17 Retaining Walls

17.1 Function and Types of Retaining Walls

Retaining walls are used to hold back masses of earth or other loose material where conditions make it impossible to let those masses assume their natural slopes. Such conditions occur when the width of an excavation, cut, or embankment is restricted by conditions of ownership, use of the structure, or economy. For example, in railway or highway construction the width of the right of way is fixed, and the cut or embankment must be contained within that width. Similarly, the basement walls of buildings must be located within the property and must retain the soil surrounding the basement.

Free-standing retaining walls, as distinct from those that form parts of structures, such as basement walls, are of various types, the most common of which are shown in Fig. 17.1. The gravity wall (Fig. 17.1*a*) retains the earth entirely by its own weight and generally contains no reinforcement. The reinforced concrete cantilever wall (Fig. 17.1*b*) consists of the vertical arm that retains the earth and is held in position by a footing or base slab. In this case, the weight of the fill on top of the heel, in addition to the weight of the wall, contributes to the stability of the structure. Since the arm represents a vertical cantilever, its required thickness increases rapidly with increasing height. To reduce the bending moments in vertical walls of great height, counterforts are used spaced at distances from each other equal to or slightly larger than one-half of the height (Fig. 17.1*c*). Property rights or other restrictions sometimes make it necessary to place the wall at the forward edge of the base slab, i.e., to omit the toe. Whenever it is possible, toe extensions of one-third to one-fourth of the width of the base provide a more economical solution.

Which of the three types of walls is appropriate in a given case depends on a variety of conditions, such as local availability and price of construction materials and property rights. In general, gravity walls are economical only for relatively low walls, possibly up to about 10 ft. Cantilever walls are economical for heights from 10 to 20 ft, while counterforts are used for greater heights.

17.2 Earth Pressure

In terms of physical behavior, soils and other granular masses occupy a position intermediate between liquids and solids. If sand is poured from a dump truck, it flows, but, unlike a frictionless liquid, it will not assume a horizontal surface. It maintains itself in a stable heap with sides reaching an *angle of repose*, the tangent of which is roughly equal to the coefficient of intergranular friction. If a pit is dug in clay soil, its sides can

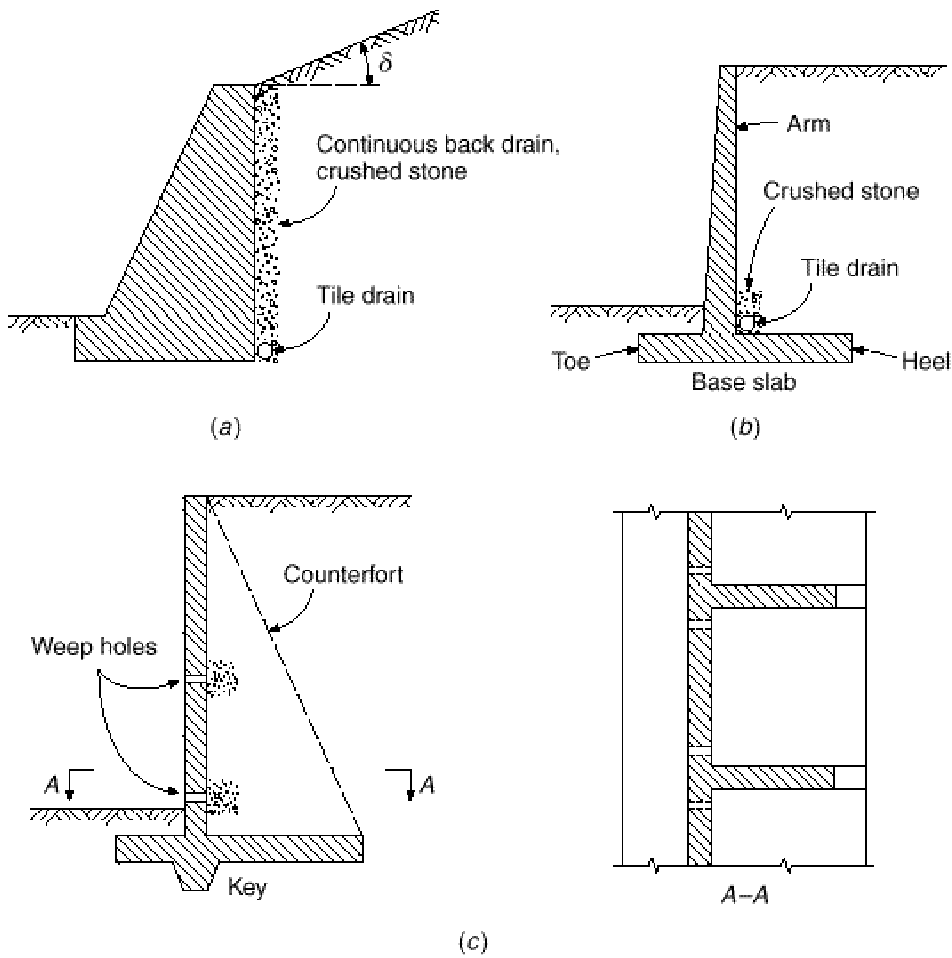


FIGURE 17.1
Types of retaining walls and back drains: (*a*) gravity wall; (*b*) cantilever wall; (*c*) counterfort wall.

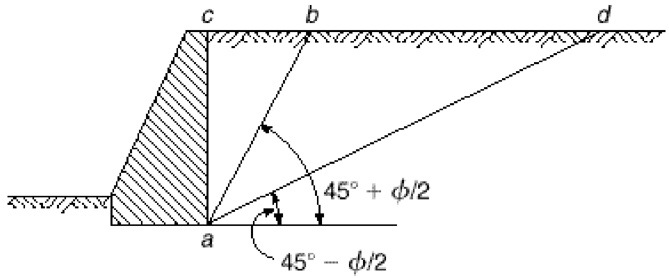
usually be made vertical over considerable depths without support; i.e., the clay will behave like a solid and will retain the shape it is given. If, however, the pit is flooded, the sides will give way, and, in many cases, the saturated clay will be converted nearly into a true liquid. The clay is capable of maintaining its shape by means of its internal cohesion, but flooding reduces that cohesion greatly, often to zero.

If a wall is built in contact with a solid, such as a rock face, no pressure is exerted on it. If, on the other hand, a wall retains a liquid, as in a reservoir, it is subject at any level to the hydrostatic pressure $w_w h$, where w_w is the unit weight of the liquid and h is the distance from the surface. If a vertical wall retains soil, the earth pressure similarly increases proportionally to the depth, but its magnitude is

$$p_h = K_0 wh \tag{17.1}$$

where w is the unit weight of the soil and K_0 is a constant known as the *coefficient of earth pressure at rest*. The value of K_0 depends not only on the nature of the backfill but also on the method of depositing and compacting it. It has been determined experimentally that, for uncompacted noncohesive soils such as sands and gravels, K_0 ranges between 0.4 and 0.5, while it may be as high as 0.8 for the same soils in a highly compacted state (Refs. 17.1 through 17.3). For cohesive soils, K_0 may be on the order of 0.7 to 1.0. Clean sands and gravels are considered superior to all other soils

FIGURE 17.2
Basis of active and passive earth pressure determination.



because they are free-draining and are not susceptible to frost action and because they do not become less stable with the passage of time. For this reason, noncohesive backfills are usually specified.

Usually, walls move slightly under the action of the earth pressure. Since walls are constructed of elastic material, they deflect under the action of the pressure, and because they generally rest on compressible soils, they tilt and shift away from the fill. (For this reason, the wall is often constructed with a slight batter toward the fill on the exposed face so that, if and when such tilting takes place, it does not appear evident to the observer.) Even if this movement at the top of the wall is only a fraction of a percent of the wall height ($\frac{1}{2}$ to $\frac{1}{10}$ percent according to Ref. 17.2), the rest pressure is materially decreased by it.

If the wall moves away from the fill, a sliding plane *ab* (Fig. 17.2) forms in the soil mass, and the wedge *abc*, sliding along that plane, exerts pressure against the wall. Here the angle · is known as the *angle of internal friction*; i.e., its tangent is equal to the coefficient of intergranular friction, which can be determined by appropriate laboratory tests. The corresponding pressure is known as the *active earth pressure*. If, on the other hand, the wall is pushed against the fill, a sliding plane *ad* is formed, and the wedge *acd* is pushed upward by the wall along that plane. The pressure that this larger wedge exerts against the wall is known as the *passive earth pressure*. (This latter case will also occur at the left face of the gravity wall in Fig. 17.1*a* when this wall yields slightly to the left under the pressure of the fill.)

The magnitude of these pressures has been analyzed by Rankine, Coulomb, and others. If the soil surface makes an angle · with the horizontal (Fig. 17.1*a*), then, according to Rankine, the *coefficient for active earth pressure* is

$$K_a = \cos \cdot \; \frac{\cos \cdot - \sqrt{\cos^2 \cdot - \cos^2 \cdot}}{\cos \cdot + \sqrt{\cos^2 \cdot - \cos^2 \cdot}} \tag{17.2}$$

and the *coefficient for passive pressure* is

$$K_p = \cos \cdot \; \frac{\cos \cdot + \sqrt{\cos^2 \cdot - \cos^2 \cdot}}{\cos \cdot - \sqrt{\cos^2 \cdot - \cos^2 \cdot}} \tag{17.3}$$

K_a and K_p replace K_0 in Eq. (17.1) to determine soil pressure p_h under active and passive conditions, respectively.

For the frequent case of a horizontal surface, that is, · = 0 (Fig. 17.2), for active pressure,

$$K_{ah} = \frac{1 - \sin \cdot}{1 + \sin \cdot} \tag{17.4}$$

and for passive pressure,

$$K_{ph} = \frac{1 + \sin \cdot}{1 - \sin \cdot} \tag{17.5}$$

Rankine's theory is valid only for noncohesive soils such as sand and gravel but, with corresponding adjustments, can also be used successfully for cohesive clay soils.

From Eqs. (17.1) to (17.5), it is seen that the earth pressure at a given depth h depends on the inclination of the surface · , the unit weight w, and the angle of friction · . The first two of these are easily determined, while little agreement has yet been reached as to the proper values of · . For the ideal case of a dry, noncohesive fill, · could be determined by laboratory tests and then used in the formulas. This is impossible for clays, only part of whose resistance is furnished by intergranular friction, while the rest is due to internal cohesion. For this reason, their actual · values are often increased by an arbitrary amount to account implicitly for the added cohesion. However, this is often unsafe since, as was shown by the example of the flooded pit, cohesion may vanish almost completely due to saturation and inundation.

In addition, fills behind retaining walls are rarely uniform, and, what is more important, they are rarely dry. Proper drainage of the fill is vitally important to reduce pressures (see Section 17.6), but even in a well-drained fill, the pressure will temporarily increase during heavy storms or sudden thaws. This is due to the fact that even though the drainage may successfully remove the water as fast as it appears, its movement through the fill toward the drains causes additional pressure (seepage pressure). In addition, frost action and other influences may temporarily increase its value over that of the theoretical active pressure. Many walls that were designed without regard to these factors have failed, been displaced, or cracked.

It is good practice, therefore, to select conservative values for · , considerably smaller than the actual test values, in all cases except where extraordinary and usually expensive precautions are taken to keep the fill dry under all conditions. An example of recommended earth-pressure values, which are quite conservative, though based on extensive research and practical experience, can be found in Ref. 17.2. Less conservative values are often used in practical designs, but these should be employed (1) with caution in view of the fact that occasional trouble has been encountered with walls so designed and (2) preferably with the advice of a geotechnical engineer.

Table 17.1 gives representative values for w and · often used in engineering practice. (Note that the · values do not account for probable additional pressures due

TABLE 17.1
Unit weights · , effective angles of internal friction · , and coefficients of friction with concrete ·

Soil	Unit Weight · , pcf	· , degrees	·
1. Sand or gravel without fine particles, highly permeable	110–120	33–40	0.5–0.6
2. Sand or gravel with silt mixture, low permeability	120–130	25–35	0.4–0.5
3. Silty sand, sand and gravel with high clay content	110–120	23–30	0.3–0.4
4. Medium or stiff clay	100–120	25–35[a]	0.2–0.4
5. Soft clay, silt	90–110	20–25[a]	0.2–0.3

[a] For saturated conditions, · for clays and silts may be close to zero.

to porewater, seepage, frost, etc.) The table also contains values for the coefficient of friction f between concrete and various soils. The values of ϕ for soils 3 through 5 may be quite unconservative; under saturated conditions, clays and silts may become entirely liquid (that is, $\phi = 0$). Soils of type 1 or 2 should be used as backfill for retaining walls wherever possible.

17.3 Earth Pressure for Common Conditions of Loading

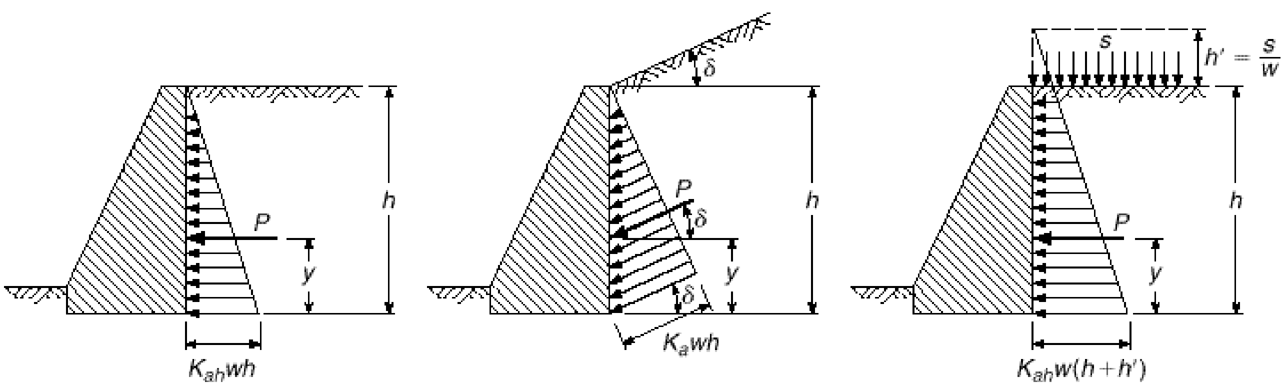
In computing earth pressures on walls, three common conditions of loading are most often met: (1) horizontal surface of fill at the top of the wall, (2) inclined surface of fill sloping up and back from the top of the wall, and (3) horizontal surface of fill carrying a uniformly distributed additional load (surcharge), such as from goods in a storage yard or traffic on a road.

The increase in pressure caused by uniform surcharge s (case 3) is computed by converting its load into an equivalent, imaginary height of earth h' above the top of the wall such that

$$h' = \frac{s}{w} \tag{17.6}$$

and measuring the depth to a given point on the wall from this imaginary surface. This amounts to replacing h with $(h + h')$ in Eq. (17.1).

The distributions of pressure for cases 1 to 3 are shown in Fig. 17.3. The total earth thrust P per linear foot of wall is equal to the area under the pressure distribution figure, and its line of action passes through the centroid of the pressure. Figure 17.3 gives information, computed in this manner, on magnitude, point of action, and direction of P for these three cases.



(a) $y = \frac{h}{3}$, $P = \frac{1}{2}K_{ah}wh^2$

(b) $y = \frac{h}{3}$ $P = \frac{1}{2}K_a wh^2$; For $\delta = \phi$, $K_a = \cos\phi$

(c) $y = \frac{h^2 + 3hh'}{3(h + 2h')}$, $P = \frac{1}{2}K_{ah}wh(h + 2h')$

FIGURE 17.3
Earth pressures for (*a*) horizontal surface; (*b*) sloping surface; (*c*) horizontal surface with surcharge *s*.

Occasionally retaining walls must be built for conditions in which the groundwater level is above the base of the wall, either permanently or seasonally. In that case, the pressure of the soil *above* groundwater is determined as usual. The part of the wall *below* groundwater is subject to the sum of the water pressure and the earth pressure. The former is equal to the full hydrostatic pressure $p_w = w_w h_w$, where w_w and h_w are, respectively, the unit weight of water and the distance from the groundwater level to the point on the wall. The additional pressure of the soil below the groundwater level is computed from Eq. (17.1), where, however, for the portion of the soil below water, w is replaced with $w - w_w$, while h, as usual, is measured from the soil surface. That is, for submerged soil, buoyancy reduces the effective weight in the indicated manner. Pressures of this magnitude, which are considerably larger than those of drained soil, will also occur temporarily after heavy rainstorms or thaws in walls without provision for drainage, or if drains have become clogged.

The seeming simplicity of the determination of earth pressure, as indicated here, should not lull the designer into a false sense of security and certainty. No theory is more accurate than the assumptions on which it is based. Actual soil pressures are affected by irregularities of soil properties, porewater and drainage conditions, and climatic and other factors that cannot be expressed in formulas. This situation, on the one hand, indicates that involved refinements of theoretical earth pressure determinations, as sometimes attempted, are of little practical value. On the other hand, the design of a retaining wall is seldom a routine procedure, since the local conditions that affect pressures and safety vary from one locality to another.

17.4 External Stability

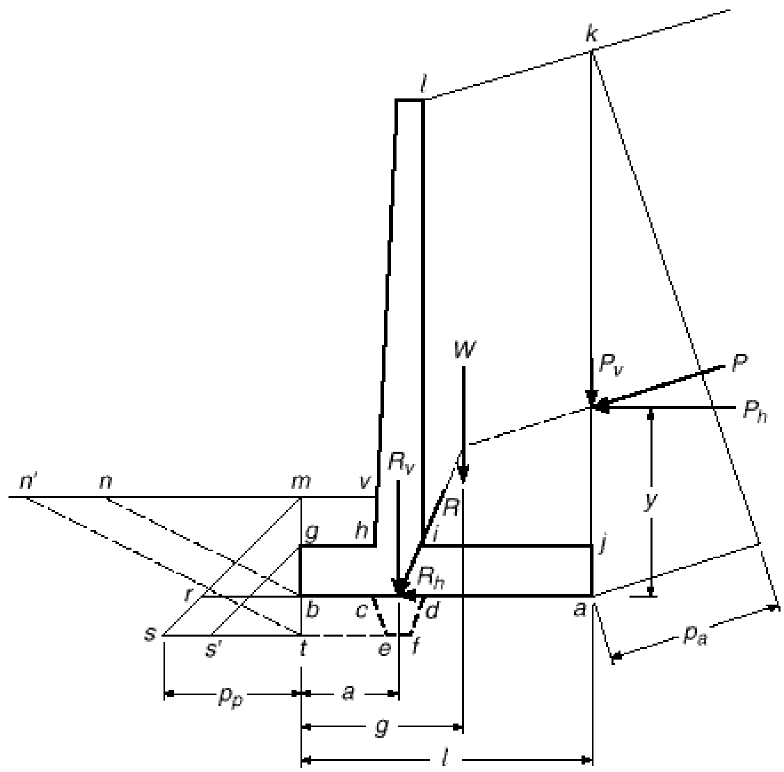
A wall may fail in two different ways: (1) its individual parts may not be strong enough to resist the acting forces, such as when a vertical cantilever wall is cracked by the earth pressure acting on it, and (2) the wall as a whole may be bodily displaced by the earth pressure, without breaking up internally. To design against the first possibility requires the determination of the necessary dimensions, thicknesses, and reinforcement to resist the moments and shears; this procedure, then, is in no way different from that of determining required dimensions and reinforcement of other types of concrete structures. The usual load factors and strength reduction factors of the ACI Code may be applied (see Section 17.5).

To safeguard the wall against bodily displacements, i.e., to ensure its external stability, requires special consideration. Consistent with current practice in geotechnical engineering, the stability investigation is based on actual earth pressures (as nearly as they may be determined) and on computed or estimated service dead and live loads, all without load factors. Computed bearing pressures are compared with allowable values, and overall factors of safety evaluated by comparing resisting forces to maximum loads acting under service conditions.

A wall, such as that in Fig. 17.4, together with the soil mass *ijkl* that rests on the base slab, may be bodily displaced by the earth thrust P that acts on the plane *ak* by *sliding* along the plane *ab*. Such sliding is resisted by the friction between the soil and footing along the same plane. To forestall motion, the forces that resist sliding must exceed those that tend to produce sliding; a factor of safety of 1.5 is generally assumed satisfactory in this connection.

In Fig. 17.4, the force that tends to produce sliding is the horizontal component P_h of the total earth thrust P. The resisting friction force is fR_v, where f is the coefficient of friction between the concrete and soil (see Table 17.1) and R_v is the vertical

FIGURE 17.4
External stability of a cantilever wall.



component of the total resultant R; that is, $R_v = W + P_v$ (W = weight of wall plus soil resting on the footing, P_v = vertical component of P). Hence, to provide sufficient safety,

$$f \cdot W + P_v \cdot \geq 1.5P_h \tag{17.7}$$

Actually, for the wall to slide to the left, it must push with it the earth *nmb*, which gives rise to the passive earth pressure indicated by the triangle *rmb*. This passive pressure represents a further resisting force that could be added to the left side of Eq. (17.7). However, this should be done only if the proper functioning of this added resistance is ensured. For that purpose, the fill *ghmv* must be placed before the backfill *ijkl* is put in place and must be secure against later removal by scour or other means throughout the lifetime of the wall. If these conditions are not met, it is better not to count on the additional resistance of the passive pressure.

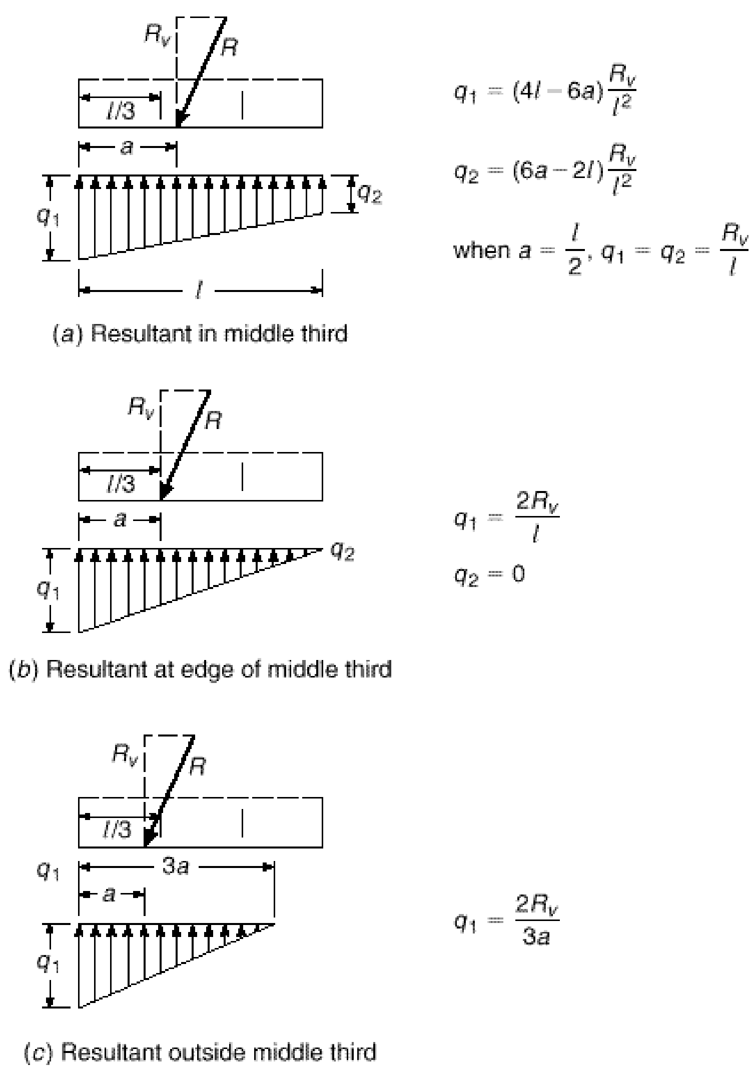
If the required sliding resistance cannot be developed by these means, a key wall *cdef* can be used to increase horizontal resistance. In this case, sliding, if it occurs, takes place along the planes *ad* and *tf*. While along *ad* and *ef*, the friction coefficient *f* applies, sliding along *te* occurs within the soil mass. The coefficient of friction that applies in this portion is consequently tan · , where the value of · may be taken from the next to last column in Table 17.1. In this situation sliding of the front soil occurs upward along *tn′* so that, if the front fill is secure, the corresponding resistance from passive soil pressure is represented by the pressure triangle *stm*. If doubt exists as to the reliability of the fill above the toe, the free surface should more conservatively be assumed at the top level of the footing, in which case the passive pressure is represented by the triangle *s′tg*.

Next, it is necessary to ensure that the pressure under the footing does not exceed the *permissible bearing pressure* for the particular soil. Let a (Fig. 17.4) be the distance from the front edge b to the intersection of the resultant with the base plane, and let R_v be the vertical component of R. (This intersection need not be located beneath the vertical arm, as shown, even though an economical wall generally results if it is so located.) Then the base plane ab, 1 ft wide longitudinally, is subject to a normal force R_v and to a moment about the centroid $(l/2 - a)R_v$. When these values are substituted in the usual formula for bending plus axial force

$$q_{\substack{max \\ min}} = \frac{N}{A} \pm \frac{Mc}{I} \tag{17.8}$$

it will be found that if the resultant is located within the middle third ($a > l/3$), compression will act throughout the section, and the maximum and minimum pressures can be computed from the equations in Fig. 17.5a. If the resultant is located just at the

FIGURE 17.5
Bearing pressures for different locations of resultant.



edge of the middle third ($a = l/3$), the pressure distribution is as shown in Fig. 17.5*b*, and Eq. (17.8) results in the formula given there.

If the resultant were located outside the middle third ($a < l/3$), Eq. (17.8) would indicate tension at and near point *a*. Obviously, tension cannot be developed between soil and a concrete footing that merely rests on it. Hence, in this case the pressure distribution of Fig. 17.5*c* will develop, which implies a slight lifting off the soil of the rear part of the footing. Equilibrium requires that R_v pass through the centroid of the pressure distribution triangle, from which the formula for q_1 for this case can easily be derived.

It is good practice, in general, to have the resultant located within the middle third. This will not only reduce the magnitude of the maximum bearing pressure but will also prevent too large a nonuniformity of pressure. If the wall is founded on a highly compressible soil, such as certain clays, a pressure distribution as in Fig. 17.5*b* would result in a much larger settlement of the toe than of the heel, with a corresponding tilting of the wall. In a foundation on such a soil, the resultant, therefore, should strike at or very near the center of the footing. If the foundation is on very incompressible soil, such as well-compacted gravel or rock, the resultant can be allowed to fall outside the middle third (Fig. 17.5*c*).

A third mode of failure is the possibility of the wall *overturning* bodily around the front edge *b* (Fig. 17.4). For this to occur, the overturning moment yP_h about point *b* would have to be larger than the restoring moment ($Wg + P_v l$) in Fig. 17.4, which is the same as saying that the resultant would have to strike outside the edge *b*. If, as is mostly the case, the resultant strikes within the middle third, adequate safety against overturning exists, and no special check need be made. If the resultant is located outside the middle third, a factor of safety of at least 1.5 should be maintained against overturning; i.e., the restoring moment should be at least 1.5 times the overturning moment.

17.5 Basis of Structural Design

In the investigation of a retaining wall for external stability, described in Section 17.4, it is the current practice to base the calculations on actual earth pressures, and on computed or estimated service dead and live loads, all with load factors of 1.0 (i.e., without load increase to account for a hypothetical overload condition). Computed soil bearing pressures, for service load conditions, are compared with allowable values set suitably lower than ultimate bearing values. Factors of safety against overturning and sliding are established, based on service load conditions.

On the other hand, the structural design of a retaining wall should be consistent with methods used for all other types of members, and thus should be based on factored loads in recognition of the possibility of an increase above service loading. ACI Code load factors relating to structural design of retaining walls are summarized as follows:

1. If resistance to earth pressure *H* is included in the design, together with dead loads *D* and live loads *L*, the required strength *U* shall be at least equal to

$$U = 1.2D + 1.6L + 1.6H$$

2. Where *D* or *L* reduce the effect of *H*, the required strength *U* shall be at least equal to

$$U = 0.9D + 1.6H$$

3. For any combination of D, L, and H, the required strength shall not be less than

$$U = 1.2D + 1.6L$$

While the ACI Code approach to load factor design is logical and relatively easy to apply to members in buildings, its application to structures that are to resist earth pressures is not so easy. Many alternative combinations of factored dead and live loads and lateral pressures are possible. Dead loads such as the weight of the concrete should be multiplied by 0.9 where they reduce design moments, such as for the toe slab of a cantilevered retaining wall, but should be multiplied by 1.2 where they increase moments, such as for the heel slab. The vertical load of the earth over the heel should be multiplied by 1.6. Obviously, no two factored load states could be obtained concurrently. For each combination of factored loads, different reactive soil pressures will be produced under the structure, requiring a new determination of those pressures for each alternative combination. Furthermore, there is no reason to believe that soil pressure would continue to be linearly distributed at the overload stage, or would increase in direct proportion to the load increase; knowledge of soil pressure distributions at incipient failure is incomplete. Necessarily, a somewhat simplified view of load factor design must be adopted in designing retaining walls.

Following the ACI Code, lateral earth pressures are multiplied by a load factor of 1.6. In general, the reactive pressure of the soil under the structure at the factored load stage is taken equal to 1.6 times the soil pressure found for service load conditions in the stability analysis.[†] For cantilever retaining walls, the calculated dead load of the toe slab, which causes moments acting in the opposite sense to those produced by the upward soil reaction, is multiplied by a factor of 0.9. For the heel slab, the required moment capacity is based on the dead load of the heel slab itself and is multiplied by 1.2, while the downward load of the earth is multiplied by 1.6. Surcharge, if present, is treated as live load with a load factor of 1.6. The upward pressure of the soil under the heel slab is taken equal to zero, recognizing that for the severe overload stage a nonlinear pressure distribution will probably be obtained, with most of the reaction concentrated near the toe. Similar assumptions appear to be reasonable in designing counterfort walls.

In accordance with ACI Code 14.1.2, cantilever retaining walls are designed following the flexural design provisions covered in Chapter 3, with minimum horizontal reinforcement provided in accordance with ACI Code 14.3.3, which stipulates a minimum ratio of

> 0.0020 for deformed bars not larger than No. 5 (No. 16) with a specified yield strength not less than 60,000 psi; or 0.0025 for other deformed bars; or 0.0020 for welded wire reinforcement not larger than W31 or D31.

17.6 DRAINAGE AND OTHER DETAILS

Such failures or damage to retaining walls as have occasionally occurred were due, in most cases, to one of two causes: overloading of the soil under the wall with consequent forward tipping or insufficient drainage of the backfill. In the latter case, hydrostatic pressure from porewater accumulated during or after rainstorms greatly increases the thrust on the wall; in addition, in subfreezing weather, ice pressure of

[†] These reactions are caused by the assumed factored load condition and have no direct relationship to ultimate soil bearing values or pressure distributions.

considerable magnitude can develop in such poorly drained soils. The two causes are often interconnected, since large thrusts correspondingly increase the bearing pressure under the footing.

Allowable bearing pressures should be selected with great care. It is necessary, for this purpose, to investigate not only the type of soil immediately underlying the footing, but also the deeper layers. Unless reliable information is available at the site, subsurface borings should be made to a depth at least equal to the height of the wall. The foundation must be laid below *frost depth*, which amounts to 4 to 5 ft and more in the northern states, to ensure against heaving by the freezing of soils containing moisture.

Drainage can be provided in various ways. *Weep holes* consisting of 6 or 8 in. pipe embedded in the wall, as shown in Fig. 17.1*c*, are usually spaced horizontally at 5 to 10 ft. In addition to the bottom row, additional rows should be provided in walls of substantial height. To facilitate drainage and prevent clogging, 1 ft^3 or more of crushed stone is placed at the rear end of each weeper. Care must be taken that the outflow from the weep holes is carried off safely so as not to seep into and soften the soil underneath the wall. To prevent this, instead of weepers, *longitudinal drains* embedded in crushed stone or gravel can be provided along the rear face of the wall (Fig. 17.1*b*) at one or more levels; the drains discharge at the ends of the wall or at a few intermediate points. The most efficient drainage is provided by a *continuous backdrain* consisting of a layer of gravel or crushed stone covering the entire rear face of the wall (Fig. 17.1*a*), with discharge at the ends. Such drainage is expensive, however, unless appropriate material is cheaply available at the site. Wherever possible, the surface of the fill should be covered with a layer of low permeability and, in the case of a horizontal surface, should be laid with a slight slope away from the wall toward a gutter or other drainage.

In long walls, provision must be made against damage caused by *expansion* or *contraction* from temperate changes and shrinkage. The AASHTO *Standard Specifications for Highway Bridges* require that for gravity walls, as well as reinforced concrete walls, expansion joints be placed at intervals of 90 ft or less, and contraction joints at not more than 30 ft (Ref. 17.4). The same specifications provide that, in reinforced concrete walls, horizontal temperature reinforcement of not less than $\frac{1}{8}$ in^2 per foot of depth be provided adjacent to the exposed surface. Similar provisions are found in Ref. 17.5.

17.7 EXAMPLE: DESIGN OF A GRAVITY RETAINING WALL

A gravity wall is to retain a bank 11 ft 6 in. high whose horizontal surface is subject to a live load surcharge of 400 psf. The soil is a sand and gravel mixture with a rather moderate amount of fine, silty particles. It can, therefore, be assumed to be in class 2 of Table 17.1 with the following characteristics: unit weight $w = 120$ pcf, $\phi = 30°$ (with adequate drainage to be provided), and base friction coefficient $f = 0.5$. With sin 30° = 0.5, from Eqs. (17.4) and (17.5), the soil pressure coefficients are $K_{ah} = 0.333$ and $K_{ph} = 3.0$. The allowable bearing pressure is assumed to be 8000 psf. This coarse-grained soil has little compressibility, so that the resultant can be allowed to strike near the outer-third point (see Section 17.4). The weight of the concrete is $w_c = 150$ pcf.

The optimum design of any retaining wall is a matter of successive approximation. Reasonable dimensions are assumed based on experience, and the various conditions of stability are checked for these dimensions. On the basis of a first trial,

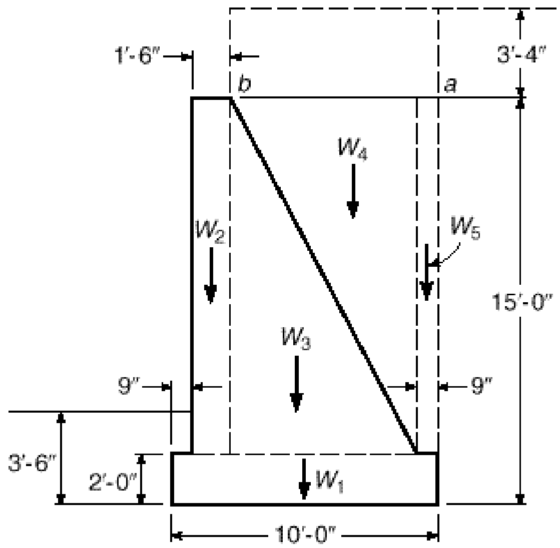


FIGURE 17.6
Gravity retaining wall.

dimensions are readjusted, and one or two additional trials usually result in a favorable design. In the following, only the final design is analyzed in detail. The final dimensions are shown in Fig. 17.6.

The equivalent height of surcharge is $h' = 400 \cdot 120 = 3.33$ ft. From Fig. 17.3*c* the total earth thrust is

$$P = 1 \cdot 2 \times 0.333 \times 120 \times 15 \times 21.67 = 6500 \text{ lb}$$

and its distance from the base is $y = (225 + 150) \cdot (3 \times 21.67) = 5.77$ ft. Hence, the overturning moment $M_o = 6500 \times 5.77 = 37{,}500$ ft-lb. To compute the weight W and its restoring moment M_r about the edge of the toe, individual weights are taken, as shown in Fig. 17.6. With x representing the distance of the line of action of each subweight from the front edge, the following computation results:

Component Weights	lb	ft	ft-lb
W_1: 10 × 2 × 150	3,000	5.0	15,000
W_2: 1.5 × 13 × 150	2,930	1.5	4,400
W_3: 7·2 × 13 × 150	6,830	4.58	31,300
W_4: 7·2 × 13 × 120	5,460	6.92	37,800
W_5: 0.75 × 13 × 120	1,170	9.63	11,270
Total	19,390		99,770

The distance of the resultant from the front edge is

$$a = \frac{99{,}770 - 37{,}500}{19{,}390} = 3.21 \text{ ft}$$

which is just outside the middle third. The safety factor against overturning, $99{,}770 \cdot 37{,}500 = 2.66$, is ample. From Fig. 17.5*c* the maximum soil pressure is $q = (2 \times 19{,}390) \cdot (3 \times 3.21) = 4030$ psf.

These computations were made for the case in which the surcharge extends only to the rear edge of the wall, point a of Fig. 17.6. If the surcharge extends forward to point b, the following modifications are obtained:

$$W = 19{,}390 + 400 \times 7.75 = 22{,}490 \text{ lb}$$

$$M_r = 99{,}770 + 400 \times 7.75 \times 6.13 = 118{,}770 \text{ ft-lb}$$

$$a = \frac{118{,}770 - 37{,}500}{22{,}490} = 3.61 \text{ ft}$$

This is inside the middle third, and from Fig. 17.5a, the maximum bearing pressure is

$$q_1 = \frac{(40.0 - 21.7)22{,}490}{100} = 4120 \text{ psf}$$

The situation most conducive to sliding occurs when the surcharge extends only to point a, since additional surcharge between a and b would increase the total weight and the corresponding resisting friction. The friction force is

$$F = 0.5 \times 19{,}390 = 9695 \text{ lb}$$

Additionally, sliding is resisted by the passive earth pressure on the front of the wall. Although the base plane is 3.5 ft below grade, the top layer of soil cannot be relied upon to furnish passive pressure, since it is frequently loosened by roots and the like, or it could be scoured out by cloudbursts. For this reason, the top 1.5 ft will be discounted in computing the passive pressure, which then becomes

$$P_p = 1/2wh^2K_{ph} = 1/2 \times 120 \times 2^2 \times 3.0 = 720 \text{ lb}$$

The safety factor against sliding, $(9695 + 720)/6500 = 1.6$, is but slightly larger than the required value 1.5, indicating a favorable design. Ignoring the passive pressure gives a safety factor of 1.49, which is very close to the acceptable value.

17.8 Example: Design of a Cantilever Retaining Wall

A cantilever wall is to be designed for the situation of the gravity wall in Section 17.7. Concrete with $f'_c = 4500$ psi and steel with $f_y = 60{,}000$ psi will be used.

a. Preliminary Design

To facilitate computation of weights for checking the stability of the wall, it is advantageous first to ascertain the thickness of the arm and the footing.† For this purpose the thickness of the footing is roughly estimated, and then the required thickness of the arm is determined at its bottom section. With the bottom of the footing at 3.5 ft below grade and an estimated footing thickness of 1.5 ft, the free height of the arm is 13.5 ft. Hence, with respect to the bottom of the arm (see Fig. 17.3c),

$$P = 1/2 \times 0.333 \times 120 \times 13.5 \times 20.16 = 5440 \text{ lb}$$

$$y = \frac{183 + 135}{3 \times 20.16} = 5.25 \text{ ft}$$

$$M_u = 1.6 \times 5440 \times 5.25 = 45{,}700 \text{ ft-lb}$$

† Valuable guidance is provided for the designer in tabulated designs such as those found in Ref. 17.6 and by the sample calculations in Ref. 17.7.

For the given grades of concrete and steel, the maximum permitted reinforcement ratio $\rho_{max} = 0.0200$. For economy and ease of bar placement, a ratio of about 40 percent of the maximum, or 0.008, will be used. Then from Graph A.1*b* of Appendix A,

$$\frac{M_u}{\phi bd^2} = 430$$

For a unit length of the wall ($b = 12$ in.), with $\phi = 0.90$, the required effective depth is

$$d = \sqrt{\frac{45{,}700 \times 12}{0.90 \times 12 \times 430}} = 10.9 \text{ in.}$$

A protective cover of 2 in. is required for concrete exposed to earth. Thus, estimating the bar diameter to be 1 in., the minimum required thickness of the arm at the base is 13.4 in. This will be increased to 16 in., because the cost of the extra concrete in such structures is usually more than balanced by the simultaneous saving in steel and ease of concrete placement. The arm is then checked for shear at a distance d above the base, or 12.5 ft below the top of the wall:

$$P = \tfrac{1}{2} \times 0.333 \times 120 \times 12.5 \times 19.16 = 4800 \text{ lb}$$

$$V_u = 1.6 \times 4800 = 7680 \text{ lb}$$

$$\phi V_c = 2\phi\sqrt{f'_c}\,bd$$

$$= 2 \times 0.75\sqrt{4500} \times 12 \times 13.5$$

$$= 16{,}300 \text{ lb}$$

confirming that the arm is more than adequate to resist the factored shear force.

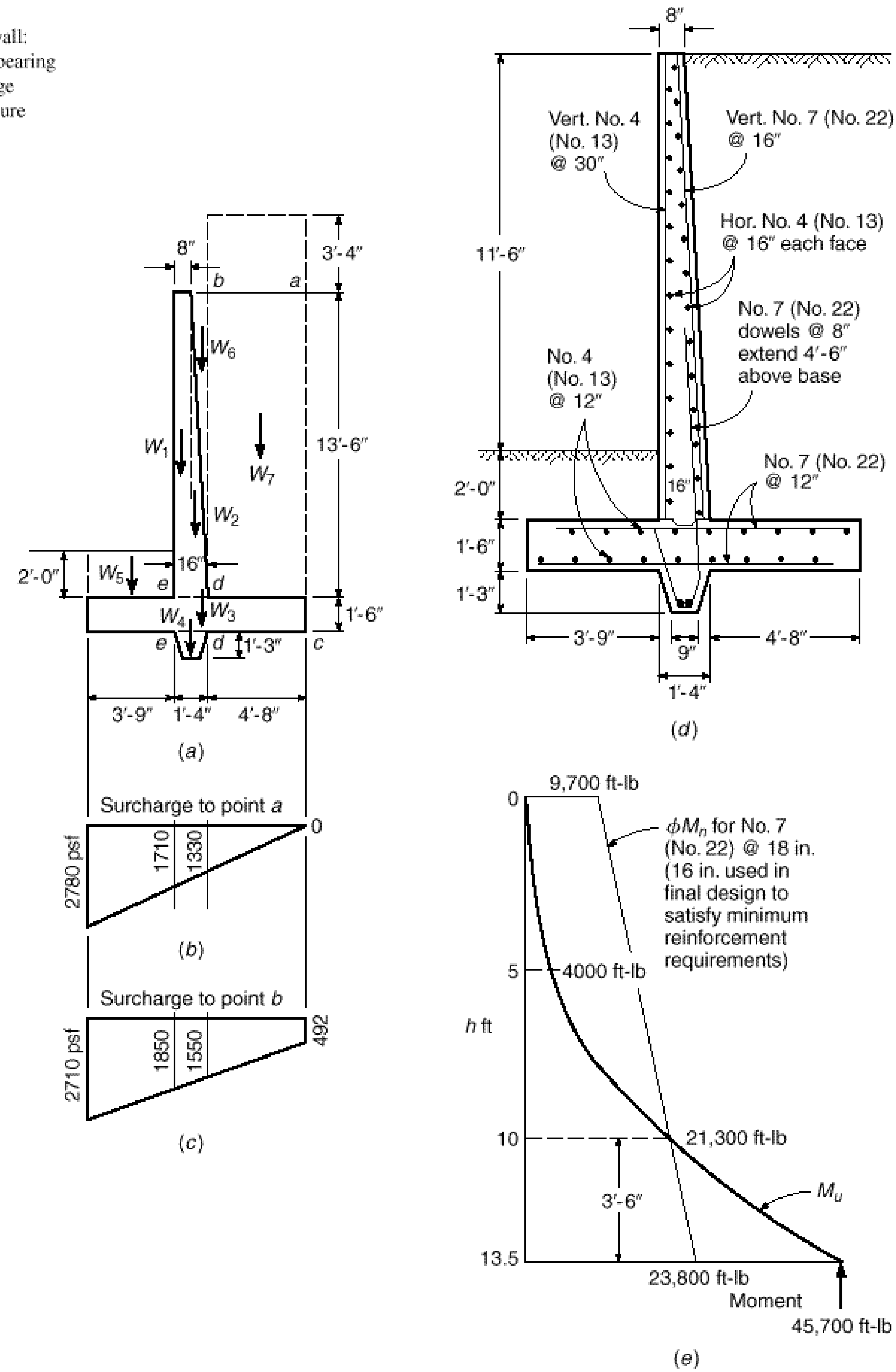
The thickness of the base is usually the same or slightly larger than that at the bottom of the arm. Hence, the estimated 1.5 ft need not be revised. Since the moment in the arm decreases with increasing distance from the base and is zero at the top, the arm thickness at the top will be made 8 in. It is now necessary to assume lengths of heel and toe slabs and to check the stability for these assumed dimensions. Intermediate trials are omitted here, and the final dimensions are shown in Fig. 17.7*a*. Trial computations have shown that safety against sliding can be achieved only by an excessively long heel or by a key. The latter, requiring the smaller concrete volume, has been adopted.

b. Stability Investigation

Weights and moments about the front edge are as follows:

Component Weights	lb	ft	ft-lb
W_1: 0.67 × 13.5 × 150	1,360	4.08	5,550
W_2: 0.67 × 0.5 × 13.5 × 150	680	4.67	3,180
W_3: 9.75 × 1.5 × 150	2,190	4.88	10,700
W_4: 1.33 × 1.25 × 150	250	4.42	1,100
W_5: 3.75 × 2 × 120	900	1.88	1,690
W_6: 0.67 × 0.5 × 13.5 × 120	540	4.86	2,620
W_7: 4.67 × 13.5 × 120	7,570	7.42	56,200
Total	13,490		81,040

FIGURE 17.7
Cantilever retaining wall: (*a*) cross section; (*b*) bearing pressure with surcharge to *a*; (*c*) bearing pressure with surcharge to *b*; (*d*) reinforcement; (*e*) moment variation with height.



The total soil pressure on the plane *ac* is the same as for the gravity wall designed in Section 17.7, that is, $P = 6500$ lb, and the overturning moment is

$$M_o = 37{,}500 \text{ ft-lb}$$

The distance of the resultant from the front edge is

$$a = \frac{81{,}040 - 37{,}500}{13{,}490} = 3.23 \text{ ft}$$

which locates the resultant barely outside of the middle third. The corresponding maximum soil pressure at the toe, from Fig. 17.5*c*, is

$$q_1 = \frac{2 \times 13{,}470}{3 \times 3.23} = 2780 \text{ psf}$$

The factor of safety against overturning, 81,040·37,500 = 2.16, is ample.

To check the safety against sliding, remember (Section 17.4) that if sliding occurs it proceeds between concrete and soil along the heel and key (i.e., length *ae* in Fig. 17.4), but takes place within the soil in front of the key (i.e., along length *te* in Fig. 17.4). Consequently, the coefficient of friction that applies for the former length is $f = 0.5$, while for the latter it is equal to the internal soil friction, i.e., tan 30° = 0.577.

The bearing pressure distribution is shown in Fig. 17.7*b*. Since the resultant is at a distance $a = 3.23$ ft from the front, i.e., nearly at the middle third, it is assumed that the bearing pressure becomes zero exactly at the edge of the heel, as shown in Fig. 17.7*b*.

The resisting force is then computed as the sum of the friction forces of the rear and front portion, plus the passive soil pressure in front of the wall. For the latter, as in Section 17.7, the top 1.5 ft layer of soil will be discounted as unreliable. Hence,

Friction, toe:	$(2780 + 1710) \times 0.5 \times 3.75 \times 0.577 = 4860$ lb
Friction, heel and key:	$1710 \times 0.5 \times 6 \times 0.5 = 2570$ lb
Passive earth pressure:	$0.5 \times 120 \times 3.25^2 \times 3.0 = 1900$ lb
Total resistance to sliding:	$= 9330$ lb

The factor of safety against sliding, 9330·6500 = 1.44, is only 4 percent below the recommended value of 1.5 and can be regarded as adequate.

The computations hold for the case in which the surcharge extends from the right to point *a* above the edge of the heel. The other case of load distribution, in which the surcharge is placed over the entire surface of the fill up to point *b*, evidently does not change the earth pressure on the plane *ac*. It does, however, add to the sum of the vertical forces and increases both the restoring moment M_r and the friction along the base. Consequently, the danger of sliding or overturning is greater when the surcharge extends only to *a*, for which situation these two cases have been checked and found adequate. In view of the added vertical load, however, the bearing pressure is largest when the surface is loaded to *b*. For this case,

$$W = 13{,}490 + 400 \times 5.33 = 15{,}600 \text{ lb}$$

$$M_r = 81{,}040 + 400 \times 5.33 \times 7.09 = 96{,}200 \text{ ft-lb}$$

$$a = \frac{96{,}200 - 37{,}500}{15{,}600} = 3.76 \text{ ft}$$

which places the resultant inside the middle third. Hence, from Fig. 17.5*a*,

$$q_1 = \cdot 39.0 - 22.5 \cdot \frac{15{,}600}{9.75^2} = 2710 \text{ psf}$$

$$q_2 = \cdot 22.5 - 19.5 \cdot \frac{15{,}600}{9.75^2} = 492 \text{ psf}$$

which is far below the allowable pressure of 8000 psf. The corresponding bearing pressure distribution is shown in Fig. 17.7*c*.

The external stability of the wall has now been ascertained, and it remains to determine the required reinforcement and to check internal resistances.

c. Arm and Key

The moment at the bottom section of the arm has previously been determined as M_u = 45,700 ft-lb, and a wall thickness of 16 in. at the bottom and 8 in. at the top has been selected. With a concrete cover of 2 in. clear, $d = 16.0 - 2.0 - 0.5 = 13.5$ in. Then

$$\frac{M_u}{\cdot\, bd^2} = \frac{45{,}700 \times 12}{0.90 \times 12 \times 13.5^2} = 279$$

Interpolating from Graph A.1*b* of Appendix A, with f_y = 60,000 psi and f_c' = 4500 psi, the required reinforcement ratio · is 0.0049 and $A_s = 0.0049 \times 12 \times 13.5 = 0.79$ in²/ft. The required area of steel is provided by No. 7 (No. 22) bars at 9 in. on centers.

The bending moment in the arm decreases rapidly with increasing distance from the bottom. For this reason, only part of the main reinforcement is needed at higher elevations, and alternate bars will be discontinued where no longer needed. To determine the cutoff point, the moment diagram for the arm has been drawn by computing bending moments at two intermediate levels, 10 ft and 5 ft from the top. These two moments, determined in the same manner as that at the base of the arm, were found to be 21,300 and 4000 ft-lb, respectively. The resisting moment provided by alternate bars, i.e., by No. 7 (No. 22) bars at 18 in. center to center, at the bottom of the arm is

$$\cdot\, M_n = \frac{0.90 \times 0.40 \times 60{,}000}{12} \cdot 13.50 - 0.26 \cdot = 23{,}800 \text{ ft-lb}$$

At the top, $d = 8.0 - 2.5 = 5.5$ in., and the resisting moment of the same bars is only $\cdot\, M_n = 23{,}800(5.5 \cdot 13.5) = 9700$ ft-lb. Hence, the straight line drawn in Fig. 17.7*e* indicates the resisting moment provided at any elevation by half the number of main bars. The intersection of this line with the moment diagram at a distance of 3 ft 6 in. from the bottom represents the point above which alternate bars are no longer needed. ACI Code 12.10.3 specifies that any bar shall be extended beyond the point at which it is no longer needed to carry flexural stress for a distance equal to d or 12 bar diameters, whichever is greater. In the arm, at a distance of 3 ft 6 in. from the bottom, d = 11.4 in., while 12 bar diameters for No. 7 (No. 22) bars are equal to 10.5 in. Hence, half the bars can be discontinued 12 in. above the point where no longer needed, or a distance of 4 ft 6 in. above the base. This exceeds the required development length of 39 in. above the base.

To facilitate construction, the footing is placed first, and a construction joint is provided at the base of the arm, as shown in Fig. 17.7*d*. The main bars of the arm, therefore, end at the top of the base slab, and dowels are placed in the latter to be spliced with them; the integrity of the arm depends entirely on the strength of the splices used for these tension bars. Splicing all tension bars in one section by simple contact splices can easily lead to splitting of the concrete owing to the stress concentrations at the ends of the spliced bars. One way to avoid this difficulty is to weld all splices; this will entail considerable extra cost.

In this particular wall, another way of placing the reinforcing offers a more economical solution. Because alternate bars in the arm can be discontinued at a distance of 4 ft 6 in. above the base, the dowels will be carried up 4 ft 6 in. from the top of the base. These need not be spliced at all, because above that level only alternate No. 7 (No. 22) bars, 18 in. on centers, are needed. These latter bars are placed full length over the entire height of the arm and are spliced at the bottom with alternate shorter dowels. By this means, only 50 percent of the bars needed at the bottom of the arm are spliced; this is not objectionable.

For splices of deformed bars in tension, at sections where the ratio of steel provided to steel required is less than 2 and where no more than 50 percent of the steel is spliced, the ACI Code requires a Class B splice with a length equal to 1.3 times the development length of the bar (see Section 5.11a). The development length of the No. 7 (No. 22) bars for the given material strengths is 39 in., and so the required splice length is $1.3 \times 39 = 50.7$ in., which is less than the 4 ft 6 in. available.

According to the ACI Code, main flexural reinforcement is not to be terminated in a tension zone unless one of three conditions is satisfied: (1) shear at the cutoff point does not exceed two-thirds that permitted, (2) certain excess shear reinforcement is provided, or (3) the continuing reinforcement provides double the area required for flexure at the cutoff point and the factored shear does not exceed three-fourths of the design shear. It is easily confirmed that the shear 4 ft 6 in. above the base is well below two-thirds the value that can be carried by the concrete; thus main bars can be terminated as planned.

Prior to completing the design of the arm, the minimum tensile reinforcement ratio specified by the ACI Code must be checked. The actual ratio provided by the No. 7 (No. 22) bars at 18 in. spacing, with $d = 10.8$ in. just above the cutoff point, is 0.0031, about 10 percent below the minimum value of $3\sqrt{4500}/60{,}000 = 0.0034$. To handle this, the spacing of the No. 7 (No. 22) bars will be reduced to 8 in., giving a spacing of 16 in. above the cutoff. This will increase the amount of steel, but by less than would be needed if the bars were extended to a height where the decreasing value of d allowed the minimum reinforcement ratio to be satisfied. A final ACI Code requirement is that the maximum spacing of the primary flexural reinforcement exceed neither 3 times the wall thickness nor 18 in.; these restrictions are satisfied as well.

Since the dowels had to be extended at least partly into the key to produce the necessary length of embedment, they were bent as shown to provide both reinforcement for the key and anchorage for the arm reinforcement. The exact force that the key must resist is difficult to determine, since probably the major part of the force acting on the portion of the soil in front of the key is transmitted to it through friction along the base of the footing. The relatively strong reinforcement of the key by means of the extended dowels is considered sufficient to prevent separation from the footing.

The sloping sides of the key were provided to facilitate excavation without loosening the adjacent soil. This is necessary to ensure proper functioning of the key.

In addition to the main steel in the stem, reinforcement must be provided in the horizontal direction to control shrinkage and temperature cracking, in accordance with ACI Code 14.3.3. Calculations will be based on the average wall thickness of 12 in. The required steel area is 0.0020 times the gross concrete area. No. 4 (No. 13) bars 16 in. on centers, each face, will be used, as shown in Fig. 17.7*d*. Although not required by the Code for cantilever retaining walls, vertical steel equal to 0.0012 times the gross concrete area will also be provided (to limit horizontal surface cracking), with at least one-half of this value provided on the exposed face, as specified for other walls under ACI Code 14.3.2. No. 4 (No. 13) bars 30 in. on centers will satisfy this requirement.

d. Toe Slab

The toe slab acts as a cantilever projecting outward from the face of the stem. It must resist the upward pressures shown in Fig. 17.7*b* or *c* and the downward load of the toe slab itself, each multiplied by appropriate load factors. The downward load of the earth fill over the toe will be neglected because it is subject to possible erosion or removal. A load factor of 1.6 will be applied to the service load bearing pressures. Comparison of the pressures of Fig. 17.7*b* and *c* indicates that for the toe slab, the more severe loading case results from surcharge to *b*. Because the self-weight of the toe slab tends to reduce design moments and shears, it will be multiplied by a load factor of 0.9. Thus the factored load moment at the outer face of the stem is

$$M_u = 1.6\left(\frac{2710}{2} \times 3.75^2 \times \frac{2}{3} + \frac{1850}{2} \times 3.75^2 \times \frac{1}{3}\right) - 0.9\left(225 \times 3.75^2 \times \frac{1}{2}\right)$$

$$= 25{,}800 \text{ ft-lb}$$

For concrete cast against and permanently exposed to earth, a minimum protective cover for steel of 3 in. is required; if the bar diameter is about 1 in., the effective depth will be 18.0 − 3.0 − 0.5 = 14.5 in. Thus, for a 12 in. strip of toe slab,

$$\frac{M_u}{\phi bd^2} = \frac{25{,}800 \times 12}{0.90 \times 12 \times 14.5^2} = 136$$

Graph A.1*b* of Appendix A shows that, for this value, the required reinforcement ratio would be below the minimum of $3\sqrt{4500}/60{,}000 = 0.0034$. A somewhat thinner base slab appears possible. However, moments in the heel slab are yet to be investigated, as well as shears in both the toe and heel, and the trial depth of 18 in. will be retained tentatively. The required flexural steel

$$A_s = 0.0034 \times 12 \times 14.5 = 0.59 \text{ in}^2/\text{ft}$$

is provided by No. 7 (No. 22) bars 12 in. on centers. The required length of embedment for these bars past the exterior face of the stem is the full development length of 39 in. Thus, they will be continued 39 in. past the face of the wall, as shown in Fig. 17.7*d*.

Shear will be checked at a distance $d = 1.21$ ft from the face of the stem (2.54 ft from the end of the toe), according to the usual ACI Code procedures. The service load bearing pressure at that location (with reference to Fig. 17.7*c*) is 2130 psf, and the factored load shear is

$$V_u = 1.6\left(2710 \times \tfrac{1}{2} \times 2.54 + 2130 \times \tfrac{1}{2} \times 2.54\right) - 0.9(225 \times 2.54)$$

$$= 9320 \text{ lb}$$

The design shear strength of the concrete is

$$\phi V_c = 2 \times 0.75\sqrt{4500} \times 12 \times 14.5 = 17{,}500 \text{ lb}$$

well above the required value V_u.

e. Heel Slab

The heel slab, too, acts as a cantilever, projecting in this case from the back face of the stem and loaded by surcharge, earth fill, and its own weight. The upward reaction of the soil will be neglected here, for reasons given earlier. Applying appropriate load factors, the moment to be resisted is

$$M_u = 1.2 \times 225 \times 4.67^2 \times 1\cdot 2 + 1.6\cdot 400 \times 4.67^2 \times 1\cdot 2 + 1620 \times 4.67^2 \times 1\cdot 2\cdot$$
$$= 38{,}200 \text{ ft-lb}$$

Thus,

$$\frac{M_u}{\cdot\, bd^2} = \frac{38{,}200 \times 12}{0.90 \times 12 \times 14.5^2} = 202$$

Interpolating from Graph A.1*b*, the required reinforcement ratio is 0.0035, just above the minimum value of 0.0034. The required steel area will again be provided using No. 7 (No. 22) bars 12 in. on centers, as shown in Fig. 17.7*d*. These bars are classed as top bars, as they have more than 12 in. of concrete below; thus the required length of embedment to the left of the inside face of the stem is 39 × 1.3 = 51 in.

According to normal ACI Code procedures, the first critical section for shear would be a distance *d* from the face of support. However, the justification for this provision of the ACI Code is the presence, in the usual case, of vertical compressive stress near a support which tends to decrease the likelihood of shear failure in that region. However, the cantilevered heel slab is essentially hung from the bottom of the stem by the flexural tensile steel in the stem, and the concrete compression normally found near a support is absent here. Consequently, the critical section for shear in the heel slab will be taken at the back face of the stem. At that location,

$$V_u = 1.2\cdot 225 \times 4.67\cdot + 1.6\cdot 2020 \times 4.67\cdot$$
$$= 16{,}350 \text{ lb}$$

The design shear strength provided by the concrete is the same as for the toe slab:

$$\cdot\, V_c = 17{,}500 \text{ lb}$$

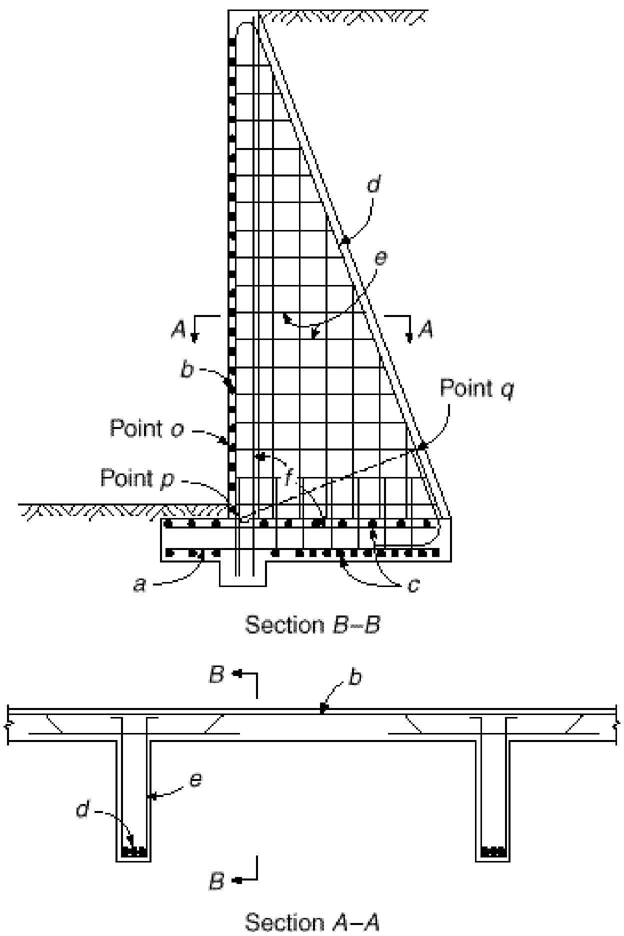
Because this is only 7 percent in excess of the required value V_u, no reduction in thickness of the base slab, considered earlier, will be made.

The base slab is well below grade and will not be subjected to the extremes of temperature that will be imposed on the stem concrete. Consequently, crack control steel in the direction perpendicular to the main reinforcement is not a major consideration. No. 4 (No. 13) bars 12 in. on centers will be provided, at one face only, placed as shown in Fig. 17.7*d*. These bars serve chiefly as spacers for the main flexural reinforcement.

17.9 COUNTERFORT RETAINING WALLS

The external stability of a counterfort retaining wall is determined in the same manner as in the examples of Sections 17.7 and 17.8. The toe slab represents a cantilever built in along the front face of the wall, loaded upward by the bearing pressure, exactly as in the cantilever wall described in Section 17.8. Reinforcement is provided by bars *a* in Fig. 17.8.

FIGURE 17.8
Details of counterfort retaining wall.



A panel of the vertical wall between two counterforts is a slab acted upon by horizontal earth pressure and supported along three sides, i.e., at the two counterforts and the base slab, while the fourth side, the top edge, is not supported. The earth pressure increases with distance from the free surface. The determination of moments and shears in such a slab supported on three sides and nonuniformly loaded is rather involved. It is customary in the design of such walls to disregard the support of the vertical wall by the base slab and to design it as if it were a continuous slab spanning horizontally between counterforts. This procedure is conservative, because the moments obtained by this approximation are larger than those corresponding to the actual conditions of support, particularly in the lower part of the wall. Hence, for very large installations, significant savings may be achieved by a more accurate analysis. The best computational tool for this is the Hillerborg *strip method*, a plasticity-based theory for design of slabs described in detail in Chapter 15. Alternatively, results of elastic analysis are tabulated for a range of variables in Ref. 17.8.

Slab moments are determined for strips 1 ft wide spanning horizontally, usually for the strip at the bottom of the wall and for three or four equally spaced additional strips at higher elevations. The earth pressure on the different strips decreases with increasing elevation and is determined using Eq. (17.1). Moment values for the bottom strips may be reduced to account for the fact that additional support is provided by the base slab. Horizontal bars b (Fig. 17.8) are provided, as required, with increased spacing or decreased diameter corresponding to the smaller moments. Alternate bars are bent to provide for the negative moments in the wall at the counterforts, or additional straight bars are used as negative reinforcement, as shown in Section *A-A* of Fig. 17.8.

The heel slab is supported, as is the wall slab, i.e., by the counterforts and at the wall. It is loaded downward by the weight of the fill resting on it, its own weight, and such surcharge as there may be. This load is partially counteracted by the upward bearing pressure on the underside of the heel. As in the vertical wall, a simplified analysis consists in neglecting the influence of the support along the third side and in determining moments and shears for strips parallel to the wall, each strip representing a continuous beam supported at the counterforts. For a horizontal soil surface, the downward load is constant for the entire heel, whereas the upward load from the bearing pressure is usually smallest at the rear edge and increases frontward. For this reason, the span moments are positive (compression on top) and the support moments negative in the rear portion of the heel. Near the wall, the bearing pressure often exceeds the vertical weights, resulting in a net upward load. The signs of the moments are correspondingly reversed, and steel must be placed accordingly. Bars c are provided for these moments.

The counterforts are wedge-shaped cantilevers built in at the bottom in the base slab. They support the wall slab and, therefore, are loaded by the total soil pressure over a length equal to the distance center to center between counterforts. They act as a T beam of which the wall slab is the flange and the counterfort the stem. The maximum bending moment is that of the total earth pressure, taken about the bottom of the wall slab. This moment is held in equilibrium by the force in the bars d, and hence, the effective depth for bending is the perpendicular distance pq from the center of bars d to the center of the bottom section of the wall slab. Since the moment decreases rapidly in the upper parts of the counterfort, part of the bars d can be discontinued.

In regard to shear, the authors suggest the horizontal section oq as a conservative location for checking adequacy. Modification of the customary shear computation is required for wedge-shaped members (see Section 4.7). Usually concrete alone is sufficient to carry the shear, although bars e act as stirrups and can be used for resisting excess shear.

The main purpose of bars e is to counteract the pull of the wall slab, and they are thus designed for the full reaction of this slab.

The remaining bars of Fig. 17.8 serve as shrinkage reinforcement, except that bars f have an important additional function. It will be recalled that the wall and heel slabs are supported on three sides. Even though they were designed as if supported only by the counterforts, they develop moments where they join. The resulting tension in and near the reentrant corner should be provided for by bars f.

The question of reinforcing bar details, always important, is particularly so for corners subject to substantial bending moments, such as are present for both cantilever and counterfort retaining walls. Valuable suggestions are found in Ref. 17.9.

17.10 PRECAST RETAINING WALLS

Largely because of the high cost of forming cast-in-place retaining walls, there has been increasing use in recent years of various forms of precast concrete walls. Sections can be mass produced under controlled factory conditions using standardized forms, with excellent quality control. On-site construction time is greatly reduced, and generally only a small crew using light equipment is required. Weather becomes much less of a factor in completion of the work than for cast-in-place walls.

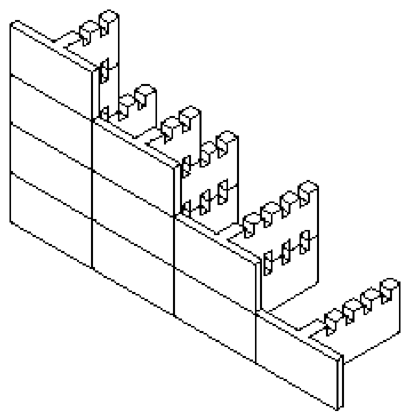


FIGURE 17.9
Precast T-Wall[R] retaining wall system. (*Courtesy Concrete Systems Inc., Hudson, New Hampshire.*)

One type of precast wall is shown in Fig. 17.9. Precast T sections are used, each 2.5 ft high and 5 ft wide, with T stems varying according to requirements from 4 ft to 20 ft. Individual units are stacked, using shear keys in the space created where teeth of a top and bottom unit come together, at approximately a 6 ft spacing perpendicular to the face of the wall. Calculations for stability against sliding and overturning and for bearing pressures are the same as for cast-in-place cantilever or counterfort walls, with stability provided by the combined weight of the concrete wall and compacted select backfill. Such walls can be constructed with vertical face or battered section, with heights up to 25 ft.

Walls of the type shown have been used for highways, parking lots, commercial and industrial sites, bank stabilization, wing walls, and similar purposes.

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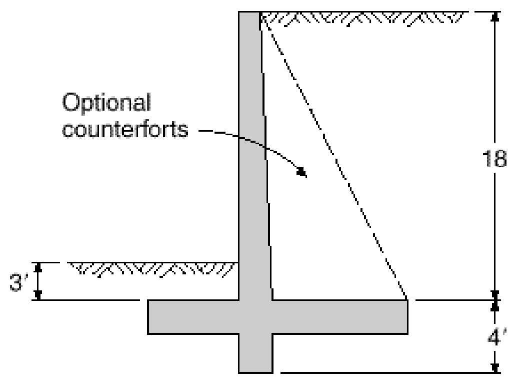
Problems

17.1. A cantilever retaining wall is to be designed with geometry as indicated in Fig. P17.1. Backfill material will be well-drained gravel having unit weight w = 120 pcf, internal friction angle ϕ = 33°, and friction factor against the concrete base f = 0.55. Backfill placed in front of the toe will have the same properties and will be well compacted. The final grade behind the wall will be level with the top of the wall, with no surcharge. At the lower level, it will be 3 ft above the top of the base slab. To improve sliding resistance, a key will be used, tentatively projecting to a depth 4 ft below the top of the base slab. (This dimension may be modified if necessary.)

(*a*) Based on a stability investigation, select wall geometry suitable for the specified conditions. For a first trial, place the outer face of the wall $\frac{1}{3}$ the width of the base slab back from the toe.

(*b*) Prepare the complete structural design, specifying size, placement, and cutoff points for all reinforcement. Materials have strengths $f_c' = 4000$ psi and $f_y = 60{,}000$ psi. Allowable soil bearing pressure is 5000 psf.

FIGURE P17.1



17.2. Redesign the wall of Problem 17.1 as a counterfort retaining wall. Counterforts are tentatively spaced 12 ft on centers, although this may be modified if desirable. Include all reinforcement details, including reinforcement in the counterforts.