moment is transferred by flexure and 40 percent by shear, in accordance with the available data. If b_2 is very large relative to b_1, nearly all of the moment is transferred by flexure.

The moment M_{ub} can be accommodated by concentrating a suitable fraction of the slab column-strip reinforcement near the column. According to ACI Code 13.5.3, this steel must be placed within a width between lines 1.5h on each side of the column or capital, where h is the total thickness of the slab or drop panel.

The moment M_{uv}, together with the vertical reaction delivered to the column, causes shear stresses assumed to vary linearly with distance from the centroid of the critical section, as indicated for an interior column by Fig. 13.31*c*. The stresses can be calculated from

$$v_l = \frac{V_u}{A_c} - \frac{M_{uv}c_l}{J_c} \tag{13.17a}$$

$$v_r = \frac{V_u}{A_c} + \frac{M_{uv}c_r}{J_c} \tag{13.17b}$$

where A_c = area of critical section = $2d[(c_1 + d) + (c_2 + d)]$

c_l, c_r = distances from centroid of critical section to left and right face of section, respectively

J_c = property of critical section analogous to polar moment of inertia

For an interior column, the quantity J_c is

$$J_c = \frac{2d(c_1 + d)^3}{12} + \frac{2(c_1 + d)d^3}{12} + 2d(c_2 + d)\left(\frac{c_1 + d}{2}\right)^2 \tag{13.18}$$

Note the implication, in the use of the parameter J_c in the form of a polar moment of inertia, that shear stresses indicated on the near and far faces of the critical section in Fig. 13.31*c* have horizontal as well as vertical components.

According to ACI Code 11.12.6, the maximum shear stress calculated by Eq. (13.17) must not exceed · v_n. For slabs without shear reinforcement, · v_n = · V_c· $b_o d$, where V_c is the smallest value given by Eqs. (13.11*a*), (13.11*b*), or (13.11*c*). For slabs with shear reinforcement other than shearheads, · v_n = · (V_c 1 V_s)· $b_o d$, where V_c and V_s are as established in Section 13.10c, e, or f. Where shearhead reinforcement (see Section 13.10d) is used, the sum of the shear stresses due to vertical load on the second critical section, near the end of the shearhead arms, and the shear stresses resulting from moment transfer about the centroid of the first critical section d·2 from the support faces must not exceed 4· · $\overline{f_c}$. In support of the last calculation, ACI Code Commentary R11.12.6.3 notes that tests indicate the first critical section is appropriate for calculation of stresses caused by transfer of moments even when shearheads are used. Even though the critical sections for direct shear transfer and shear due to moment transfer differ, they coincide or are in close proximity at the column corners where failures initiate, and it is conservative to take the maximum shear as the sum of the two components.

Equations similar to those above can be derived for the edge columns shown in Fig. 13.31*d* and *e* or for a corner column. Although the centroidal distances c_l and c_r are equal for the interior column, this is not true for the edge column of Fig. 13.31*d* or for a corner column.

According to ACI Code 13.6.3.6, when the direct design method is used, the moment to be transferred between the slab and an edge column by shear is to be taken equal to $0.30M_o$, where M_o is found from Eq. (13.1). This is intended to compensate for assigning a high proportion of the static moment to the positive and interior negative moment regions according to Table 13.3, and to ensure that adequate shear strength is provided between the slab and the edge column, where unbalanced moment is high and the critical section width is reduced.

The application of moment to a column from a slab or beam introduces shear to the column also, as is clear from Fig. 13.31*a*. This shear must be considered in the design of lateral column reinforcement.

As pointed out in Section 13.10, most flat plate structures, if they are overloaded, fail in the region close to the column, where large shear and bending forces must be transferred. There has been much research aimed at developing improved design details for this region. The design engineer should consult Refs. 13.25 through 13.27 for additional specific information.

13.12 OPENINGS IN SLABS

Almost invariably, slab systems must include openings. These may be of substantial size, as required by stairways and elevator shafts, or they may be of smaller dimensions, such as those needed to accommodate heating, plumbing, and ventilating risers; floor and roof drains; and access hatches.

Relatively small openings usually are not detrimental in *beam-supported slabs*. As a general rule, the equivalent of the interrupted reinforcement should be added at the sides of the opening. Additional diagonal bars should be included at the corners to control the cracking that will almost inevitably occur there. The importance of small openings in *slabs supported directly by columns* (flat slabs and flat plates) depends upon the location of the opening with respect to the columns. From a structural point of view, they are best located away from the columns, preferably in the area common to the slab middle strips. Unfortunately, architectural and functional considerations usually cause them to be located close to the columns. In this case, the reduction in effective shear perimeter is the major concern, because such floors are usually shear-critical.

According to ACI Code 11.12.5, if the opening is close to the column (within 10 slab thicknesses or within the column strips), then that part of b_o included within the radial lines projecting from the opening to the centroid of the column should be considered ineffective. This is shown in Fig. 13.32, along with the effect of free edges on the perimeter of the critical section. If shearheads (see Section 13.10d) are used under such circumstances, the reduction in width of the critical section is found in the same way, except that only one-half the perimeter included within the radial lines need be deducted.

With regard to flexural requirements, the total amount of steel required by calculation must be provided regardless of openings. Any steel interrupted by holes should be matched with an equivalent amount of supplementary reinforcement on either side, properly lapped to transfer stress by bond. Concrete compression area to provide the required strength must be maintained; usually this would be restrictive only near the columns. According to ACI Code 13.4.2, openings of any size may be located in the area common to intersecting middle strips. In the area common to intersecting column strips, not more than one-eighth of the width of the column strip in either span can be interrupted by openings. In the area common to one middle strip and one column strip, not more than one-quarter of the reinforcement in either strip may be interrupted by the opening.

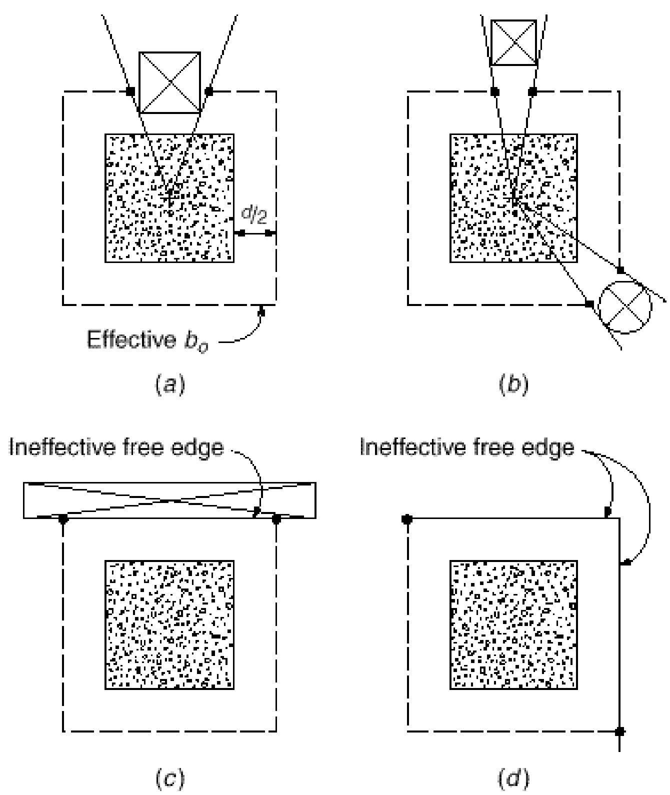


FIGURE 13.32
Effect of openings and free edges on the determination of the perimeter of the critical section for shear b_o.

ACI Code 13.4.1 permits openings of *any* size if it can be shown by analysis that the strength of the slab is at least equal to that required and that all serviceability conditions, i.e., cracking and deflection limits, are met. The *strip method* of analysis and design for openings in slabs, by which specially reinforced integral beams, or *strong bands*, of depth equal to the slab depth are used to frame the openings, will be described in detail in Chapter 15. Very large openings should preferably be framed by beams or slab bands of increased depth to restore, as nearly as possible, the continuity of the slab. The beams must be designed to carry a portion of the floor load, in addition to loads applied directly by partition walls, elevator support beams, or stair slabs.

13.13 Deflection Calculations

The deflection of a uniformly loaded flat plate, flat slab, or two-way slab supported by beams on column lines can be calculated by an equivalent frame method that corresponds with the method for moment analysis described in Section 13.9 (Ref. 13.28). The definition of column and middle strips, the longitudinal and transverse moment distribution coefficients, and many other details are the same as for the moment analysis. Following the calculation of deflections by this means, they can be compared directly with limiting values like those of Table 6.2, which are applicable to slabs as well as to beams, according to the ACI Code.

A slab region bounded by column centerlines is shown in Fig. 13.33. While no column-line beams, drop panels, or column capitals are shown, the presence of any of these introduces no fundamental complication.

The deflection calculation considers the deformation of such a typical region in one direction at a time, after which the contributions from each direction are added to obtain the total deflection at any point of interest.

In reference to Fig. 13.33*a*, the slab is considered to act as a broad, shallow beam of width equal to the panel dimension l_y and having the span l_x. Initially the slab is considered to rest on unyielding support lines at $x = 0$ and $x = l_x$. Because of variation of moment as well as flexural stiffness across the width of the slab, all unit strips in the X direction will not deform identically. Typically the slab curvature in the middle-strip region will be less than that in the region of the column strips because the middle-strip moments are less. The result is as indicated in Fig. 13.33*a*.

Next the slab is analyzed for bending in the Y direction (Fig. 13.33*b*). Once again the effect of transverse variation of bending moment and flexural rigidity is seen.

The actual deformed shape of the panel is represented in Fig. 13.33*c*. The midpanel deflection is the sum of the midspan deflection of the column strip in one direction and that of the middle strip in the other direction; i.e.,

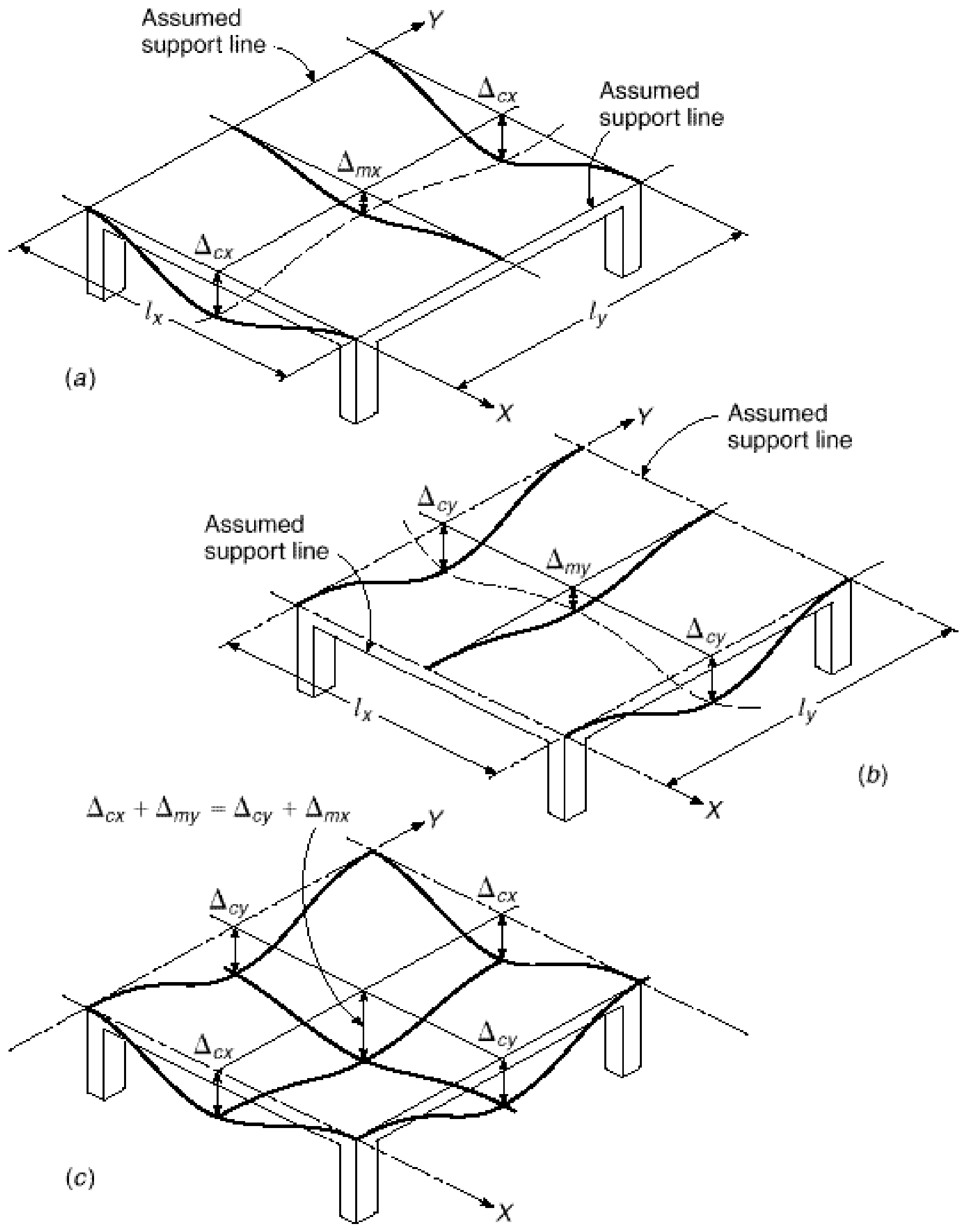
$$\Delta_{max} = \Delta_{cx} + \Delta_{my} \tag{13.19a}$$

or

$$\Delta_{max} = \Delta_{cy} + \Delta_{mx} \tag{13.19b}$$

In calculations of the deformation of the slab panel in either direction, it is convenient first to assume that it deforms into a cylindrical surface, as it would if the bending moment at all sections were uniformly distributed across the panel width and if lateral bending of the panel were suppressed. The supports are considered to be fully

FIGURE 13.33
Basis of equivalent frame method for deflection analysis: (*a*) *X* direction bending; (*b*) *Y* direction bending; (*c*) combined bending.



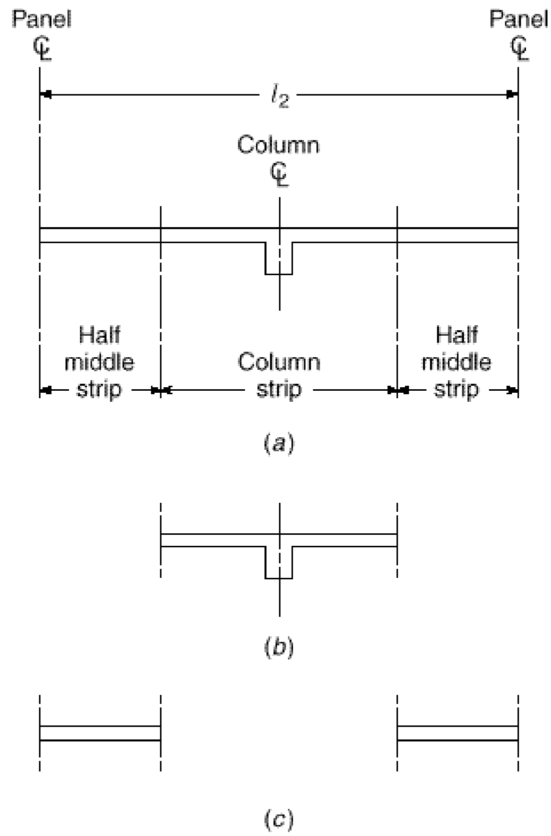
fixed against both rotation and vertical displacement at this stage. Thus, a *reference deflection* is computed:

$$_{f,ref} = \frac{wl^4}{384E_cI_{frame}} \tag{13.20}$$

where w is the load per foot along the span of length l and I_{frame} is the moment of inertia of the full-width panel (Fig. 13.34*a*) including the contribution of the column-line beam or drop panels and column capitals if present.

The effect of the actual moment variation across the width of the panel and the variation of stiffness due to beams, variable slab depth, etc., are accounted for by mul-

FIGURE 13.34
Effective cross sections for deflection calculations: (*a*) full-width frame; (*b*) column strip; (*c*) middle strips.



tiplying the reference deflection by the ratio of $M \cdot EI$ for the respective strips to that of the full-width frame:

$$\Delta_{f,col} = \Delta_{f,ref} \frac{M_{col}}{M_{frame}} \frac{E_c I_{frame}}{E_c I_{col}} \tag{13.21a}$$

$$\Delta_{f,mid} = \Delta_{f,ref} \frac{M_{mid}}{M_{frame}} \frac{E_c I_{frame}}{E_c I_{mid}} \tag{13.21b}$$

The subscripts relate the deflection Δ, the bending moment M, or the moment of inertia I to the full-width frame, column strip, or middle strip, as shown in Fig. 13.34*a, b,* and *c,* respectively.

The moment ratios $M_{col} \cdot M_{frame}$ and $M_{mid} \cdot M_{frame}$ are identical to the lateral moment distribution factors already found for the flexural analysis (see Table 13.4). A minor complication results from the fact that the lateral distribution of bending moments, according to the ACI Code, is not the same at the negative and positive-moment sections. However, it appears consistent with the degree of accuracy usually required, as well as consistent with deflection methods endorsed elsewhere in the ACI Code, to use a simple average of lateral distribution coefficients for the negative and positive portions of each strip.

The presence of drop panels or column capitals in the column strip of a flat slab floor requires consideration of the variation of the moment of inertia in the span

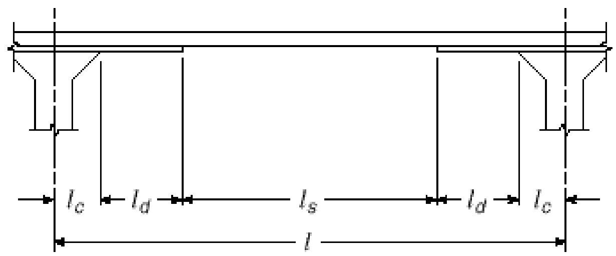


FIGURE 13.35
Flat slab span with variable moment of inertia.

direction (see Fig. 13.35). It is suggested in Ref. 13.29 that a weighted average moment of inertia be used in such cases:

$$I_{av} = 2\frac{l_c}{l}I_c + 2\frac{l_d}{l}I_d + \frac{l_s}{l}I_s \quad (13.22)$$

where I_c = moment of inertia of slab including both drop panel and capital
I_d = moment of inertia of slab with drop panel only
I_s = moment of inertia of slab alone

Span distances are defined in Fig. 13.35.

Next it is necessary to correct for the rotations of the equivalent frame at the supports, which until now were considered fully fixed. If the ends of the columns are considered fixed at the floor above and floor below, as usual for frame analysis, the rotation of the column at the floor divided by the stiffness of the equivalent column is

$$\cdot = \frac{M_{net}}{K_{ec}} \quad (13.23)$$

where · = angle change, radians
M_{net} = difference in floor moments to left and right of column
K_{ec} = stiffness of equivalent column (see Section 13.9c)

In some cases, the connection between the floor slab and column transmits negligible moment, as for lift slabs; thus $K_{ec} = 0$. The flexural analysis will indicate that the net moment is zero. The support rotation can be found in such cases by applying the moment-area theorems, taking moments of the $M \cdot EI$ area about the far end of the span, and dividing by the span length.

Once the rotation at each end is known, the associated midspan deflection of the equivalent frame can be calculated. It is easily confirmed that the midspan deflection of a member experiencing an end rotation of · radians, the far end being fixed, is

$$\cdot_{\cdot} = \frac{\cdot l}{8} \quad (13.24)$$

Thus the total deflection at midspan of the column strip or middle strip is the sum of the three parts

$$\cdot_{col} = \cdot_{f,col} + \cdot_{\cdot l} + \cdot_{\cdot r} \quad (13.25a)$$

$$\cdot_{mid} = \cdot_{f,mid} + \cdot_{\cdot l} + \cdot_{\cdot r} \quad (13.25b)$$

where the subscripts l and r refer to the left and right ends of the span, respectively.

The calculations described are repeated for the equivalent frame in the second direction of the structure, and the total deflection at midpanel is obtained by summing the column-strip deflection in one direction and the middle-strip deflection in the other, as indicated by Eqs. (13.19).

The midpanel deflection should be the same whether calculated by Eq. (13.19*a*) or Eq. (13.19*b*). Actually, a difference will usually be obtained because of the approximate nature of the calculations. For very rectangular panels, the main contribution to midpanel deflection is that of the long-direction column strip. Consequently, the midpanel deflection is best found by summing that of the long-direction column strip and the short-direction middle strip. However, for exterior panels, the important contribution is from the column strips perpendicular to the discontinuous edge, even though the long side of the panel may be parallel to that edge.

In slabs, as in beams, the effect of concrete cracking is to reduce the flexural stiffness. According to ACI Code 9.5.3, the effective moment of inertia given by Eq. (6.8) is applicable to slabs as well as beams, although other values may be used if results are in reasonable agreement with tests. In most cases, two-way slabs will be essentially uncracked at service loads, and it is satisfactory to base deflection calculations on the uncracked moment of inertia I_g (see Ref. 13.28 for comparison with tests). In Ref. 13.30, Branson suggests the following refinements: (1) for slabs without beams, use I_g for all dead load deflections; for dead plus live load deflections, use I_g for middle strips and I_e for column strips; (2) for slabs with beams, use I_g for all dead load deflections; for dead plus live load deflections, use I_g for column strips and I_e for middle strips. For continuous spans, I_e can be based on the midspan positive moment without serious error.

The deflections calculated using the procedure described are short-term deflections. Long-term slab deflections can be calculated by multiplying the short-term deflections by the factor · of Eq. (6.11), as for beams. Because compression steel is seldom used in slabs, a multiplier of 2.0 results. Test evidence and experience with actual structures indicates that this may seriously underestimate long-term slab deflections, and multipliers for long-term deflection from 2.5 to 4.0 have been recommended (Refs. 13.30 to 13.32). A multiplier of 3.0 gives acceptable results in most cases.

It should be recognized that the prediction of slab deflections, both initial elastic and long-term, is complicated by the many uncertainties associated with actual building construction. Loading history, particularly during construction, has a profound effect on final deflections (Ref. 13.33). Construction loads can equal or exceed the service live load. Such loads may include the weight of stacked building material and usually include the weight of slabs above the one cast earlier, applied through shoring and reshoring to the lower slab. Because construction loads are applied to immature concrete in the slabs, the immediate elastic deflections are large, and, upon removal of the construction loads, elastic recovery is less than the initial elastic deflection because E_c increases with age. Cracking resulting from construction loading does not disappear with removal of the temporary load and may result in live load deflections greater than expected. Creep during construction loading may be greater than expected because of the early age of the concrete when loaded. Shrinkage deflections of thin slabs are often of the same order of magnitude as the elastic deflections, and some cases must be calculated separately.

It is important to recognize that both initial and time-dependent slab deflections are subject to a high degree of variability. Calculated deflections are an estimate, at best, and considerable deviation from calculated values is to be expected in actual structures.

EXAMPLE 13.7 **Calculation of deflections.** Find the deflections at the center of the typical exterior panel of the two-way floor designed in Example 13.2 due to dead and live loads. The live load may be considered a short-term load and will be distributed uniformly over all panels. The floor will support nonstructural elements that are likely to be damaged by large deflections. Take $E_c = 3.6 \times 10^6$ psi.

Solution. The elastic deflection due to the self-weight of 88 psf will be found, after which the additional long-term dead load deflection can be found by applying the factor $\lambda = 3.0$, and the short-term live load deflection due to 144 psf by direct proportion.

The effective concrete cross sections, upon which moment-of-inertia calculations will be based, are shown in Fig. 13.36 for the full-width frame, the column strip, and the middle strips, for the short-span and long-span directions. Note that the width of the column strip in both directions is based on the shorter panel span, according to the ACI Code. The values of moment of inertia are as follows:

	Short Direction	Long Direction
I_{frame}	27,900 in^4	25,800 in^4
I_{col}	21,000 in^4	21,000 in^4
I_{mid}	5,150 in^4	3,430 in^4

First calculating the deflections of the floor in the *short-span direction* of the panel, from Eq. (13.20) the reference deflection is

$$\Delta_{f,ref} = \frac{88 \times 25(20 \times 12)^4}{12 \times 384 \times 3.6 \times 10^6 \times 27{,}900} = 0.016 \text{ in.}$$

(Note that the centerline span distance is used here, although clear span was used in the moment analysis to approximate the moment reduction due to support width, according to ACI Code procedures.) From the moment analysis in the short-span direction, it was concluded that 68 percent of the moment at both negative and positive sections was taken by the

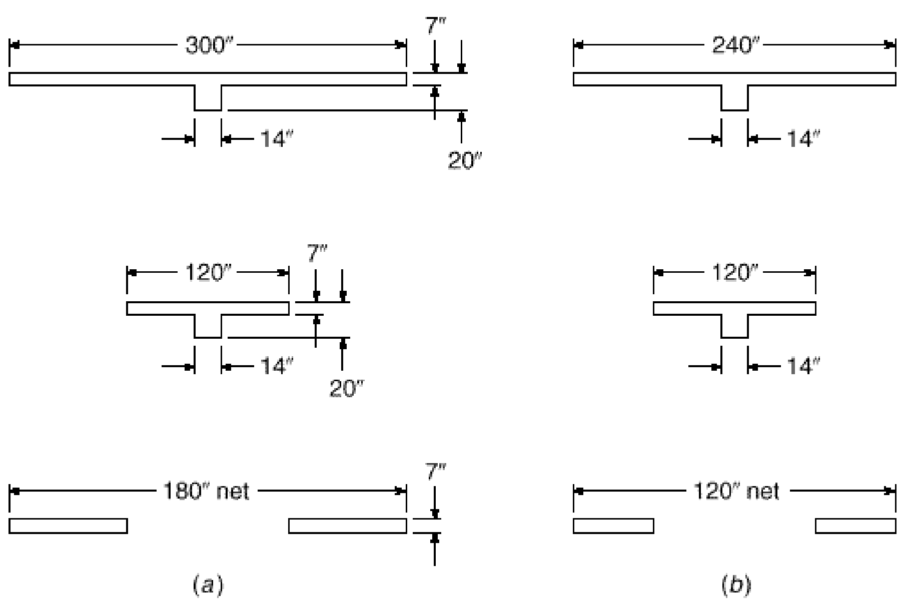


FIGURE 13.36
Cross-sectional dimensions for deflection example: (*a*) short-span direction frame, column strip, and middle strip; (*b*) long-span direction frame, column strip, and middle strip.

column strip and 32 percent by the middle strips. Accordingly, from Eqs. (13.21*a*) and (13.21*b*),

$$\cdot_{f,col} = 0.016 \times 0.68 \times \frac{27{,}900}{21{,}000} = 0.014 \text{ in.}$$

$$\cdot_{f,mid} = 0.016 \times 0.32 \times \frac{27{,}900}{5150} = 0.028 \text{ in.}$$

For the panel under investigation, which is fully continuous over both supports in the short direction, it may be assumed that support rotations are negligible; consequently, $\Delta_{\cdot l}$ and $\Delta_{\cdot r} = 0$, and from Eqs. (13.25*a*) and (13.25*b*),

$$\cdot_{col} = 0.014 \text{ in.}$$

$$\cdot_{mid} = 0.028 \text{ in.}$$

Now calculating the deformations in the *long direction* of the panel gives the reference deflection

$$\cdot_{f,ref} = \frac{88 \times 20 \cdot 25 \times 12^{\cdot 4}}{12 \times 384 \times 3.6 \times 10^6 \times 25{,}800} = 0.033 \text{ in.}$$

From the moment analysis it was found that the column strip would take 93 percent of the exterior negative moment, 81 percent of the positive moment, and 81 percent of the interior negative moment. Thus the average lateral distribution factor for the column strip is

$$\cdot \frac{0.93 + 0.81}{2} + 0.81 \cdot \frac{1}{2} = 0.84$$

or 84 percent, while the middle strips are assigned 16 percent. Then from Eqs. (13.21*a*) and (13.21*b*),

$$\cdot_{f,col} = 0.033 \times 0.84 \times \frac{25{,}800}{21{,}000} = 0.034 \text{ in.}$$

$$\cdot_{f,mid} = 0.033 \times 0.16 \times \frac{25{,}800}{3430} = 0.040 \text{ in.}$$

While rotation at the interior column may be considered negligible, rotation at the exterior column cannot. For the dead load of the slab, the full static moment is

$$M_o = \tfrac{1}{8} \times 0.088 \times 20 \times 25^2 = 137.5 \text{ ft-kips}$$

It was found that 16 percent of the static moment, or 22.0 ft-kips, should be assigned to the exterior support section. The resulting rotation is found from Eq. (13.23). It is easily confirmed that the stiffness of the equivalent column (see Section 13.9c) is $169 \times 3.6 \times 10^6$ in-lb/rad; hence

$$\cdot = \frac{22{,}000 \times 12}{169 \times 3.6 \times 10^6} = 0.00043 \text{ rad}$$

From Eq. (13.24), the corresponding midpanel deflection component is

$$\cdot_{\cdot l} = \frac{0.00043 \times 25 \times 12}{8} = 0.016 \text{ in.}$$

Thus, from Eqs. (13.25*a*) and (13.25*b*), the deflections of the column and middle strips in the long direction are

$$\cdot_{col} = 0.034 + 0.016 = 0.050 \text{ in.}$$

$$\cdot_{mid} = 0.040 + 0.016 = 0.056 \text{ in.}$$

and from Eq. (13.19*a*) the short-term midpanel deflection due to self-weight is

$$\cdot_{max} = 0.050 + 0.028 = 0.078 \text{ in.}$$

The long-term deflection due to dead load is 3.0 × 0.078 = 0.234 in., and the short-term live load deflection is 0.078 × 144·88 = 0.128.

The ACI limiting value for the present case is found to be 1·480 times the span, or 20 × 12·480 = 0.500 in., based on the sum of the long-time deflection due to sustained load and the immediate deflection due to live load. The sum of these deflection components in the present case is

$$\cdot_{max} = 0.234 + 0.128 = 0.362 \text{ in.}$$

well below the permissible value.

13.14 Analysis for Horizontal Loads

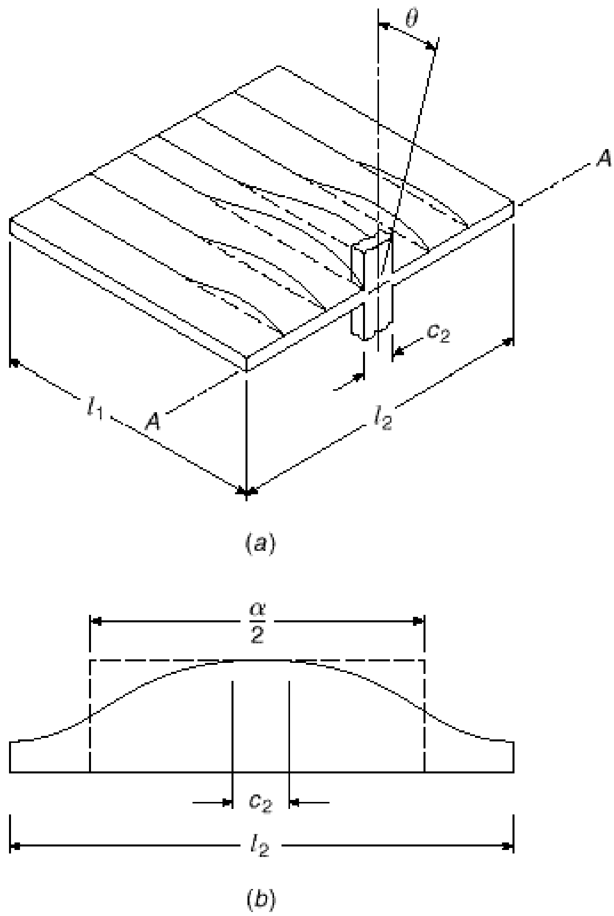
Either the direct design method or the equivalent frame method, described in the preceding sections of this chapter, may be used for the analysis of two-way slab systems for gravity loads, according to ACI Code 13.5.1. However, the ACI Code provisions are not meant to apply to the analysis of buildings subject to lateral loads, such as loads caused by wind or earthquake. For lateral load analysis, the designer may select any method that is shown to satisfy equilibrium and geometric compatibility, and to give results that are in reasonable agreement with available test data. The results of the lateral load analysis may then be combined with those from the vertical load analysis, according to ACI Code 13.5.1.

Plane frame analysis, with the building assumed to consist of parallel frames each bounded laterally by the panel centerlines on either side of the column lines, has often been used in analyzing unbraced buildings for horizontal loads, as well as vertical. For vertical load analysis by the equivalent frame method, a single floor is usually studied as a substructure with attached columns assumed fully fixed at the floors above and below, but for horizontal frame analysis the equivalent frame includes all floors and columns, extending from the bottom to the top of the structure.

The main difficulty in equivalent frame analysis for horizontal loads lies in modeling the stiffness of the region at the beam-column (or slab-beam-column) connections. Transfer of forces in this region involves bending, torsion, shear, and axial load, and is further complicated by the effects of concrete cracking in reducing stiffness, and reinforcement in increasing it. Frame moments are greatly influenced by horizontal displacements at the floors, and a conservatively low value of stiffness should be used to ensure that a reasonable estimate of drift is included in the analysis.

While a completely satisfactory basis for modeling the beam-column joint stiffness has not been developed, at least two methods have been used in practice (Ref. 13.34). The first is based on an equivalent beam width · l_2, less than the actual width, to reduce the stiffness of the slab for purposes of analysis. Figure 13.37*a* shows a plate fixed at the far edge and supported by a column of width c_2 at the near side. If a rotation · is imposed at the column, the plate rotation along the axis *A* will vary as shown by Fig. 13.37*a*, from · at the column to smaller values away from the column. An equivalent width factor · is obtained from the requirement that the stiffness of a prismatic beam of width · l_2 must equal the stiffness of the plate of width l_2. This equality is obtained if the areas under the two rotation diagrams of Fig. 13.37*b* are equal. Thus the frame analysis is based on a reduced slab (or slab-beam) stiffness found

FIGURE 13.37
Equivalent beam width for horizontal load analysis.



using · l_2 rather than l_2. Comparative studies indicate that, for flat plate floors, a value for · between 0.25 and 0.50 may be used (Ref. 13.34).

Alternatively, the beam-column stiffness can be modeled based on a transverse torsional member corresponding to that used in deriving the stiffness of the equivalent column for the vertical load analysis of two-way slabs by the equivalent frame method (see Section 13.9c). Rotational stiffness of the joint is a function of the flexural stiffness of the columns framing into the joint from above and below and the torsional stiffness of the transverse strip of slab or slab beam at the column. The equivalent column stiffness is found from Eq. (13.9) and the torsional stiffness from Eq. (13.10), as before.

Finally, for frames in which two-way systems act as primary members resisting lateral loads, ACI Code 13.3.8 requires that the lengths of reinforcement be determined by analysis because the lengths shown in Fig. 13.14 may not be adequate. The values in Fig. 13.14, however, are retained as minimum values.

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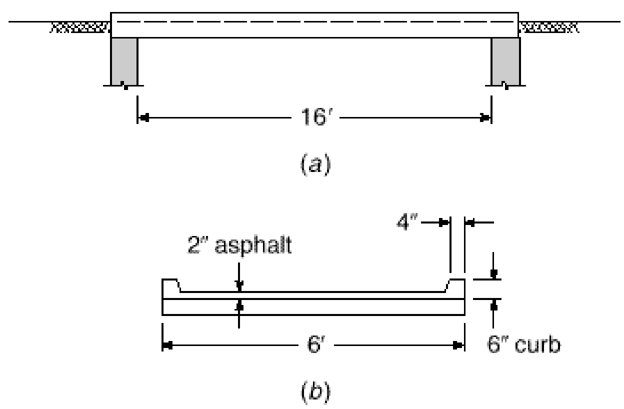
PROBLEMS

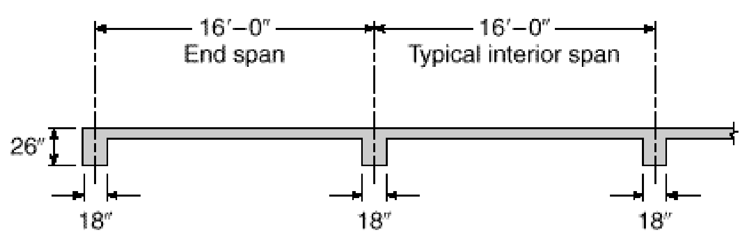
13.1. A footbridge is to be built, consisting of a one-way solid slab spanning 16 ft between masonry abutments, as shown in Fig. P13.1. A service live load of 100 psf must be carried. In addition, a 2000 lb concentrated load, assumed to be uniformly distributed across the bridge width, may act at any location on the span. A 2 in. asphalt wearing surface will be used, weighing 20 psf. Precast concrete curbs are attached so as to be nonstructural. Prepare a design for the slab, using material strengths $f_y = 60{,}000$ psi and $f_c' = 4000$ psi, and summarize your results in the form of a sketch showing all concrete dimensions and reinforcement.

FIGURE P13.1



13.2. A reinforced concrete building floor system consists of a continuous one-way slab built monolithically with its supporting beams, as shown in cross section in Fig. P13.2. Service live load will be 125 psf. Dead loads include a 10 psf allowance for nonstructural lightweight concrete floor fill and surface, and a 10 psf allowance for suspended loads, plus the self-weight of the floor. Using ACI coefficients from Chapter 12, calculate the design moments and shears and design the slab, using a maximum tensile reinforcement ratio of 0.006. Use all straight bar reinforcement. One-half of the positive-moment bars will be discontinued where no longer required; the other half will be continued into the supporting beams as specified by the ACI Code. All negative steel will be discontinued at the same distance from the support face in each case. Summarize your design with a sketch showing concrete dimensions, and size, spacing, and cutoff points for all reinforcement. Material strengths are $f_y = 60{,}000$ psi and $f_c' = 3000$ psi.

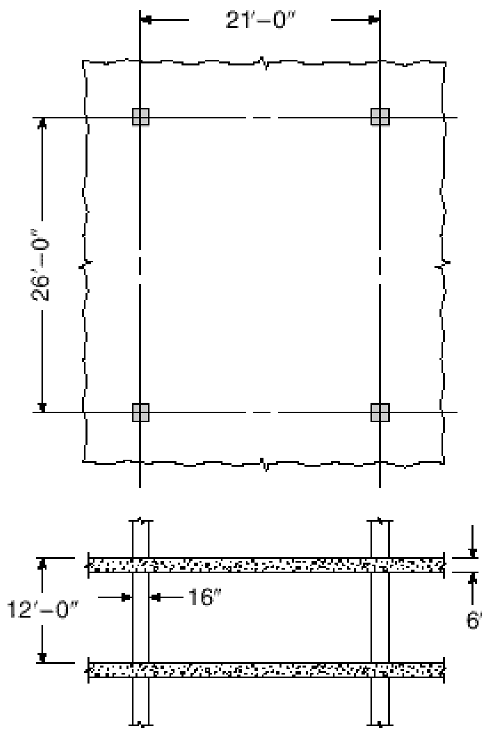
FIGURE P13.2



13.3. For the one-way slab floor in Problem 13.2, calculate the immediate and long-term deflection due to dead loads. Assume that all dead loads are applied when the construction shoring is removed. Also determine the deflection due to application of the full service live load. Assuming that sensitive equipment will be installed 6 months after the shoring is removed, calculate the relevant deflection components and compare the total with maximum values recommended in the ACI Code.

13.4. A monolithic reinforced concrete floor consists of rectangular bays measuring 21 × 26 ft, as shown in Fig. P13.4. The floor is designed to carry a service live load of 125 psf uniformly distributed over its surface in addition to its own weight, using a concrete strength of 5000 psi and reinforcement having f_y = 60,000 psi. Design a typical interior panel using the ACI direct design method of Sections 13.6 through 13.8.

FIGURE P13.4



13.5. Redesign the typical interior panel described in Problem 13.4 using the ACI equivalent frame method of Section 13.9. Compare your results with those for Problem 13.4, and comment.

13.6. Redesign the typical exterior panel of the floor of Example 13.2 as a part of a flat plate structure, with no beams between interior columns but with beams provided along the outside edge to stiffen the slab. No drop panels or column capitals are permitted, but shear reinforcement similar to Fig. 13.24*b* may be incorporated if necessary. Column size is 20 × 20 in., and the floor-to-floor height is 12 ft. Use either the direct design method or the equivalent frame method. Summarize your design by means of a sketch showing plan and typical cross sections.

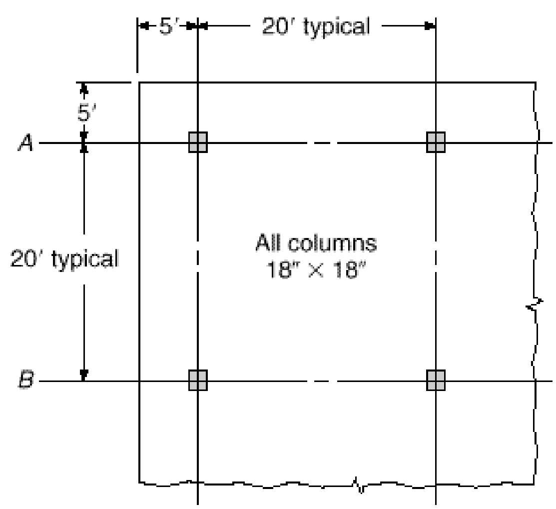
13.7. A multistory commercial building is to be designed as a flat plate system with floors of uniform thickness having no beams or drop panels. Columns are laid out on a uniform 20 ft spacing in each direction and have a 16 in. square section and a vertical dimension 10 ft from floor to floor. Specified service live load is 100 psf including partition allowance. Using the direct design method, design a typical interior panel, determining the required floor thickness, size and spacing of reinforcing bars, and bar details including cutoff points. To simplify construction, the reinforcement in each direction will be the same; use an average effective depth in the calculations. Use all straight bars. For moderate spans such as this, it has been determined that supplementary shear reinforcement would not be economical, although column capitals may be used if needed. Thus, slab thickness may be based on Eqs. (13.11*a*), (13.11*b*), and (13.11*c*), or column capital dimensions can be selected using those equations if slab thickness is based on the equations in Section 13.8. Material strengths are $f_y = 60{,}000$ psi and $f'_c = 4000$ psi.

13.8. Prepare alternative designs for shear reinforcement at the supports of the slab described in Example 13.5 (*a*) using bent bar reinforcement similar to Fig. 13.24*b*, and (*b*) using integral beams with vertical stirrups similar to Fig. 13.24*e*.

13.9. Prepare an alternative design for shear reinforcement at the supports of the slab described in Example 13.4, using a shearhead similar to Fig. 13.24*a*. As an alternative to shear reinforcement of any kind, calculate the smallest acceptable dimensions for a 45° column capital (see Fig. 13.1*e*) that would permit the concrete slab to resist the entire shear force. Drop panels are not permitted.

13.10. Figure P13.10 shows a flat plate floor designed to carry a factored load of 325 psf. The total slab thickness $h = 7\frac{1}{2}$ in. and the average effective depth $d = 6$ in. Material strengths are $f_y = 60{,}000$ psi and $f'_c = 4000$ psi. The design for punching shear at a typical interior column *B*2 provided the basis for Example 13.4. To provide a full perimeter b_o at the exterior column *B*1, the slab is cantilevered past the columns as shown. A total shear force $V_u = 105$ kips must be transmitted to the column, along with a bending moment $M_u = 120$ ft-kips about an axis parallel to the edge of the slab. Check for punching shear at column *B*1 and, if ACI Code restrictions are not met, suggest appropriate modifications in the proposed design. Edge beams are not permitted.

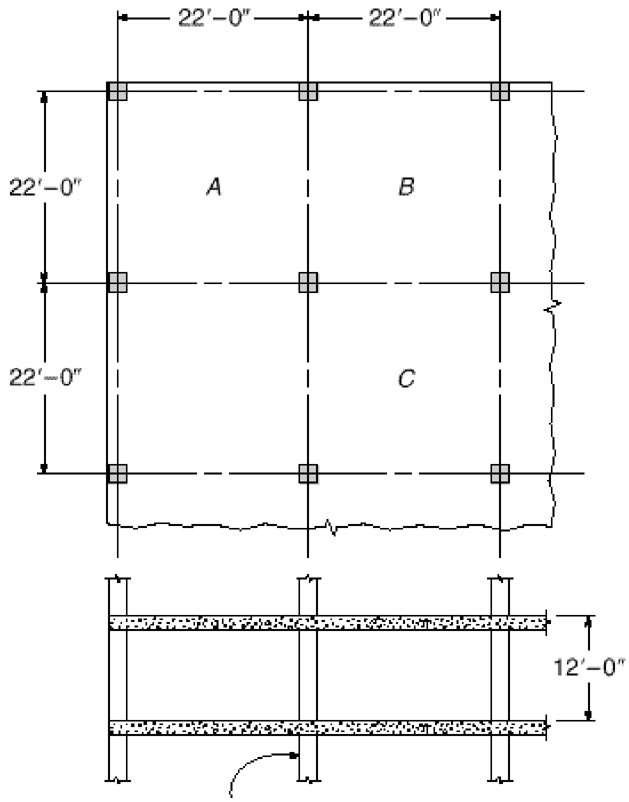
FIGURE P13.10



13.11. For the flat plate floor in Example 13.3, find the following deflection components at the center of panel *C*: (*a*) immediate deflection due to total dead load; (*b*) additional dead load deflection after a long period of time, due to total dead load; (*c*) immediate deflection due to three-quarters full live load. The moment of inertia of the cross concrete sections I_g may be used for all calculations. It may be assumed that maximum deflection will be obtained for the same loading pattern that would produce maximum positive moment in the panel. Check predicted deflection against ACI limitations, assuming that nonstructural attached elements would be damaged by excessive deflections.

13.12. A parking garage is to be designed using a two-way flat slab on the column lines, as shown in Fig. P13.12. A live load of 100 psf is specified. Find the required slab thickness, using a reinforcement ratio of approximately 0.005, and design the reinforcement for a typical corner panel *A*, edge panel *B*, and interior panel *C*. Check shear capacity. Detail the reinforcement, showing size, spacing, and length. All straight bars will be used. Material strengths will be $f_y = 60{,}000$ psi and $f'_c = 5000$ psi. Specify the design method selected and comment on your results.

FIGURE P13.12



13.13. For the typical interior panel *C* of the parking garage in Problem 13.12, (*a*) compute the immediate and long-term deflections due to dead load, and (*b*) compute the deflection due to the full service live load. Compare with ACI Code maximum permissible values, given that there are no elements attached that would be damaged by large deflections.