16 Footings and Foundations

16.1 Types and Functions

The substructure, or foundation, is the part of a structure that is usually placed below the surface of the ground and that transmits the load to the underlying soil or rock. All soils compress noticeably when loaded and cause the supported structure to settle. The two essential requirements in the design of foundations are that the total settlement of the structure be limited to a tolerably small amount and that differential settlement of the various parts of the structure be eliminated as nearly as possible. With respect to possible structural damage, the elimination of differential settlement, i.e., different amounts of settlement within the same structure, is even more important than limitations on uniform overall settlement.

To limit settlements as indicated, it is necessary (1) to transmit the load of the structure to a soil stratum of sufficient strength and (2) to spread the load over a sufficiently large area of that stratum to minimize bearing pressure. If adequate soil is not found immediately below the structure, it becomes necessary to use deep foundations such as piles or caissons to transmit the load to deeper, firmer layers. If satisfactory soil directly underlies the structure, it is merely necessary to spread the load, by footings or other means. Such substructures are known as *spread* foundations, and it is mainly this type that will be discussed. Information on the more special types of deep foundations can be found in texts on foundation engineering, e.g., Refs. 16.1 to 16.4.

16.2 Spread Footings

Spread footings can be classified as wall and column footings. The horizontal outlines of the most common types are given in Fig. 16.1. A wall footing is simply a strip of reinforced concrete, wider than the wall, that distributes its pressure. Single-column footings are usually square, sometimes rectangular, and represent the simplest and most economical type. Their use under exterior columns meets with difficulties if property rights prevent the use of footings projecting beyond the exterior walls. In this case, combined footings or strap footings are used that enable one to design a footing that will not project beyond the wall column. Combined footings under two or more columns are also used under closely spaced, heavily loaded interior columns where single footings, if they were provided, would completely or nearly merge.

Such individual or combined column footings are the most frequently used types of spread foundations on soils of reasonable bearing capacity. If the soil is weak and/or column loads are great, the required footing areas become so large as to be uneconomical. In this case, unless a deep foundation is called for by soil conditions, a mat

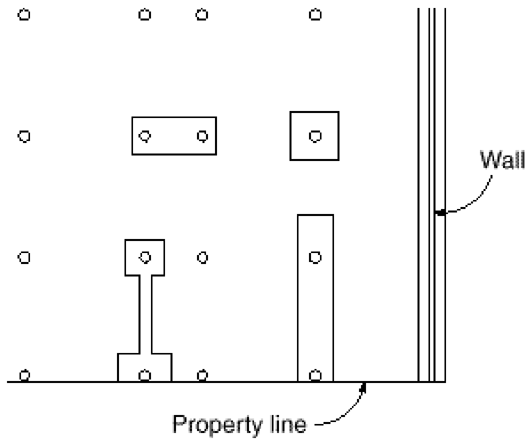


FIGURE 16.1
Types of spread footing.

or raft foundation is resorted to. This consists of a solid reinforced concrete slab that extends under the entire building and, consequently, distributes the load of the structure over the maximum available area. Such a foundation, in view of its own rigidity, also minimizes differential settlement. It consists, in its simplest form, of a concrete slab reinforced in both directions. A form that provides more rigidity consists of an inverted girder floor. Girders are located in the column lines in each direction, and the slab is provided with two-way reinforcement, spanning between girders. Inverted flat slabs, with capitals at the bottoms of the columns, are also used for mat foundations.

16.3 Design Factors

In ordinary construction, the load on a wall or column is transmitted vertically to the footing, which in turn is supported by the upward pressure of the soil on which it rests. If the load is symmetrical with respect to the bearing area, the bearing pressure is assumed to be uniformly distributed (Fig. 16.2*a*). It is known that this is only approximately true. Under footings resting on coarse-grained soils, the pressure is larger at the center of the footing and decreases toward the perimeter (Fig. 16.2*b*). This is so because the individual grains in such soils are somewhat mobile, so that the soil located close to the perimeter can shift very slightly outward in the direction of lower soil stresses. In contrast, in clay soils pressures are higher near the edge than at the center of the footing, since in such soils the load produces a shear resistance around

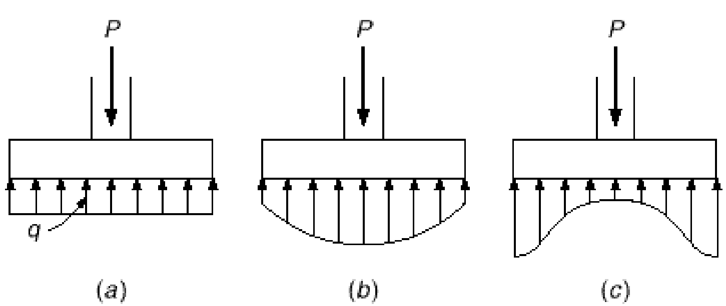


FIGURE 16.2
Bearing pressure distribution: (*a*) as assumed; (*b*) actual, for granular soils; (*c*) actual, for cohesive soils.

the perimeter that adds to the upward pressure (Fig. 16.2*c*). It is customary to disregard these nonuniformities (1) because their numerical amount is uncertain and highly variable, depending on types of soil, and (2) because their influence on the magnitudes of bending moments and shearing forces in the footing is relatively small.

On compressible soils, footings should be loaded concentrically to avoid tilting, which will result if bearing pressures are significantly larger under one side of the footing than under the opposite side. This means that single footings should be placed concentrically under the columns and wall footings concentrically under the walls and that, for combined footings, the centroid of the footing area should coincide with the resultant of the column loads. Eccentrically loaded footings can be used on highly compacted soils and on rock. It follows that one should count on rotational restraint of the column by a single footing only when such favorable soil conditions are present and when the footing is designed both for the column load and the restraining moment. Even then, less than full fixity should be assumed, except for footings on rock.

The accurate determination of stresses in foundation elements of all kinds is difficult, partly because of the uncertainties in determining the actual distribution of upward pressures but also because the structural elements themselves represent relatively massive blocks or thick slabs subject to heavy concentrated loads from the structure above. Design procedures for single-column footings are based largely on the results of experimental investigations by Talbot (Ref. 16.5) and Richart (Ref. 16.6). These tests and the recommendations resulting from them have been reevaluated in the light of more recent research, particularly that focusing on shear and diagonal tension (Refs. 16.7 to 16.9). Combined footings and mat foundations also can be designed by simplified methods, although increasing use is made of more sophisticated tools, such as finite element analysis and strut-and-tie models.

16.4 Loads, Bearing Pressures, and Footing Size

Allowable bearing pressures are established from principles of soil mechanics, on the basis of load tests and other experimental determinations (see, for example, Refs. 16.1 to 16.4). Allowable bearing pressures q_a under service loads are usually based on a safety factor of 2.5 to 3.0 against exceeding the bearing capacity of the particular soil and to keep settlements within tolerable limits. Many local building codes contain allowable bearing pressures for the types of soils and soil conditions found in the particular locality.

For concentrically loaded footings, the required area is determined from

$$A_{req} = \frac{D + L}{q_a} \tag{16.1}$$

In addition, most codes permit a 33 percent increase in allowable pressure when the effects of wind W or earthquake E are included, in which case,

$$A_{req} = \frac{D + L + W}{1.33q_a} \qquad \text{or} \qquad \frac{D + L + E}{1.33q_a} \tag{16.2}$$

It should be noted that footing sizes are determined for unfactored service loads and soil pressures, in contrast to the strength design of reinforced concrete members, which utilizes factored loads and factored nominal strengths. This is because, for footing design, safety is provided by the overall safety factors just mentioned, in contrast to the separate load and strength reduction factors used to dimension members.

The required footing area A_{req} is the larger of those determined by Eqs. (16.1) and (16.2). The loads in the numerators of Eqs. (16.1) and (16.2) must be calculated at the level of the base of the footing, i.e., at the contact plane between soil and footing. This means that the weight of the footing and surcharge (i.e., fill and possible liquid pressure on top of the footing) must be included. Wind loads and other lateral loads cause a tendency to overturn. In checking for overturning of a foundation, only those live loads that contribute to overturning should be included, and dead loads that stabilize against overturning should be multiplied by 0.9. A safety factor of at least 1.5 should be maintained against overturning, unless otherwise specified by the local building code (Ref. 16.8).

A footing is eccentrically loaded if the supported column is not concentric with the footing area or if the column transmits at its juncture with the footing not only a vertical load but also a bending moment. In either case, the load effects at the footing base can be represented by the vertical load P and a bending moment M. The resulting bearing pressures are again assumed to be linearly distributed. As long as the resulting eccentricity $e = M/P$ does not exceed the kern distance k of the footing area, the usual flexure formula

$$q_{\substack{max \\ min}} = \frac{P}{A} \pm \frac{Mc}{I} \tag{16.3}$$

permits the determination of the bearing pressures at the two extreme edges, as shown in Fig. 16.3*a*. The footing area is found by trial and error from the condition $q_{max} \leq q_a$. If the eccentricity falls outside the kern, Eq. (16.3) gives a negative value (tension) for q along one edge of the footing. Because no tension can be transmitted at the contact area between soil and footing, Eq. (16.3) is no longer valid and bearing pressures are distributed as shown in Fig. 16.3*b*. For rectangular footings of size $l \times b$, the maximum pressure can be found from

$$q_{max} = \frac{2P}{3bm} \tag{16.4}$$

which, again, must be no larger than the allowable pressure q_a. For nonrectangular footing areas of various configurations, kern distances and other aids for calculating bearing pressures can be found in Refs. 16.1 and 16.8 and elsewhere.

FIGURE 16.3
Assumed bearing pressures under eccentrically loaded footing.

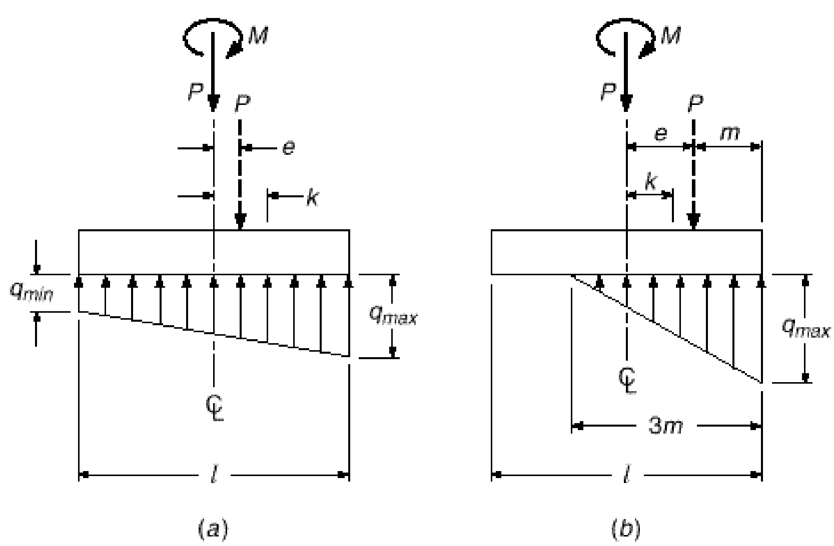
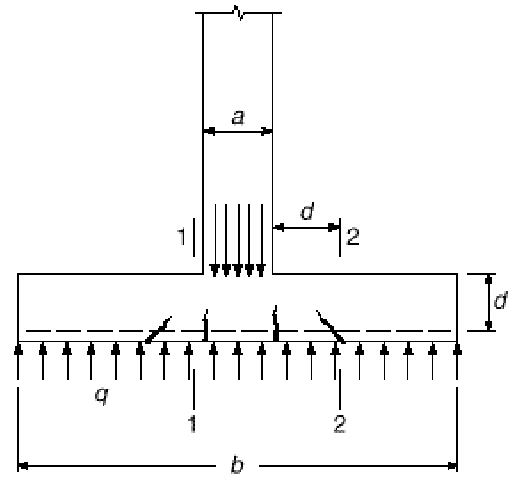


FIGURE 16.4
Wall footing.



EXAMPLE 16.1

Design of wall footing. A 16 in. concrete wall supports a dead load $D = 14$ kips/ft and a live load $L = 10$ kips/ft. The allowable bearing pressure is $q_a = 4.5$ kips/ft^2 at the level of the bottom of the footing, which is 4 ft below grade. Design a footing for this wall using 4000 psi concrete and Grade 60 steel.

SOLUTION. With a 12 in. thick footing, the footing weight per square foot is 150 psf, and the weight of the 3 ft fill on top of the footing is $3 \times 100 = 300$ psf. Consequently, the portion of the allowable bearing pressure that is available or effective for carrying the wall load is

$$q_e = 4500 - (150 + 300) = 4050 \text{ psf}$$

The required width of the footing is therefore $b = 24{,}000/4050 = 5.93$ ft. A 6 ft wide footing will be assumed.

The bearing pressure for strength design of the footing, caused by the factored loads, is

$$q_u = \frac{1.2 \times 14 + 1.6 \times 10}{6} \times 10^3 = 5470 \text{ psf}$$

From this, the factored moment for strength design is

$$M_u = \frac{1}{8} \times 5470(6 - 1.33)^2 \times 12 = 178{,}900 \text{ in-lb/ft}$$

and assuming $d = 9$ in., the shear at section 2-2 is

$$V_u = 5470\left[\frac{1}{2}(6 - 1.33) - \frac{9}{12}\right] = 8670 \text{ lb/ft}$$

Shear usually governs the depth of footings, particularly since the use of shear reinforcements in footings is generally avoided as uneconomical. The design shear strength per foot [see Eq. (4.12*b*)] is

$$\phi V_c = \phi\left(2\sqrt{f'_c}\,bd\right) = 0.75\left(2\sqrt{4000} \times 12d\right) = 1138d \text{ lb/ft}$$

from which

$$d = \frac{8670}{1138} = 7.6 \text{ in.}$$

Once the required footing area has been determined, the footing must then be designed to develop the necessary strength to resist all moments, shears, and other internal actions caused by the applied loads. For this purpose, the load factors of ACI Code 9.2 apply to footings as to all other structural components. Correspondingly, for strength design, the footing is dimensioned for the effects of the following external loads (see Table 1.2):

$$U = 1.2D + 1.6L$$

or if wind effects are to be included,

$$U = 1.2D + 1.6L_r + 1.0L + 0.8W$$

In seismic zones, earthquake forces E must be considered according to Table 1.2. The requirement that

$$U = 0.9D + 1.6W$$

will hardly ever govern the strength design of a footing, but will affect overturning and stability. Lateral earth pressure H may, on occasion, affect footing design, in which case

$$U = 1.2D + 1.6 \cdot L + H \cdot + 0.5L_r$$

For pressures F from liquids, such as groundwater, $1.2F$ must be added to the first equation.

These factored loads must be counteracted and equilibrated by corresponding bearing pressures in the soil. Consequently, once the footing area is determined, the bearing pressures are recalculated for the factored loads for purposes of strength computations. These are fictitious pressures that are needed only to determine the factored loads for use in design. To distinguish them from the actual pressures q under service loads, the soil pressures that equilibrate the factored loads U will be designated q_u.

16.5 WALL FOOTINGS

The simple principles of beam action apply to wall footings with only minor modifications. Figure 16.4 shows a wall footing with the forces acting on it. If bending moments were computed from these forces, the maximum moment would be found to occur at the middle of the width. Actually, the very large rigidity of the wall modifies this situation, and the tests cited in Section 16.3 show that, for footings under concrete walls, it is satisfactory to compute the moment at the face of the wall (section 1-1). Tension cracks in these tests formed at the locations shown in Fig. 16.4, i.e., under the face of the wall rather than in the middle. For footings supporting masonry walls, the maximum moment is computed midway between the middle and the face of the wall, because masonry is generally less rigid than concrete. The maximum bending moment in footings under concrete walls is therefore given by

$$M_u = \frac{1}{8} q_u \cdot b - a \cdot^2 \tag{16.5}$$

For determining shear stresses, the vertical shear force is computed on section 2-2, located, as in beams, at a distance d from the face of the wall. Thus,

$$V_u = q_u \cdot \frac{b - a}{2} - d \cdot \tag{16.6}$$

The calculation of development length is based on the section of maximum moment, i.e., section 1-1.

In computing bending moments and shears, only the upward pressure q_u that is caused by the factored column loads is considered. The weight of the footing proper does not cause moments or shears, just as no moments or shears are present in a book lying flat on a table.

a. Shear

Once the required footing area A_{req} has been established from the allowable bearing pressure q_a and the most unfavorable combination of service loads, including weight of footing and overlying fill (and such surcharge as may be present), the thickness h of the footing must be determined. In single footings, the effective depth d is mostly governed by shear. Since such footings are subject to two-way action, i.e., bending in both major directions, their performance in shear is much like that of flat slabs in the vicinity of columns (see Section 13.10). However, in contrast to two-way floor and roof slabs, it is generally not economical in footings to use shear reinforcement. For this reason, only the design of footings in which all shear is carried by the concrete will be discussed here. For the rare cases where the thickness is restricted so that shear reinforcement must be used, the information in Section 13.10 about slabs applies also to footings.

Two different types of shear strength are distinguished in footings: two-way, or punching, shear and one-way, or beam, shear.

A column supported by the slab shown in Fig. 16.6 tends to punch through that slab because of the shear stresses that act in the footing around the perimeter of the column. At the same time, the concentrated compressive stresses from the column spread out into the footing so that the concrete adjacent to the column is in vertical or slightly inclined compression, in addition to shear. As a consequence, if failure occurs, the fracture takes the form of the truncated pyramid shown in Fig. 16.7 (or of a truncated cone for a round column), with sides sloping outward at an angle approaching 45°. The average shear stress in the concrete that fails in this manner can be taken as that acting on vertical planes laid through the footing around the column on a perime-

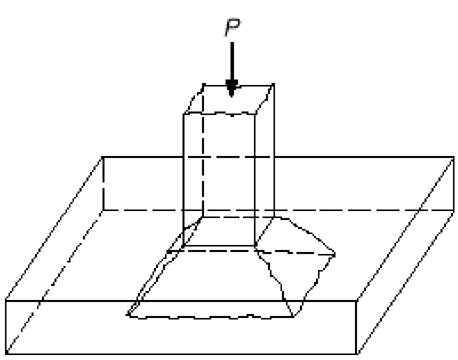


FIGURE 16.6
Punching-shear failure in single footing.

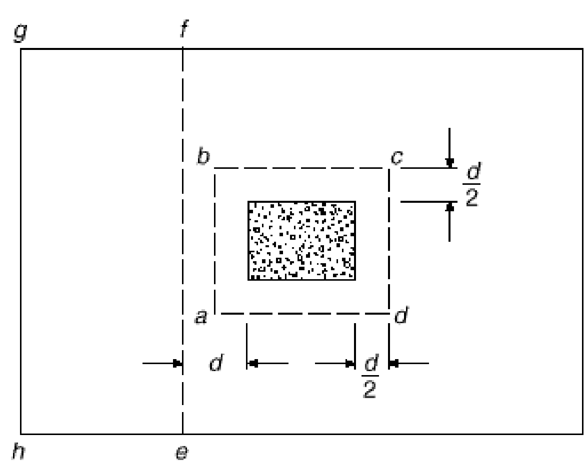


FIGURE 16.7
Critical sections for shear.

Since ACI Code 7.7.1 calls for a 3 in. clear cover on bars, a 12 in. thick footing will be selected, giving $d = 8.5$ in. This is sufficiently close to the assumed values, and the calculations need not be revised.

To determine the required steel area, $M_u \cdot \cdot\ bd^2 = 178{,}900 \cdot (0.90 \times 12 \times 8.5^2) = 229$ is used to enter Graph A.1b of Appendix A. For this value, the curve 60·4 gives the reinforcement ratio · $= 0.0038$. The required steel area is then $A_s = 0.0038 \times 8.5 \times 12 = 0.39$ in^2/ft. No. 5 (No. 16), $9\frac{1}{2}$ in. on centers, furnish $A_s = 0.39$ in^2/ft. The required development length according to Table A.10 of Appendix A is 24 in. This length is to be furnished from section 1-1 outward. The length of each bar, if end cover is 3 in., is $72 - 6 = 66$ in., and the actual development length from section 1-1 to the nearby end is $\frac{1}{2} \cdot 66 - 16 \cdot = 25$ in., which is more than the required development length.

Longitudinal shrinkage and temperature reinforcement, according to ACI Code 7.12, must be at least $0.002 \times 12 \times 12 = 0.29$ in^2/ft. No. 5 (No. 16) bars on 12 in. centers will furnish 0.31 in^2/ft.

16.6 Column Footings

In plan, single-column footings are usually square. Rectangular footings are used if space restrictions dictate this choice or if the supported columns have a strongly elongated rectangular cross section. In the simplest form, they consist of a single slab (Fig. 16.5*a*). Another type is that shown in Fig. 16.5*b*, where a pedestal or cap is interposed between the column and the footing slab; the pedestal provides for a more favorable transfer of load and in many cases is required to provide the necessary development length for dowels. This form is also known as a *stepped* footing. All parts of a stepped footing must be cast at one time to provide monolithic action. Sometimes sloped footings like those shown in Fig. 16.5*c* are used. They require less concrete than stepped footings, but the additional labor necessary to produce the sloping surfaces (formwork, etc.) usually makes stepped footings more economical. In general, single-slab footings (Fig. 16.5*a*) are most economical for thicknesses up to 3 ft.

Single-column footings can be represented as cantilevers projecting out from the column in both directions and loaded upward by the soil pressure. Corresponding tension stresses are caused in both of these directions at the bottom surface. Such footings are, therefore, reinforced by two layers of steel, perpendicular to each other and parallel to the edges.

The required bearing area is obtained by dividing the total load, including the weight of the footing, by the selected bearing pressure. Weights of footings, at this stage, must be estimated and usually amount to 4 to 8 percent of the column load, the former value applying to the stronger types of soils.

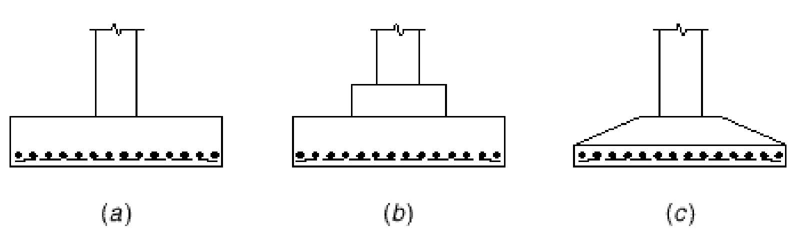


FIGURE 16.5
Types of single-column footings.

ter a distance $d/2$ from the faces of the column (vertical section through *abcd* in Fig. 16.7). The concrete subject to this shear stress v_{u1} is also in vertical compression from the stresses spreading out from the column, and in horizontal compression in both major directions because of the biaxial bending moments in the footing. This triaxiality of stress increases the shear strength of the concrete. Tests of footings and of flat slabs have shown, correspondingly, that for punching-type failures the shear stress computed on the critical perimeter area is larger than in one-way action (e.g., beams).

As discussed in Section 13.10, the ACI Code equations (13.11*a,b,c*) give the nominal punching-shear strength on this perimeter:

$$V_c = 4\sqrt{f'_c}\,b_o d \qquad (16.7a)$$

except for columns of elongated cross section, for which

$$V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f'_c}\,b_o d \qquad (16.7b)$$

For cases in which the ratio of critical perimeter to slab depth b_o/d is very large,

$$V_c = \left(\frac{\alpha_s d}{b_o} + 2\right)\sqrt{f'_c}\,b_o d \qquad (16.7c)$$

where b_o is the perimeter *abcd* in Fig. 16.7; $\beta_c = a/b$ is the ratio of the long to short sides of the column cross section; and α_s is 40 for interior loading, 30 for edge loading, and 20 for corner loading of a footing. The punching-shear strength of the footing is to be taken as the smallest of the values given by Eqs. (16.7*a*), (16.7*b*), and (16.7*c*), and the design strength is ϕV_c, as usual, where $\phi = 0.75$ for shear.

The application of Eqs. (16.7) to punching shear in footings under columns with other than a rectangular cross section is shown in Fig. 13.23. For such situations, ACI Code 11.12.1 indicates that the perimeter b_o must be of minimum length but need not approach closer than $d/2$ to the perimeter of the actual loaded area. The manner of defining a and b for such irregular loaded areas is also shown in Fig. 13.23. If a moment is transferred from the column to the footing, the criteria discussed in Section 13.11 for the transfer of moment by bending and shear at slab-column connections must be satisfied.

Shear failures can also occur, as in beams or one-way slabs, at a section a distance d from the face of the column, such as section *ef* of Fig. 16.7. Just as in beams and one-way slabs, the nominal shear strength is given by Eq. (4.12*a*), that is,

$$V_c = \left(1.9\sqrt{f'_c} + 2500\,\rho_w\frac{V_u d}{M_u}\right) bd \le 3.5\sqrt{f'_c}\,bd \qquad (16.8a)$$

where b = width of footing at distance d from face of column
= *ef* in Fig. 16.7
V_u = total factored shear force on that section
= q_u times footing area outside that section (area *efgh* in Fig. 16.7)
M_u = moment of V_u about *ef*

In footing design, the simpler and somewhat more conservative Eq. (4.12*b*) is generally used, i.e.,

$$V_c = 2\sqrt{f'_c}\,bd \qquad (16.8b)$$

The required depth of footing d is then calculated from the usual equation

$$V_u \le \phi V_c \qquad (16.9)$$

applied separately in connection with Eqs. (16.7) and (16.8). For Eq. (16.7), $V_u = V_{u1}$ is the total upward pressure caused by q_u on the area outside the perimeter *abcd* in Fig. 16.7. For Eq. (16.8), $V_u = V_{u2}$ is the total upward pressure on the area *efgh* outside the section *ef* in Fig. 16.7. The required depth is then the larger of those calculated from either Eq. (16.7) or (16.8). For shear, · = 0.75.

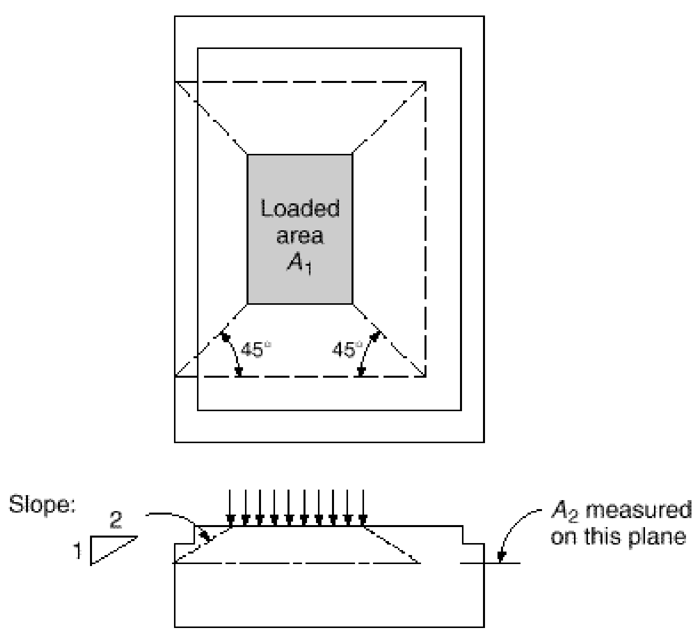
b. Bearing: Transfer of Forces at Base of Column

When a column rests on a footing or pedestal, it transfers its load to only a part of the total area of the supporting member. The adjacent footing concrete provides lateral support to the directly loaded part of the concrete. This causes triaxial compressive stresses that increase the strength of the concrete that is loaded directly under the column. Based on tests, ACI Code 10.17.1 provides that when the supporting area is wider than the loaded area on all sides, the design bearing strength is

$$\cdot\ P_n = 0.85 \cdot f'_c A_1 \sqrt{\frac{A_2}{A_1}} \leq 0.85 \cdot f'_c A_1 \times 2 \qquad (16.10)$$

For bearing on concrete, · = 0.65, f'_c is the cylinder strength of the footing concrete, which frequently is less than that of the column, and A_1 is the loaded area. A_2 is the area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the support and having for its upper base the loaded area and having side slopes of 1 vertical to 2 horizontal. The meaning of this definition of A_2 may be clarified by Fig. 16.8. For the somewhat unusual case shown, where the top of the support is stepped, a step that is deeper or closer to the loaded area than that shown may result in reduction in the value of A_2. A footing for which the top surface is sloped

FIGURE 16.8
Definition of areas A_1 and A_2.



away from the loaded area more steeply than 1 to 2 will result in a value of A_2 equal to A_1. In most usual cases, for which the top of the footing is flat and the sides are vertical, A_2 is simply the maximum area of the portion of the supporting surface that is geometrically similar to, and concentric with, the loaded area.

All axial forces and bending moments that act at the bottom section of a column must be transferred to the footing at the bearing surface by compression in the concrete and by reinforcement. With respect to the reinforcement, this may be done either by extending the column bars into the footing or by providing dowels that are embedded in the footing and project above it. In the latter case, the column bars merely rest on the footing and in most cases are tied to dowels. This results in a simpler construction procedure than extending the column bars into the footing. To ensure the integrity of the junction between column and footing, ACI Code 15.8.2 requires that the minimum area of reinforcement that crosses the bearing surface (dowels or column bars) be 0.005 times the gross area of the supported column. The length of the dowels or bars of diameter d_b must be sufficient on both sides of the bearing surface to provide the required development length for compression bars (see Section 5.7), that is, $l_d \ge 0.02 f_y d_b / \sqrt{f'_c}$ and $\ge 0.0003 f_y d_b$. In addition, if dowels are used, the lapped length must be at least that required for a lap splice in compression (see Section 5.11b); i.e., the length of lap must not be less than the usual development length in compression and must not be less than $0.0005 f_y d_b$. Where bars of different sizes are lap-spliced, the splice length should be the larger of the development length of the larger bar or the splice length of the smaller bar, according to the ACI Code.

The two largest bar sizes, Nos. 14 (No. 43) and 18 (No. 57), are frequently used in columns with large axial forces. Under normal circumstances, the ACI Code specifically prohibits the lap splicing of these bars because tests have shown that welded splices or other positive connections are necessary to develop these heavy bars fully. However, a specific exception is made for dowels for Nos. 14 (No. 43) and 18 (No. 57) column bars. Relying on long-standing successful use, ACI Code 12.16.2 permits these heavy bars to be spliced to dowels of lesser diameter [i.e., No. 11 (No. 36) or smaller], provided that the dowels have a development length into the column corresponding to that of the column bar [i.e., Nos. 14 or 18 (Nos. 43 or 57), as the case may be] and into the footing as prescribed for the particular dowel size [i.e., No. 11 (No. 36) or smaller, as the case may be].

c. Bending Moments, Reinforcement, and Bond

If a vertical section is passed through a footing, the bending moment that is caused in the section by the net upward soil pressure (i.e., factored column load divided by bearing area) is obtained from simple statics. Figure 16.9 shows such a section *cd* located along the face of the column. The bending moment about *cd* is that caused by the upward pressure q_u on the area to one side of the section, i.e., the area *abcd*. The reinforcement perpendicular to that section, i.e., the bars running in the long direction, is calculated from this bending moment. Likewise, the moment about section *ef* is caused by the pressure q_u on the area *befg*, and the reinforcement in the short direction, i.e., perpendicular to *ef*, is calculated for this bending moment. In footings that support reinforced concrete columns, these critical sections for bending are located at the faces of the loaded area, as shown.

In footings supporting steel columns, the sections *cd* and *ef* are located not at the edge of the steel base plate but halfway between the edge of the column and that of the steel base plate, according to ACI Code 15.4.2.

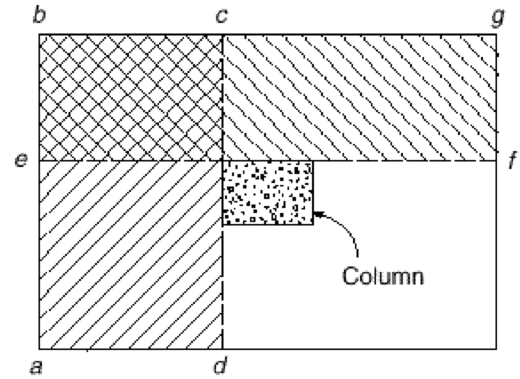


FIGURE 16.9
Critical sections for bending and bond.

In footings with *pedestals*, the width resisting compression in sections *cd* and *ef* is that of the pedestal; the corresponding depth is the sum of the thickness of pedestal and footing. Further sections parallel to *cd* and *ef* are passed at the edge of the pedestal, and the moments are determined in the same manner, to check the strength at locations in which the depth is that of the footing only.

For footings with relatively small pedestals, the latter are often discounted in moment and shear computation, and bending is checked at the face of the column, with width and depth equal to that of the footing proper.

In *square footings*, the reinforcement is uniformly distributed over the width of the footing in each of the two layers; i.e., the spacing of the bars is constant. The moments for which the two layers are designed are the same. However, the effective depth d for the upper layer is less by 1 bar diameter than that of the lower layer. Consequently, the required A_s is larger for the upper layer. Instead of using different spacings or different bar diameters in each of the two layers, it is customary to determine A_s based on average depth and to use the same arrangement of reinforcement for both layers.

In *rectangular footings*, the reinforcement *in the long direction* is again uniformly distributed over the pertinent (shorter) width. In locating the bars in the short direction, one has to consider that the support provided to the footing by the column is concentrated near the middle. Consequently, the curvature of the footing is sharpest, i.e., the moment per foot largest, immediately under the column, and it decreases in the long direction with increasing distance from the column. For this reason, a larger steel area per longitudinal foot is needed in the central portion than near the far ends of the footing. ACI Code 15.4.4, therefore, provides the following:

> For reinforcement in the short direction, a portion of the total reinforcement [given by Eq. (16.11)] shall be distributed uniformly over a band width (centered on the centerline of the column or pedestal) equal to the length of the short side of the footing. The remainder of the reinforcement required in the short direction shall be distributed uniformly outside the center band width of the footing.

$$\frac{\text{Reinforcement in band width}}{\text{Total reinforcement in short direction}} = \frac{2}{\cdot + 1} \tag{16.11}$$

where · is the ratio of the long side to the short side of the footing.

According to the ACI Code 10.5.4, the usual minimum flexural reinforcement ratios of Section 3.4d need not be applied to either slabs or footings. Instead, the minimum steel requirements for shrinkage and temperature crack control for structural slabs are to be imposed, as given in Table 13.2. The maximum spacing of bars in the direction of the span is reduced to the lesser of 3 times the footing thickness h and 18 in., rather than $5h$ as is normal for shrinkage and temperature steel. These requirements for minimum steel and maximum spacing are to be applied to mat foundations as well as individual footings.

Earlier editions of the ACI Code, through 1989, were somewhat ambiguous as to whether or not minimum steel requirements for flexural members were to be applied to slabs and footings. For slabs, the argument was presented that an overload would be distributed laterally and that a sudden failure is therefore less likely than for beams; therefore the usual requirement could be relaxed. Although that reasoning may apply to highly indeterminate building floors, the possibility for redistribution in a footing is much more limited. Because of this, and because of the importance of a footing to the safety of the structure, many engineers apply the minimum flexural reinforcement ratio of Eq. (3.41) to footings as well as beams. This seems prudent, and the following design examples use the more conservative minimum flexural steel requirements of Eq. (3.41).

The critical sections for development length of footing bars are the same as those for bending. Development length may also have to be checked at all vertical planes in which changes of section or of reinforcement occur, as at the edges of pedestals or where part of the reinforcement may be terminated.

EXAMPLE 16.2

Design of a square footing. A column 18 in. square, with $f'_c = 4$ ksi, reinforced with eight No. 8 (No. 25) bars of $f_y = 60$ ksi, supports a dead load of 225 kips and a live load of 175 kips. The allowable soil pressure q_a is 5 kips/ft^2. Design a square footing with base 5 ft below grade, using $f'_c = 4$ ksi and $f_y = 60$ ksi.

Solution. Since the space between the bottom of the footing and the surface will be occupied partly by concrete and partly by soil (fill), an average unit weight of 125 pcf will be assumed. The pressure of this material at the 5 ft depth is $5 \times 125 = 625$ psf, leaving a bearing pressure of $q_e = 5000 - 625 = 4375$ psf available to carry the column service load. Hence, the required footing area $A_{req} = (225 + 175)/4.375 = 91.5$ ft^2. A base 9 ft 6 in. square is selected, furnishing a footing area of 90.3 ft^2, which differs from the required area by about 1 percent.

For strength design, the upward pressure caused by the factored column loads is $q_u = (1.2 \times 225 + 1.6 \times 175)/9.5^2 = 6.10$ kips/ft^2.

The footing depth in square footings is usually determined based on two-way or punching shear on the critical perimeter *abcd* in Fig. 16.10. Trial calculations suggest $d = 19$ in. Hence, the length of the critical perimeter is

$$b_o = 4(18 + d) = 148 \text{ in.}$$

The shear force acting on this perimeter, being equal to the total upward pressure minus that acting within the perimeter *abcd*, is

$$V_{u1} = 6.10\left[9.5^2 - \left(\frac{37}{12}\right)^2\right] = 492 \text{ kips}$$

The corresponding nominal shear strength [Eq. (13.11*a*)] is

$$V_c = 4\sqrt{4000} \times 148 \times \frac{19}{1000} = 711 \text{ kips}$$

and

$$\phi V_c = 0.75 \times 711 = 534 \text{ kips}$$

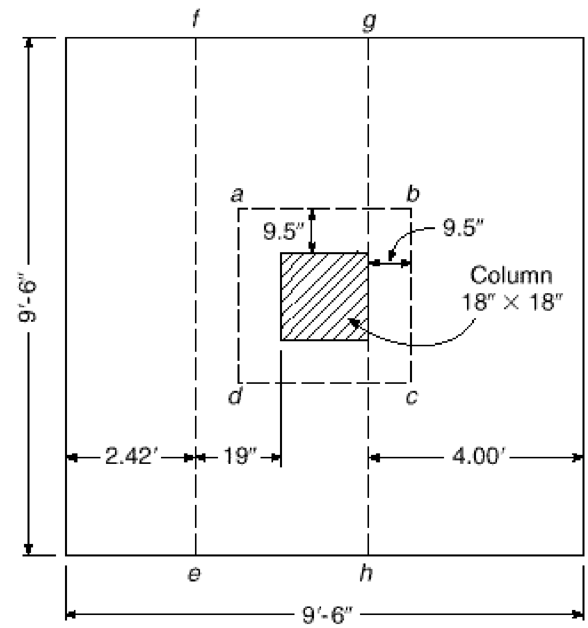


FIGURE 16.10
Critical sections for Example 16.2.

Since the design strength exceeds the factored shear V_{u1}, the depth $d = 19$ in. is adequate for punching shear. The selected value $d = 19$ in. will now be checked for one-way or beam shear on section *ef*. The factored shear force acting on that section is

$$V_{u2} = 6.10 \times 2.42 \times 9.5 = 140 \text{ kips}$$

and the nominal shear strength is

$$V_c = 2\sqrt{4000} \times 9.5 \times 12 \times \frac{19}{1000} = 274 \text{ kips}$$

The design shear strength $0.75 \times 274 = 205$ kips is larger than the factored shear V_{u2}, so that $d = 19$ in. is also adequate for one-way shear.

The bending moment on section *gh* of Fig. 16.10 is

$$M_u = 6.10 \times 9.5 \frac{4.0^2}{2} 12 = 5560 \text{ in-kips}$$

Because the depth required for shear is greatly in excess of that required for bending, the reinforcement ratio will be low and the corresponding depth of the rectangular stress block small. If $a = 2$ in., the required steel area is

$$A_s = \frac{5560}{0.90 \times 60(19 - 1)} = 5.72 \text{ in}^2$$

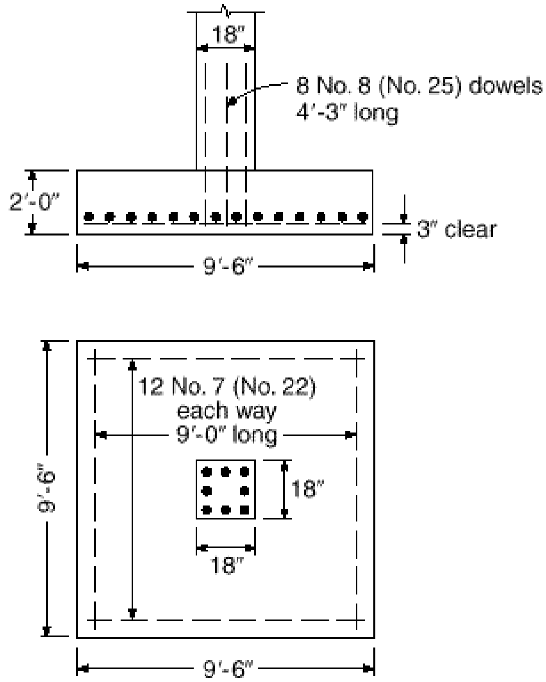
Checking the minimum reinforcement ratio using Eq. (3.41) results in

$$A_{s,min} = \frac{3\sqrt{4000}}{60{,}000} \times 114 \times 19 = 6.85 \text{ in}^2$$

but not less than

$$A_{s,min} = \frac{200}{60{,}000} \times 114 \times 19 = 7.22 \text{ in}^2$$

FIGURE 16.11
Footing in Example 16.2.



The controlling value of 7.22 in^2 is larger than the 5.72 in^2 calculated for bending. Twelve No. 7 (No. 22) bars furnishing 7.20 in^2 will be used in each direction. The required development length beyond section gh is found from Table A.10 to be 41 in., which is more than adequately met by the actual length of bars beyond section gh, namely $48 - 3 = 45$ in.

Checking for transfer of forces at the base of the column shows that the footing concrete, which has the same f_c' as the column concrete and for which the strength is enhanced according to Eq. (16.10), is clearly capable of carrying that part of the column load transmitted by the column concrete. The force in the column carried by the steel will be transmitted to the footing using dowels to match the column bars. These must extend into the footing the full development length in compression, which is found from Table A.11 of Appendix A to be 19 in. for No. 8 (No. 25) bars. This is accommodated in a footing with $d = 19$ in. Above the top surface of the footing, the No. 8 (No. 25) dowels must extend into the column that same development length, but not less than the requirement for a lapped splice in compression (see Section 5.11b). The minimum lap splice length for the No. 8 (No. 25) bars is $0.0005 \times 1.0 \times 60{,}000 = 30$ in., which is seen to control here. Thus the bars will be carried 30 in. into the column, requiring a total dowel length of 49 in. This will be rounded upward for practical reasons to 4.25 ft, as shown in Fig. 16.11. It is easily confirmed that the minimum dowel steel requirement of $0.005 \times 18 \times 18 = 1.62$ in^2 does not control here.

For concrete in contact with ground, a minimum cover of 3 in. is required for corrosion protection. With $d = 19$ in., measured from the top of the footing to the center of the upper layer of bars, the total thickness of the footing that is required to provide 3 in. clear cover for the lower steel layer is

$$h = 19 + 1.5 \times 1 + 3 = 23.5 \text{ in.}$$

The footing, with 24 in. thickness, is shown in Fig. 16.11.

16.7 Combined Footings

Spread footings that support more than one column or wall are known as *combined footings*. They can be divided into two categories: those that support two columns and those that support more than two (generally large numbers of) columns.

Examples of the first type, i.e., two-column footings, are shown in Fig. 16.1. In buildings where the allowable soil pressure is large enough for single footings to be adequate for most columns, two-column footings are seen to become necessary in two situations: (1) if columns are so close to the property line that single-column footings cannot be made without projecting beyond that line, and (2) if some adjacent columns are so close to each other that their footings would merge. Both situations are shown in Fig. 16.1.

When the bearing capacity of the subsoil is low so that large bearing areas become necessary, individual footings are replaced by *continuous strip footings* that support more than two columns and usually all columns in a row. Sometimes such strips are arranged in both directions, in which case a *grid foundation* is obtained, as shown in Fig. 16.12. Strip footings can be made to develop a much larger bearing area much more economically than can be done by single footings because the individual strips represent continuous beams whose moments are much smaller than the cantilever moments in large single footings that project far out from the column in all four directions.

In many cases, the strips are made to merge, resulting in a mat foundation, as shown in Fig. 16.13. That is, the foundation consists of a solid reinforced concrete slab under the entire building. In structural action, such a mat is very similar to a flat slab or a flat plate, upside down, i.e., loaded upward by the bearing pressure and downward by the concentrated column reactions. The mat foundation evidently develops the maximum available bearing area under the building. If the soil's capacity is so low that even this large bearing area is insufficient, some form of deep foundation, such as piles

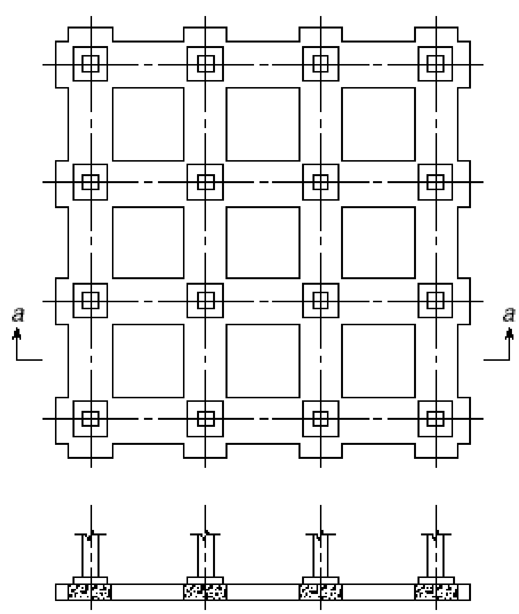


FIGURE 16.12
Grid foundation.

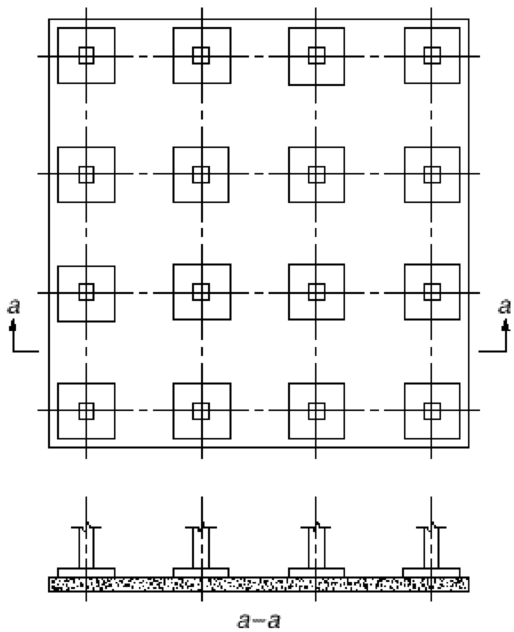


FIGURE 16.13
Mat foundation.

or caissons, must be used. These are discussed in texts on foundation design and fall outside the scope of the present volume.

Mat foundations may be designed with the column pedestals, as shown in Figs. 16.12 and 16.13, or without them, depending on whether or not they are necessary for shear strength and the development length of dowels.

Apart from developing large bearing areas, another advantage of strip and mat foundations is that their continuity and rigidity help in reducing differential settlements of individual columns relative to each other, which may otherwise be caused by local variations in the quality of subsoil, or other causes. For this purpose, continuous foundations are frequently used in situations where the superstructure or the type of occupancy provides unusual sensitivity to differential settlement.

Much useful and important design information pertaining to combined footings and mats is found in Refs. 16.10 and 16.11.

16.8 Two-Column Footings

It is desirable to design combined footings so that the centroid of the footing area coincides with the resultant of the two column loads. This produces uniform bearing pressure over the entire area and forestalls a tendency for the footings to tilt. In plan, such footings are rectangular, trapezoidal, or T shaped, the details of the shape being arranged to produce coincidence of centroid and resultant. The simple relationships shown in Fig. 16.14 facilitate the determination of the shape of the bearing area (from Ref. 16.8). In general, the distances m and n are given, the former being the distance from the center of the exterior column to the property line and the latter the distance from that column to the resultant of both column loads.

Another expedient that is used if a single footing cannot be centered under an exterior column is to place the exterior column footing eccentrically and to connect it

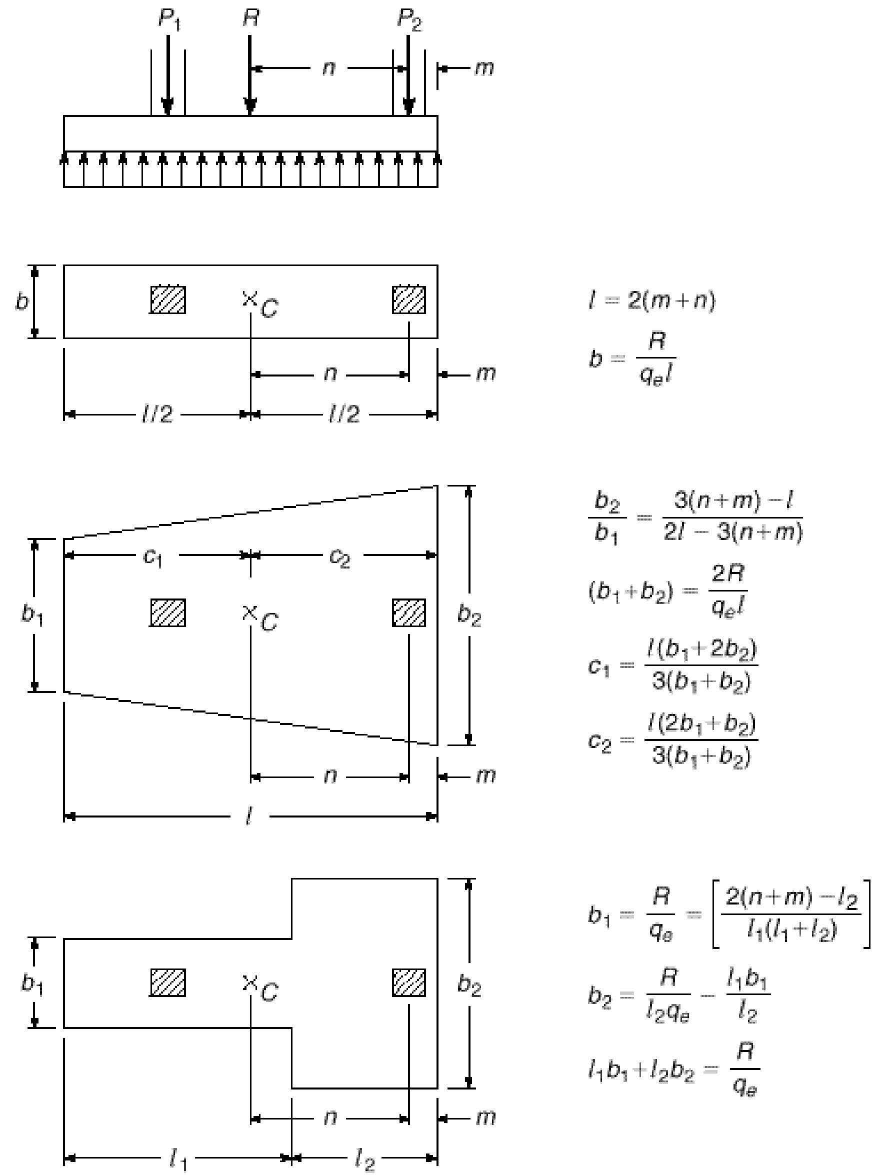


FIGURE 16.14
Two-column footing.
(*Adapted from Ref. 16.8.*)

with the nearest interior column footing by a beam or strap. This strap, being counterweighted by the interior column load, resists the tilting tendency of the eccentric exterior footing and equalizes the pressure under it. Such foundations are known as *strap, cantilever*, or *connected footings*.

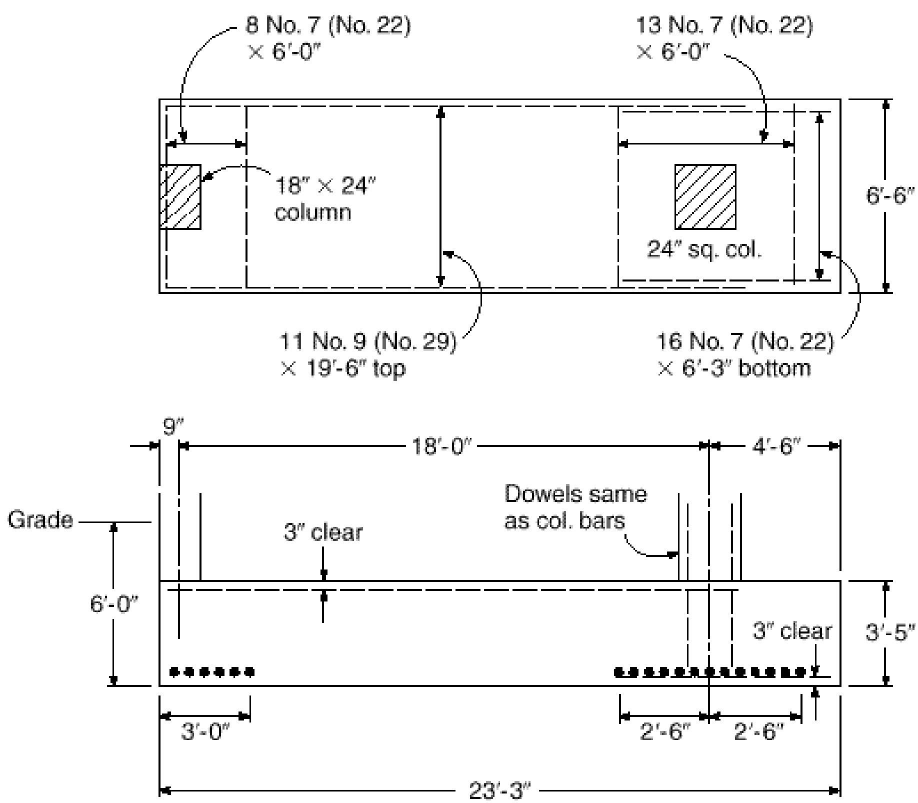
The two examples that follow demonstrate some of the peculiarities of the design of two-column footings.

EXAMPLE 16.3 **Design of a combined footing supporting one exterior and one interior column.** An exterior 24 × 18 in. column with $D = 170$ kips, $L = 130$ kips, and an interior 24 × 24 in. column with $D = 250$ kips, $L = 200$ kips are to be supported on a combined rectangular footing whose outer end cannot protrude beyond the outer face of the exterior column (see Fig. 16.1). The distance center to center of columns is 18 ft 0 in., and the allowable bearing pressure of the soil is 6000 psf. The bottom of the footing is 6 ft below grade, and a surcharge of 100 psf is specified on the surface. Design the footing for f'_c = 3000 psi, f_y = 60,000 psi.

Solution. The space between the bottom of the footing and the surface will be occupied partly by concrete (footing, concrete floor) and partly by backfill. An average unit weight of 125 pcf can be assumed. Hence, the effective portion of the allowable bearing pressure that is available for carrying the column loads is $q_e = q_a -$ (weight of fill and concrete + surcharge) = $6000 - (6 \times 125 + 100) = 5150$ psf. Then the required area A_{req} = sum of column loads/q_e = $750/5.15 = 145.5$ ft^2. The resultant of the column loads is located from the center of the exterior column a distance $450 \times 18/750 = 10.8$ ft. Hence, the length of the footing must be $2(10.8 + 0.75) = 23.1$ ft. A length of 23 ft 3 in. is selected. The required width is then $145.5/23.25 = 6.3$ ft. A width of 6 ft 6 in. is selected (see Fig. 16.15).

Longitudinally, the footing represents a beam, loaded from below, spanning between columns and cantilevering beyond the interior column. Since this beam is considerably wider than the columns, the column loads are distributed crosswise by transverse beams, one under each column. In the present relatively narrow and long footing, it will be found that the required minimum depth for the transverse beams is smaller than is required for the footing in the longitudinal direction. These "beams," therefore, are not really distinct members but merely represent transverse strips in the main body of the footing, reinforced so that they are capable of resisting the transverse bending moments and the corresponding shears. It then becomes necessary to decide how large the effective width of this transverse beam can be assumed to be. Obviously, the strip directly under the column does not deflect independently and is strengthened by the adjacent parts of the footing. The effective width of the

FIGURE 16.15
Combined footing in Example 16.3.



transverse beams is therefore evidently larger than that of the column. In the absence of definite rules for this case, or of research results on which to base such rules, the authors recommend conservatively that the load be assumed to spread outward from the column into the footing at a slope of 2 vertical to 1 horizontal. This means that the effective width of the transverse beam is assumed to be equal to the width of the column plus $d/2$ on either side of the column, d being the effective depth of the footing.

Strength design in longitudinal direction

The net upward pressure caused by the factored column loads is

$$q_u = \frac{1.2(170 + 250) + 1.6(130 + 200)}{23.25 \times 6.5} = 6.83 \text{ kips/ft}^2$$

Then the net upward pressure per linear foot in the longitudinal direction is $6.83 \times 6.5 = 44.4$ kips/ft. The maximum negative moment between the columns occurs at the section of zero shear. Let x be the distance from the outer edge of the exterior column to this section. Then (see Fig. 16.16)

$$V_u = 44{,}400x - 412{,}000 = 0$$

results in $x = 9.28$ ft. The moment at this section is

$$M_u = \left(44{,}400 \frac{9.28^2}{2} - 412{,}000(9.28 - 0.75)\right)12 = -19{,}230{,}000 \text{ in-lb}$$

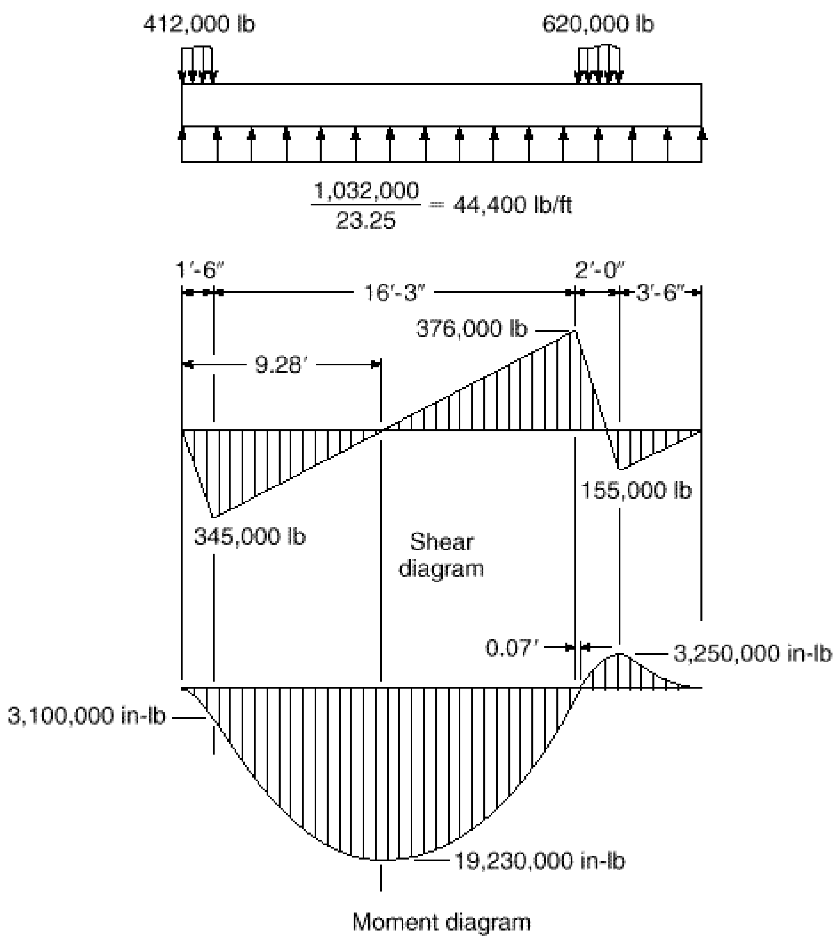


FIGURE 16.16
Moment and shear diagrams for footing in Example 16.3.

per linear foot of the transverse beam is 620,000/6.5 = 95,400 lb/ft. The moment at the edge of the interior column is

$$M_u = 95{,}400 \frac{2.25^2}{2} 12 = 2{,}900{,}000 \text{ in-lb}$$

Since the transverse bars are placed on top of the longitudinal bars (see Fig. 16.15), the actual value of d furnished is 37.5 − 1.0 = 36.5 in. The minimum required steel area controls; i.e.,

$$A_s = \frac{200}{60{,}000} 61.5 \times 36.5 = 7.48 \text{ in}^2$$

Thirteen No. 7 (No. 22) bars are selected and placed within the 61.5 in. effective width of the transverse beam.

Punching shear at the perimeter a distance $d/2$ from the column has been checked before. The critical section for regular flexural shear, at a distance d from the face of the column, lies beyond the edge of the footing, and therefore no further check on shear is needed.

The design of the transverse beam under the exterior column is the same as the design of that under the interior column, except that the effective width is 36.75 in. The details of the calculations are not shown. It can be easily checked that eight No. 7 (No. 22) bars, placed within the 36.75 in. effective width, satisfy all requirements. Design details are shown in Fig. 16.15.

EXAMPLE 16.4

Design of a strap footing. In a strap or connected footing, the exterior footing is placed eccentrically under its column so that it does not project beyond the property line. Such an eccentric position would result in a strongly uneven distribution of bearing pressure, which could lead to tilting of the footing. To counteract this eccentricity, the footing is connected by a beam or strap to the nearest interior footing.

Both footings are so proportioned that under service load the pressure under each of them is uniform and the same under both footings. To achieve this, it is necessary, as in other combined footings, that the centroid of the combined area for the two footings coincide with the resultant of the column loads. The resulting forces are shown schematically in Fig. 16.17. They consist of the loads P_e and P_i of the exterior and interior columns, respectively, and of the net upward pressure q, which is uniform and equal under both footings. The resultants R_e and R_i of these upward pressures are also shown. Since the interior footing is concentric with the interior column, R_i and P_i are collinear. This is not the case for the exterior forces R_e and P_e where the resulting couple just balances the effect of the eccentricity of the column relative to the center of the footing. The strap proper is generally constructed

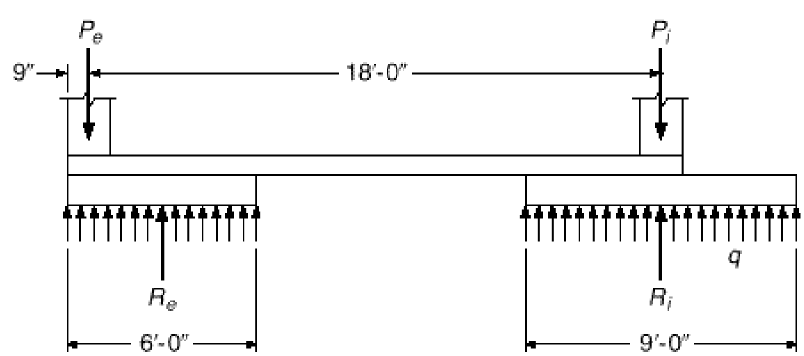


FIGURE 16.17
Forces and reactions on the strap footing in Example 16.4.

The moment at the right edge of the interior column is

$$M_u = 44{,}400 \frac{3.5^2}{2} 12 = 3{,}260{,}000 \text{ in-lb}$$

and the details of the moment diagram are as shown in Fig. 16.16. Try $d = 37.5$ in.

From the shear diagram in Fig. 16.16, it is seen that the critical section for flexural shear occurs at a distance d to the left of the left face of the interior column. At that point, the factored shear is

$$V_u = 376{,}000 - \frac{37.5}{12} 44{,}400 = 237{,}000 \text{ lb}$$

and the design shear strength

$$\phi V_c = 0.75 \times 2\sqrt{3000} \times 78 \times 37.5 = 240{,}000 \text{ lb} > V_u$$

indicating that $d = 37.5$ in. is adequate.

Additionally, as in single footings, punching shear should be checked on a perimeter section a distance $d/2$ around the column, on which the nominal shear stress $v_c = 4\sqrt{3000} = 220$ psi. Of the two columns, the exterior one with a three-sided perimeter a distance $d/2$ from the column is more critical in regard to this punching shear. The perimeter is

$$b_o = 2\left(1.5 + \frac{37.5/12}{2}\right) + \left(2.0 + \frac{37.5}{12}\right) = 11.25 \text{ ft}$$

and the shear force, being the column load minus the soil pressure within the perimeter, is

$$V_u = 412{,}000 - 3.06 \times 5.12(6830) = 305{,}000 \text{ lb}$$

On the other hand, the design shear strength on the perimeter section is

$$\phi V_c = 0.75 \times 220 \times 11.25 \times 12 \times 37.5 = 835{,}000 \text{ lb}$$

considerably larger than the factored shear V_u.

With $d = 37.5$ in., and with 3.5 in. cover from the center of the bars to the top surface of the footing, the total thickness is 41 in.

To determine the required steel area, $M_u/\phi bd^2 = 19{,}230{,}000/(0.9 \times 78 \times 37.5^2) = 195$ is used to enter Graph A.1*b* of Appendix A. For this value, the curve 60/3 gives the reinforcement ratio $\rho = 0.0035$. The required steel area is $A_s = 0.0035 \times 37.5 \times 78 = 10.3$ in^2. Eleven No. 9 (No. 29) bars furnish 11.00 in^2. The required development length is found to be 6.7 ft. From Fig. 16.16, the distance from the point of maximum moment to the nearer left end of the bars is seen to be $9.30 - \frac{3}{12} = 9.05$ ft, much larger than the required minimum development length. The selected reinforcement is therefore adequate for both bending and bond.

For the portion of the longitudinal beam that cantilevers beyond the interior column, the minimum required steel area controls. Here,

$$A_{s,min} = \frac{3\sqrt{3000}}{60{,}000} \times 78 \times 37.5 = 8.01 \text{ in}^2$$

but not less than

$$A_{s,min} = \frac{200}{60{,}000} \times 78 \times 37.5 = 9.75 \text{ in}^2$$

Sixteen No. 7 (No. 22) bars with $A_s = 9.62$ in^2 are selected; their development length is computed and for bottom bars is found satisfactory.

Design of transverse beam under interior column

The width of the transverse beam under the interior column can now be established as previously suggested and is $24 + 2(d/2) = 24 + 2 \times 18.75 = 61.5$ in. The net upward load

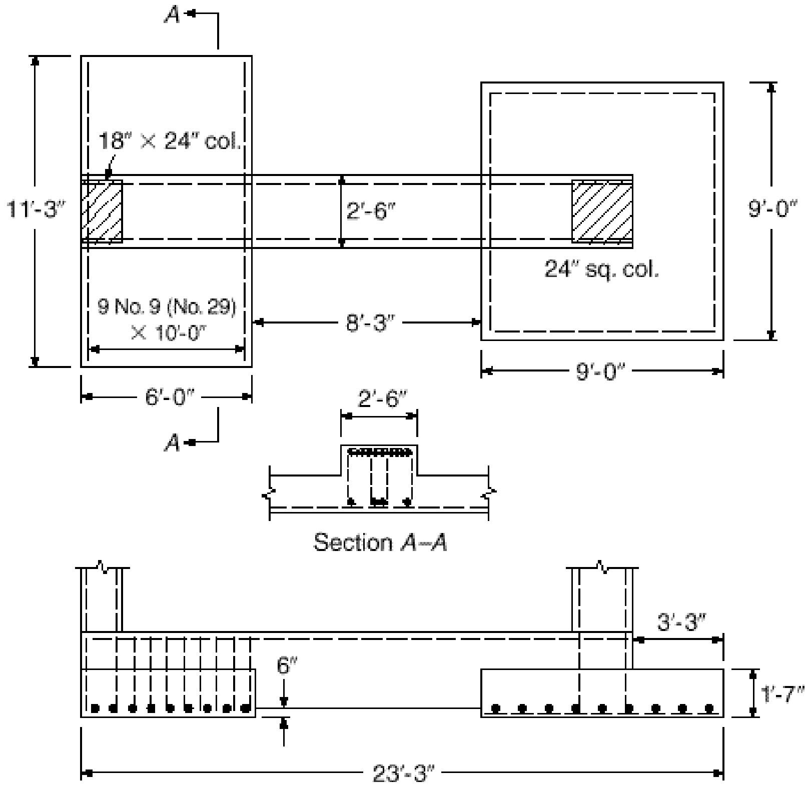


FIGURE 16.18
Strap footing in Example 16.4.

so that it will not bear on the soil. This can be achieved by providing formwork not only for the sides but also for the bottom face and by withdrawing it before backfilling.

To illustrate this design, the columns in Example 16.3 will now be supported on a strap footing. Its general shape, plus dimensions as determined only subsequently by calculations, is seen in Fig. 16.18. With an allowable bearing pressure of $q_a = 6.0$ kips/ft^2 and a depth of 6 ft to the bottom of the footing as before, the bearing pressure available for carrying the external loads applied to the footing is $q_e = 5.15$ kips/ft^2, as in Example 16.3. These external loads, for the strap footing, consist of the column loads and of the weight plus fill and surcharge of that part of the strap that is located between the footings. (The portion of the strap located directly on top of the footing displaces a corresponding amount of fill and therefore is already accounted for in the determination of the available bearing pressure q.) If the bottom of the strap is 6 in. above the bottom of the footings to prevent bearing on soil, the total depth to grade is 5.5 ft. If the strap width is estimated to be 2.5 ft, its estimated weight plus fill and surcharge is $2.5 \times 5.5 \times 0.125 + 0.100 \times 2.5 = 2$ kips/ft. If the gap between footings is estimated to be 8 ft, the total weight of the strap is 16 kips. Hence, for purposes of determining the required footing area, 8 kips will be added to the dead load of each column. The required total area of both footings is then $(750 + 16)/5.15 = 149$ ft^2. The distance of the resultant of the two column loads plus the strap load from the axis of the exterior column, with sufficient accuracy, is $458 \times 18/766 = 10.75$ ft, or 11.50 ft from the outer edge, almost identical to that calculated for Example 16.3. Trial calculations show that a rectangular footing 6 ft 0 in. × 11 ft 3 in. under the exterior column and a square footing 9 × 9 ft under the interior column have a combined area of 149 ft^2 and a distance from the outer edge to the centroid of the combined areas of $(6 \times 11.25 \times 3 + 9 \times 9 \times 18.75)/149 = 11.55$ ft, which is almost exactly equal to the previously calculated distance to the resultant of the external forces.

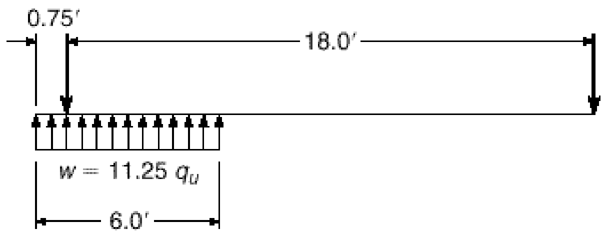


FIGURE 16.19
Forces acting on strap in Example 16.4.

For *strength calculations*, the bearing pressure caused by the factored external loads, including that of the strap with its fill and surcharge, is

$$q_u = \frac{1.2 \cdot 170 + 250 + 16 \cdot + 1.6 \cdot 130 + 200 \cdot}{149} = 7.06 \text{ kips} \cdot \text{ft}^2$$

Design of footings

The exterior footing performs exactly like a wall footing with a length of 6 ft. Even though the column is located at its edge, the balancing action of the strap results in uniform bearing pressure, the downward load being transmitted to the footing uniformly by the strap. Hence, the design is carried out exactly as it is for a wall footing (see Section 16.5).

The interior footing, even though it merges in part with the strap, can safely be designed as an independent, square single-column footing (see Section 16.6). The main difference is that, because of the presence of the strap, punching shear cannot occur along the truncated pyramid surface shown in Fig. 16.6. For this reason, two-way or punching shear, according to Eq. (16.7), should be checked along a perimeter section located at a distance $d \cdot 2$ outward from the longitudinal edges of the strap and from the free face of the column, d being the effective depth of the footing. Flexural or one-way shear, as usual, is checked at a section a distance d from the face of the column.

Design of strap

Even though the strap is in fact monolithic with the interior footing, the effect on the strap of the soil pressure under this footing can safely be neglected because the footing has been designed to withstand the entire upward pressure as if the strap were absent. In contrast, because the exterior footing has been designed as a wall footing that receives its load from the strap, the upward pressure from the wall footing becomes a load that must be resisted by the strap. With this simplification of the actually somewhat more complex situation, the strap represents a single-span beam loaded upward by the bearing pressure under the exterior footing and supported by downward reactions at the centerlines of the two columns (Fig. 16.19). A width of 30 in. is selected. For a column width of 24 in., this permits beam and column bars to be placed without interference where the two members meet and allows the column forms to be supported on the top surface of the strap. The maximum moment, as determined by equating the shear force to zero, occurs close to the inner edge of the exterior footing. Shear forces are large in the vicinity of the exterior column. Stirrup design is completed using a strut-and-tie model. The footing is drawn approximately to scale in Fig. 16.18, which also shows the general arrangement of the reinforcement in the footings and the strap.

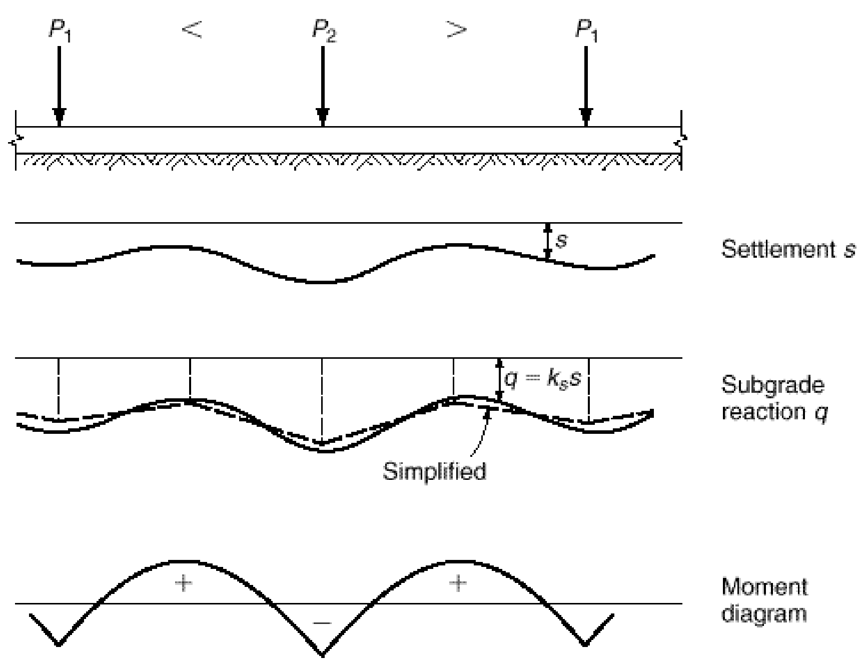
16.9 STRIP, GRID, AND MAT FOUNDATIONS

As mentioned in Section 16.7, continuous foundations are often used to support heavily loaded columns, especially when a structure is located on relatively weak or uneven soil. The foundation may consist of a *continuous strip footing* supporting all columns in a given row, or of two sets of such strip footings intersecting at right angles so that they form one continuous *grid foundation* (Fig. 16.12). For even larger loads or weaker soils, the strips are made to merge, resulting in a *mat foundation* (Fig. 16.13).

For the design of such continuous foundations, it is essential that reasonably realistic assumptions be made regarding the distribution of bearing pressures that act as upward loads on the foundation. For compressible soils, it can be assumed, as a first approximation, that the deformation or settlement of the soil at a given location and the bearing pressure at that location are proportional to each other. If columns are spaced at moderate distances and if the strip, grid, or mat foundation is quite rigid, the settlements in all portions of the foundation will be substantially the same. This means that the bearing pressure, also known as *subgrade reaction*, will be the same, provided that the centroid of the foundation coincides with the resultant of the loads. If they do not coincide, then for such rigid foundations the subgrade reaction can be assumed to vary linearly. Bearing pressures can be calculated based on statics, as discussed for single footings (see Fig. 16.3). In this case, all loads, the downward column loads as well as the upward-bearing pressures, are known. Hence, moments and shear forces in the foundation can be found by statics alone. Once these are determined, the design of strip and grid foundations is similar to that of inverted continuous beams and that of mat foundations to that of inverted flat slabs or plates.

On the other hand, if the foundation is relatively flexible and the column spacing large, settlements will no longer be uniform or linear. For one thing, the more heavily loaded columns will cause larger settlements, and thereby larger subgrade reactions, than the lighter ones. Also, since the continuous strip or slab midway between columns will deflect upward relative to the nearby columns, the soil settlement, and thereby the subgrade reaction, will be smaller midway between columns than directly at the columns. This is shown schematically for a strip footing in Fig. 16.20; the subgrade reaction can no longer be assumed to be uniform. Mat foundations

FIGURE 16.20
Strip footing. (*Adapted from Ref. 16.8.*)



likewise require different approaches, depending on whether or not they can be assumed to be rigid when calculating the soil reaction.

Criteria have been established as a measure of the relative stiffness of the structure versus the stiffness of the soil (Refs. 16.10 and 16.11). If the relative stiffness is low, the foundation should be designed as a flexible member with a nonlinear upward reaction from the soil. For strip footings, a reasonably accurate but fairly complex analysis can be done using the theory of beams on elastic foundations (Ref. 16.12). Kramrisch (Ref. 16.8) has suggested simplified procedures, based on the assumption that contact pressures vary linearly between load points, as shown in Fig. 16.20.

For nonrigid mat foundations, great advances in analysis have been made using finite element methods, which can account specifically for the stiffnesses of both the structure and the soil. There are a large number of commercially available programs (e.g., PCAMats, Portland Cement Association, Skokie, Illinois) based on the finite element method, permitting quick modeling and analysis of combined footings, strip footings, and mat foundations.

16.10 Pile Caps

If the bearing capacity of the upper soil layers is insufficient for a spread foundation, but firmer strata are available at greater depth, piles are used to transfer the loads to these deeper strata. Piles are generally arranged in groups or clusters, one under each column. The group is capped by a spread footing or cap that distributes the column load to all piles in the group. These pile caps are in most ways very similar to footings on soil, except for two features. For one, reactions on caps act as concentrated loads at the individual piles, rather than as distributed pressures. For another, if the total of all pile reactions in a cluster is divided by the area of the footing to obtain an equivalent uniform pressure (for purposes of comparison only), it is found that this equivalent pressure is considerably higher in pile caps than for spread footings. This means that moments, and particularly shears, are also correspondingly larger, which requires greater footing depths than for a spread footing of similar horizontal dimensions. To spread the load evenly to all piles, it is in any event advisable to provide ample rigidity, i.e., depth, for pile caps.

Allowable bearing capacities of piles R_a are obtained from soil exploration, pile-driving energy, and test loadings, and their determination is not within the scope of the present book (see Refs. 16.1 to 16.4). As in spread footings, the effective portion of R_a available to resist the unfactored column loads is the allowable pile reaction less the weight of footing, backfill, and surcharge per pile. That is,

$$R_e = R_a - W_f \qquad (16.12)$$

where W_f is the total weight of footing, fill, and surcharge divided by the number of piles.

Once the available or effective pile reaction R_e is determined, the number of piles in a concentrically loaded cluster is the integer next larger than

$$n = \frac{D + L}{R_e}$$

As far as the effects of wind, earthquake moments at the foot of the columns, and safety against overturning are concerned, design considerations are the same as described in Section 16.4 for spread footings. These effects generally produce an eccentrically loaded pile cluster in which different piles carry different loads. The number and location of piles in such a cluster are determined by successive approximation based on the requirement that the load on the most heavily loaded pile must not exceed the allowable pile reaction R_a. With a linear distribution of pile loads due to bending, the maximum pile reaction is

$$R_{max} = \frac{P}{n} + \frac{M}{I_{pg} \cdot c} \tag{16.13}$$

where P is the maximum load (including weight of cap, backfill, etc.) and M the moment to be resisted by the pile group, both referred to the bottom of the cap; I_{pg} is the moment of inertia of the entire pile group about the centroidal axis about which bending occurs; and c is the distance from that axis to the extreme pile. $I_{pg} = \Sigma_1^n \cdot 1 \times y_i^2 \cdot$; i.e., it is the moment of inertia of n piles, each counting as one unit and located a distance y_i from the described centroidal axis.

Piles are generally arranged in tight patterns, which minimizes the cost of the caps, but they cannot be placed closer than conditions of driving and of undisturbed carrying capacity will permit. A spacing of about 3 times the butt (top) diameter of the pile but no less than 2 ft 6 in. is customary. Commonly, piles with allowable reactions of 30 to 70 tons are spaced at 3 ft 0 in. (Ref. 16.8).

The *design* of footings on piles is similar to that of single-column footings. One approach is to design the cap for the pile reactions calculated for the factored column loads. For a concentrically loaded cluster, this would give $R_u = (1.2D + 1.6L) \cdot n$. However, since the number of piles was taken as the next larger integral according to Eq. (16.13), determining R_u in this manner can lead to a design where the strength of the cap is less than the capacity of the pile group. It is therefore recommended that the pile reaction for strength design be taken as

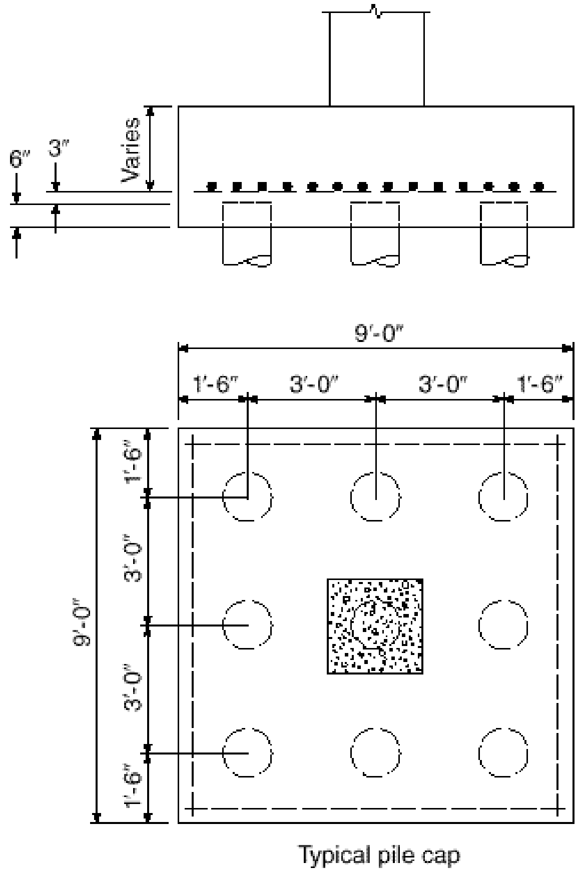
$$R_u = R_e \times \text{average load factor} \tag{16.14}$$

where the average load factor $= (1.2D + 1.6L) \cdot (D + L)$. In this manner, the cap is designed to be capable of developing the full allowable capacity of the pile group. Details of a typical pile cap are shown in Fig. 16.21.

As in single-column spread footings, the depth of the pile cap is usually governed by shear. ACI Code 15.5.3 specifies that, when the distance between the axis of a pile and the axis of a column is more than 2 times the distance from the top of the pile cap and the top of the pile, shear design must follow the procedures for flat slabs and footings, as described in Section 16.6a. For closer spacings between piles and columns, the Code specifies either the use of the procedures described in Section 16.6a or the use of a three-dimensional strut-and-tie model (ACI Code Appendix A) based on the principles described in Chapter 10. In the latter case, the struts must be designed as bottle-shaped without transverse reinforcement (Table 10.1) because of the difficulty of providing such reinforcement in a pile cap. The use of strut-and-tie models to design pile caps is discussed in Ref. 16.13.

When the procedures for flat slabs and footings are used, both punching or two-way shear and flexural or one-way shear need to be considered. The critical sections are the same as given in Section 16.6a. The difference is that shear in caps is caused by concentrated pile reactions rather than by distributed bearing pressures. This poses the question of how to calculate shear if the critical section intersects the circumference of one or more piles. For this case ACI Code 15.5.4 accounts for the fact that a

FIGURE 16.21
Typical single-column footing on piles (pile cap).



pile reaction is not really a point load, but is distributed over the pile-bearing area. Correspondingly, for piles with diameters d_p, it stipulates as follows:

> Computation of shear on any section through a footing on piles shall be in accordance with the following:
> (*a*) The entire reaction from any pile whose center is located $d_p/2$ or more outside this section shall be considered as producing shear on that section.
> (*b*) The reaction from any pile whose center is located $d_p/2$ or more inside the section shall be considered as producing no shear on that section.
> (*c*) For intermediate positions of the pile center, the portion of the pile reaction to be considered as producing shear on the section shall be based on straight-line interpolation between the full value at $d_p/2$ outside the section and zero at $d_p/2$ inside the section.

In addition to checking two-way and one-way shear, as just discussed, punching shear must also be investigated for the individual pile. Particularly in caps on a small number of heavily loaded piles, it is this possibility of a pile punching upward through the cap that may govern the required depth. The critical perimeter for this action, again, is located at a distance $d/2$ outside the upper edge of the pile. However, for relatively deep caps and closely spaced piles, critical perimeters around adjacent piles may overlap. In this case, fracture, if any, would undoubtedly occur along an outward-

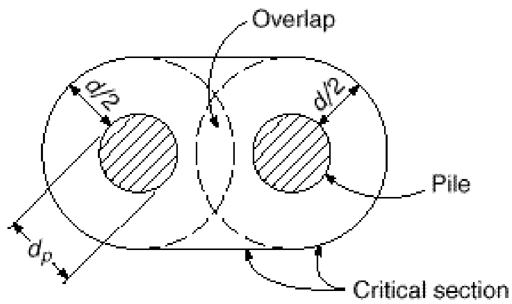


FIGURE 16.22
Critical section for punching shear with closely spaced piles.

slanting surface around both adjacent piles. For such situations the critical perimeter is so located that its length is a minimum, as shown for two adjacent piles in Fig. 16.22.

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PROBLEMS

16.1. A continuous strip footing is to be located concentrically under a 12 in. wall that delivers service loads $D = 25{,}000$ lb/ft and $L = 15{,}000$ lb/ft to the top of the footing. The bottom of the footing will be 4 ft below the final ground surface. The soil has a density of 120 pcf and allowable bearing capacity of 8000 psf. Material strengths are $f'_c = 3000$ psi and $f_y = 60{,}000$ psi. Find (*a*) the required width of the footing, (*b*) the required effective and total depths, based on shear, and (*c*) the required flexural steel area.

16.2. An interior column for a tall concrete structure carries total service loads $D = 500$ kips and $L = 514$ kips. The column is 22×22 in. in cross section and is reinforced with twelve No. 11 (No. 36) bars centered 3 in. from the column faces (equal number of bars each face). For the column, $f'_c = 4000$ psi and $f_y = 60{,}000$ psi. The column will be supported on a square footing, with the bottom of the footing 6 ft below grade. Design the footing, determining all concrete dimensions and amount and placement of all reinforcement, including length and placement of dowel steel. No shear reinforcement is permitted. The allowable soil-bearing pressure is 8000 psf. Material strengths for the footing are $f'_c = 3000$ psi and $f_y = 60{,}000$ psi.

16.3. Two interior columns for a high-rise concrete structure are spaced 15 ft apart, and each carries service loads $D = 500$ kips and $L = 514$ kips. The columns are to be 22 in. square in cross section, and will each be reinforced with twelve No. 11 (No. 36) bars centered 3 in. from the column faces, with an equal number of bars at each face. For the columns, $f'_c = 4000$ psi and $f_y = 60{,}000$ psi. The columns will be supported on a rectangular combined footing with a long-side dimension twice that of the short side. The allowable soil-bearing pressure is 8000 psf. The bottom of the footing will be 6 ft below grade. Design the footing for these columns, using $f'_c = 3000$ psi and $f_y = 60{,}000$ psi. Specify all reinforcement, including length and placement of footing bars and dowel steel.

16.4. A pile cap is to be designed to distribute a concentric force from a single column to a nine-pile group, with geometry as shown in Fig. 16.21. The cap will carry calculated dead load and service live load of 280 kips and 570 kips, respectively, from a 19 in. square concrete column reinforced with six No. 14 (No. 43) bars. The permissible load per pile at service load is 100 kips, and the pile diameter is 16 in. Find the required effective and total depths of the pile cap and the required reinforcement. Check all relevant aspects of the design, including the development length for the reinforcement and transfer of forces at the base of the column. Material strengths for the column are $f'_c = 4000$ psi and $f_y = 60{,}000$ psi, and for the pile cap are $f'_c = 3000$ psi and $f_y = 60{,}000$ psi.

16.5. Complete the design of the strap footing in Example 16.4 and determine all dimensions and reinforcement. Compare the total volume of concrete in the strap footing in Example 16.4 with that of the rectangular combined footing in Example 16.3. It will be found that the strap footing is significantly more economical in terms of material (although forming would be more costly). This economy of material would increase with increasing distance between the columns.