3 Flexural Analysis and Design of Beams

3.1 Introduction

The fundamental assumptions upon which the analysis and design of reinforced concrete members are based were introduced in Section 1.8, and the application of those assumptions to the simple case of axial loading was developed in Section 1.9. The student should review Sections 1.8 and 1.9 at this time. In developing methods for the analysis and design of beams in this chapter, the same assumptions apply, and identical concepts will be used. This chapter will include analysis and design for flexure, including the dimensioning of the concrete cross section and the selection and placement of reinforcing steel. Other important aspects of beam design including shear reinforcement, bond and anchorage of reinforcing bars, and the important questions of serviceability (e.g., limiting deflections and controlling concrete cracking) will be treated in Chapters 4, 5, and 6.

3.2 Bending of Homogeneous Beams

Reinforced concrete beams are nonhomogeneous in that they are made of two entirely different materials. The methods used in the analysis of reinforced concrete beams are therefore different from those used in the design or investigation of beams composed entirely of steel, wood, or any other structural material. The fundamental principles involved are, however, essentially the same. Briefly, these principles are as follows.

At any cross section there exist internal forces that can be resolved into components normal and tangential to the section. Those components that are normal to the section are the *bending* stresses (tension on one side of the neutral axis and compression on the other). Their function is to resist the bending moment at the section. The tangential components are known as the *shear* stresses, and they resist the transverse or shear forces.

Fundamental assumptions relating to flexure and flexural shear are as follows:

1. A cross section that was plane before loading remains plane under load. This means that the unit strains in a beam above and below the neutral axis are proportional to the distance from that axis.
2. The bending stress f at any point depends on the strain at that point in a manner given by the stress-strain diagram of the material. If the beam is made of a homogeneous material whose stress-strain diagram in tension and compression is that of Fig. 3.1a, the following holds. If the maximum strain at the outer fibers is smaller than the strain ϵ_p up to which stress and strain are proportional for the

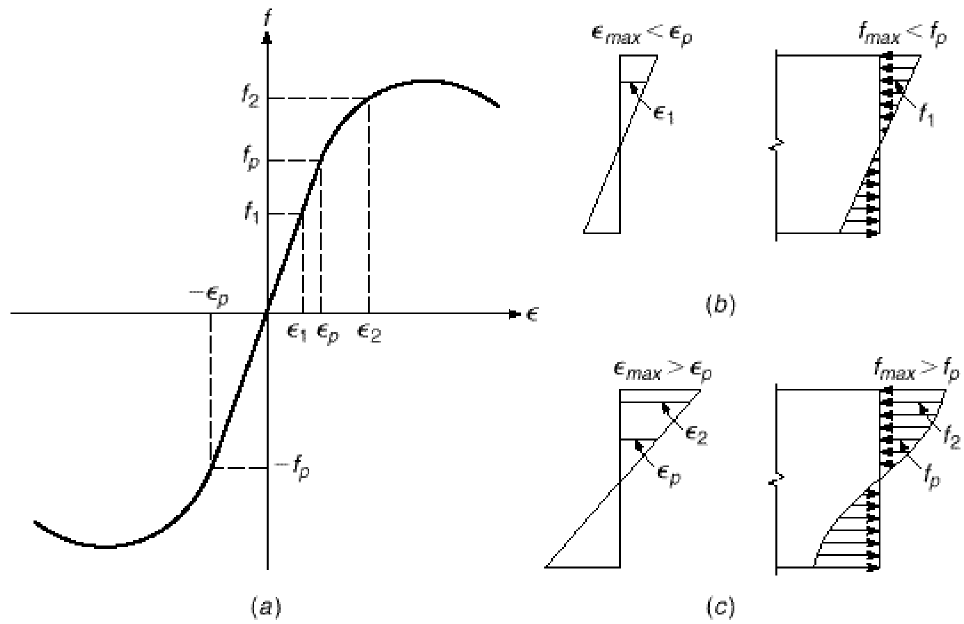


FIGURE 3.1
Elastic and inelastic stress distributions in homogeneous beams.

given material, then the compression and tension stresses on either side of the axis are proportional to the distance from the axis, as shown in Fig. 3.1*b*. However, if the maximum strain at the outer fibers is larger than ϵ_p, this is no longer true. The situation that then occurs is shown in Fig. 3.1*c*; i.e., in the outer portions of the beam, where $\epsilon > \epsilon_p$, stresses and strains are no longer proportional. In these regions, the magnitude of stress at any level, such as f_2 in Fig. 3.1*c*, depends on the strain ϵ_2 at that level in the manner given by the stress-strain diagram of the material. In other words, for a given strain in the beam, the stress at a point is the same as that given by the stress-strain diagram for the same strain.

3. The distribution of the shear stresses v over the depth of the section depends on the shape of the cross section and of the stress-strain diagram. These shear stresses are largest at the neutral axis and equal to zero at the outer fibers. The shear stresses on horizontal and vertical planes through any point are equal.

4. Owing to the combined action of shear stresses (horizontal and vertical) and flexure stresses, at any point in a beam there are inclined stresses of tension and compression, the largest of which form an angle of 90° with each other. The intensity of the inclined maximum or principal stress at any point is given by

$$t = \frac{f}{2} \pm \sqrt{\frac{f^2}{4} + v^2} \tag{3.1}$$

where f = intensity of normal fiber stress
v = intensity of tangential shearing stress

The inclined stress makes an angle α with the horizontal such that $\tan 2\alpha = 2v/f$.

5. Since the horizontal and vertical shearing stresses are equal and the flexural stresses are zero at the neutral plane, the inclined tensile and compressive stresses at any point in that plane form an angle of 45° with the horizontal, the intensity of each being equal to the unit shear at the point.

6. When the stresses in the outer fibers are smaller than the proportional limit f_p, the beam behaves *elastically*, as shown in Fig. 3.1*b*. In this case the following pertains:

(a) The neutral axis passes through the center of gravity of the cross section.

(b) The intensity of the bending stress normal to the section increases directly with the distance from the neutral axis and is a maximum at the extreme fibers. The stress at any given point in the cross section is represented by the equation

$$f = \frac{My}{I} \tag{3.2}$$

where f = bending stress at a distance y from neutral axis
M = external bending moment at section
I = moment of inertia of cross section about neutral axis

The maximum bending stress occurs at the outer fibers and is equal to

$$f_{max} = \frac{Mc}{I} = \frac{M}{S} \tag{3.3}$$

where c = distance from neutral axis to outer fiber
$S = I \cdot c$ = section modulus of cross section

(c) The shear stress (horizontal equals vertical) · at any point in the cross section is given by

$$\cdot = \frac{VQ}{Ib} \tag{3.4}$$

where V = total shear at section
Q = statical moment about neutral axis of that portion of cross section lying between a line through point in question parallel to neutral axis and nearest face (upper or lower) of beam
I = moment of inertia of cross section about neutral axis
b = width of beam at a given point

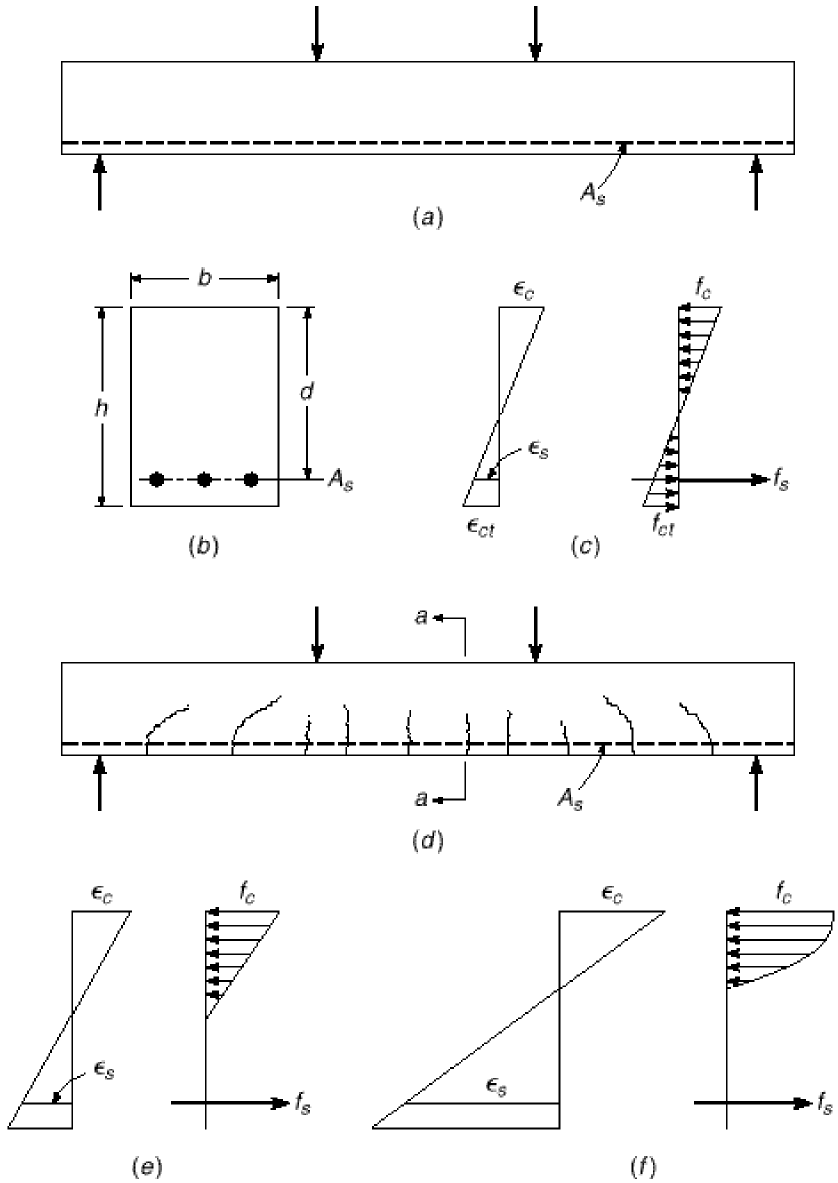
(d) The intensity of shear along a vertical cross section in a rectangular beam varies as the ordinates of a parabola, the intensity being zero at the outer fibers of the beam and a maximum at the neutral axis. For a total depth h, the maximum is $\frac{3}{2}V \cdot bh$, since at the neutral axis $Q = bh^2 \cdot 8$ and $I = bh^3 \cdot 12$ in Eq. (3.4).

The remainder of this chapter deals only with bending stresses and their effects on reinforced concrete beams. Shear stresses and their effects are discussed separately in Chapter 4.

3.3 Reinforced Concrete Beam Behavior

Plain concrete beams are inefficient as flexural members because the tensile strength in bending (modulus of rupture, see Section 2.9) is a small fraction of the compressive strength. As a consequence, such beams fail on the tension side at low loads long before the strength of the concrete on the compression side has been fully utilized. For this reason, steel reinforcing bars are placed on the tension side as close to the extreme tension fiber as is compatible with proper fire and corrosion protection of the steel. In such a reinforced concrete beam, the tension caused by the bending moments is chiefly

FIGURE 3.2
Behavior of reinforced concrete beam under increasing load.



resisted by the steel reinforcement, while the concrete alone is usually capable of resisting the corresponding compression. Such joint action of the two materials is assured if relative slip is prevented. This is achieved by using deformed bars with their high bond strength at the steel-concrete interface (see Section 2.14) and, if necessary, by special anchorage of the ends of the bars. A simple example of such a beam, with the customary designations for the cross-sectional dimensions, is shown in Fig. 3.2. For simplicity, the discussion that follows will deal with beams of rectangular cross section, even though members of other shapes are very common in most concrete structures.

When the load on such a beam is gradually increased from zero to the magnitude that will cause the beam to fail, several different stages of behavior can be clearly distinguished. At low loads, as long as the maximum tensile stress in the concrete is smaller than the modulus of rupture, the entire concrete is effective in resisting stress, in compression on one side and in tension on the other side of the neutral axis. In addition, the reinforcement, deforming the same amount as the adjacent concrete, is also subject to tensile stresses. At this stage, all stresses in the concrete are of small magnitude and are proportional to strains. The distribution of strains and stresses in concrete and steel over the depth of the section is as shown in Fig. 3.2*c*.

When the load is further increased, the tensile strength of the concrete is soon reached, and at this stage tension cracks develop. These propagate quickly upward to or close to the level of the neutral plane, which in turn shifts upward with progressive cracking. The general shape and distribution of these tension cracks is shown in Fig. 3.2*d*. In well-designed beams, the width of these cracks is so small (hairline cracks) that they are not objectionable from the viewpoint of either corrosion protection or appearance. Their presence, however, profoundly affects the behavior of the beam under load. Evidently, in a cracked section, i.e., in a cross section located at a crack such as *a-a* in Fig. 3.2*d*, the concrete does not transmit any tensile stresses. Hence, just as in tension members (Section 1.9b), the steel is called upon to resist the entire tension. At moderate loads, if the concrete stresses do not exceed approximately $f_c' \cdot 2$, stresses and strains continue to be closely proportional (see Fig. 1.16). The distribution of strains and stresses at or near a cracked section is then that shown in Fig. 3.2*e*. When the load is still further increased, stresses and strains rise correspondingly and are no longer proportional. The ensuing nonlinear relation between stresses and strains is that given by the concrete stress-strain curve. Therefore, just as in homogeneous beams (see Fig. 3.1), the distribution of concrete stresses on the compression side of the beam is of the same shape as the stress-strain curve. Figure 3.2*f* shows the distribution of strains and stresses close to the ultimate load.

Eventually, the carrying capacity of the beam is reached. Failure can be caused in one of two ways. When relatively moderate amounts of reinforcement are employed, at some value of the load the steel will reach its yield point. At that stress, the reinforcement yields suddenly and stretches a large amount (see Fig. 2.15), and the tension cracks in the concrete widen visibly and propagate upward, with simultaneous significant deflection of the beam. When this happens, the strains in the remaining compression zone of the concrete increase to such a degree that crushing of the concrete, the *secondary compression failure*, ensues at a load only slightly larger than that which caused the steel to yield. Effectively, therefore, attainment of the yield point in the steel determines the carrying capacity of moderately reinforced beams. Such yield failure is gradual and is preceded by visible signs of distress, such as the widening and lengthening of cracks and the marked increase in deflection.

On the other hand, if large amounts of reinforcement or normal amounts of steel of very high strength are employed, the compressive strength of the concrete may be exhausted before the steel starts yielding. Concrete fails by crushing when strains become so large that they disrupt the integrity of the concrete. Exact criteria for this occurrence are not yet known, but it has been observed that rectangular beams fail in compression when the concrete strains reach values of about 0.003 to 0.004. Compression failure through crushing of the concrete is sudden, of an almost explosive nature, and occurs without warning. For this reason it is good practice to dimension beams in such a manner that should they be overloaded, failure would be initiated by yielding of the steel rather than by crushing of the concrete.

The analysis of stresses and strength in the different stages just described will be discussed in the next several sections.

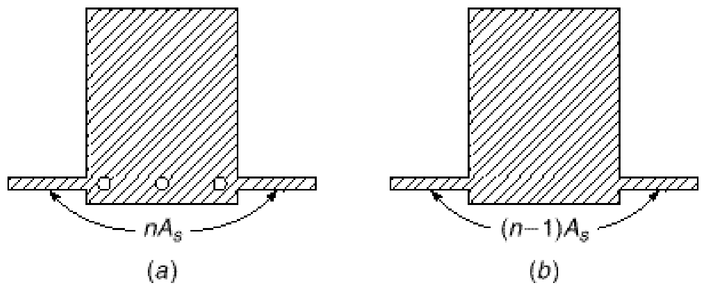


FIGURE 3.3
Uncracked transformed beam section.

a. Stresses Elastic and Section Uncracked

As long as the tensile stress in the concrete is smaller than the modulus of rupture, so that no tension cracks develop, the strain and stress distribution as shown in Fig. 3.2*c* is essentially the same as in an elastic, homogeneous beam (Fig. 3.1*b*). The only difference is the presence of another material, the steel reinforcement. As shown in Section 1.9a, in the elastic range, for any given value of strain, the stress in the steel is n times that of the concrete [Eq. (1.6)]. In the same section, it was shown that one can take account of this fact in calculations by replacing the actual steel-and-concrete cross section with a fictitious section thought of as consisting of concrete only. In this "transformed section," the actual area of the reinforcement is replaced with an equivalent concrete area equal to nA_s located at the level of the steel. The transformed, uncracked section pertaining to the beam of Fig. 3.2*b* is shown in Fig. 3.3.

Once the transformed section has been obtained, the usual methods of analysis of elastic homogeneous beams apply. That is, the section properties (location of neutral axis, moment of inertia, section modulus, etc.) are calculated in the usual manner, and, in particular, stresses are computed with Eqs. (3.2) to (3.4).

EXAMPLE 3.1

A rectangular beam has the dimensions (see Fig. 3.2*b*) $b = 10$ in., $h = 25$ in., and $d = 23$ in., and is reinforced with three No. 8 (No. 25) bars so that $A_s = 2.37$ in^2. The concrete cylinder strength f'_c is 4000 psi, and the tensile strength in bending (modulus of rupture) is 475 psi. The yield point of the steel f_y is 60,000 psi, the stress-strain curves of the materials being those of Fig. 1.16. Determine the stresses caused by a bending moment $M = 45$ ft-kips.

Solution. With a value $n = E_s/E_c = 29{,}000{,}000/3{,}600{,}000 = 8$, one has to add to the rectangular outline an area $(n - 1)A_s = 7 \times 2.37 = 16.59$ in^2, disposed as shown on Fig. 3.4, to obtain the uncracked, transformed section. Conventional calculations show that the location of the neutral axis of this section is given by $\bar{y} = 13.2$ in., and its moment of inertia about this axis is 14,740 in^4. For $M = 45$ ft-kips $= 540{,}000$ in-lb, the concrete compression stress at the top fiber is, from Eq. (3.3),

$$f_c = 540{,}000 \frac{13.2}{14{,}740} = 484 \text{ psi}$$

and, similarly, the concrete tension stress at the bottom fiber is

$$f_{ct} = 540{,}000 \frac{11.8}{14{,}740} = 432 \text{ psi}$$

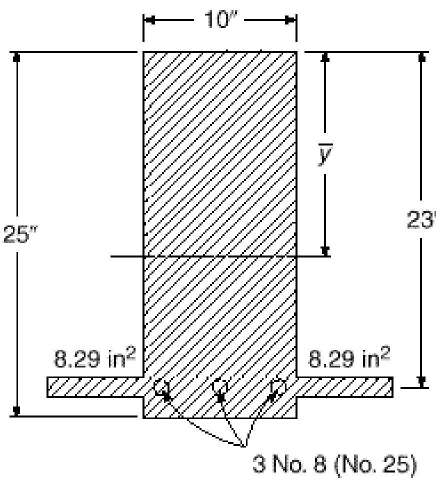


FIGURE 3.4
Transformed beam section of Example 3.1.

Since this value is below the given tensile bending strength of the concrete, 475 psi, no tension cracks will form, and calculation by the uncracked, transformed section is justified. The stress in the steel, from Eqs. (1.6) and (3.2), is

$$f_s = n\frac{My}{I} = 8 \cdot 540{,}000\frac{9.8}{14{,}740} = 2870 \text{ psi}$$

By comparing f_c and f_s with the cylinder strength and the yield point respectively, it is seen that at this stage the actual stresses are quite small compared with the available strengths of the two materials.

b. Stresses Elastic and Section Cracked

When the tensile stress f_{ct} exceeds the modulus of rupture, cracks form, as shown in Fig. 3.2*d*. If the concrete compressive stress is less than approximately $\frac{1}{2}f_c'$ and the steel stress has not reached the yield point, both materials continue to behave elastically, or very nearly so. This situation generally occurs in structures under normal service conditions and loads, since at these loads the stresses are generally of the order of magnitude just discussed. At this stage, for simplicity and with little if any error, it is assumed that tension cracks have progressed all the way to the neutral axis and that sections plane before bending are plane in the deformed member. The situation with regard to strain and stress distribution is then that shown in Fig. 3.2*e*.

To compute stresses, and strains if desired, the device of the transformed section can still be used. One need only take account of the fact that all of the concrete that is stressed in tension is assumed cracked, and therefore effectively absent. As shown in Fig. 3.5*a*, the transformed section then consists of the concrete in compression on one side of the axis and n times the steel area on the other. The distance to the neutral axis, in this stage, is conventionally expressed as a fraction kd of the effective depth d. (Once the concrete is cracked, any material located below the steel is ineffective, which is why d is the effective depth of the beam.) To determine the location of the

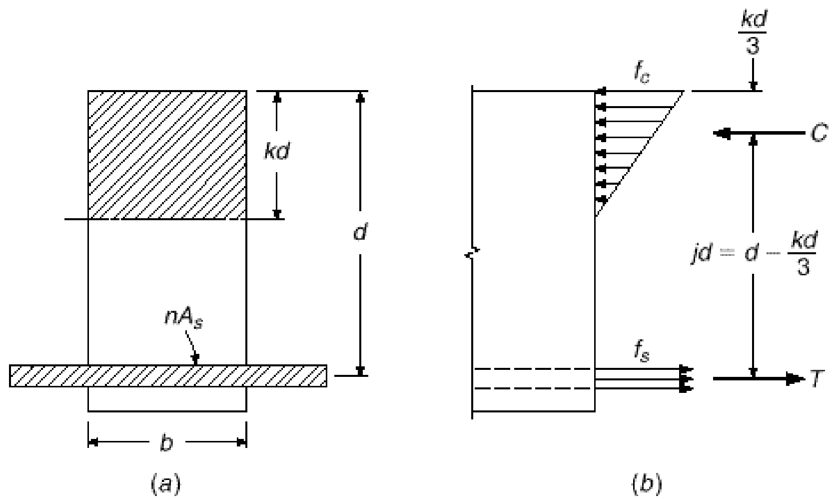


FIGURE 3.5
Cracked transformed section.

neutral axis, the moment of the tension area about the axis is set equal to the moment of the compression area, which gives

$$b\frac{(kd)^2}{2} - nA_s(d - kd) = 0 \tag{3.5}$$

Having obtained kd by solving this quadratic equation, one can determine the moment of inertia and other properties of the transformed section as in the preceding case. Alternatively, one can proceed from basic principles by accounting directly for the forces that act on the cross section. These are shown in Fig. 3.5*b*. The concrete stress, with maximum value f_c at the outer edge, is distributed linearly as shown. The entire steel area A_s is subject to the stress f_s. Correspondingly, the total compression force C and the total tension force T are

$$C = \frac{f_c}{2}bkd \qquad \text{and} \qquad T = A_sf_s \tag{3.6}$$

The requirement that these two forces be equal numerically has been taken care of by the manner in which the location of the neutral axis has been determined.

Equilibrium requires that the couple constituted by the two forces C and T be equal numerically to the external bending moment M. Hence, taking moments about C gives

$$M = Tjd = A_sf_sjd \tag{3.7}$$

where jd is the internal lever arm between C and T. From Eq. (3.7), the steel stress is

$$f_s = \frac{M}{A_sjd} \tag{3.8}$$

Conversely, taking moments about T gives

$$M = Cjd = \frac{f_c}{2}bkdjd = \frac{f_c}{2}kjbd^2 \tag{3.9}$$

from which the concrete stress is

$$f_c = \frac{2M}{kjbd^2} \tag{3.10}$$

In using Eqs. (3.6) through (3.10), it is convenient to have equations by which k and j may be found directly, to establish the neutral axis distance kd and the internal lever arm jd. First defining the *reinforcement ratio*

$$\rho = \frac{A_s}{bd} \tag{3.11}$$

then substituting $A_s = \rho bd$ into Eq. (3.5) and solving for k, one obtains

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n \tag{3.12}$$

From Fig. 3.5b it is seen that $jd = d - kd/3$, or

$$j = 1 - \frac{k}{3} \tag{3.13}$$

Values of k and j for elastic cracked section analysis, for common reinforcement ratios and modular ratios, are found in Table A.6 of Appendix A.

EXAMPLE 3.2

The beam of Example 3.1 is subject to a bending moment $M = 90$ ft-kips (rather than 45 ft-kips as previously). Calculate the relevant properties and stresses.

Solution. If the section were to remain uncracked, the tensile stress in the concrete would now be twice its previous value, that is, 864 psi. Since this exceeds by far the modulus of rupture of the given concrete (475 psi), cracks will have formed and the analysis must be adapted appropriately. Equation (3.5), with the known quantities b, n, and A_s inserted, gives the distance to the neutral axis $kd = 7.6$ in., or $k = 7.6/23 = 0.33$. From Eq. (3.13), $j = 1 - 0.33/3 = 0.89$. With these values the steel stress is obtained from Eq. (3.8) as $f_s =$ 22,300 psi, and the maximum concrete stress from Eq. (3.10) as $f_c = 1390$ psi.

Comparing the results with the pertinent values for the same beam when subject to one-half the moment, as previously calculated, one notices that (1) the neutral plane has migrated upward so that its distance from the top fiber has changed from 13.2 to 7.6 in.; (2) even though the bending moment has only been doubled, the steel stress has increased from 2870 to 22,300 psi, or about 7.8 times, and the concrete compression stress has increased from 484 to 1390 psi, or 2.9 times; (3) the moment of inertia of the cracked transformed section is easily computed to be 5,910 in^4, compared with 14,740 in^4 for the uncracked section. This affects the magnitude of the deflection, as discussed in Chapter 6. Thus, it is seen how radical is the influence of the formation of tension cracks on the behavior of reinforced concrete beams.

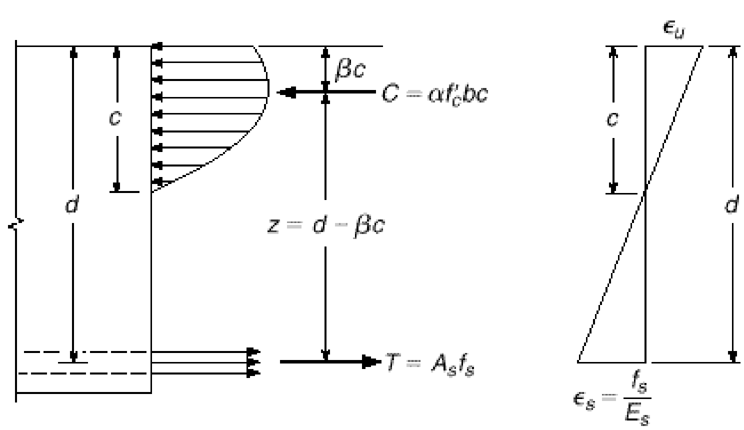
c. Flexural Strength

It is of interest in structural practice to calculate those stresses and deformations that occur in a structure in service under design load. For reinforced concrete beams, this can be done by the methods just presented, which assume elastic behavior of both materials. It is equally, if not more, important that the structural engineer be able to predict with satisfactory accuracy the strength of a structure or structural member. By making this strength larger by an appropriate amount than the largest loads that can be expected during the lifetime of the structure, an adequate margin of safety is assured. In the past, methods based on elastic analysis, like those just presented or variations thereof, have been used for this purpose. It is clear, however, that at or near the ultimate load, stresses are no longer proportional to strains. In regard to axial compression, this has been discussed in detail in Section 1.9, and in regard to bending, it has been pointed out that at high loads, close to failure, the distribution of stresses and strains is that of Fig. 3.2*f* rather than the elastic distribution of Fig. 3.2*e*. More realistic methods of analysis, based on actual inelastic rather than assumed elastic behavior of the materials and on results of extremely extensive experimental research, have been developed to predict the member strength. They are now used almost exclusively in structural design practice.

If the distribution of concrete compressive stresses at or near ultimate load (Fig. 3.2*f*) had a well-defined and invariable shape—parabolic, trapezoidal, or otherwise—it would be possible to derive a completely rational theory of bending strength, just as the theory of elastic bending with its known triangular shape of stress distribution (Figs. 3.1*b* and 3.2*c* and *e*) is straightforward and rational. Actually, inspection of Figs. 2.3, 2.4, and 2.6, and of many more concrete stress-strain curves that have been published, shows that the geometrical shape of the stress distribution is quite varied and depends on a number of factors, such as the cylinder strength and the rate and duration of loading. For this and other reasons, a wholly rational flexural theory for reinforced concrete has not yet been developed (Refs. 3.1 to 3.3). Present methods of analysis, therefore, are based in part on known laws of mechanics and are supplemented, where needed, by extensive test information.

Let Fig. 3.6 represent the distribution of internal stresses and strains when the beam is about to fail. One desires a method to calculate that moment M_n (nominal moment) at which the beam will fail either by tension yielding of the steel or by crushing of the concrete in the outer compression fiber. For the first mode of failure, the criterion is that the steel stress equal the yield point, $f_s = f_y$. It has been mentioned before that an exact criterion for concrete compression failure is not yet known, but that for rectangular beams strains of 0.003 to 0.004 have been measured immediately preceding failure. If one assumes, usually slightly conservatively, that the concrete is about to crush when the maximum strain reaches $\epsilon_u = 0.003$, comparison with a great many tests of beams and columns of a considerable variety of shapes and conditions of loading shows that a satisfactorily accurate and safe strength prediction can be made

FIGURE 3.6
Stress distribution at ultimate load.



(Ref. 3.4). In addition to these two criteria (yielding of the steel at a stress of f_y and crushing of the concrete at a strain of 0.003), it is not really necessary to know the exact shape of the concrete stress distribution in Fig. 3.6. What is necessary is to know, for a given distance c of the neutral axis, (1) the total resultant compression force C in the concrete and (2) its vertical location, i.e., its distance from the outer compression fiber.

In a rectangular beam, the area that is in compression is bc, and the total compression force on this area can be expressed as $C = f_{av}bc$, where f_{av} is the average compression stress on the area bc. Evidently, the average compressive stress that can be developed before failure occurs becomes larger, the higher the cylinder strength f_c' of the particular concrete. Let

$$\cdot = \frac{f_{av}}{f_c'} \tag{3.14}$$

Then

$$C = \cdot f_c' bc \tag{3.15}$$

For a given distance c to the neutral axis, the location of C can be defined as some fraction · of this distance. Thus, as indicated in Fig. 3.6, for a concrete of given strength it is necessary to know only · and · to completely define the effect of the concrete compressive stresses.

Extensive direct measurements, as well as indirect evaluations of numerous beam tests, have shown that the following values for · and · are satisfactorily accurate (see Ref. 3.5, where · is designated as k_1k_3 and · as k_2):

· equals 0.72 for $f_c' \leq 4000$ psi and decreases by 0.04 for every 1000 psi above 4000 up to 8000 psi. For $f_c' > 8000$ psi, · $= 0.56$.

· equals 0.425 for $f_c' \leq 4000$ psi and decreases by 0.025 for every 1000 psi above 4000 up to 8000 psi. For $f_c' > 8000$ psi, · $= 0.325$.

The decrease in · and · for high-strength concretes is related to the fact that such concretes are more brittle; i.e., they show a more sharply curved stress-strain plot with a smaller near-horizontal portion (see Figs. 2.3 and 2.4). Figure 3.7 shows these simple relations.

If this experimental information is accepted, the maximum moment can be calculated from the laws of equilibrium and from the assumption that plane cross sections remain plane. Equilibrium requires that

$$C = T \qquad \text{or} \qquad \cdot f_c' bc = A_s f_s \tag{3.16}$$

Also, the bending moment, being the couple of the forces C and T, can be written as either

$$M = Tz = A_s f_s \cdot d - \cdot c \cdot \tag{3.17}$$

or

$$M = Cz = \cdot f_c' bc \cdot d - \cdot c \cdot \tag{3.18}$$

For failure initiated by yielding of the tension steel, $f_s = f_y$. Substituting this value in Eq. (3.16), one obtains the distance to the neutral axis

$$c = \frac{A_s f_y}{\cdot f_c' b} \tag{3.19a}$$

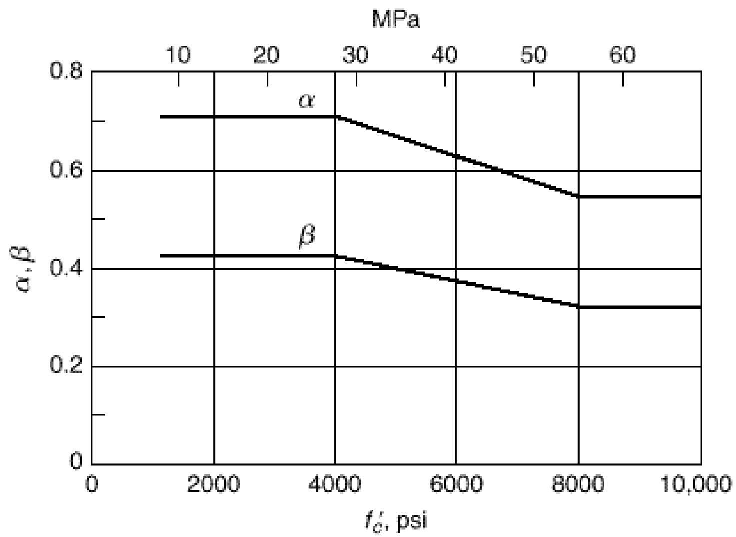


FIGURE 3.7
Variation of α and β with concrete strength f'_c.

Alternatively, using $A_s = \rho bd$, the neutral axis distance is

$$c = \frac{\rho f_y d}{\alpha f'_c} \tag{3.19b}$$

giving the distance to the neutral axis when tension failure occurs. The nominal moment M_n is then obtained from Eq. (3.17) with the value for c just determined, and $f_s = f_y$; that is,

$$M_n = \rho f_y b d^2 \left(1 - \frac{\beta \rho f_y}{\alpha f'_c}\right) \tag{3.20a}$$

With the specific, experimentally obtained values for α and β given previously, this becomes

$$M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f'_c}\right) \tag{3.20b}$$

If, for larger reinforcement ratios, the steel does not reach yield at failure, then the strain in the concrete becomes $\epsilon_u = 0.003$, as previously discussed. The steel stress f_s, not having reached the yield point, is proportional to the steel strain ϵ_s; i.e., according to Hooke's law,

$$f_s = \epsilon_s E_s$$

From the strain distribution of Fig. 3.6, the steel strain ϵ_s can be expressed in terms of the distance c by evaluating similar triangles, after which it is seen that

$$f_s = \epsilon_u E_s \frac{d - c}{c} \tag{3.21}$$

Then, from Eq. (3.16),

$$\alpha f'_c b c = A_s \epsilon_u E_s \frac{d - c}{c} \tag{3.22}$$

and this quadratic may be solved for c, the only unknown for the given beam. With both c and f_s known, the nominal moment of the beam, so heavily reinforced that failure occurs by crushing of the concrete, may be found from either Eq. (3.17) or Eq. (3.18).

Whether or not the steel has yielded at failure can be determined by comparing the actual reinforcement ratio with the *balanced reinforcement ratio* ρ_b, representing that amount of reinforcement necessary for the beam to fail by crushing of the concrete at the same load that causes the steel to yield. This means that the neutral axis must be so located that at the load at which the steel starts yielding, the concrete reaches its compressive strain limit ε_u. Correspondingly, setting $f_s = f_y$ in Eq. (3.21) and substituting the yield strain ε_y for f_y/E_s, one obtains the value of c defining the unique position of the neutral axis corresponding to simultaneous crushing of the concrete and initiation of yielding in the steel,

$$c = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} d \tag{3.23}$$

Substituting that value of c into Eq. (3.16), with $A_s f_s = \rho b d f_y$, one obtains for the balanced reinforcement ratio

$$\rho_b = \frac{\alpha f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} \tag{3.24}$$

EXAMPLE 3.3 Determine the nominal moment M_n at which the beam of Examples 3.1 and 3.2 will fail.

Solution. For this beam the reinforcement ratio $\rho = A_s/bd = 2.37/(10 \times 23) = 0.0103$. The balanced reinforcement ratio is found from Eq. (3.24) to be 0.0284. Since the amount of steel in the beam is less than that which would cause failure by crushing of the concrete, the beam will fail in tension by yielding of the steel. Its nominal moment, from Eq. (3.20*b*), is

$$M_n = 0.0103 \times 60{,}000 \times 10 \times 23^2 \left(1 - 0.59 \frac{0.0103 \times 60{,}000}{4000}\right)$$

$$= 2{,}970{,}000 \text{ in-lb} = 248 \text{ ft-kips}$$

When the beam reaches M_n, the distance to its neutral axis, from Eq. (3.19*b*), is

$$c = \frac{0.0103 \times 60{,}000 \times 23}{0.72 \times 4000} = 4.94$$

It is interesting to compare this result with those of Examples 3.1 and 3.2. In the previous calculations, it was found that at low loads, when the concrete had not yet cracked in tension, the neutral axis was located at a distance of 13.2 in. from the compression edge; at higher loads, when the tension concrete was cracked but stresses were still sufficiently small to be elastic, this distance was 7.6 in. Immediately before the beam fails, as has just been shown, this distance has further decreased to 4.9 in. For these same stages of loading, the stress in the steel increased from 2870 psi in the uncracked section, to 22,300 psi in the cracked elastic section, and to 60,000 psi at the nominal moment capacity. This migration of the neutral axis toward the compression edge and the increase in steel stress as load is increased is a graphic illustration of the differences between the various stages of behavior through which a reinforced concrete beam passes as its load is increased from zero to the value that causes it to fail. The examples also illustrate the fact that nominal moments cannot be determined accurately by elastic calculations.

3.4 Design of Tension-Reinforced Rectangular Beams

For reasons that were explained in Chapter 1, the present design of reinforced concrete structures is based on the concept of providing sufficient strength to resist hypothetical overloads. The *nominal strength* of a proposed member is calculated, based on the best current knowledge of member and material behavior. That nominal strength is modified by a *strength reduction factor* · , less than unity, to obtain the *design strength*. The *required strength*, should the hypothetical overload stage actually be realized, is found by applying *load factors* · , greater than unity, to the loads actually expected. These expected *service loads* include the calculated dead load, the calculated or legally specified live load, and environmental loads such as those due to wind, seismic action, or temperature. Thus reinforced concrete members are proportioned so that, as shown in Eq. (1.5),

$$M_u \leq \cdot\, M_n$$

$$P_u \leq \cdot\, P_n$$

$$V_u \leq \cdot\, V_n$$

where the subscripts n denote the nominal strengths in flexure, thrust, and shear respectively, and the subscripts u denote the factored load moment, thrust, and shear. The strength reduction factors · normally differ, depending upon the type of strength to be calculated, the importance of the member in the structure, and other considerations discussed in detail in Chapter 1.

A member proportioned on the basis of adequate strength at a hypothetical overload stage must also perform in a satisfactory way under normal service load conditions. In specific terms, the deflection must be limited to an acceptable value, and concrete tensile cracks, which inevitably occur, must be of narrow width and well distributed throughout the tensile zone. Therefore, after proportioning for adequate strength, deflections are calculated and compared against limiting values (or otherwise controlled), and crack widths limited by specific means. This approach to design, referred to in Europe, and to some extent in U.S. practice, as *limit states design*, is the basis of the 2002 ACI Code, and it is the approach that will be followed in this and later chapters.

a. Equivalent Rectangular Stress Distribution

The method presented in Section 3.3c for calculating the flexural strength of reinforced concrete beams, derived from basic concepts of structural mechanics and pertinent experimental research information, also applies to situations other than the case of rectangular beams reinforced on the tension side. It can be used and gives valid answers for beams of other cross-sectional shapes, reinforced in other manners, and for members subject not only to simple bending but also to the simultaneous action of bending and axial force (compression or tension). However, the pertinent equations for these more complex cases become increasingly cumbersome and lengthy. What is more important, it becomes increasingly difficult for the designer to visualize the physical basis for the design methods and formulas; this could lead to a blind reliance on formulas, with a resulting lack of actual understanding. This is not only undesirable on general grounds but, practically, is more likely to lead to numerical errors in design work than when the designer at all times has a clear picture of the physical situation in the member being dimensioned or analyzed. Fortunately, it is possible, essentially by a conceptual trick, to formulate the strength analysis of reinforced concrete members in a different manner, which gives the same answers as the general analysis just developed but which is much more easily visualized and much more easily applied to cases of greater complexity than that of the simple rectangular beam. Its consistency is shown, and its application to more complex cases has been checked against the results of a vast number of tests on a great variety of types of members and conditions of loading (Ref. 3.4).

It was noted in the preceding section that the actual geometrical shape of the concrete compressive stress distribution varies considerably and that, in fact, one need not know this shape exactly, provided one does know two things: (1) the magnitude C of the resultant of the concrete compressive stresses and (2) the location of this resultant. Information on these two quantities was obtained from the results of experimental research and expressed in the two parameters · and · .

Evidently, then, one can think of the actual complex stress distribution as replaced by a fictitious one of some simple geometric shape, provided that this fictitious distribution results in the same total compression force C applied at the same location as in the actual member when it is on the point of failure. Historically, a number of simplified, fictitious equivalent stress distributions has been proposed by investigators in various countries. The one generally accepted in this country, and increasingly abroad, was first proposed by C. S. Whitney (Ref. 3.4) and was subsequently elaborated and checked experimentally by others (see, for example, Refs. 3.5 and 3.6). The actual stress distribution immediately before failure and the fictitious equivalent distribution are shown in Fig. 3.8.

It is seen that the actual stress distribution is replaced by an equivalent one of simple rectangular outline. The intensity $\cdot f_c'$ of this equivalent constant stress and its depth $a = \cdot_1 c$ are easily calculated from the two conditions that (1) the total compression force C and (2) its location, i.e., distance from the top fiber, must be the same in the equivalent rectangular as in the actual stress distribution. From Fig. 3.8*a* and *b* the first condition gives

$$C = \cdot f_c' cb = \cdot f_c' ab \qquad \text{from which} \qquad \cdot = \cdot \frac{c}{a}$$

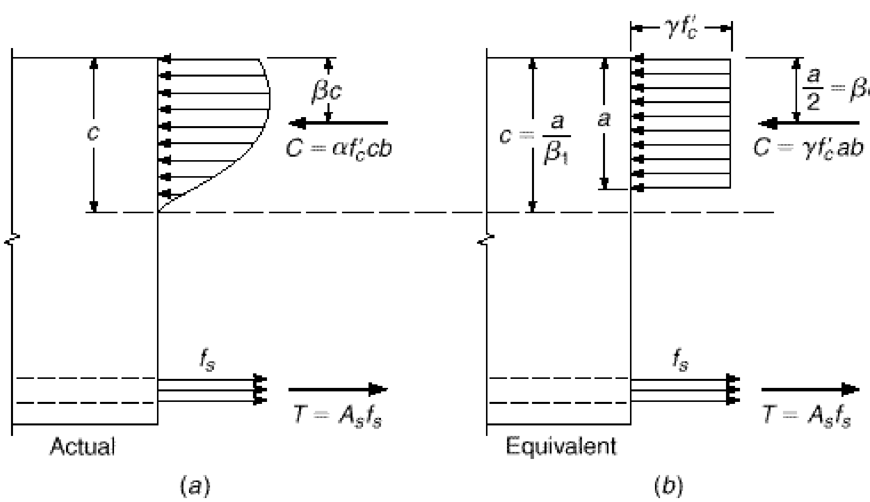


FIGURE 3.8
Actual and equivalent rectangular stress distributions at ultimate load.

TABLE 3.1
Concrete stress block parameters

	f_c', psi				
	≤4000	5000	6000	7000	≥8000
α	0.72	0.68	0.64	0.60	0.56
β	0.425	0.400	0.375	0.350	0.325
$\beta_1 = 2\beta$	0.85	0.80	0.75	0.70	0.65
$\gamma = \alpha/\beta_1$	0.85	0.85	0.85	0.86	0.86

With $a = \beta_1 c$, this gives $\gamma = \alpha/\beta_1$. The second condition simply requires that in the equivalent rectangular stress block, the force C be located at the same distance βc from the top fiber as in the actual distribution. It follows that $\beta_1 = 2\beta$.

To supply the details, the upper two lines of Table 3.1 present the experimental evidence of Fig. 3.7 in tabular form. The lower two lines give the just-derived parameters β_1 and γ for the rectangular stress block. It is seen that the stress intensity factor γ is essentially independent of f_c' and can be taken as 0.85 throughout. Hence, regardless of f_c', the concrete compression force at failure in a rectangular beam of width b is

$$C = 0.85 f_c' ab \tag{3.25}$$

Also, for the common concretes with $f_c' \le 4000$ psi, the depth of the rectangular stress block is $a = 0.85c$, c being the distance to the neutral axis. For higher-strength concretes, this distance is $a = \beta_1 c$, with the β_1 values shown in Table 3.1. This is expressed in ACI Code 10.2.7.3 as follows: β_1 shall be taken as 0.85 for concrete strengths up to and including 4000 psi; for strengths above 4000 psi, β_1 shall be reduced continuously at a rate of 0.05 for each 1000 psi of strength in excess of 4000 psi, but β_1 shall not be taken less than 0.65. In mathematical terms, the relationship between β_1 and f_c' can be expressed as

$$\beta_1 = 0.85 - 0.05\,\frac{f_c' - 4000}{1000} \qquad \text{and} \qquad 0.65 \le \beta_1 \le 0.85 \tag{3.26}$$

The equivalent rectangular stress distribution can be used for deriving the equations that have been developed in Section 3.3c. The failure criteria, of course, are the same as before: yielding of the steel at $f_s = f_y$ or crushing of the concrete at $\epsilon_u = 0.003$. Because the rectangular stress block is easily visualized and its geometric properties are extremely simple, many calculations are carried out directly without reference to formally derived equations, as will be seen in the following sections.

b. Balanced Strain Condition

A reinforcement ratio ρ_b producing balanced strain conditions can be established based on the condition that, at balanced failure, the steel strain is exactly equal to ϵ_y when the strain in the concrete simultaneously reaches the crushing strain of $\epsilon_u = 0.003$. Referring to Fig. 3.6,

$$c = \frac{\epsilon_u}{\epsilon_u + \epsilon_y}\,d \tag{3.27}$$

which is seen to be identical to Eq. (3.23). Then, from the equilibrium requirement that $C = T$

$$\rho_b f_y bd = 0.85 f'_c ab = 0.85 \beta_1 f'_c bc$$

from which

$$\rho_b = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_y} \qquad (3.28)$$

This is easily shown to be equivalent to Eq. (3.24).

c. Underreinforced Beams

A compression failure in flexure, should it occur, gives little if any warning of distress, while a tension failure, initiated by yielding of the steel, typically is gradual. Distress is obvious from observing the large deflections and widening of concrete cracks associated with yielding of the steel reinforcement, and measures can be taken to avoid total collapse. In addition, most beams for which failure initiates by yielding possess substantial strength based on strain-hardening of the reinforcing steel, which is not accounted for in the calculations of M_n.

Because of these differences in behavior, it is prudent to require that beams be designed such that failure, if it occurs, will be by yielding of the steel, not by crushing of the concrete. This can be done, theoretically, by requiring that the reinforcement ratio ρ be less than the balance ratio ρ_b given by Eq. (3.28).

In actual practice, the upper limit on ρ should be below ρ_b for the following reasons: (1) for a beam with ρ exactly equal to ρ_b, the compressive strain limit of the concrete would be reached, theoretically, at precisely the same moment that the steel reaches its yield stress, without significant yielding before failure, (2) material properties are never known precisely, (3) strain-hardening of the reinforcing steel, not accounted for in design, may lead to a brittle concrete compression failure even though ρ may be somewhat less than ρ_b, (4) the actual steel area provided, considering standard reinforcing bar sizes, will always be equal to or larger than required, based on selected reinforcement ratio ρ, tending toward overreinforcement, and (5) the extra ductility provided by beams with lower values of ρ increases the deflection capability substantially and, thus, provides warning prior to failure.

d. ACI Code Provisions for Underreinforced Beams

While the nominal strength of a member may be computed based on principles of mechanics, the mechanics alone cannot establish safe limits for maximum reinforcement ratios. These limits are defined by the ACI Code. The limitations take two forms. First, the Code addresses the minimum tensile reinforcement strain allowed at nominal strength in the design of beams. Second, the Code defines strength reduction factors that may depend on the tensile strain at nominal strength. Both limitations are based on the *net tensile strain* ϵ_t of the reinforcement farthest from the compression face of the concrete at the depth d_t. The net tensile strain is exclusive of prestress, temperature, and shrinkage effects. For beams with a single layer of reinforcement, the depth to the centroid of the steel d is the same as d_t. For beams with multiple layers of reinforcement,

d_t is greater than the depth to the centroid of the reinforcement d. Substituting d_t for d and ϵ_t for ϵ_y in Eq. (3.27), the net tensile strain may be represented as

$$\epsilon_t = \epsilon_u \frac{d_t - c}{c} \tag{3.29}$$

Then based on Eq. (3.28), the reinforcement ratio to produce a selected value of net tensile strain is

$$\rho = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t} \tag{3.30a}$$

To ensure underreinforced behavior, ACI Code 10.3.5 establishes a minimum net tensile strain ϵ_t at the nominal member strength of 0.004 for members subjected to axial loads less than $0.10 f'_c A_g$, where A_g is the gross area of the cross section. By way of comparison ϵ_y, the steel strain at the balanced condition, is 0.00207 for $f_y = 60{,}000$ psi and 0.00259 for $f_y = 75{,}000$ psi.

Using $\epsilon_t = 0.004$ in Eq. (3.30*a*) provides the maximum reinforcement ratio allowed by the ACI Code for beams.

$$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} \tag{3.30b}$$

The maximum reinforcement ratio is exact for beams with a single layer of reinforcement and slightly conservative for beams with multiple layers of reinforcement where d_t is greater than d. Because $\epsilon_t \geq 0.004$ ensures that steel is yielding in tension, $f_s = f_y$ at failure, and the nominal flexural strength (referring to Fig. 3.11) is given by

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) \tag{3.31}$$

where

$$a = \frac{A_s f_y}{0.85 f'_c b} \tag{3.32}$$

The ACI Code further encourages the use of lower reinforcement ratios by allowing higher strength reduction factors in such beams. The Code defines a *tension-controlled member* as one with a net tensile strain greater than or equal to 0.005. The corresponding strength reduction factor is $\phi = 0.9$.[†] The Code additionally defines a *compression-controlled member* as having a net tensile strain of less than 0.002. The strength reduction factor for compression-controlled members is 0.65. A value of 0.70 may be used if the members are spirally reinforced. A value of $\epsilon_t = 0.002$ corresponds approximately to the yield strain for steel with f_y 60,000 psi yield strength. Between net tensile strains of 0.002 and 0.005, the strength reduction factor varies linearly, and the ACI Code allows a linear interpolation of ϕ based on ϵ_t, as shown in Fig. 3.9.

Calculation of the nominal moment capacity frequently involves determination of the depth of the equivalent rectangular stress block a. Since $c = a/\beta_1$, it is sometimes more convenient to compute c/d ratios than the net tensile strain. The assumption that plane sections remain plane ensures a direct correlation between net tensile strain and the c/d ratio, as shown in Fig. 3.10.

[†] The selection of a net tensile strain of 0.005 is intended to encompass the yield strain of all reinforcing steel including high-strength rods and prestressing tendons.

FIGURE 3.9
Variation of strength reduction factor with net tensile strain.

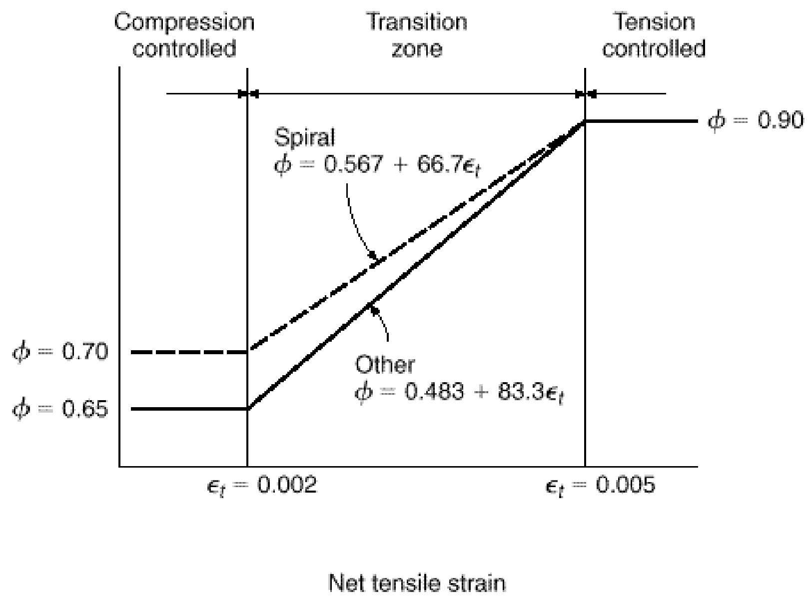
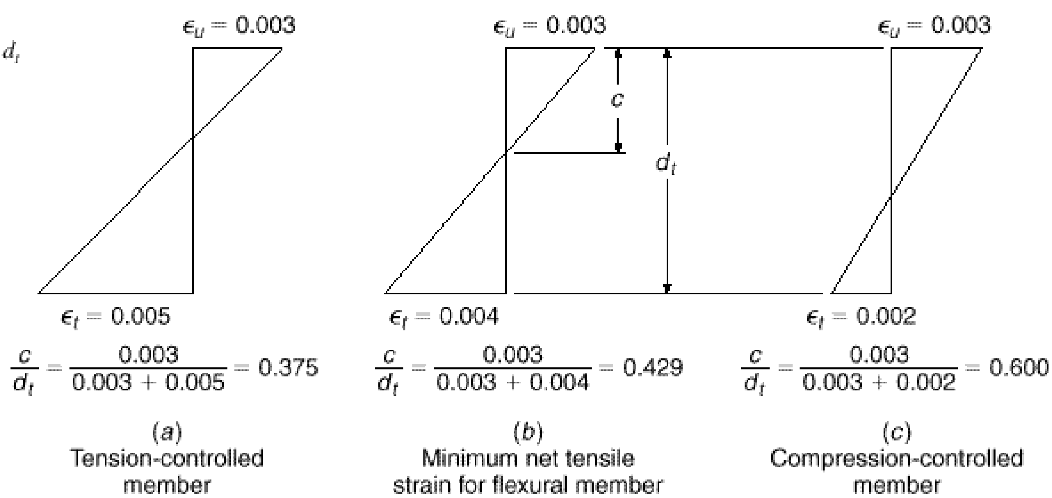


FIGURE 3.10
Net tensile strain and c/d_t ratios.



EXAMPLE 3.4 Using the equivalent rectangular stress distribution, directly calculate the nominal strength of the beam previously analyzed in Example 3.3.

Solution. The distribution of stresses, internal forces, and strains is as shown in Fig. 3.11. The maximum reinforcement ratio is calculated from Eq. (3.30*b*) as

$$\rho_{max} = 0.85 \times 0.85 \frac{4000}{60{,}000} \frac{0.003}{0.003 + 0.004} = 0.0206$$

and comparison with the actual reinforcement ratio of 0.0103 confirms that the member is underreinforced and will fail by yielding of the steel. The depth of the equivalent stress block is found from the equilibrium condition that $C = T$. Hence $0.85 f'_c ab = A_s f_y$, or

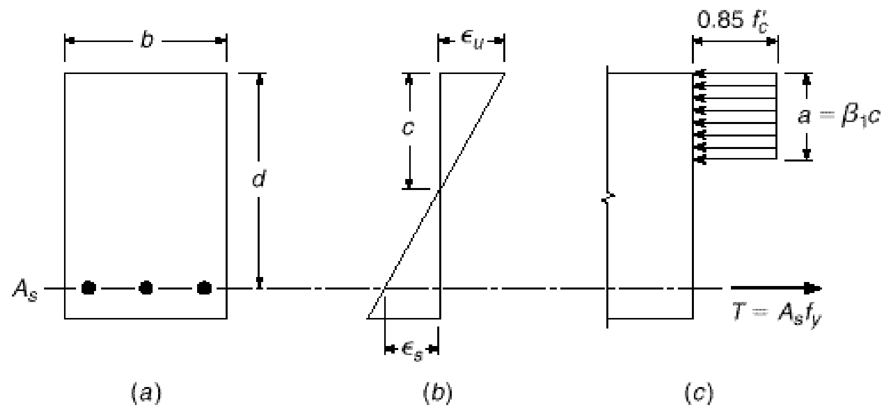


FIGURE 3.11
Singly reinforced rectangular beam.

$a = 2.37 \times 60{,}000/(0.85 \times 4000 \times 10) = 4.18$. The distance to the neutral axis, by definition of the rectangular stress block, is $c = a/\beta_1 = 4.18/0.85 = 4.92$. The nominal moment is

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 2.37 \times 60{,}000\,(23 - 2.09) = 2{,}970{,}000 \text{ in-lb} = 248 \text{ ft-kips}$$

The results of this simple and direct numerical analysis, based on the equivalent rectangular stress distribution, are identical with those previously determined from the general strength analysis described in Section 3.3c.

It is convenient for everyday design to combine Eqs. (3.31) and (3.32) as follows. Noting that $A_s = \rho bd$, Eq. (3.32) can be rewritten as

$$a = \frac{\rho f_y d}{0.85 f_c'} \tag{3.33}$$

This is then substituted into Eq. (3.31) to obtain

$$M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right) \tag{3.34}$$

which is identical to Eq. (3.20*b*) derived in Section 3.3c. This basic equation can be simplified further as follows:

$$M_n = R b d^2 \tag{3.35}$$

in which

$$R = \rho f_y \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right) \tag{3.36}$$

The *flexural resistance factor* R depends only on the reinforcement ratio and the strengths of the materials and is easily tabulated. Tables A.5a and A.5b of Appendix A give R values for ordinary combinations of steel and concrete and the full practical range of reinforcement ratios.

In accordance with the safety provisions of the ACI Code, the nominal flexural strength M_n is reduced by imposing the strength reduction factor ϕ to obtain the *design strength:*

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right) \tag{3.37}$$

or, alternatively,

$$\phi M_n = \phi \rho f_y bd^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right) \tag{3.38}$$

or

$$\phi M_n = \phi Rbd^2 \tag{3.39}$$

EXAMPLE 3.4 (continued) Calculate the design moment capacity for the beam analyzed in Example 3.4.

Solution. For a distance to the neutral axis of $c = 4.92$, $\epsilon_t = 0.003(23 - 4.92)/4.92 = 0.011$ from Eq. (3.28). $\epsilon_t > 0.005$, so $\phi = 0.90$ and the design capacity is

$$\phi M_n = 0.9 \times 248 = 223 \text{ ft-kips}$$

e. Minimum Reinforcement Ratio

Another mode of failure may occur in very lightly reinforced beams. If the flexural strength of the cracked section is less than the moment that produced cracking of the previously uncracked section, the beam will fail immediately and without warning of distress upon formation of the first flexural crack. To ensure against this type of failure, a *lower limit* can be established for the reinforcement ratio by equating the cracking moment, computed from the concrete modulus of rupture (Section 2.9), to the strength of the cracked section.

For a rectangular section having width b, total depth h, and effective depth d (see Fig. 3.2b), the section modulus with respect to the tension fiber is $bh^2/6$. For typical cross sections, it is satisfactory to assume that $h/d = 1.1$ and that the internal lever arm at flexural failure is $0.95d$. If the modulus of rupture is taken as $f_r = 7.5\sqrt{f_c'}$, as usual, then an analysis equating the cracking moment to the flexural strength results in

$$A_{s,min} = \frac{1.8\sqrt{f_c'}}{f_y} bd \tag{3.40a}$$

This development can be generalized to apply to beams having a T cross section (see Section 3.8 and Fig. 3.15). The corresponding equations depend on the proportions of the cross section and on whether the beam is bent with the flange (slab) in tension or in compression. For T beams of typical proportions that are bent with the flange in compression, analysis will confirm that the minimum steel area should be

$$A_{s,min} = \frac{2.7\sqrt{f_c'}}{f_y} b_w d \tag{3.40b}$$

where b_w is the width of the web, or stem, projecting below the slab. For T beams that are bent with the flange in tension, from a similar analysis, the minimum steel area is

$$A_{s,min} = \frac{6.2\sqrt{f_c'}}{f_y} b_w d \tag{3.40c}$$

The ACI Code requirements for minimum steel area are based on the results just discussed, but there are some differences. According to ACI Code 10.5, at any section where tensile reinforcement is required by analysis, with some exceptions as noted below, the area A_s provided must not be less than

$$A_{s,min} = \frac{3\sqrt{f'_c}}{f_y} b_w d \geq \frac{200 b_w d}{f_y} \tag{3.41}$$

This applies to both positive and negative bending sections. The inclusion of the additional limit of $200b_w d/f_y$ is merely for historical reasons; it happens to give the same minimum reinforcement ratio of 0.005 that was imposed in earlier codes for then-common material strengths. Note that in Eq. (3.41) the section width b_w is used; it is understood that for rectangular sections, $b_w = b$. Note further that the ACI coefficient of 3 is a conservatively rounded value compared with 2.7 in Eq. (3.40*b*) for T beams with the flange in compression, and is very conservative when applied to rectangular beam sections, for which a rational analysis gives 1.8 in Eq. (3.40*a*). This probably reflects the view that the minimum steel for the negative bending sections of a continuous T beam (which are, in effect, rectangular sections, as discussed in Section 3.8c) should be no less than for the positive bending sections, where the moment is generally smaller.

ACI Code 10.5 treats *statically determinate* T beams with the flange in *tension* as a special case, for which the minimum steel area is equal to or greater than the value given by Eq. (3.41) with b_w replaced by either $2b_w$ or the width of the *flange*, whichever is smaller.

Note that ACI Code Eq. (3.41) is conveniently expressed in terms of a *minimum tensile reinforcement ratio* ρ_{min} by dividing both sides by $b_w d$.

According to ACI Code 10.5, the requirements of Eq. (3.41) need not be imposed if, at every section, the area of tensile reinforcement provided is at least one-third greater than that required by analysis. This provides sufficient reinforcement for large members such as grade beams, where the usual equations would require excessive amounts of steel.

For structural slabs and footings of uniform thickness, the minimum area of tensile reinforcement in the direction of the span is that required for shrinkage and temperature steel (see Section 13.3 and Table 13.2), and the above minimums need not be imposed. The maximum spacing of such steel is the smaller of 3 times the total slab thickness or 18 in.

f. Examples of Rectangular Beam Analysis and Design

Flexural problems can be classified broadly as *analysis problems* or *design problems*. In analysis problems, the section dimensions, reinforcement, and material strengths are known, and the moment capacity is required. In the case of design problems, the required moment capacity is given, as are the material strengths, and it is required to find the section dimensions and reinforcement. Examples 3.5 and 3.6 illustrate analysis and design, respectively.

EXAMPLE 3.5

Flexural strength of a given member. A rectangular beam has width 12 in. and effective depth 17.5 in. It is reinforced with four No. 9 (No. 29) bars in one row. If $f_y = 60{,}000$ psi and $f'_c = 4000$ psi, what is the nominal flexural strength, and what is the maximum moment that can be utilized in design, according to the ACI Code?

Solution. From Table A.2 of Appendix A, the area of four No. 9 (No. 29) bars is 4.00 in². Thus, the actual reinforcement ratio is $\rho = 4.00/(12 \times 17.5) = 0.0190$. This is below the maximum value from Eq. (3.30*b*) of

$$\rho_{max} = 0.85 \times 0.85 \left(\frac{4}{60}\right)\left(\frac{0.003}{0.003 + 0.004}\right) = 0.0206$$

so failure by tensile yielding would be obtained. For this underreinforced beam, from Eq. (3.32),

$$a = \frac{4.00 \times 60}{0.85 \times 4 \times 12} = 5.88 \text{ in.}$$

and, from Eq. (3.31),

$$M_n = 4.00 \times 60\left(17.5 - \frac{5.88}{2}\right) = 3490 \text{ in-kips}$$

the depth to the neutral axis is $c = a/\beta_1 = 5.88/0.85 = 6.92$. The net tensile strain is $\epsilon_t = \epsilon_u(d - c)/c = 0.003 \times (17.5 - 6.92)/6.92 = 0.00458 > 0.004$ but less than 0.005; thus, the strength reduction factor must be adjusted. Using a linear interpolation from Fig. 3.9, $\phi = 0.87$, and the design strength is taken as

$$\phi M_n = 0.87 \times 3490 = 3040 \text{ in-kips}$$

The ACI Code limits on the reinforcement ratio,

$$\rho_{max} = 0.0206$$

$$\rho_{min} = \frac{3\sqrt{4000}}{60{,}000} \geq \frac{200}{60{,}000} = 0.0033$$

are satisfied for this beam.

EXAMPLE 3.6 **Concrete dimensions and steel area to resist a given moment.** Find the cross section of concrete and area of steel required for a simply supported rectangular beam with a span of 15 ft that is to carry a computed dead load of 1.27 kips/ft and a service live load of 2.15 kips/ft. Material strengths are $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

Solution. Load factors are first applied to the given service loads to obtain the factored load for which the beam is to be designed, and the corresponding moment:

$$w_u = 1.2 \times 1.27 + 1.6 \times 2.15 = 4.96 \text{ kips/ft}$$

$$M_u = \frac{1}{8} \times 4.96 \times 15^2 \times 12 = 1670 \text{ in-kips}$$

The concrete dimensions will depend on the designer's choice of reinforcement ratio. To minimize the concrete section, it is desirable to select the maximum permissible reinforcement ratio. To maintain $\phi = 0.9$, the maximum reinforcement ratio corresponding to a net tensile strain of 0.005 will be selected (see Fig. 3.9). Then, from Eq. (3.30*a*)

$$\rho = 0.85\beta_1 \frac{f'_c}{f_y}\frac{\epsilon_u}{\epsilon_u + 0.005} = 0.85 \times 0.85\left(\frac{4}{60}\right)\left(\frac{0.003}{0.003 + 0.005}\right) = 0.0181$$

Using Eq. (3.30*b*) gives $\rho_{max} = 0.0206$, but would require a lower strength reduction factor.

Setting the required flexural strength equal to the design strength from Eq. (3.38), and substituting the selected values for · and material strengths,

$$M_u = \cdot\ M_n$$

$$1670 = 0.90 \times 0.0181 \times 60bd^2 \cdot\ 1 - 0.59\,\frac{0.0181 \times 60}{4}\cdot$$

from which

$$bd^2 = 2040 \text{ in}^3$$

A beam with width $b = 10$ in. and $d = 14.3$ in. will satisfy this requirement. The required steel area is found by applying the chosen reinforcement ratio to the required concrete dimensions:

$$A_s = 0.0181 \times 10 \times 14.3 = 2.59 \text{ in}^2$$

Two No. 10 (No. 32) bars provide 2.54 in^2 and is very close to the required area.

Assuming 2.5 in. concrete cover from the centroid of the bars, the required total depth is $h = 16.8$ in. In actual practice, however, the concrete dimensions b and h are always rounded upward to the nearest inch, and often to the nearest multiple of 2 in. (see Section 3.5). The actual d is then found by subtracting the required concrete cover dimension from h. For the present example, $b = 10$ in. and $h = 18$ in. will be selected, resulting in effective depth $d =$ 15.5 in. Improved economy then may be possible, refining the steel area based on the actual, larger, effective depth. One can obtain the revised steel requirement directly by solving Eq. (3.38) for · , with · $M_n = M_u$. A quicker solution can be obtained by iteration. First a reasonable value of a is assumed, and A_s is found from Eq. (3.37). From Eq. (3.32) a revised estimate of a is obtained, and A_s is revised. This method converges very rapidly. For example, assume $a = 5$ in. Then

$$A_s = \frac{1670}{0.90 \times 60 \cdot 15.5 - 2.5 \cdot} = 2.38 \text{ in}^2$$

Checking the assumed a:

$$a = \frac{2.38 \times 60}{0.85 \times 4 \times 10} = 4.20 \text{ in.}$$

This is close enough to the assumed value that no further calculation is required. The required steel area of 2.38 in^2 could be provided using three No. 8 (No. 25) bars, but for simplicity of construction, two No. 10 (No. 32) bars will be used as before.

A somewhat larger beam cross section using less steel may be more economical, and will tend to reduce deflections. As an alternative solution, the beam will be redesigned with a lower reinforcement ratio of · $= 0.60\cdot_{max} = 0.60 \times 0.0206 = 0.0124$. Setting the required strength equal to the design strength [Eq. (3.38)] as before:

$$1670 = 0.90 \times 0.0124 \times 60bd^2 \cdot\ 1 - 0.59\,\frac{0.0124 \times 60}{4}\cdot$$

and

$$bd^2 = 2800 \text{ in}^3$$

A beam with $b = 10$ in. and $d = 16.7$ in. will meet the requirement, for which

$$A_s = 0.0124 \times 10 \times 16.7 = 2.07 \text{ in}^2$$

Two No. 9 (No. 29) bars are almost sufficient, providing an area of 2.00 in^2. If the total concrete height is rounded upward to 20 in., a 17.5 in. effective depth results, reducing the required steel area to 1.96 in^2. Two No. 9 (No. 29) bars remain the best choice.

It is apparent that an infinite number of solutions to the stated problem are possible, depending upon the reinforcement ratio selected. That ratio may vary from an upper limit of ρ_{max} to a lower limit of $3\sqrt{f_c'}/f_y \geq 200/f_y$ for beams, according to the ACI Code. To compare the two solutions (using the theoretical dimensions, unrounded for the comparison, and assuming h is 2.5 in. greater than d in each case), increasing the concrete section area by 14 percent achieves a steel saving of 20 percent. The second solution would certainly be more economical and would be preferred, unless beam dimensions must be minimized for architectural or functional reasons. Economical designs will typically have reinforcement ratios between $0.50\rho_{max}$ and $0.75\rho_{max}$.

There is a type of problem, occurring frequently, that does not fall strictly into either the analysis or design category. The concrete dimensions are given and are known to be adequate to carry the required moment, and it is necessary only to find the steel area. Typically, this is the situation at critical design sections of continuous beams, in which the concrete dimensions are often kept constant, although the steel reinforcement varies along the span according to the required flexural resistance. Dimensions b, d, and h are determined at the maximum moment section, usually at one of the supports. At other supports, and at midspan locations, where moments are usually smaller, the concrete dimensions are known to be adequate and only the tensile steel remains to be found. An identical situation was encountered in the design problem of Example 3.6, in which concrete dimensions were rounded upward from the minimum required values, and the required steel area was to be found. In either case, the iterative approach demonstrated in Example 3.6 is convenient.

EXAMPLE 3.7

Determination of steel area. Using the same concrete dimensions as were used for the second solution of Example 3.6 ($b = 10$ in., $d = 17.5$ in., and $h = 20$ in.) and the same material strengths, find the steel area required to resist a moment M_u of 1300 in-kips.

Solution. Assume $a = 4.0$ in. Then

$$A_s = \frac{1300}{0.90 \times 60(17.5 - 2.0)} = 1.55 \text{ in}^2$$

Checking the assumed a:

$$a = \frac{1.55 \times 60}{0.85 \times 4 \times 10} = 2.74 \text{ in.}$$

Next assume $a = 2.6$ in. and recalculate A_s:

$$A_s = \frac{1300}{0.90 \times 60(17.5 - 1.3)} = 1.49 \text{ in}^2$$

No further iteration is required. Use $A_s = 1.49$ in^2. Two No. 8 (No. 25) bars will be used. A check of the reinforcement ratio shows $\rho < \rho_{max}$ and $\phi = 0.9$.

As seen in Example 3.5, the strength reduction factor becomes a variable at high reinforcement ratios. Example 3.8 demonstrates how the variation in strength reduction factor affects the design process.

EXAMPLE 3.8

Determination of steel area and variable strength reduction factor. Architectural considerations limit the height of a 20 ft long simple span beam to 16 in. and the width to 12 in. The following loads and material properties are given: $w_d = 0.79$ kips/ft, $w_l = 1.65$ kips/ft, $f_c' = 5000$ psi, and $f_y = 60{,}000$ psi. Determine the reinforcement for the beam.

SOLUTION. Calculating the factored loads gives

$$w_u = 1.2 \times 0.79 + 1.6 \times 1.65 = 3.59 \text{ kips/ft}$$

$$M_u = 3.59 \times \frac{20^2}{8} = 179 \text{ ft-kips} = 2150 \text{ in-kips}$$

Assume $a = 4.0$ in. and $\phi = 0.90$. The structural depth is $16 - 2.5$ in. $= 13.5$ in. Calculating A_s:

$$A_s = \frac{M_u/\phi}{f_y(d - a/2)} = \frac{2150/0.90}{60(13.5 - 2.0)} = 3.46 \text{ in}^2$$

Try two No. 10 (No. 32) and one No. 9 (No. 29) bar, $A_s = 3.54$ in^2.

Check $a = 3.54 \times 60/(0.85 \times 5 \times 12) = 4.16$ in. from Eq. (3.32). This is more than assumed; therefore, continue to check the moment capacity.

$$M_n = 3.54 \times 60(13.5 - 4.16/2) = 2426 \text{ in-kips}$$

Using a ϕ of 0.90 gives $\phi M_n = 2183$ in-kips, which is adequate; however, the net tensile strain must be checked to validate the selection of $\phi = 0.9$. In this case $c = a/\beta_1 = 4.16/0.80 = 5.20$ in. The c/d ratio is $0.385 > 0.375$, so $\epsilon_t \geq 0.005$ is not satisfied. The corresponding net tensile strain is

$$\epsilon_t = 0.003 \frac{13.5 - 5.2}{5.2} = 0.00479$$

A value of $\epsilon_t = 0.00479$ is allowed by the ACI Code, but only if the strength reduction factor is adjusted. A linear interpolation from Fig. 3.9 gives $\phi = 0.88$ and $M_u = \phi M_n = 2140$ in-kips, which is less than the required capacity. Try increasing the reinforcement to three No. 10 (No. 32) bars, $A_s = 3.81$ in^2. Repeating the calculations:

$$a = \frac{3.81 \times 60}{0.85 \times 5 \times 12} = 4.48 \text{ in.}$$

$$c = \frac{4.48}{0.80} = 5.60 \text{ in.}$$

$$M_n = 3.81 \times 60\left(13.5 - \frac{4.48}{2}\right) = 2574 \text{ in-kips}$$

$$\epsilon_t = \frac{0.003(13.5 - 5.60)}{5.60} = 0.00423$$

$$\phi = 0.483 + 83.3 \times 0.00423 = 0.835$$

$$M_u = \phi M_n = 0.835 \times 2574 = 2150 \text{ in-kips}$$

which meets the design requirements.

In actuality, the first solution deviates less than 1 percent from the desired value and would likely be acceptable. The remaining portion of the example demonstrates the design implications of requiring a variable strength reduction factor when the net tensile strain falls between 0.005 and 0.004. In this example, the reinforcement increased nearly 8 percent, yet the design moment capacity ϕM_n only increased 0.5 percent due to the decreasing strength reduction factor.

In solving these examples, the basic equations have been used to develop familiarity with them. In actual practice, however, design aids such as Table A.4 of Appendix A, giving values of maximum and minimum reinforcement ratios, and Table A.5, providing values of flexural resistance factor R, are more convenient. The example problems will be repeated in Section 3.5 to demonstrate use of these aids.

g. Overreinforced Beams

According to the ACI Code, all beams are to be designed for yielding of the tension steel with ϵ_t not less than 0.004 and, thus, $\rho \le \rho_{max}$. Occasionally, however, such as when analyzing the capacity of existing construction, it may be necessary to calculate the flexural strength of an overreinforced compression-controlled member, for which f_s is less than f_y at flexural failure.

In this case, the steel strain, in Fig. 3.11*b*, will be less than the yield strain, but can be expressed in terms of the concrete strain ϵ_u and the still-unknown distance c to the neutral axis:

$$\epsilon_s = \epsilon_u \frac{d - c}{c} \tag{3.42}$$

From the equilibrium requirement that $C = T$, one can write

$$0.85\beta_1 f'_c bc = \rho \epsilon_s E_s bd$$

Substituting the steel strain from Eq. (3.42) in the last equation, and defining $k_u = c/d$, one obtains a quadratic equation in k_u as follows:

$$k_u^2 + m\rho k_u - m\rho = 0$$

Here, $\rho = A_s/bd$ as usual and m is a material parameter given by

$$m = \frac{E_s \epsilon_u}{0.85\beta_1 f'_c} \tag{3.43}$$

Solving the quadratic equation for k_u,

$$k_u = \sqrt{m\rho + \left(\frac{m\rho}{2}\right)^2} - \frac{m\rho}{2} \tag{3.44}$$

The neutral axis depth for the overreinforced beam can then easily be found from $c = k_u d$, after which the stress-block depth $a = \beta_1 c$. With steel strain ϵ_s then computed from Eq. (3.42), and with $f_s = E_s \epsilon_s$, the nominal flexural strength is

$$M_n = A_s f_s \left(d - \frac{a}{2}\right) \tag{3.45}$$

The strength reduction factor ϕ will equal 0.65 for beams in this range.

3.5 Design Aids

Basic equations were developed in Section 3.4 for the analysis and design of reinforced concrete beams, and these were used directly in the examples. In practice, the design of beams and other reinforced concrete members is greatly facilitated by the use of aids such as those in Appendix A of this text and in Refs. 3.7 through 3.9. Tables A.1, A.2, A.4 through A.7, and Graph A.1 of Appendix A relate directly to this chapter, and the student can scan this material to become familiar with the coverage. Other aids will be discussed, and their use demonstrated, in later chapters.

Equation (3.39) gives the flexural design strength ϕM_n of an underreinforced rectangular beam with a reinforcement ratio at or below ρ_{max}. The flexural resistance factor R, from Eq. (3.36), is given in Table A.5a for lower reinforcement ratios or Table A.5b for higher reinforcement ratios. Alternatively, R can be obtained from Graph A.1. For *analysis* of the capacity of a section with known concrete dimensions b and d, having known reinforcement ratio ρ, and with known materials strengths, the design strength ϕM_n can be obtained directly by Eq. (3.39).

For *design* purposes, where concrete dimensions and reinforcement are to be found and the factored load moment M_u is to be resisted, there are two possible approaches. One starts with selecting the optimum reinforcement ratio, and then calculating concrete dimensions, as follows:

1. Set the required strength M_u equal to the design strength ϕM_n from Eq. (3.39):
$$M_u = \phi R b d^2$$
2. With the aid of Table A.4, select an appropriate reinforcement ratio between ρ_{max} and ρ_{min}. Often a ratio of about $0.60\rho_{max}$ will be an economical and practical choice. Selection of $\rho \leq \rho$ for $\epsilon_t = 0.005$ assures that ϕ will remain equal to 0.90. For $\rho \le \rho_{max}$ and above ρ for $\epsilon_t = 0.005$, an iterative solution will be necessary.
3. From Table A.5, for the specified material strengths and selected reinforcement ratio, find the flexural resistance factor R. Then
$$bd^2 = \frac{M_u}{\phi R}$$
4. Choose b and d to meet that requirement. Unless construction depth must be limited or other constraints exist (see Section 12.6), an effective depth about 2 to 3 times the width is often appropriate.
5. Calculate the required steel area
$$A_s = \rho b d$$
Then, referring to Table A.2, choose the size and number of bars, giving preference to the larger bar sizes to minimize placement costs.
6. Refer to Table A.7 to ensure that the selected beam width will provide room for the bars chosen, with adequate concrete cover and spacing. (These points will be discussed further in Section 3.6.)

The alternative approach starts with selecting concrete dimensions, after which the required reinforcement is found, as follows:

1. Select beam width b and effective depth d. Then calculate the required R:
$$R = \frac{M_u}{\phi b d^2}$$
2. Using Table A.5 for specified material strengths, find the reinforcement ratio $\rho \leq \rho_{max}$ that will provide the required value of R and verify the selected value of ϕ.
3. Calculate the required steel area
$$A_s = \rho b d$$
and from Table A.2 select the size and number of bars.
4. Using Table A.7, confirm that the beam width is sufficient to contain the selected reinforcement.

Use of design aids to solve the example problems of Section 3.4 will be illustrated as follows.

EXAMPLE 3.9

Flexural strength of a given member. Find the nominal flexural strength and design strength of the beam in Example 3.5, which has $b = 12$ in. and $d = 17.5$ in. and is reinforced with four No. 9 (No. 29) bars. Make use of the design aids of Appendix A. Material strengths are $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

Solution. From Table A.2, four No. 9 (No. 29) bars provide $A_s = 4.00$ in^2, and with $b = 12$ in. and $d = 17.5$ in., the reinforcement ratio is $\rho = 4.00/(12 \times 17.5) = 0.0190$. According to Table A.4, this is below $\rho_{max} = 0.0206$ and above $\rho_{min} = 0.0033$. Then from Table A.5b, with $f'_c = 4000$ psi, $f_y = 60{,}000$ psi, and $\rho = 0.019$, the value $R = 949$ psi is found. The nominal and design strengths are (with $\phi = 0.86$ from Example 3.5) respectively

$$M_n = Rbd^2 = 949 \times 12 \times \frac{17.5^2}{1000} = 3490 \text{ in-kips}$$

$$\phi M_n = 0.86 \times 3490 = 3000 \text{ in-kips}$$

as before.

EXAMPLE 3.10

Concrete dimensions and steel area to resist a given moment. Find the cross section of concrete and the area of steel required for the beam in Example 3.6, making use of the design aids of Appendix A. $M_u = 1670$ in-kips, $f'_c = 4000$ psi, and $f_y = 60{,}000$ psi. Use a reinforcement ratio of $0.60\rho_{max}$.

Solution. From Table A.4, the maximum reinforcement ratio is $\rho_{max} = 0.0206$. For economy, a value of $\rho = 0.60\rho_{max} = 0.0124$ will be used. For that value, by interpolation from Table A.5a, the required value of R is 663. Then

$$bd^2 = \frac{M_u}{\phi R} = \frac{1670 \times 1000}{0.90 \times 663} = 2800 \text{ in}^3$$

Concrete dimensions $b = 10$ in. and $d = 16.7$ in. will satisfy this, but the depth will be rounded to 17.5 in. to provide a total beam depth of 20.0 in. It follows that

$$R = \frac{M_u}{\phi bd^2} = \frac{1670 \times 1000}{0.90 \times 10 \times 17.5^2} = 606 \text{ psi}$$

and from Table A.5a, by interpolation, $\rho = 0.0112$. This leads to a steel requirement of $A_s = 0.0112 \times 10 \times 17.5 = 1.96$ in^2 as before.

EXAMPLE 3.11

Determination of steel area. Find the steel area required for the beam in Example 3.7, with concrete dimensions $b = 10$ in. and $d = 17.5$ in. known to be adequate to carry the factored load moment of 1300 in-lb. Material strengths are $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

Solution. Note that in cases in which the concrete dimensions are known to be adequate and only the reinforcement must be found, the iterative method used earlier is not required. The necessary flexural resistance factor is

$$R = \frac{M_u}{\phi bd^2} = \frac{1300 \times 1000}{0.90 \times 10 \times 17.5^2} = 472 \text{ psi}$$

According to Table A.5a, with the specified material strengths, this corresponds to a reinforcement ratio of · = 0.0085, giving a steel area of

$$A_s = 0.0085 \times 10 \times 17.5 = 1.49 \text{ in}^2$$

as before. Two No. 8 (No. 25) bars will be used.

The tables and graphs of Appendix A give basic information and are used extensively throughout this text for illustrative purposes. The reader should be aware, however, of the greatly expanded versions of these tables, plus many other useful aids, that are found in Refs. 3.7 through 3.9 and elsewhere.

3.6 Practical Considerations in the Design of Beams

To focus attention initially on the basic aspects of flexural design, the preceding examples were carried out with only minimum regard for certain practical considerations that always influence the actual design of beams. These relate to optimal concrete proportions for beams, rounding of dimensions, standardization of dimensions, required cover for main and auxiliary reinforcement, and selection of bar combinations. Good judgment on the part of the design engineer is particularly important in translating from theoretical requirements to practical design. Several of the more important aspects are discussed here; much additional guidance is provided by the publications of ACI (Refs. 3.7 and 3.8) and CRSI (Refs. 3.9 to 3.11).

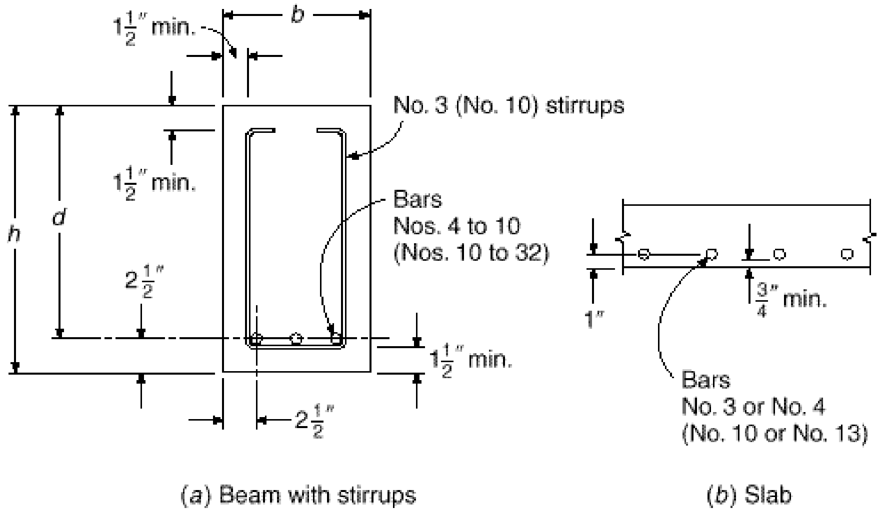
a. Concrete Protection for Reinforcement

To provide the steel with adequate concrete protection against fire and corrosion, the designer must maintain a certain minimum thickness of concrete cover outside of the outermost steel. The thickness required will vary, depending upon the type of member and conditions of exposure. According to ACI Code 7.7, for cast-in-place concrete, concrete protection at surfaces not exposed directly to the ground or weather should be not less than $\frac{3}{4}$ in. for slabs and walls and $1\frac{1}{2}$ in. for beams and columns. If the concrete surface is to be exposed to the weather or in contact with the ground, a protective covering of at least 2 in. is required [$1\frac{1}{2}$ in. for No. 5 (No. 16) and smaller bars], except that, if the concrete is cast in direct contact with the ground without the use of forms, a cover of at least 3 in. must be furnished.

In general, the centers of main flexural bars in beams should be placed $2\frac{1}{2}$ to 3 in. from the top or bottom surface of the beam to furnish at least $1\frac{1}{2}$ in. of clear cover for the bars and the stirrups (see Fig. 3.12). In slabs, 1 in. to the center of the bar is ordinarily sufficient to give the required $\frac{3}{4}$ in. cover.

To simplify construction and thereby to reduce costs, the overall concrete dimensions of beams, b and h, are almost always rounded upward to the nearest inch, and often to the next multiple of 2 in. As a result, the actual effective depth d, found by subtracting the sum of cover distance, stirrup diameter, and half the main reinforcing bar diameter from the total depth h, is seldom an even dimension. For slabs, the total depth is generally rounded upward to the nearest $\frac{1}{2}$ in. up to 6 in. in depth, and to the nearest inch above that thickness. The differences between h and d shown in Fig. 3.12 are not exact, but are satisfactory for design purposes for beams with No. 3 (No. 10) stirrups and No. 10 (No. 32) longitudinal bars or smaller, and for slabs using No. 4 (No. 13) or

FIGURE 3.12
Requirements for concrete cover in beams and slabs.



smaller bars. If larger bars are used for the main flexural reinforcement or for the stirrups, as is frequently the case, the corresponding dimensions are easily calculated.

Recognizing the closer tolerances that can be maintained under plant-control conditions, ACI Code 7.7.3 permits some reduction in concrete protection for reinforcement in precast concrete.

b. Concrete Proportions

Reinforced concrete beams may be wide and shallow, or relatively narrow and deep. Consideration of maximum material economy often leads to proportions with effective depth d in the range from about 2 to 3 times the width b (or web width b_w for T beams). However, constraints may dictate other choices and, as will be discussed in Section 12.6, maximum material economy may not translate into maximum structural economy. For example, with one-way concrete joists supported by monolithic beams (see Chapter 18), use of beams and joists with the same total depth will permit use of a single flat-bottom form, resulting in fast, economical construction and permitting level ceilings. The beams will generally be wide and shallow, with heavier reinforcement than otherwise, but the result will be an overall saving in construction cost. In other cases, it may be necessary to limit the total depth of floor or roof construction for architectural or other reasons. An advantage of reinforced concrete is its adaptability to such special needs.

c. Selection of Bars and Bar Spacing

As noted in Section 2.14, common reinforcing bar sizes range from No. 3 to No. 11 (No. 10 to No. 36), the bar number corresponding closely to the number of eighth-inches (millimeters) of bar diameter. The two larger sizes, No. 14 (No. 43) [$1\frac{3}{4}$ in. (43 mm) diameter] and No. 18 (No. 57) [$2\frac{1}{4}$ in. (57 mm) diameter] are used mainly in columns.

It is often desirable to mix bar sizes to meet steel area requirements more closely. In general, mixed bars should be of comparable diameter, for practical as well as theoretical reasons, and generally should be arranged symmetrically about the vertical centerline. Many designers limit the variation in diameter of bars in a single layer to two bar sizes, using, say, No. 10 and No. 8 (No. 32 and No. 25) bars together, but not Nos. 11 and 6 (Nos. 36 and 19). There is some practical advantage to minimizing the number of different bar sizes used for a given structure.

Normally, it is necessary to maintain a certain minimum distance between adjacent bars to ensure proper placement of concrete around them. Air pockets below the steel are to be avoided, and full surface contact between the bars and the concrete is desirable to optimize bond strength. ACI Code 7.6 specifies that the minimum clear distance between adjacent bars shall not be less than the nominal diameter of the bars, or 1 in. (For columns, these requirements are increased to $1\frac{1}{2}$ bar diameters and $1\frac{1}{2}$ in.) Where beam reinforcement is placed in two or more layers, the clear distance between layers must not be less than 1 in., and the bars in the upper layer should be placed directly above those in the bottom layer.

The maximum number of bars that can be placed in a beam of given width is limited by bar diameter and spacing requirements and is also influenced by stirrup diameter, by concrete cover requirement, and by the maximum size of concrete aggregate specified. Table A.7 of Appendix A gives the maximum number of bars that can be placed in a single layer in beams, assuming $1\frac{1}{2}$ in. concrete cover and the use of No. 4 (No. 13) stirrups. When using the minimum bar spacing in conjunction with a large number of bars in a single plane of reinforcement, the designer should be aware that problems may arise in the placement and consolidation of concrete, especially when multiple layers of bars are used or when the bar spacing is smaller than the size of the vibrator head.

There are also restrictions on the *minimum* number of bars that can be placed in a single layer, based on requirements for the distribution of reinforcement to control the width of flexural cracks (see Section 6.3). Table A.8 gives the minimum number of bars that will satisfy ACI Code requirements, which will be discussed in Chapter 6.

In large girders and columns, it is sometimes advantageous to "bundle" tensile or compressive reinforcement with two, three, or four bars in contact to provide for better deposition of concrete around and between adjacent bundles. These bars may be assumed to act as a unit, with not more than four bars in any bundle, provided that stirrups or ties enclose the bundle. No more than two bars should be bundled in one plane; typical bundle shapes are triangular, square, or L-shaped patterns. Individual bars in a bundle, cut off within the span of flexural members, should terminate at different points. ACI Code 7.6.6 requires at least 40 bar diameters stagger between points of cutoff. Where spacing limitations and minimum concrete cover requirements are based on bar diameter, a unit of bundled bars is treated as a single bar with a diameter that provides the same total area.

ACI Code 7.6.6 states that bars larger than No. 11 (No. 36) shall not be bundled in beams, although the AASHTO Specifications permit bundling of Nos. 14 and 18 (Nos. 43 and 57) bars in highway bridges.

3.7 Rectangular Beams with Tension and Compression Reinforcement

If a beam cross section is limited because of architectural or other considerations, it may happen that the concrete cannot develop the compression force required to resist the given bending moment. In this case, reinforcement is added in the compression zone, resulting in a so-called *doubly reinforced* beam, i.e., one with compression as

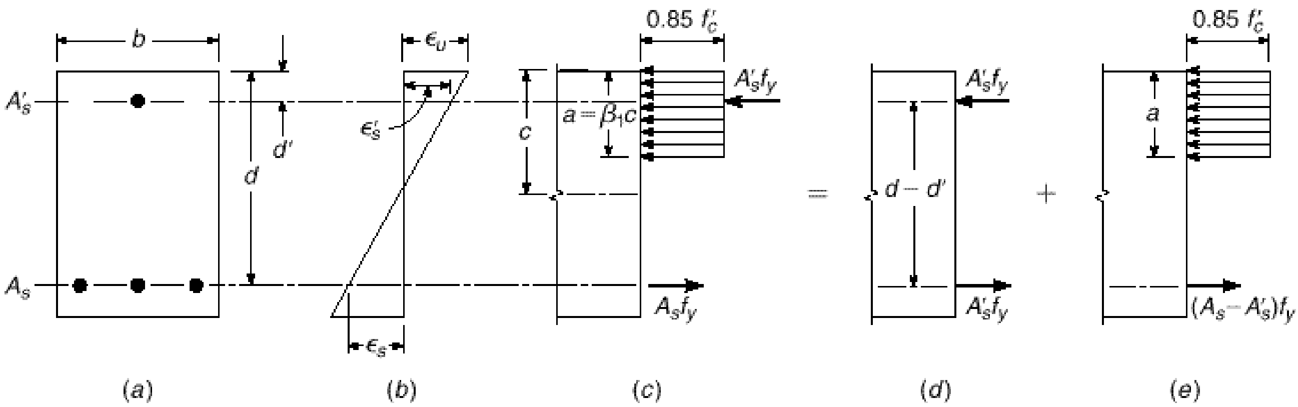


FIGURE 3.13
Doubly reinforced rectangular beam.

well as tension reinforcement (see Fig. 3.13). The use of compression reinforcement has decreased markedly with the use of strength design methods, which account for the full strength potential of the concrete on the compressive side of the neutral axis. However, there are situations in which compressive reinforcement is used for reasons other than strength. It has been found that the inclusion of some compression steel will reduce the long-term deflections of members (see Section 6.5). In addition, in some cases, bars will be placed in the compression zone for minimum-moment loading (see Section 12.2) or as stirrup-support bars continuous throughout the beam span (see Chapter 4). It may be desirable to account for the presence of such reinforcement in flexural design, although in many cases they are neglected in flexural calculations.

a. Tension and Compression Steel Both at Yield Stress

If, in a doubly reinforced beam, the tensile reinforcement ratio ρ is less than or equal to ρ_b, the strength of the beam may be approximated within acceptable limits by disregarding the compression bars. The strength of such a beam will be controlled by tensile yielding, and the lever arm of the resisting moment will ordinarily be but little affected by the presence of the compression bars.

If the tensile reinforcement ratio is larger than ρ_b, a somewhat more elaborate analysis is required. In Fig. 3.13*a*, a rectangular beam cross section is shown with compression steel A'_s placed a distance d' from the compression face and with tensile steel A_s at effective depth d. It is assumed initially that both A'_s and A_s are stressed to f_y at failure. The total resisting moment can be thought of as the sum of two parts. The first part, M_{n1}, is provided by the couple consisting of the force in the compression steel A'_s and the force in an equal area of tension steel

$$M_{n1} = A'_s f_y (d - d') \tag{3.46a}$$

as shown in Fig. 3.13*d*. The second part, M_{n2}, is the contribution of the remaining tension steel $A_s - A'_s$ acting with the compression concrete:

$$M_{n2} = (A_s - A'_s) f_y \left(d - \frac{a}{2}\right) \tag{3.46b}$$

as shown in Fig. 3.13*e*, where the depth of the stress block is

$$a = \frac{(A_s - A'_s)f_y}{0.85f'_c b} \tag{3.47a}$$

With the definitions $\rho = A_s/bd$ and $\rho' = A'_s/bd$, this can be written

$$a = \frac{(\rho - \rho')f_y d}{0.85f'_c} \tag{3.47b}$$

The total nominal resisting moment is then

$$M_n = M_{n1} + M_{n2} = A'_s f_y(d - d') + (A_s - A'_s)f_y\left(d - \frac{a}{2}\right) \tag{3.48}$$

In accordance with the safety provisions of the ACI Code, the net tensile strain is checked, and if $\epsilon_t \geq 0.005$, this nominal capacity is reduced by the factor $\phi = 0.90$ to obtain the design strength. For ϵ_t between 0.005 and 0.004, ϕ must be adjusted, as discussed earlier.

It is highly desirable, for reasons given earlier, that failure, should it occur, be precipitated by tensile yielding rather than crushing of the concrete. This can be ensured by setting an *upper limit* on the tensile reinforcement ratio. By setting the tensile steel strain in Fig. 3.13*b* equal to ϵ_y to establish the location of the neutral axis for the failure condition and then summing horizontal forces shown in Fig. 3.13*c* (still assuming the compressive steel to be at the yield stress at failure), it is easily shown that the balanced reinforcement ratio $\bar{\rho}_b$ for a doubly reinforced beam is

$$\bar{\rho}_b = \rho_b + \rho' \tag{3.49}$$

where ρ_b is the balanced reinforcement ratio for the corresponding singly reinforced beam and is calculated from Eq. (3.28). The ACI Code limits the net tensile strain, not the reinforcement ratio. To provide the same margin against brittle failure as for singly reinforced beams, the maximum reinforcement ratio should be limited to

$$\bar{\rho}_{max} = \rho_{max} + \rho' \tag{3.50}$$

Because ρ_{max} establishes the location of the neutral axis, the limitation in Eq. (3.50) will provide acceptable net tensile strains. A check of ϵ_t is required to determine the strength reduction factor ϕ and verify net tensile strain requirements are satisfied. Substituting ρ for $\epsilon_t \geq 0.005$ for ρ_{max} in Eq. (3.50) will give $\phi = 0.90$.

b. Compression Steel below Yield Stress

The preceding equations, through which the fundamental analysis of doubly reinforced beams is developed clearly and concisely, are valid *only* if the compression steel has yielded when the beam reaches its nominal capacity. In many cases, such as for wide, shallow beams, beams with more than the usual concrete cover over the compression bars, beams with high yield strength steel, or beams with relatively small amounts of tensile reinforcement, the compression bars will be below the yield stress at failure. It is necessary, therefore, to develop more generally applicable equations to account for the possibility that the compression reinforcement has not yielded when the doubly reinforced beam fails in flexure.

Whether or not the compression steel will have yielded at failure can be determined as follows. Referring to Fig. 3.13*b*, and taking as the limiting case $\epsilon_s' = \epsilon_y$, one obtains, from geometry,

$$\frac{c}{d'} = \frac{\epsilon_u}{\epsilon_u - \epsilon_y} \qquad \text{or} \qquad c = \frac{\epsilon_u}{\epsilon_u - \epsilon_y} d'$$

Summing forces in the horizontal direction (Fig. 3.13*c*) gives the *minimum* tensile reinforcement ratio $\bar{\rho}_{cy}$ that will ensure yielding of the compression steel at failure:

$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{d'}{d} \frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho' \tag{3.51}$$

If the *tensile reinforcement ratio* is *less than* this limiting value, the neutral axis is sufficiently high that the compression steel stress at failure is less than the yield stress. In this case, it can easily be shown on the basis of Fig. 3.13*b* and *c* that the balanced reinforcement ratio is

$$\bar{\rho}_b = \rho_b + \rho' \frac{f_s'}{f_y} \tag{3.52}$$

where

$$f_s' = E_s \epsilon_s' = E_s \left[\epsilon_u - \frac{d'}{d}(\epsilon_u + \epsilon_y)\right] \le f_y \tag{3.53a}$$

To determine ρ_{max}, $\epsilon_t = 0.004$ is substituted for ϵ_y in Eq. (3.53*a*), giving

$$f_s' = E_s \left[\epsilon_u - \frac{d'}{d}(\epsilon_u + 0.004)\right] \le f_y \tag{3.53b}$$

Hence, the maximum reinforcement ratio permitted by the ACI Code is

$$\bar{\rho}_{max} = \rho_{max} + \rho' \frac{f_s'}{f_y} \tag{3.54}$$

where f_s' is given in Eq. (3.53*b*). A simple comparison shows that Eqs. (3.52) and (3.54), with f_s' given by Eqs. (3.53*a*) and (3.53*b*), respectively, are the generalized forms of Eqs. (3.49) and (3.50).

It should be emphasized that Eqs. (3.53*a*) and (3.53*b*) for compression steel stress apply *only for beams with exact strain values in the extreme tensile steel of* ϵ_y *or* $\epsilon_t = 0.004$.

If the tensile reinforcement ratio is less than $\bar{\rho}_b$, as given by Eq. (3.52), and less than $\bar{\rho}_{cy}$ given by Eq. (3.51), then the tensile steel is at the yield stress at failure but the compression steel is not, and new equations must be developed for compression steel stress and flexural strength. The compression steel stress can be expressed in terms of the still-unknown neutral axis depth as

$$f_s' = \epsilon_u E_s \frac{c - d'}{c} \tag{3.55}$$

Consideration of horizontal force equilibrium (Fig. 3.13*c* with compression steel stress equal to f_s') then gives

$$A_s f_y = 0.85\beta_1 f_c' bc + A_s' \epsilon_u E_s \frac{c - d'}{c} \tag{3.56}$$

TABLE 3.2
Minimum beam depths for compression reinforcement to yield

	$\epsilon_t = 0.004$		$\epsilon_t = 0.005$	
f_y, psi	Maximum d'/d	Minimum d for $d' = 2.5$ in., in.	Maximum d'/d	Minimum d for $d' = 2.5$ in., in.
40,000	0.23	10.8	0.20	12.3
60,000	0.13	18.8	0.12	21.5
75,000	0.06	42.7	0.05	48.8

This is a quadratic equation in c, the only unknown, and is easily solved for c. The nominal flexural strength is found using the value of f_s' from Eq. (3.55), and $a = \beta_1 c$ in the expression

$$M_n = 0.85 f_c' ab \left(d - \frac{a}{2}\right) + A_s' f_s' (d - d') \tag{3.57}$$

This nominal capacity is reduced by the strength reduction factor ϕ to obtain the design strength.

If compression bars are used in a flexural member, precautions must be taken to ensure that these bars will not buckle outward under load, spalling off the outer concrete. ACI Code 7.11.1 imposes the requirement that such bars be anchored in the same way that compression bars in columns are anchored by lateral ties (Section 8.2). Such ties must be used throughout the distance where the compression reinforcement is required.

For the compression steel to yield, the reinforcement ratio must lie below $\bar\rho_{max}$ and above $\bar\rho_{cy}$. The ratio between d' and the steel centroidal depth d to allow yielding of the compression reinforcement can be found by equating $\bar\rho_{cy}$ to $\bar\rho_{max}$ (or $\bar\rho$ for ϵ_t = 0.005) and solving for d'/d. Furthermore, if d' is assumed to be 2.5 in., as is often the case, the minimum depth of beam necessary for the compression steel to yield may be found for each grade of steel. The ratios and minimum beam depths are summarized in Table 3.2. Values are included for $\epsilon_t = 0.004$, the minimum tensile yield strain permitted for flexural members, and $\epsilon_t = 0.005$, the net tensile strain needed to ensure that $\phi = 0.90$. For beams with less than the minimum depth, the compression reinforcement cannot yield unless the tensile reinforcement exceeds ρ_{max}. The compression reinforcement may yield in beams that exceed the minimum depth in Table 3.2, depending on the relative distribution of the tensile and compressive reinforcement.

c. Examples of Analysis and Design of Beams with Tension and Compression Steel

As was the case for beams with only tension reinforcement, doubly reinforced beam problems can be placed in one of two categories: analysis problems or design problems. For *analysis*, in which the concrete dimensions, reinforcement, and material strengths are given, one can find the flexural strength directly from the equations in Section 3.7a or Section 3.7b. First, it must be confirmed that the tensile reinforcement ratio is less than $\bar\rho_b$ given by Eq. (3.52), with compression steel stress from

Eq. (3.53*a*). Once it is established that the tensile steel has yielded, the tensile reinforcement ratio defining compression steel yielding is calculated from Eq. (3.51), and the actual tensile reinforcement ratio is compared. If it is greater than $\bar{\rho}_{cy}$, then $f_s' = f_y$, and M_n is found from Eq. (3.48). If it is less than $\bar{\rho}_{cy}$, then $f_s' < f_y$. In this case, c is calculated by solving Eq. (3.56), f_s' comes from Eq. (3.55), and M_n is found from Eq. (3.57).

For the *design* case, in which the factored load moment M_u to be resisted is known and the section dimensions and reinforcement are to be found, a direct solution is impossible. The steel areas to be provided depend on the steel stresses, which are not known before the section is proportioned. It can be assumed that the compression steel stress is equal to the yield stress, but this must be confirmed; if it is not so, the design must be adjusted. The design procedure can be outlined as follows:

1. Calculate the maximum moment that can be resisted by the underreinforced section with $\rho = \rho_{max}$ or ρ for $\epsilon_t = 0.005$ to ensure that $\phi = 0.90$. The corresponding tensile steel area is $A_s = \rho_{max}bd$, and, as usual,

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

with

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

2. Find the excess moment, if any, that must be resisted, and set $M_2 = M_n$, as calculated in step 1.

$$M_1 = \frac{M_u}{\phi} - M_2$$

A_s from step 1 is now defined as A_{s2}, i.e., that part of the tension steel area in the doubly reinforced beam that works with the compression force in the concrete. In Fig. 3.13*e*, $(A_s - A_s') = A_{s2}$.

3. Tentatively assume that $f_s' = f_y$. Then

$$A_s' = \frac{M_1}{f_y(d - d')}$$

Alternatively, if from Table 3.2, the compression reinforcement is known not to yield, go to step 6.

4. Add an additional amount of tensile steel $A_{s1} = A_s'$. Thus, the total tensile steel area A_s is A_{s2} from step 2 plus A_{s1}.
5. Analyze the doubly reinforced beam to see if $f_s' = f_y$; that is, check the tensile reinforcement ratio against $\bar{\rho}_{cy}$.
6. If $\rho < \bar{\rho}_{cy}$, then the compression steel stress is less than f_y and the compression steel area must be increased to provide the needed force. This can be done as follows. The stress block depth is found from the requirement of horizontal equilibrium (Fig. 3.13*e*),

$$a = \frac{(A_s - A_s')f_y}{0.85 f_c' b}$$

and the neutral axis depth is $c = a/\beta_1$. From Eq. (3.55),

$$f_s' = \epsilon_u E_s \frac{c - d'}{c}$$

The revised compression steel area, acting at f_s', must provide the same force as the trial steel area that was assumed to act at f_y. Therefore,

$$A'_{s,revised} = A'_{s,trial} \frac{f_y}{f_s'}$$

The tensile steel area need not be revised, because it acts at f_y as assumed.

EXAMPLE 3.12

Flexural strength of a given member. A rectangular beam has a width of 12 in. and an effective depth to the centroid of the tension reinforcement of 24 in. The tension reinforcement consists of six No. 10 (No. 32) bars in two rows. Compression reinforcement consisting of two No. 8 (No. 25) bars is placed 2.5 in. from the compression face of the beam. If $f_y = 60{,}000$ psi and $f_c' = 5000$ psi, what is the design moment capacity of the beam?

Solution. The steel areas and ratios are

$$A_s = 7.62 \text{ in}^2 \qquad \rho = \frac{7.62}{12 \times 24} = 0.0265$$

$$A_s' = 1.58 \text{ in}^2 \qquad \rho' = \frac{1.58}{12 \times 24} = 0.0055$$

Check the beam first as a singly reinforced beam to see if the compression bars can be disregarded,

$$\rho_{max} = 0.0243 \qquad \text{from Table A.4 of Appendix A}$$

The actual $\rho = 0.0265$ is larger than ρ_{max}, so the beam must be analyzed as doubly reinforced. From Eq. (3.51),

$$\bar{\rho}_{cy} = 0.85 \times 0.80 \times \frac{5}{60} \times \frac{2.5}{24} \times \frac{0.003}{0.003 - 0.00207} + 0.0055 = 0.0245$$

The tensile reinforcement ratio is greater than this, so the compression bars will yield when the beam fails. The maximum reinforcement ratio thus can be found from Eq. (3.50),

$$\bar{\rho}_{max} = 0.0243 + 0.0055 = 0.0298$$

The actual tensile reinforcement ratio is below the maximum value, as required. Then, from Eq. (3.47*a*),

$$a = \frac{(7.62 - 1.58)60}{0.85 \times 5 \times 12} = 7.11 \text{ in.}$$

$$c = \frac{7.11}{0.80} = 8.89$$

$$\epsilon_t = 0.003\left(\frac{24 - 8.89}{8.89}\right) = 0.0051$$

and

$$\phi = 0.90$$

and from Eq. (3.48),

$$M_n = 1.58 \times 60(24 - 2.5) + 6.04 \times 60\left(24 - \frac{7.11}{2}\right) = 9450 \text{ in-kips}$$

The design strength is

$$\phi M_n = 0.90 \times 9450 = 8500 \text{ in-kips}$$

EXAMPLE 3.13 **Design of a doubly reinforced beam.** A rectangular beam that must carry a service live load of 2.47 kips/ft and a calculated dead load of 1.05 kips/ft on an 18 ft simple span is limited in cross section for architectural reasons to 10 in. width and 20 in. total depth. If f_y = 60,000 psi and f'_c = 4000 psi, what steel area(s) must be provided?

SOLUTION. The service loads are first increased by load factors to obtain the factored load of $1.2 \times 1.05 + 1.6 \times 2.47 = 5.21$ kips/ft. Then $M_u = 5.21 \times 18^2 \cdot 8 = 211$ ft-kips $=$ 2530 in-kips. To satisfy spacing and cover requirements (see Section 3.6), assume that the tension steel centroid will be 4 in. above the bottom face of the beam and that compression steel, if required, will be placed 2.5 in. below the beam's top surface. Then $d = 16$ in. and $d' = 2.5$ in.

First, check the capacity of the section if singly reinforced. Table A.4 shows the maximum · for $\cdot_t = 0.005$, the strain associated with · $= 0.90$, to be 0.0181. While the maximum reinforcement ratio is slightly higher, Example 3.8 demonstrated there was no economic efficiency of using $\cdot_t \le 0.005$. So, $A_s = 10 \times 16 \times 0.0181 = 2.90$ in^2. Then, with

$$a = \frac{2.90 \times 60}{0.85 \times 4 \times 10} = 5.12 \text{ in.}$$

$c = a \cdot 0.85 = 6.02$ in. and the maximum nominal moment that can be developed is

$$M_n = 2.90 \times 60 \cdot 16 - 5.12 \cdot 2 \cdot = 2340 \text{ in-kips}$$

Alternatively, $R = 913$ from Table A.5b, the nominal moment is $M_n = 913 \times 10 \times 16^2 \cdot 1000$ $= 2340$ in-kips. Because the corresponding design moment, · $M_n = 2100$ in-kips, is less than the required capacity, 2530 in-kips, compression steel is needed as well as additional tension steel.

The remaining moment to be carried by the compression steel couple is

$$M_1 = \frac{2530}{0.90} - 2340 = 470 \text{ in-kips}$$

As d is less than the value required to develop the compression reinforcement yield stress (Table 3.2), a reduced stress in the compression reinforcement will be used.

$$\cdot'_s = 0.003\,\frac{6.02 - 2.5}{6.02} = 0.00175 \quad \text{and} \quad f'_s = 0.00175 \times 29{,}000 = 50.9 \text{ ksi}$$

Try $f'_s = 50$ ksi for the compression reinforcement to obtain the area of steel.

$$A'_s = \frac{470}{50 \cdot 16 - 2.5 \cdot} = 0.70 \text{ in}^2$$

The total area of tensile reinforcement at 60 ksi is

$$A_s = 2.90 + 0.70 \cdot \frac{50}{60} \cdot = 3.48 \text{ ksi}$$

Two No. 6 (No. 19) bars will be used for the compression reinforcement and four No. 9 (No. 29) bars will provide the tensile steel area as shown in Fig. 3.14. To place the tension bars in a 10 in. beam width, two rows of two bars each are used.

A final check is made to ensure that the selection of reinforcement does not create a lower compressive stress than the assumed 50 ksi.

$$A_s = 4.0 - 0.88 \cdot \frac{50}{60} \cdot = 3.27 \text{ in}^2$$

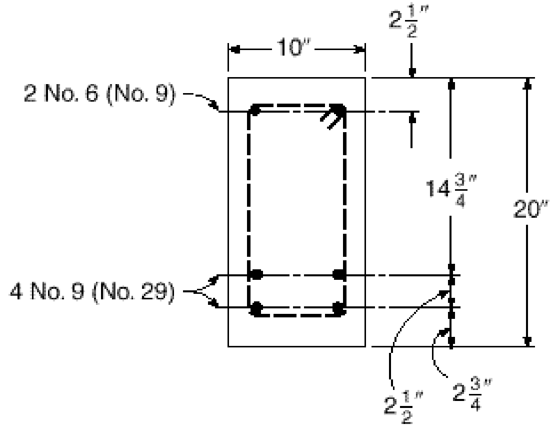


FIGURE 3.14
Doubly reinforced beam of Example 3.13.

which is greater than 2.90 in² for $\epsilon_t = 0.005$, so $\phi < 0.90$.

$$a = \frac{3.27 \times 60}{0.85 \times 4 \times 10} = 5.77 \text{ in.}$$

$$c = \frac{5.77}{0.85} = 6.79 \text{ in.}$$

$$\epsilon'_s = 0.003\,\frac{6.79 - 2.5}{6.79} = 0.0019$$

$$f'_s = 29{,}000 \times 0.0019 = 55.0 \text{ ksi}$$

which is greater than assumed. Check ϕ using $d_t = 17.25$ from Fig. 3.13 and compute the revised M_n. For simplicity, the area of tensile reinforcement is not modified.

$$\epsilon_t = 0.003\,\frac{17.25 - 6.79}{6.79} = 0.0046$$

for which $\phi = 0.87$. Then

$$M_u = 0.87\left[3.27 \times 60\left(16.0 - \frac{5.77}{2}\right)\right.$$

$$\left. + 0.88 \times 55.0(16 - 2.5)\right] = 2810 \text{ in-kips}$$

This is greater than M_u, so no further refinement is necessary.

d. Tensile Steel below the Yield Stress

All doubly reinforced beams designed according to the ACI Code must be underreinforced, in the sense that the tensile reinforcement ratio is limited to ensure yielding at beam failure. Two cases were considered in Sections 3.7a and 3.7b, respectively: (a) both tension steel and compression steel yield, and (b) tension steel yields but compression steel does not. Two other combinations may be encountered in analyzing the capacity of existing beams: (c) tension steel does not yield, but compression steel does, and (d) neither tension steel nor compression steel yields. The last two cases are unusual, and in fact, it would be difficult to place sufficient tension reinforcement to create such conditions, but it is possible. The solution in such cases is obtained as a simple extension of the treatment of Section 3.7b. An equation for horizontal equilibrium is written, in which both tension and compression steel stress are expressed in terms of the unknown neutral axis depth c. The resulting quadratic equation is solved for c, after which steel stresses can be calculated and the nominal flexural strength determined.

3.8 T Beams

With the exception of precast systems, reinforced concrete floors, roofs, decks, etc., are almost always monolithic. Forms are built for beam soffits and sides and for the underside of slabs, and the entire construction is cast at once, from the bottom of the deepest beam to the top of the slab. Beam stirrups and bent bars extend up into the slab. It is evident, therefore, that a part of the slab will act with the upper part of the beam to resist longitudinal compression. The resulting beam cross section is T-shaped rather than rectangular. The slab forms the beam flange, while the part of the beam projecting below the slab forms what is called the *web* or *stem*. The upper part of such a T beam is stressed laterally due to slab action in that direction. Although transverse compression at the level of the bottom of the slab may increase the longitudinal compressive strength by as much as 25 percent, transverse tension at the top surface reduces the longitudinal compressive strength (see Section 2.10). Neither effect is usually taken into account in design.

a. Effective Flange Width

The next question to be resolved is that of the effective width of flange. In Fig. 3.15*a*, it is evident that if the flange is but little wider than the stem width, the entire flange can be considered effective in resisting compression. For the floor system shown in Fig. 3.15*b*, however, it may be equally obvious that elements of the flange midway between the beam stems are less highly stressed in longitudinal compression than those elements directly over the stem. This is so because of shearing deformation of the flange, which relieves the more remote elements of some compressive stress.

Although the actual longitudinal compression varies because of this effect, it is convenient in design to make use of an *effective flange width*, which may be smaller than the actual flange width but is considered to be uniformly stressed at the maximum value. This effective width has been found to depend primarily on the beam span and on the relative thickness of the slab.

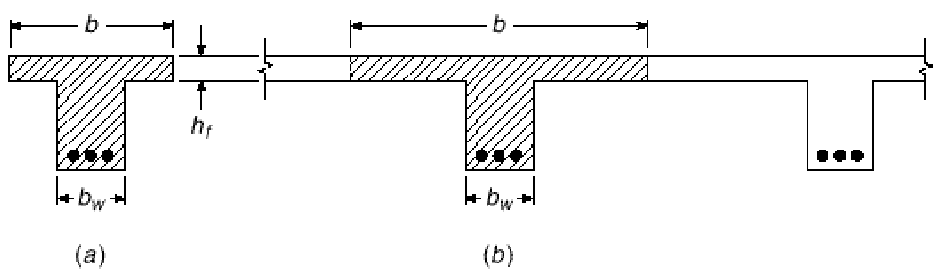


FIGURE 3.15 Effective flange width of T beams.

The criteria for effective width given in ACI Code 8.10 are as follows:

1. For symmetrical T beams, the effective width b shall not exceed one-fourth the span length of the beam. The overhanging slab width on either side of the beam web shall not exceed 8 times the thickness of the slab nor go beyond one-half the clear distance to the next beam.
2. For beams having a slab on one side only, the effective overhanging slab width shall not exceed one-twelfth the span length of the beam, 6 times the slab thickness, or one-half the clear distance to the next beam.
3. For isolated beams in which the flange is used only for the purpose of providing additional compressive area, the flange thickness shall not be less than one-half the width of the web, and the total flange width shall not be more than 4 times the web width.

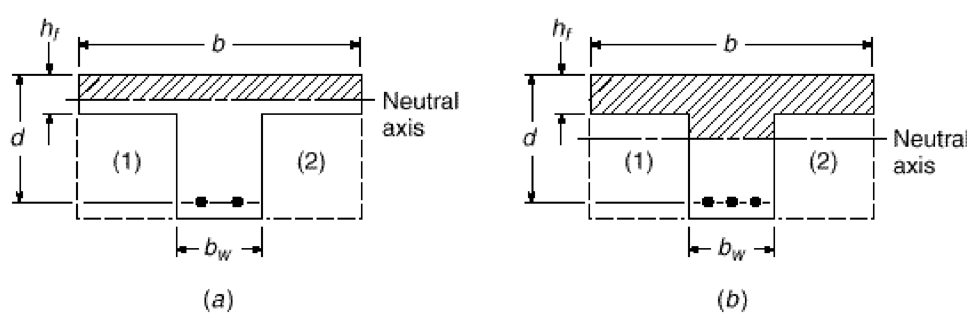
b. Strength Analysis

The neutral axis of a T beam may be either in the flange or in the web, depending upon the proportions of the cross section, the amount of tensile steel, and the strengths of the materials. If the calculated depth to the neutral axis is less than or equal to the slab thickness h_f, the beam can be analyzed as if it were a rectangular beam of width equal to b, the effective flange width. The reason for this is illustrated in Fig. 3.16*a*, which shows a T beam with the neutral axis in the flange. The compressive area is indicated by the shaded portion of the figure. If the additional concrete indicated by areas 1 and 2 had been added when the beam was cast, the physical cross section would have been rectangular with a width b. No bending strength would have been added because areas 1 and 2 are entirely in the tension zone, and tension concrete is disregarded in flexural calculations. The original T beam and the rectangular beam are equal in flexural strength, and rectangular beam analysis for flexure applies.

When the neutral axis is in the web, as in Fig. 3.16*b*, the preceding argument is no longer valid. In this case, methods must be developed to account for the actual T-shaped compressive zone.

In treating T beams, it is convenient to adopt the same equivalent stress distribution that is used for beams of rectangular cross section. The rectangular stress block, having a uniform compressive-stress intensity 0.85 f'_c, was devised originally on the basis of tests of rectangular beams (see Section 3.4a), and its suitability for T beams may be questioned. However, extensive calculations based on actual stress-strain curves (reported in Ref. 3.12) indicate that its use for T beams, as well as for beams of circular or triangular cross section, introduces only minor error and is fully justified.

FIGURE 3.16
Effective cross sections of T beams.



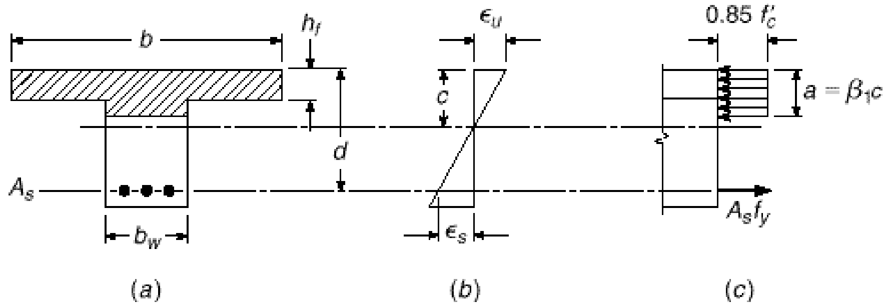


FIGURE 3.17.
Strain and equivalent stress distributions for T beams.

Accordingly, a T beam may be treated as a rectangular beam if the depth of the equivalent stress block is less than or equal to the flange thickness. Figure 3.17 shows a tensile-reinforced T beam with effective flange width b, web width b_w, effective depth to the steel centroid d, and flange thickness h_f. If for trial purposes the stress block is assumed to be completely within the flange,

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{\rho f_y d}{0.85 f'_c} \tag{3.58}$$

where $\rho = A_s/bd$. If a is less than or equal to the flange thickness h_f, the member may be treated as a rectangular beam of width b and depth d. If a is greater than h_f, a T beam analysis is required as follows.

It will be assumed that the strength of the T beam is controlled by yielding of the tensile steel. This will nearly always be the case because of the large compressive concrete area provided by the flange. In addition, an upper limit can be established for the reinforcement ratio to ensure that this is so, as will be shown.

As a computational device, it is convenient to divide the total tensile steel into two parts. The first part, A_{sf}, represents the steel area which, when stressed to f_y, is required to balance the longitudinal compressive force in the overhanging portions of the flange that are stressed uniformly at $0.85 f'_c$. Thus,

$$A_{sf} = \frac{0.85 f'_c (b - b_w) h_f}{f_y} \tag{3.59}$$

The force $A_{sf} f_y$ and the equal and opposite force $0.85 f'_c (b - b_w) h_f$ act with a lever arm $d - h_f/2$ to provide the nominal resisting moment:

$$M_{n1} = A_{sf} f_y \left(d - \frac{h_f}{2}\right) \tag{3.60}$$

The remaining steel area, $A_s - A_{sf}$, at a stress f_y, is balanced by the compression in the rectangular portion of the beam. The depth of the equivalent rectangular stress block in this zone is found from horizontal equilibrium:

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f'_c b_w} \tag{3.61}$$

An additional moment M_{n2} is thus provided by the forces $(A_s - A_{sf}) f_y$ and $0.85 f'_c a b_w$ acting at the lever arm $d - a/2$:

$$M_{n2} = (A_s - A_{sf}) f_y \left(d - \frac{a}{2}\right) \tag{3.62}$$

and the total nominal resisting moment is the sum of the parts:

$$M_n = M_{n1} + M_{n2} = A_{sf} f_y \left(d - \frac{h_f}{2}\right) + (A_s - A_{sf}) f_y \left(d - \frac{a}{2}\right) \tag{3.63}$$

This moment is reduced by the strength reduction factor ϕ in accordance with the safety provisions of the ACI Code to obtain the design strength.

As for rectangular beams, the tensile steel should yield prior to sudden crushing of the compression concrete, as assumed in the preceding development. Yielding of the tensile reinforcement and Code compliance are ensured if the net tensile strain is greater than 0.004. From the geometry of the section,

$$\frac{c}{d_t} \le \frac{\epsilon_u}{\epsilon_u + \epsilon_t} \tag{3.64}$$

Setting $\epsilon_u = 0.003$ and $\epsilon_t = 0.004$ provides a maximum c/d_t ratio of 0.429, as seen in Fig. 3.10. Thus, as long as the depth to the neutral axis is less than $0.429d_t$, the net tensile strain requirements are satisfied, as they are for rectangular beam sections. This will occur if $\rho_w = A_s/b_w d$ is less than

$$\rho_{w,max} = \rho_{max} + \rho_f \tag{3.65}$$

where $\rho_f = A_{sf}/b_w d$ and ρ_{max} is as previously defined for a rectangular cross section [Eq. (3.30*b*)]. For c/d_t ratios between 0.429 and 0.375, equivalent to ρ_w between the $\rho_{w,max}$ from Eq. (3.65) and $\rho_{w,max}$ calculated by substituting ρ from Eq. (3.30*a*) with $\epsilon_t = 0.005$ in place of ρ_{max}, the strength reduction factor ϕ must be adjusted for ϵ_t, as shown in Fig. 3.9.

The practical result of applying Eq. (3.65) is that the stress block of T beams will almost always be within the flange, except for unusual geometry or combinations of material strength. Consequently, rectangular beam equations may be applied in most cases.

The ACI Code restriction that the tensile reinforcement ratio for beams must not be less than $\rho_{min} = 3\sqrt{f'_c}/f_y$ and $\ge 200/f_y$ (see Section 3.4d) applies to T beams as well as rectangular beams. For T beams, the ratio ρ should be computed for this purpose based on the web width b_w.

c. Proportions of Cross Section

When designing T beams, in contrast to analyzing the capacity of a given section, normally the slab dimensions and beam spacing will have been established by transverse flexural requirements. Consequently, the only additional section dimensions that must be determined from flexural considerations are the width and depth of the web and the area of the tensile steel.

If the stem dimensions were selected on the basis of concrete stress capacity in compression, they would be very small because of the large compression flange width furnished by the presence of the slab. Such a design would not represent the optimum solution because of the large tensile steel requirement resulting from the small effective depth, because of the excessive web reinforcement that would be required for shear, and because of large deflections associated with such a shallow member. It is better practice to select the proportions of the web (1) so as to keep an arbitrarily low web-reinforcement ratio ρ_w or (2) so as to keep web-shear stress at desirably low limits or (3) for continuous T beams, on the basis of the flexural requirements at the supports, where the effective cross section is rectangular and of width b_w.

In addition to the main reinforcement calculated according to the preceding requirements, it is necessary to ensure the integrity of the compressive flange of T beams by providing steel in the flange in the direction transverse to the main span. In typical construction, the slab steel serves this purpose. In other cases, separate bars must be added to permit the overhanging flanges to carry, as cantilever beams, the loads directly applied. According to ACI Code 8.10.5, the spacing of such bars must not exceed 5 times the thickness of the flange nor in any case exceed 18 in.

d. Examples of Analysis and Design of T Beams

For *analyzing* the capacity of a T beam with known concrete dimensions and tensile steel area, it is reasonable to start with the assumption that the stress block depth a does not exceed the flange thickness h_f. In that case, all ordinary rectangular beam equations (see Section 3.4) apply, with beam width taken equal to the effective width of the flange. If, upon checking that assumption, a proves to exceed h_f, then T beam analysis must be applied. Equations (3.59) through (3.63) can be used, in sequence, to obtain the nominal flexural strength, after which the design strength is easily calculated.

For *design*, the following sequence of calculations may be followed:

1. Establish flange thickness h_f based on flexural requirements of the slab, which normally spans transversely between parallel T beams.
2. Determine the effective flange width b according to ACI limits.
3. Choose web dimensions b_w and d based on either of the following:
   (a) negative bending requirements at the supports, if a continuous T beam
   (b) shear requirements, setting a reasonable upper-limit on the nominal unit shear stress v_u in the beam web (see Chapter 4)
4. With all concrete dimensions thus established, calculate a trial value of A_s, assuming that a does not exceed h_f, with beam width equal to flange width b. Use ordinary rectangular beam design methods.
5. For the trial A_s, check the depth of stress block a to confirm that it does not exceed h_f. If it should exceed that value, revise A_s using the T beam equations.
6. Check to ensure that $\epsilon_t \geq 0.004$ or $c/d \leq 0.429$. (This will almost invariably be the case.)
7. Check to ensure that $\rho_w \geq \rho_{w,min}$.

EXAMPLE 3.14 **Moment capacity of a given section.** An isolated T beam is composed of a flange 28 in. wide and 6 in. deep cast monolithically with a web of 10 in. width that extends 24 in. below the bottom surface of the flange to produce a beam of 30 in. total depth. Tensile reinforcement consists of six No. 10 (No. 32) bars placed in two horizontal rows. The centroid of the bar group is 26 in. from the top of the beam. It has been determined that the concrete has a strength of 3000 psi and that the yield stress of the steel is 60,000 psi. What is the design moment capacity of the beam?

Solution. It is easily confirmed that the flange dimensions are satisfactory according to the ACI Code for an isolated beam. The entire flange can be considered effective. For six No. 10 (No. 32) bars, $A_s = 7.62$ in^2. First check the location of the neutral axis, on the assumption that rectangular beam equations may be applied,

$$\rho = \frac{7.62}{28 \times 26} = 0.0105$$

and from Eq. (3.32)

$$a = \frac{7.62 \times 60}{0.85 \times 3 \times 28} = 6.40 \text{ in.}$$

This exceeds the flange thickness, and so a T beam analysis is required. From Eq. (3.59),

$$A_{sf} = 0.85 \times \frac{3}{60} \times (28 - 10) \times 6 = 4.59 \text{ in}^2$$

Hence

$$A_s - A_{sf} = 7.62 - 4.59 = 3.03 \text{ in}^2$$

Then, from Eq. (3.60),

$$M_{n1} = 4.59 \times 60(26 - 3) = 6330 \text{ in-kips}$$

while from Eqs. (3.58) and (3.59)

$$a = \frac{3.03 \times 60}{0.85 \times 3 \times 10} = 7.13 \text{ in.}$$

$$M_{n2} = 3.03 \times 60(26 - 3.56) = 4080 \text{ in-kips}$$

The depth to the neutral axis is $c = a/\beta_1 = 7.13/0.85 = 8.39$ and $d_t = 27.5$ in. to the lowest bar. The c/d_t ratio is $8.39/27.5 = 0.305 < 0.325$, so the $\epsilon_t \geq 0.005$ requirement is met and $\phi = 0.90$. When the ACI strength reduction factor is incorporated, the design strength is

$$\phi M_n = 0.90(6330 + 4080) = 9370 \text{ in-kips}$$

EXAMPLE 3.15 **Determination of steel area for a given moment.** A floor system consists of a 3 in. concrete slab supported by continuous T beams with a 24 ft span, 47 in. on centers. Web dimensions, as determined by negative-moment requirements at the supports, are $b_w = 11$ in. and $d = 20$ in. What tensile steel area is required at midspan to resist a factored moment of 6400 in-kips if $f_y = 60{,}000$ psi and $f'_c = 3000$ psi?

Solution. First determining the effective flange width,

$$16\, h_f + b_w = (16 \times 3) + 11 = 59 \text{ in.}$$

$$\frac{\text{Span}}{4} = 24 \times \frac{12}{4} = 72 \text{ in.}$$

$$\text{Centerline beam spacing} = 47 \text{ in.}$$

The centerline T beam spacing controls in this case, and $b = 47$ in. The concrete dimensions b_w and d are known to be adequate in this case, since they have been selected for the larger negative support moment applied to the effective rectangular section $b_w d$. The tensile steel at midspan is most conveniently found by trial. Assuming the stress-block depth equal to the flange thickness of 3 in., one gets

$$d - \frac{a}{2} = 20 - 1.50 = 18.50 \text{ in.}$$

Trial:

$$A_s = \frac{M_u}{\phi f_y (d - a/2)} = \frac{6400}{0.90 \times 60 \times 18.50} = 6.41 \text{ in}^2$$

$$\rho = \frac{A_s}{bd} = \frac{6.41}{47 \times 20} = 0.00682$$

$$a = \frac{\rho f_y d}{0.85 f_c'} = \frac{0.00682 \times 60 \times 20}{0.85 \times 3} = 3.21 \text{ in.}$$

Since a is greater than h_f, a T beam design is required and $\phi = 0.90$ is assumed.

$$A_{sf} = \frac{0.85 f_c' (b - b_w) h_f}{f_y} = \frac{0.85 \times 3 \times 36 \times 3}{60} = 4.59 \text{ in}^2$$

$$\phi M_{n1} = \phi A_{sf} f_y \left(d - \frac{h_f}{2}\right) = 0.90 \times 4.59 \times 60 \times 18.50 = 4590 \text{ in-kips}$$

$$\phi M_{n2} = M_u - \phi M_{n1} = 6400 - 4590 = 1810 \text{ in-kips}$$

Assume $a = 4.00$ in.:

$$A_s - A_{sf} = \frac{\phi M_{n2}}{\phi f_y (d - a/2)} = \frac{1810}{0.90 \times 60 \times 18.00} = 1.86 \text{ in}^2$$

Check:

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' b_w} = \frac{1.86 \times 60}{0.85 \times 3 \times 11} = 3.98 \text{ in.}$$

This is satisfactorily close to the assumed value of 4 in. Then

$$A_s = A_{sf} + (A_s - A_{sf}) = 4.59 + 1.86 = 6.45 \text{ in}^2$$

Checking to ensure that the net tensile strain of 0.005 is met to allow $\phi = 0.90$,

$$c = \frac{a}{\beta_1} = \frac{3.98}{0.85} = 4.68$$

$$\frac{c}{d_t} = \frac{4.68}{20} = 0.23 < 0.325$$

indicating that the design is satisfactory.

The close agreement should be noted between the approximate tensile steel area of 6.41 in^2 found by assuming the stress-block depth equal to the flange thickness and the more exact value of 6.45 in^2 found by T beam analysis. The approximate solution would be satisfactory in most cases.

REFERENCES

3.1. H. Rusch, "Researches Toward a General Flexural Theory of Structural Concrete," *J. ACI*, vol. 32, no. 1, 1960, pp. 1–28.

3.2. L. B. Kriz, "Ultimate Strength Criteria for Reinforced Concrete," *J. Eng. Mech. Div. ASCE*, vol. 85, no. EM3, 1959, pp. 95–110.

3.3. L. B. Kriz and S. L. Lee, "Ultimate Strength of Overreinforced Beams," *Proc. ASCE*, vol. 86, no. EM3, 1960, pp. 95–106.

3.4. C. S. Whitney, "Design of Reinforced Concrete Members Under Flexure or Combined Flexure and Direct Compression," *J. ACI*, vol. 33, Mar-Apr, 1937, pp. 483–498.

3.5. A. H. Mattock, L. B. Kriz, and E. Hogenstad, "Rectangular Concrete Stress Distribution in Ultimate Strength Design," *J. ACI*, vol. 32, no. 8, 1961, pp. 875–928.

3.6. P. H. Kaar, N. W. Hanson, and H. T. Capell, "Stress-Strain Curves and Stress Block Coefficients for High-Strength Concrete," *Proceedings Douglas McHenry Symposium*, ACI Special Publication SP-55, 1978.
3.7. *ACI Design Handbook: Beams, One-Way Slabs, Brackets, Footings, Pile Caps, Columns, Two-Way Slabs, and Seismic Design in Accordance with the Strength Design Method of 318-95*, SP-17, American Concrete Institute, Farmington Hills, MI, 1997.
3.8. *ACI Detailing Manual*, ACI Special Publication SP-66, American Concrete Institute, Farmington Hills, MI, 1994.
3.9. *CRSI Design Handbook*, 9th ed., Concrete Reinforcing Steel Institute, Schaumburg, IL, 2002.
3.10. *Economical Concrete Construction*, Engineering Data Report No. 30, Concrete Reinforcing Steel Institute, Schaumburg, IL, 1988.
3.11. *Manual of Standard Practice*, 27th ed., Concrete Reinforcing Steel Institute, Schaumburg, IL, 2001.
3.12. C. W. Dolan, *Ultimate Capacity of Reinforced Concrete Sections Using a Continuous Stress-Strain Function*, MS Thesis, Cornell University, Ithaca, NY, June 1967.

Problems

3.1. A rectangular beam made using concrete with $f_c' = 4000$ psi and steel with $f_y = 60{,}000$ psi has width $b = 24$ in., total depth $h = 18$ in., and effective depth $d = 15.5$ in. Concrete modulus of rupture $f_r = 475$ psi. The elastic modulus of the steel and concrete are, respectively, 29,000,000 psi and 3,600,000 psi. The tensile steel area is A_s = five No. 11 (No. 36) bars.

(*a*) Find the maximum service load moment that can be resisted without stressing the concrete higher than $0.45\,f_c'$ or the steel above $0.40f_y$.

(*b*) Determine the nominal flexural strength of the beam section, and calculate the ratio of nominal flexural strength to service load moment.

(*c*) Determine whether this beam will show flexural cracking before reaching the service load calculated in part (*a*).

3.2. A rectangular, tension-reinforced beam is to be designed for dead load of 500 lb/ft plus self weight and service live load of 1200 lb/ft, with a 22 ft simple span. Material strengths will be $f_y = 60$ ksi and $f_c' = 3$ ksi for steel and concrete, respectively. The total beam depth must not exceed 16 in. Calculate the required beam width and tensile steel requirement, using a reinforcement ratio of 0.60· $_{max}$. Use ACI load factors and strength reduction factors. The effective depth may be assumed to be 2.5 in. less than the total depth.

3.3. A beam with a 20 ft simple span has cross-section dimensions $b = 12$ in., $d = 23$ in., and $h = 25$ in. (see Fig. 3.2*b* for notation). It carries a uniform service load of 2450 lb/ft in addition to its own weight. Material strengths are $f_c' = 4000$ psi and $f_y = 60{,}000$ psi. Assume a weight of 150 pcf for reinforced concrete.

(*a*) Check whether this beam, if reinforced with three No. 9 (No. 29) bars, is adequate to carry this load with a minimum factor of safety against flexural failure of 1.85. If this requirement is not met, select a three-bar reinforcement of diameter or diameters adequate to provide this safety.

(*b*) Determine the maximum stress in the steel and in the concrete under service load, i.e., when the beam carries its own weight and the specified uniform load.

(*c*) Will the beam show hairline cracks on the tension side under service load?

3.4. A rectangular reinforced concrete beam has dimensions $b = 12$ in., $d = 21$ in., and $h = 24$ in., and is reinforced with three No. 10 (No. 32) bars. Material strengths are $f_y = 60{,}000$ psi and $f_c' = 4000$ psi.

(*a*) Find the moment that will produce the first cracking at the bottom surface of the beam, basing your calculation on I_g, the moment of inertia of the gross concrete section.

(*b*) Repeat the calculation using I_{ut}, the moment of inertia of the uncracked transformed section.
(*c*) Determine the maximum moment that can be carried without stressing the concrete beyond $0.45 f'_c$ or the steel beyond $0.40 f_y$.
(*d*) Find the nominal flexural strength and design strength of this beam.
(*e*) Compute the ratio of design strength (*d*) to service capacity (*c*).

3.5. A tensile-reinforced beam has $b = 12$ in. and $d = 20$ in. to the center of the bars, which are placed all in one row. If $f_y = 60{,}000$ psi and $f'_c = 5000$ psi, find the nominal flexural strength M_n for (*a*) A_s = two No. 8 (No. 25) bars, (*b*) A_s = two No. 10 (No. 32) bars, (*c*) A_s = three No. 10 (No. 32) bars.

3.6. A singly reinforced rectangular beam is to be designed, with effective depth approximately 1.5 times the width, to carry a service live load of 1500 lb/ft in addition to its own weight, on a 24 ft simple span. The ACI Code load factors are to be applied as usual. With $f_y = 60{,}000$ psi and $f'_c = 4000$ psi, determine the required concrete dimensions *b*, *d*, and *h*, and steel reinforcing bars (*a*) for $\cdot = 0.50\cdot_{max}$ and (*b*) for $\cdot = \cdot_{max}$. Include a sketch of each cross section drawn to scale. Allow for No. 3 (No. 10) stirrups. Comment on your results.

3.7. A four-span continuous beam of constant rectangular section is supported at *A*, *B*, *C*, *D*, and *E*. Factored moments resulting from analysis are

At supports, ft-kips	At midspan, ft-kips
$M_a = 92$	$M_{ab} = 105$
$M_b = 147$	$M_{bc} = 92$
$M_c = 134$	$M_{cd} = 92$
$M_d = 147$	$M_{de} = 105$
$M_e = 92$	

Determine the required concrete dimensions for this beam, using $d = 1.75b$, and find the required reinforcement for all critical moment sections. Use a maximum reinforcement ratio of $\cdot = 0.60\cdot_{max}$, $f_y = 60{,}000$ psi, and $f'_c = 5000$ psi.

3.8. A two-span continuous concrete beam is to be supported by three masonry walls spaced 25 ft on centers. A service live load of 1.5 kips/ft is to be carried, in addition to the self-weight of the beam. A constant rectangular cross section is to be used, with $h = 2b$, but reinforcement is to be varied according to requirements. Find the required concrete dimensions and reinforcement at all critical sections. Allow for No. 3 (No. 10) stirrups. Include sketches, drawn to scale, of critical cross sections. Use $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

3.9. A rectangular concrete beam measures 12 in. wide and has an effective depth of 18 in. Compression steel consisting of two No. 8 (No. 25) bars is located 2.5 in. from the compression face of the beam. If $f'_c = 4000$ psi and $f_y = 60{,}000$ psi, what is the design moment capacity of the beam, according to the ACI Code, for the following alternative tensile steel areas: (*a*) A_s = three No. 10 (No. 32) bars in one layer, (*b*) A_s = four No. 10 (No. 32) bars in two layers, (*c*) A_s = six No. 9 (No. 29) bars in two layers? (*Note*: Check for yielding of compression steel in each case.) Plot M_n versus $\cdot$ and comment on your findings.

3.10. A rectangular concrete beam of width $b = 24$ in. is limited by architectural considerations to a maximum total depth $h = 16$ in. It must carry a total factored load moment $M_u = 400$ ft-kips. Design the flexural reinforcement for this member, using compression steel if necessary. Allow 3 in. to the center of the bars from the compression or tension face of the beam. Material strengths are f_y = 60,000 psi and f'_c = 4000 psi. Select reinforcement to provide the needed areas, and show a sketch of your final design, including provision for No. 4 (No. 13) stirrups.

3.11. A rectangular beam with width b = 24 in., total depth h = 14 in., and effective depth to the tensile steel d = 11.5 in. is constructed using materials with strengths f'_c = 4000 psi and f_y = 60,000 psi. Tensile reinforcement consists of two No. 11 (No. 36) bars plus three No. 10 (No. 32) bars in one row. Compression reinforcement consisting of two No. 10 (No. 32) bars is placed at distance d' = 2.5 in. from the compression face. Calculate the nominal and design strengths of the beam (*a*) neglecting the compression reinforcement, (*b*) accounting for the compression reinforcement and assuming that it acts at f_y, and (*c*) accounting for the compression reinforcement working at its actual stress f'_s, established by analysis.

3.12. A tensile-reinforced T beam is to be designed to carry a uniformly distributed load on a 20 ft simple span. The total moment to be carried is M_u = 5780 in-kips. Concrete dimensions, governed by web shear and clearance requirements, are b = 20 in., b_w = 10 in., h_f = 5 in., and d = 20 in. If f_y = 60 ksi and f'_c = 4 ksi, what tensile reinforcement is required at midspan? Select appropriate reinforcement to provide this area and check concrete cover limitations, assuming No. 3 (No. 10) stirrups. What total depth h is required? Sketch your design.

3.13. A concrete floor system consists of parallel T beams spaced 10 ft on centers and spanning 32 ft between supports. The 6 in. thick slab is cast monolithically with T beam webs having width b_w = 14 in. and total depth, measured from the top of the slab, of h = 28 in. The effective depth will be taken 3 in. less than the total depth. In addition to its own weight, each T beam must carry a superimposed dead load of 50 psf and service live load of 225 psf. Material strengths are f_y = 60,000 psi and f'_c = 4000 psi. Determine the required tensile steel area and select the reinforcement needed for a typical member.

3.14. A precast T beam is to be used as a bridge over a small roadway. Concrete dimensions are b = 48 in., b_w = 16 in., h_f = 5 in., and h = 25 in. The effective depth d = 20 in. Concrete and steel strengths are 6000 psi and 60,000 psi, respectively. Using approximately one-half the maximum tensile reinforcement permitted by the ACI Code (select the actual size of bar and number to be used), determine the design moment capacity of the girder. If the beam is used on a 30 ft simple span, and if in addition to its own weight it must support railings, curbs, and suspended loads totaling 0.475 kips/ft, what uniform service live load limit should be posted?

3.15. Compute the maximum and minimum reinforcement ratios for reinforcement with an 80 ksi yield point and f'_c = 4000 to 8000 psi in 1000 psi increments, similar to those shown in Table A.4 of Appendix A. Using the maximum and minimum reinforcement ratios, develop resistance factors and design graphs similar to Table A.5b and Graph A.1a.