# 5 Bond, Anchorage, and Development Length

## 5.1 Fundamentals of Flexural Bond

If the reinforced concrete beam of Fig. 5.1*a* were constructed using plain round reinforcing bars, and, furthermore, if those bars were to be greased or otherwise lubricated before the concrete were cast, the beam would be very little stronger than if it were built of plain concrete, without reinforcement. If a load were applied, as shown in Fig. 5.1*b*, the bars would tend to maintain their original length as the beam deflects. The bars would slip longitudinally with respect to the adjacent concrete, which would experience tensile strain due to flexure. Proposition 2 of Section 1.8, the assumption that the strain in an embedded reinforcing bar is the same as that in the surrounding concrete, would not be valid. For reinforced concrete to behave as intended, it is essential that *bond forces* be developed on the interface between concrete and steel, such as to prevent significant slip from occurring at that interface.

Figure 5.1*c* shows the bond forces that act on the concrete at the interface as a result of bending, while Fig. 5.1*d* shows the equal and opposite bond forces acting on the reinforcement. It is through the action of these interface bond forces that the slip indicated in Fig. 5.1*b* is prevented.

Some years ago, when plain bars without surface deformations were used, initial bond strength was provided only by the relatively weak chemical adhesion and mechanical friction between steel and concrete. Once adhesion and static friction were overcome at larger loads, small amounts of slip led to interlocking of the natural roughness of the bar with the concrete. However, this natural bond strength is so low that in beams reinforced with plain bars, the bond between steel and concrete was frequently broken. Such a beam will collapse as the bar is pulled through the concrete. To prevent this, end anchorage was provided, chiefly in the form of hooks, as in Fig. 5.2. If the anchorage is adequate, such a beam will not collapse, even if the bond is broken over the entire length between anchorages. This is so because the member acts as a tied arch, as shown in Fig. 5.2, with the uncracked concrete shown shaded representing the arch and the anchored bars the tie rod. In this case, over the length in which the bond is broken, bond forces are zero. This means that over the entire unbonded length the force in the steel is constant and equal to $T = M_{max} / jd$. As a consequence, the total steel elongation in such beams is larger than in beams in which bond is preserved, resulting in larger deflections and greater crack widths.

To improve this situation, deformed bars are now universally used in the United States and many other countries (see Section 2.14). With such bars, the shoulders of the projecting ribs bear on the surrounding concrete and result in greatly increased bond strength. It is then possible in most cases to dispense with special anchorage devices such as hooks. In addition, crack widths as well as deflections are reduced.

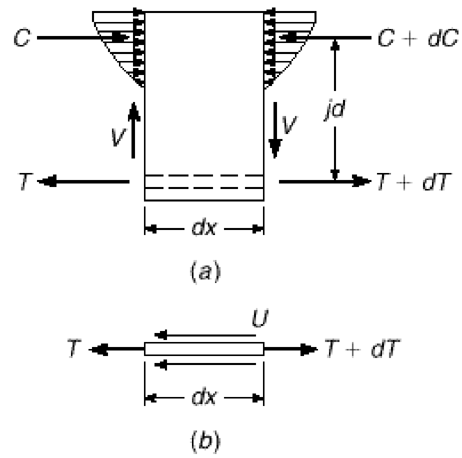



**FIGURE 5.3**
Forces acting on elemental length of beam: (*a*) free-body sketch of reinforced concrete element; (*b*) free-body sketch of steel element.

If $U$ is the magnitude of the local bond force per unit length of bar, then, by summing horizontal forces

$$U\,dx = dT \tag{b}$$

Thus

$$U = \frac{dT}{dx} \tag{5.1}$$

indicating that the local unit bond force is proportional to the rate of change of bar force along the span. Alternatively, substituting Eq. (*a*) in Eq. (5.1), the unit bond force can be written as

$$U = \frac{1}{jd}\frac{dM}{dx} \tag{c}$$

from which

$$U = \frac{V}{jd} \tag{5.2}$$

Equation (5.2) is the "elastic cracked section equation" for flexural bond force, and it indicates that the bond force per unit length is proportional to the shear at a particular section, i.e., to the rate of change of bending moment.

Note that Eq. (5.2) applies to the *tension* bars in a concrete zone that is assumed to be fully cracked, with the concrete resisting no tension. It applies, therefore, to the tensile bars in simple spans, or, in continuous spans, either to the bottom bars in the positive bending region between inflection points or to the top bars in the negative bending region between the inflection points and the supports. It does not apply to compression reinforcement, for which it can be shown that the flexural bond forces are very low.

## b. Actual Distribution of Flexural Bond Force

The actual distribution of bond force along deformed reinforcing bars is much more complex than that represented by Eq. (5.2), and Eq. (5.1) provides a better basis for

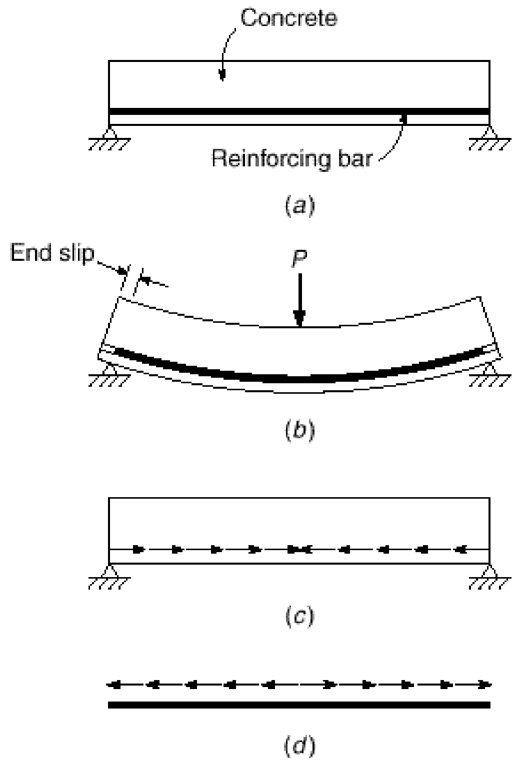



**FIGURE 5.1**
Bond forces due to flexure: (*a*) beam before loading; (*b*) unrestrained slip between concrete and steel; (*c*) bond forces acting on concrete; (*d*) bond forces acting on steel.

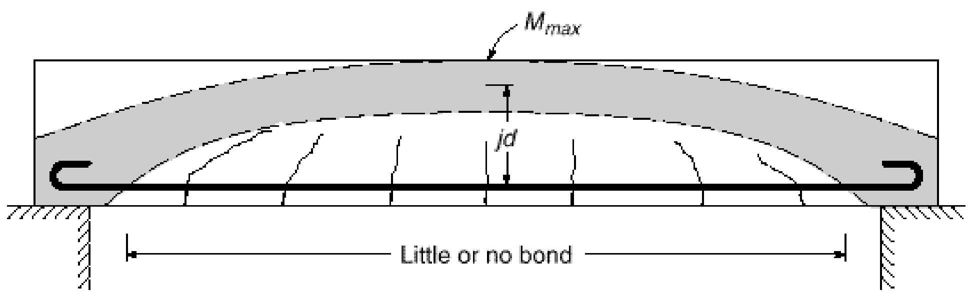



**FIGURE 5.2**
Tied-arch action in a beam with little or no bond.

## a. Bond Force Based on Simple Cracked Section Analysis

In a short piece of a beam of length $dx$, such as shown in Fig. 5.3*a*, the moment at one end will generally differ from that at the other end by a small amount $dM$. If this piece is isolated, and if one assumes that, after cracking, the concrete does not resist any tension stresses, the internal forces are those shown in Fig. 5.3*a*. The change in bending moment $dM$ produces a change in the bar force

$$dT = \frac{dM}{jd} \qquad (a)$$

where $jd$ is the internal lever arm between tensile and compressive force resultants. Since the bar or bars must be in equilibrium, this change in bar force is resisted at the contact surface between steel and concrete by an equal and opposite force produced by bond, as indicated by Fig. 5.3*b*.

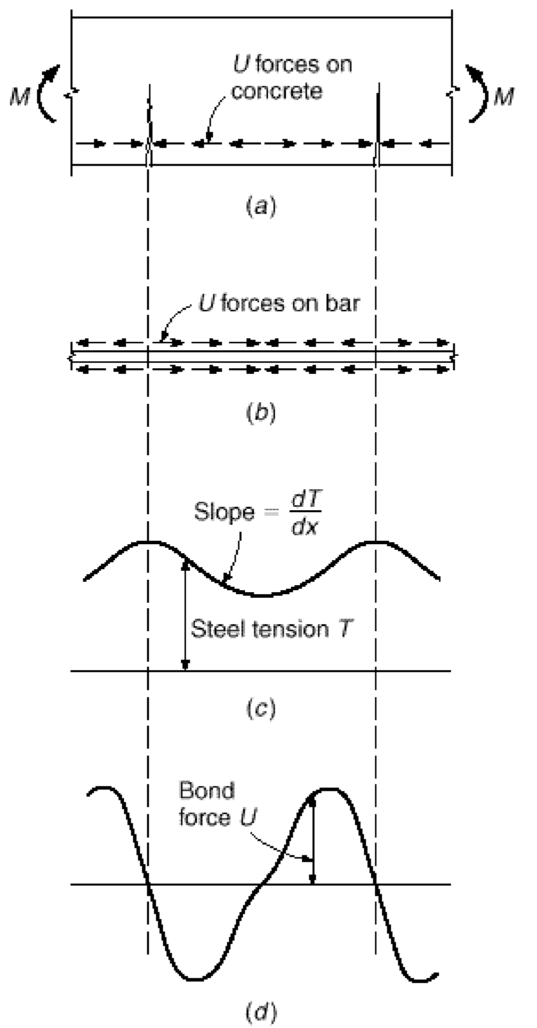



**FIGURE 5.4**
Variation of steel and bond forces in a reinforced concrete member subject to pure bending: (*a*) cracked concrete segment; (*b*) bond forces acting on reinforcing bar; (*c*) variation of tensile force in steel; (*d*) variation of bond force along steel.

understanding beam behavior. Figure 5.4 shows a beam segment subject to pure bending. The concrete fails to resist tensile stresses only where the actual crack is located; there the steel tension is maximum and has the value predicted by simple theory: $T = M{\cdot}j_d$. Between cracks, the concrete *does* resist moderate amounts of tension, introduced by bond forces acting along the interface in the direction shown in Fig. 5.4*a*. This reduces the tensile force in the steel, as illustrated by Fig. 5.4*c*. From Eq. (5.1), it is clear that $U$ is proportional to the rate of change of bar force, and thus will vary as shown in Fig. 5.4*d*; unit bond forces are highest where the slope of the steel force curve is greatest, and are zero where the slope is zero. Very high local bond forces adjacent to cracks have been measured in tests (Refs. 5.1 and 5.2). They are so high that inevitably some slip occurs between concrete and steel adjacent to each crack.

Beams are seldom subject to pure bending moment; they generally carry transverse loads producing shear and moment that vary along the span. Figure 5.5*a* shows a beam carrying a distributed load. The cracking indicated is typical. The steel force $T$ predicted by simple cracked section analysis is proportional to the moment diagram and is as shown by the dashed line in Fig. 5.5*b*. However, the actual value of $T$ is less

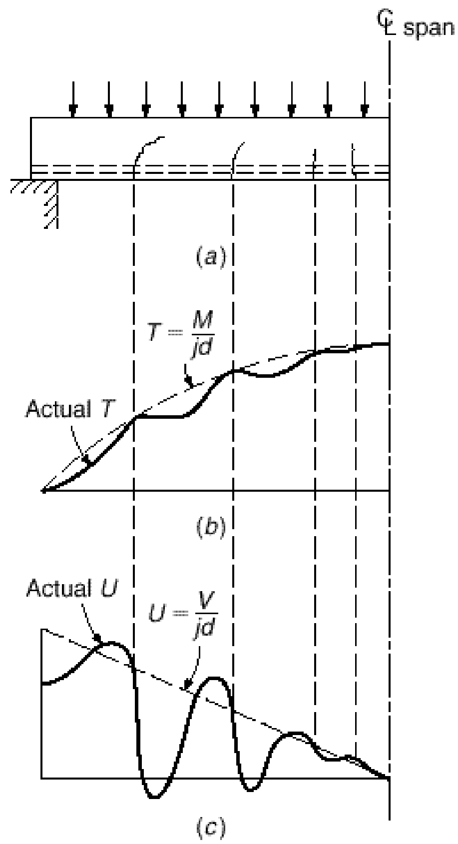



**FIGURE 5.5**
Effect of flexural cracks on bond forces in beam: (*a*) beam with flexural cracks; (*b*) variation of tensile force $T$ in steel along span; (*c*) variation of bond force per unit length $U$ along span.

than that predicted by the simple analysis everywhere except at the actual crack locations. The actual variation of $T$ is shown by the solid line of Fig. 5.5*b*. In Fig. 5.5*c*, the bond forces predicted by the simplified theory are shown by the dashed line, and the actual variation shown by the solid line. Note that the value of $U$ is equal to that given by Eq. (5.2) only at those locations where the slope of the steel force diagram equals that of the simple theory. Elsewhere, if the slope is greater than assumed, the local bond force is greater; if the slope is less, local bond force is less. Just to the left of the cracks, for the present example, $U$ is much higher than predicted by Eq. (5.2), and in all probability will result in local bond failure. Just to the right of the cracks, $U$ is much lower than predicted, and in fact is generally negative very close to the crack; i.e., the bond forces act in the reverse direction.

It is evident that actual bond forces in beams bear very little relation to those predicted by Eq. (5.2) except in the general sense that they are highest in the regions of high shear.

## 5.2 Bond Strength and Development Length

For reinforcing bars in tension, two types of bond failure have been observed. The first is *direct pullout* of the bar, which occurs when ample confinement is provided by the surrounding concrete. This could be expected when relatively small diameter bars are used with sufficiently large concrete cover distances and bar spacing. The second type of failure is *splitting* of the concrete along the bar when cover, confinement, or bar spacing is insufficient to resist the lateral concrete tension resulting from the wedging effect of the bar deformations. Present-day design methods require that both possible failure modes be accounted for.

## a. Bond Strength

If the bar is sufficiently confined by a mass of surrounding concrete, then, as the tensile force on the bar is increased, adhesive bond and friction are overcome, the concrete eventually crushes locally ahead of the bar deformations, and bar pullout results. The surrounding concrete remains intact, except for the crushing that takes place ahead of the ribs immediately adjacent to the bar interface. For modern deformed bars, adhesion and friction are much less important than the mechanical interlock of the deformations with the surrounding concrete.

Bond failure resulting from splitting of the concrete is more common in beams than direct pullout. Such splitting comes mainly from wedging action when the ribs of the deformed bars bear against the concrete (Refs. 5.3 and 5.4). It may occur either in a vertical plane as in Fig. 5.6*a* or horizontally in the plane of the bars as in Fig. 5.6*b*. The horizontal type of splitting of Fig 5.6*b* frequently begins at a diagonal crack. In this case, as discussed in connection with Fig. 4.7*b* and shown in Fig. 4.1, dowel action increases the tendency toward splitting. This indicates that shear and bond failures are often intricately interrelated.

When pullout resistance is overcome or when splitting has spread all the way to the end of an unanchored bar, complete bond failure occurs. Sliding of the steel relative to the concrete leads to immediate collapse of the beam.

If one considers the large local variations of bond force caused by flexural and diagonal cracks (see Figs. 5.4 and 5.5), it becomes clear that local bond failures immediately adjacent to cracks will often occur at loads considerably below the failure load of the beam. These local failures result in small local slips and some widening of cracks and increase of deflections, but will be harmless as long as failure does not propagate all along the bar, with resultant total slip. In fact, as discussed in connection with Fig. 5.2, when end anchorage is reliable, bond can be severed along the entire length of the bar, excluding the anchorages, without endangering the carrying capacity of the beam. End anchorage can be provided by hooks as suggested by Fig. 5.2 or,

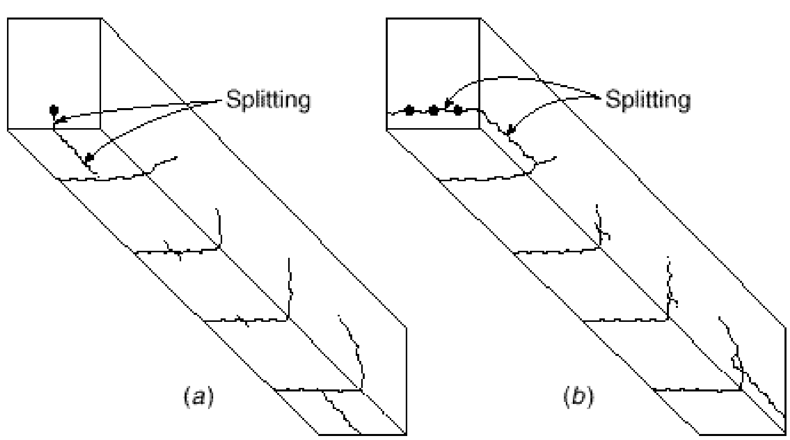
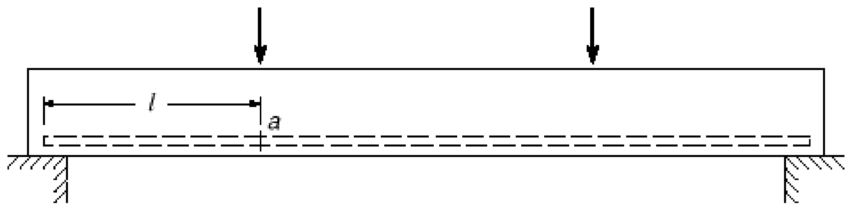



**FIGURE 5.6**
Splitting of concrete along reinforcement.

**FIGURE 5.7**
Development length.




much more commonly, by extending the straight bar a sufficient distance from the point of maximum stress.

Extensive testing (Refs. 5.5 to 5.11), using beam specimens, has established limiting values of bond strength. This testing provides the basis for current design requirements.

## b. Development Length

The preceding discussion suggests the concept of *development length* of a reinforcing bar. The development length is defined as that length of embedment necessary to develop the full tensile strength of the bar, controlled by either pullout or splitting. With reference to Fig. 5.7, the moment, and therefore the steel stress, is evidentally maximum at point $a$ (neglecting the weight of the beam) and zero at the supports. If the bar stress is $f_s$ at $a$, then the total tension force $A_b f_s$ must be transferred from the bar to the concrete in the distance $l$ by bond forces. To fully develop the strength of the bar, $A_b f_y$, the distance $l$ must be at least equal to the development length of the bar, established by tests. In the beam of Fig. 5.7, if the actual length $l$ is equal to or greater than the development length $l_d$, no premature bond failure will occur. That is, the beam will fail in bending or shear rather than by bond failure. This will be so even if in the vicinity of cracks local slip may have occurred over small regions along the beam.

It is seen that the main requirement for safety against bond failure is this: the length of the bar, from any point of given steel stress ($f_s$ or at most $f_y$) to its nearby free end must be at least equal to its development length. If this requirement is satisfied, the magnitude of the nominal flexural bond force along the beam, as given by Eq. (5.2), is of only secondary importance, since the integrity of the member is ensured even in the face of possible minor local bond failures. However, if the actual available length is inadequate for full development, special anchorage, such as by hooks, must be provided.

## c. Factors Influencing Development Length

Experimental research has identified the factors that influence development length, and analysis of the test data has resulted in the empirical equations used in present design practice. The most basic factors will be clear from review of the preceding paragraphs and include concrete tensile strength, cover distance, spacing of the reinforcing bars, and the presence of transverse steel reinforcement.

Clearly, the *tensile strength* of the concrete is important because the most common type of bond failure in beams is the type of splitting shown in Fig. 5.6. Although tensile strength does not appear explicitly in experimentally derived equations for development length (see Section 5.3), the term $\sqrt{f_c'}$ appears in the denominator of those equations and reflects the influence of concrete tensile strength.

As discussed in Section 2.9, the fracture energy of concrete plays an important role in bond failure because a splitting crack must propagate after it has formed. Since fracture energy is largely independent of compressive strength, bond strength increases more slowly than the $\sqrt{f'_c}$, and as data for higher-strength concretes has become available, $f_c'^{1/4}$ has been shown to provide a better representation of the effect of concrete strength on bond than $\sqrt{f'_c}$ (Refs. 5.12 to 5.14). This point is recognized by ACI Committee 408, Bond and Development of Reinforcement (Ref. 5.15), in proposed design expressions based on $f_c'^{1/4}$ and within the ACI Code, which sets an upper limit on the value of $\sqrt{f'_c}$ for use in design.

For lightweight concretes, the tensile strength is usually less than for normal-density concrete having the same compressive strength; accordingly, if lightweight concrete is used, development lengths must be increased. Alternatively, if split-cylinder strength is known or specified for lightweight concrete, it can be incorporated in development length equations as follows. For normal concrete, the split-cylinder tensile strength $f_{ct}$ is generally taken as $f_{ct} = 6.7\sqrt{f'_c}$. If the split-cylinder strength $f_{ct}$ is known for a particular lightweight concrete, then $\sqrt{f'_c}$ in the development length equations can be replaced by $f_{ct}/6.7$.

*Cover distance*—conventionally measured from the *center* of the bar to the nearest concrete face and measured either in the plane of the bars or perpendicular to that plane—also influences splitting. Clearly, if the vertical or horizontal cover is increased, more concrete is available to resist the tension resulting from the wedging effect of the deformed bars, resistance to splitting is improved, and development length is less.

Similarly, Fig. 5.6*b* illustrates that if the *bar spacing* is increased (e.g., if only two instead of three bars are used), more concrete per bar would be available to resist horizontal splitting (Ref. 5.16). In beams, bars are typically spaced about one or two bar diameters apart. On the other hand, for slabs, footings, and certain other types of member, bar spacings are typically much greater, and the required development length is reduced.

*Transverse reinforcement*, such as that provided by stirrups of the types shown in Fig. 4.8, improves the resistance of tensile bars to both vertical or horizontal splitting failure because the tensile force in the transverse steel tends to prevent opening of the actual or potential crack. The effectiveness of such transverse reinforcement depends on its cross-sectional area and spacing along the development length. Its effectiveness does not depend on its yield strength $f_{yt}$, because transverse reinforcement rarely yields during a bond failure (Refs. 5.12 to 5.15). The yield strength of the transverse steel $f_{yt}$, however, is presently used in the bond provisions of the ACI Code.

Based on the results of a statistical analysis of test data (Ref. 5.10), with appropriate simplifications, the length $l_d$ needed to develop stress $f_s$ in a reinforcing bar may be expressed as

$$l_d = \left(\frac{3}{40}\frac{f_s}{\sqrt{f'_c}\left(\dfrac{c + K_{tr}}{d_b}\right)}\right) d_b \qquad (5.3)$$

where $d_b$ = bar diameter

$c$ = smaller of minimum cover or one-half of bar spacing *measured to center of bar*

$K_{tr} = A_{tr}f_{yt}/(1500sn)$, which represents effect of confining reinforcement

$A_{tr}$ = area of transverse reinforcement normal to plane of splitting through the bars being developed

$s$ = spacing of transverse reinforcement
$n$ = number of bars developed or spliced at same location

Equation (5.3) captures the effects of concrete strength, concrete cover, and transverse reinforcement on $l_d$ and serves as the basis for design in the 2002 ACI Code. For full development of the bar, $f_s$ is set equal to $f_y$.

In addition to the factors just discussed, other influences have been identified. *Vertical bar location* relative to beam depth has been found to have an effect (Ref. 5.17). If bars are placed in the beam forms during construction such that a substantial depth of concrete is placed below those bars, there is a tendency for excess water, often used in the mix for workability, and for entrapped air to rise to the top of the concrete during vibration. Air and water tend to accumulate on the underside of the bars. Tests have shown a significant loss in bond strength for bars with more than 12 in. of fresh concrete cast beneath them, and accordingly the development length must be increased. This effect increases as the slump of the concrete increases and is greatest for bars cast near the upper surface of a concrete placement.

*Epoxy-coated reinforcing bars* are used regularly in projects where the structure may be subjected to corrosive environmental conditions or deicing chemicals, such as for highway bridge decks and parking garages. Studies have shown that bond strength is reduced because the epoxy coating reduces the friction between the concrete and the bar, and the required development length must be increased substantially (Refs. 5.18 to 5.22). Early evidence showed that if cover and bar spacing were large, the effect of the epoxy coating would not be so pronounced, and as a result, a smaller increase was felt justified under these conditions (Ref. 5.19). Although later research (Ref. 5.12) does not support this conclusion, provisions to allow for a smaller increase remain in the ACI Code. Since the bond strength of epoxy-coated bars is already reduced because of lack of adhesion, an upper limit has been established for the product of development length factors accounting for vertical bar location and epoxy coating.

Not infrequently, tensile reinforcement somewhat in excess of the calculated requirement will be provided, e.g., as a result of upward rounding $A_s$ when bars are selected or when minimum steel requirements govern. Logically, in this case, the required development length may be reduced by the ratio of steel area required to steel area actually provided. The modification for *excess reinforcement* should be applied only where anchorage or development for the full yield strength of the bar is not required.

Finally, based on bars with very short development lengths (most with values of $l_d/d_b < 15$), it was observed that *smaller diameter bars* required lower development lengths than predicted by Eq. (5.3). As a result, the required development lengths for No. 6 (No. 19) and smaller bars were reduced below the values required by Eq. (5.3).[†]

Reference 5.15 presents a detailed discussion of the factors that control the bond and development of reinforcing bars in tension. Except as noted, these influences are accounted for in the basic equation for development length in the 2002 ACI Code. All modification factors for development length are defined explicitly in the Code, with appropriate restrictions. Details are given next.

[†] The use of Eq. (5.3) for low values of $l_d/d_b$ greatly underestimates the actual value of bond strength and makes it appear that a lower value of $l_d$ can be used safely. An evaluation of test results for small bars with more realistic development lengths ($l_d/d_b \geq 16$), however, has shown that the special provision in the ACI Code for smaller bars is not justified (Refs. 5.14, 5.15, and 5.23). Because of the unconservative nature of the small bar provision, ACI Committee 408 (Ref. 5.15) recommends that it not be applied in design.

## 5.3 ACI Code Provisions for Development of Tension Reinforcement

The approach to bond strength incorporated in the 2002 ACI Code follows from the discussion presented in Section 5.2. The fundamental requirement is that the calculated force in the reinforcement at each section of a reinforced concrete member must be developed on each side of that section by adequate embedment length, hooks, mechanical anchorage, or a combination of these, to ensure against pullout. Local high bond forces, such as are known to exist adjacent to cracks in beams, are not considered to be significant. Generally, the force to be developed is calculated based on the yield stress in the reinforcement; i.e., the bar strength is to be fully developed.

In the 2002 ACI Code, the required development length for deformed bars in tension is based on Eq. (5.3). A single basic equation is given that includes *all* the influences discussed in Section 5.2 and thus appears highly complex because of its inclusiveness. However, it does permit the designer to see the effects of all the controlling variables and allows more rigorous calculation of the required development length when it is critical. The ACI Code also includes simplified equations that can be used for most cases in ordinary design, provided that some restrictions are accepted on bar spacing, cover values, and minimum transverse reinforcement. These alternative equations can be further simplified for normal-density concrete and uncoated bars.†

In the following presentation of development length, the basic ACI equation is given first and its terms are defined and discussed. After this, the alternative equations, also part of the 2002 ACI Code, are presented. Note that, in any case, development length $l_d$ must not be less than 12 in.

### a. Basic Equation for Development of Tension Bars

According to ACI Code 12.2.3, for deformed bars or deformed wire,

$$l_d = \left(\frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\cdots}{\left(\frac{c + K_{tr}}{d_b}\right)}\right) d_b \tag{5.4}$$

in which the term $(c + K_{tr}) / d_b$ shall not be taken greater than 2.5. In Eq. (5.4), terms are defined and values established as follows.

· = reinforcement location factor

| | |
|---|---|
| Horizontal reinforcement so placed that more than 12 in. of fresh concrete is cast in the member below the development length or splice: | 1.3 |
| Other reinforcement: | 1.0 |

· = coating factor

| | |
|---|---|
| Epoxy-coated bars or wires with cover less than $3d_b$ or clear spacing less than $6d_b$: | 1.5 |
| All other epoxy-coated bars or wires: | 1.2 |

† This two-tiered approach to development length corresponds exactly to the ACI Code treatment for $V_c$, the contribution of concrete in shear calculations. The more detailed calculation by Eq. (4.12*a*) is useful for computerized design or research but is tedious for manual calculations because of the need to recalculate the governing variables at close intervals along the span. For ordinary design, recognizing that overall economy is but little affected, the simpler but more approximate and more conservative Eq. (4.12*b*) is used.

Uncoated reinforcement: 1.0
However, the product of · · need not be taken greater than 1.7.

· = reinforcement size factor
No. 6 (No. 19) and smaller bars and deformed wires: 0.8[†]
No. 7 (No. 22) and larger bars: 1.0

· = lightweight aggregate concrete factor
When lightweight aggregate concrete is used: 1.3
However, when $f_{ct}$ is specified, · shall be permitted to be taken as $6.7 \cdot \sqrt{f_c'} \cdot f_{ct}$ but not less than 1.0.
When normal-weight concrete is used: 1.0

$c$ = spacing or cover dimension, in.
Use the smaller of either the distance from the center of the bar to the nearest concrete surface or one-half the center-to-center spacing of the bars being developed.

$K_{tr}$ = transverse reinforcement index: $A_{tr} f_{yt} \cdot (1500sn)$
where $A_{tr}$ = total cross-sectional area of all transverse reinforcement that is within the spacing $s$ and that crosses the potential plane of splitting through the reinforcement being developed, in$^2$
$f_{yt}$ = specified yield strength of transverse reinforcement, psi
$s$ = maximum spacing of transverse reinforcement within $l_d$ center-to-center, in.
$n$ = number of bars or wires being developed along the plane of splitting

It shall be permitted to use $K_{tr} = 0$ as a design simplification even if transverse reinforcement is present.

The limit of 2.5 on $(c + K_{tr}) \cdot d_b$ is imposed to avoid pullout failure. With that term taken equal to its limit of 2.5, evaluation of Eq. (5.4) results in $l_d = 0.03 d_b f_y \cdot \sqrt{f_c'}$, the experimentally derived limit found in earlier ACI Codes when pullout failure controls. Note that in Eq. (5.4) and in all other ACI Code equations relating to the development length and splices of reinforcement, *values of* $\sqrt{f_c'}$ *are not to be taken greater than 100 psi* because of the lack of experimental evidence on bond strengths obtainable with concretes having compressive strength in excess of 10,000 psi at the time that Eqs. (5.3) and (5.4) were formulated. More recent tests with concrete with values of $f_c'$ to 16,000 psi justify this limitation.

## b. Simplified Equations for Development Length

Calculation of required development length (in terms of bar diameter) by Eq. (5.4) requires that the term $(c + K_{tr}) \cdot d_b$ be calculated for each particular combination of cover, spacing, and transverse reinforcement. Alternatively, according to the Code, a simplified form of Eq. (5.4) may be used in which $(c + K_{tr}) \cdot d_b$ is set equal to 1.5, provided that certain restrictions are placed on cover, spacing, and transverse reinforcement. Two cases of practical importance are:

(*a*) Minimum clear cover of $1.0d_b$, minimum clear spacing of $1.0d_b$, and at least the Code required minimum stirrups or ties (see Section 4.5b) throughout $l_d$
(*b*) Minimum clear cover of $1.0d_b$ and minimum clear spacing of $2d_b$

[†] ACI Committee 408 recommends a value of 1.0 for all bar sizes based on experimental evidence. The ACI Code value of 0.8, however, will be used in what follows.

**TABLE 5.1**
**Simplified tension development length in bar diameters according to the 2002 ACI Code**

| | No. 6 (No. 19) and smaller bars and deformed wires | No. 7 (No. 22) and larger bars |
|---|---|---|
| Clear spacing of bars being developed or spliced $\geq d_b$, clear cover $\geq d_b$, and stirrups or ties throughout $l_d$ not less than the Code minimum | $l_d = \cdot \frac{f_y \cdot\cdot\cdot}{25 \cdot \overline{f_c}} \cdot d_b$ | $l_d = \cdot \frac{f_y \cdot\cdot\cdot}{20 \cdot \overline{f_c}} \cdot d_b$ |
| Clear spacing of bars being developed or spliced $\geq 2d_b$, and clear cover $\geq d_b$ | Same as above | Same as above |
| Other cases | $l_d = \cdot \frac{3f_y \cdot\cdot\cdot}{50 \cdot \overline{f_c}} \cdot d_b$ | $l_d = \cdot \frac{3f_y \cdot\cdot\cdot}{40 \cdot \overline{f_c}} \cdot d_b$ |

[a] For reasons discussed in Section 5.3a, ACI Committee 408 recommends that $l_d$ for No. 7 (No. 22) and larger bars be used for all bar sizes.

For either of these common cases, it is easily confirmed from Eq. (5.4) that, for No. 7 (No. 22) and larger bars:

$$l_d = \cdot \frac{f_y \cdot\cdot\cdot}{20 \cdot \overline{f_c}} \cdot d_b \tag{5.5a}$$

and for No. 6 (No. 19) bars and smaller (with · = 0.8):

$$l_d = \cdot \frac{f_y \cdot\cdot\cdot}{25 \cdot \overline{f_c}} \cdot d_b \tag{5.5b}$$

If these restrictions on spacing are not met, then, provided that Code-imposed minimum spacing requirements are met (see Section 3.6c), the term $(c + K_{tr}) \cdot d_b$ will have a value not less than 1.0 (rather than 1.5 as before) whether or not transverse steel is used. The values given by Eqs. (5.5*a*) and (5.5*b*) are then multiplied by the factor 1.5·1.0.

Thus if the designer accepts certain restrictions on bar cover, spacing, and transverse reinforcement, simplified calculation of development requirements is possible. The simplified equations are summarized in Table 5.1.

Further simplification is possible for the most common condition of normal-density concrete and uncoated reinforcement. Then · and · in Table 5.1 take the value 1.0, and the development lengths, in terms of bar diameters, are simply a function of $f_y, f'_c$, and the bar location factor · . Thus development lengths are easily tabulated for the usual combinations of material strengths and bottom or top bars and for the restrictions on bar spacing, cover, and transverse steel defined.[†] Results are given in Table A.10 of Appendix A.

Regardless of whether development length is calculated using the basic Eq. (5.4) or the more approximate Eqs. (5.5*a*) and (5.5*b*), development length may be reduced

[†] Note that, for convenient reference, the term top bar is used for any horizontal reinforcing bar placed with more than 12 in. of fresh concrete cast below the development length or splice. This definition may require that bars relatively near the bottom of a deep member be treated as top bars.

where reinforcement in a flexural member is in excess of that required by analysis, except where anchorage or development for $f_y$ is specifically required or the reinforcement is designed for a region of high seismic risk. According to the ACI Code, the reduction is made according to the ratio ($A_s$ required· $A_s$ provided).

**EXAMPLE 5.1**

**Development length in tension.** Figure 5.8 shows a beam-column joint in a continuous building frame. Based on frame analysis, the negative steel required at the end of the beam is 2.90 in$^2$; two No. 11 (No. 36) bars are used, providing $A_s$ = 3.12 in$^2$. Beam dimensions are $b$ = 10 in., $d$ = 18 in., and $h$ = 21 in. The design will include No. 3 (No. 10) stirrups spaced four at 3 in., followed by a constant 5 in. spacing in the region of the support, with 1.5 in. clear cover. Normal-density concrete is to be used, with $f_c'$ = 4000 psi, and reinforcing bars have $f_y$ = 60,000 psi. Find the minimum distance $l_d$ at which the negative bars can be cut off, based on development of the required steel area at the face of the column (*a*) using the simplified equations of Table 5.1, (*b*) using Table A.10, of Appendix A, and (*c*) using the basic Eq. (5.4).

**Solution.** Checking for lateral spacing in the No. 11 (No. 36) bars determines that the clear distance between the bars is 10 − 2(1.50 + 0.38 + 1.41) = 3.42 in., or 2.43 times the bar diameter $d_b$. The clear cover of the No. 11 (No. 36) bars to the side face of the beam is 1.50 + 0.38 = 1.88 in., or 1.33 bar diameters, and that to the top of the beam is 3.00 − 1.41· 2 = 2.30 in., or 1.63 bar diameters. These dimensions meet the restrictions stated in the second row of Table 5.1. Then for top bars, uncoated, and with normal-density concrete, we have the values of · = 1.3, · = 1.0, and · = 1.0. From Table 5.1:

$$l_d = \frac{60{,}000 \times 1.3 \times 1.0 \times 1.0}{20 \cdot \overline{4000}} 1.41 = 62 \times 1.41 = 87 \text{ in.}$$

This can be reduced by the ratio of steel required to that provided, so that the final development length is 87 × 2.90· 3.12 = 81 in.

Alternatively, from the lower portion of Table A.10, $l_d \cdot d_b$ = 62. The required length to point of cutoff is 62 × 1.41 × 2.90· 3.12 = 81 in., as before.

The more accurate Eq. (5.4) will now be used. The center-to-center spacing of the No. 11 (No. 36) bars is 10 − 2(1.50 + 0.38 + 1.41· 2) = 4.83, one-half of which is 2.42 in. The side cover to bar centerline is 1.50 + 0.38 + 1.41· 2 = 2.59 in., and the top cover is 3.00 in. The smallest of these three distances controls, and $c$ = 2.42 in. Potential splitting would be

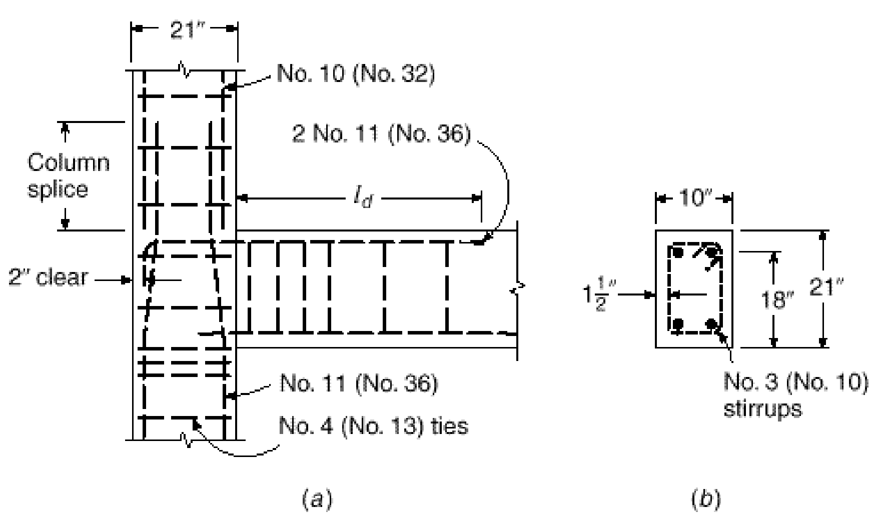



**FIGURE 5.8**
Bar details at beam-column joint for bar development examples.

in the horizontal plane of the bars, and in calculating $A_{tr}$ two times the stirrup bar area is used.[†] Based on the No. 3 (No. 10) stirrups at 5 in. spacing:

$$K_{tr} = \frac{0.11 \times 2 \times 60{,}000}{1500 \times 5 \times 2} = 0.88 \qquad \text{and} \qquad \frac{c + K_{tr}}{d_b} = \frac{2.42 + 0.88}{1.41} = 2.34$$

This is less than the limit value of 2.5. Then from Eq. (5.4):

$$l_d = \frac{3 \times 60{,}000 \times 1.3}{40 \cdot \sqrt{4000} \times 2.34} 1.41 = 40 \times 1.41 = 55.7 \text{ in.}$$

and the required development length is $55.7 \times 2.90 \cdot 3.12 = 52$ in. rather than 81 in. as before. Clearly, the use of the more accurate Eq. (5.4) permits a considerable reduction in development length. Even though its use requires much more time and effort, it is justified if the design is to be repeated many times in a structure.

## 5.4 Anchorage of Tension Bars by Hooks

### a. Standard Dimensions

In the event that the desired tensile stress in a bar cannot be developed by bond alone, it is necessary to provide special anchorage at the ends of the bar, usually by means of a 90° or a 180° hook. The dimensions and bend radii for such hooks have been standardized in ACI Code 7.1 as follows (see Fig. 5.9):

1. A 180° bend plus an extension of at least 4 bar diameters, but not less than $2\frac{1}{2}$ in. at the free end of the bar, or
2. A 90° bend plus an extension of at least 12 bar diameters at the free end of the bar, or
3. For stirrup and tie anchorage only:
   (*a*) For No. 5 (No. 16) bars and smaller, a 90° bend plus an extension of at least 6 bar diameters at the free end of the bar, or
   (*b*) For Nos. 6, 7, and 8 (Nos. 19, 22, and 25) bars, a 90° bend plus an extension of at least 12 bar diameters at the free end of the bar, or
   (*c*) For No. 8 (No. 25) bars and smaller, a 135° bend plus an extension of at least 6 bar diameters at the free end of the bar.

The minimum diameter of bend, measured on the inside of the bar, for standard hooks other than for stirrups or ties in sizes Nos. 3 through 5 (Nos. 10 through 16), should be not less than the values shown in Table 5.2. For stirrup and tie hooks, for bar sizes No. 5 (No. 16) and smaller, the inside diameter of bend should not be less than 4 bar diameters, according to the ACI Code.

When welded wire reinforcement (smooth or deformed wires) is used for stirrups or ties, the inside diameter of bend should not be less than 4 wire diameters for deformed wire larger than D6 and 2 wire diameters for all other wires. Bends with an inside diameter of less than 8 wire diameters should not be less than 4 wire diameters from the nearest welded intersection.

[†] If the top cover had controlled, the potential splitting plane would be vertical and one times the stirrup bar area would be used in calculating $A_{tr}$.

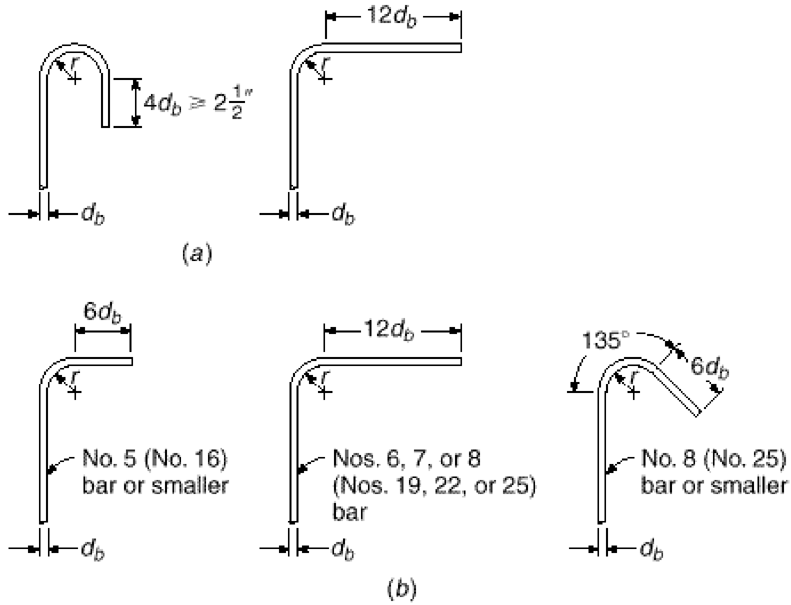



**FIGURE 5.9**
Standard bar hooks: (*a*) main reinforcement; (*b*) stirrups and ties.

**TABLE 5.2**
**Minimum diameters of bend for standard hooks**

| Bar Size | Minimum Diameter |
|---|---|
| Nos. 3 through 8 (Nos. 10 through 25) | 6 bar diameters |
| Nos. 9, 10, and 11 (Nos. 29, 32, and 36) | 8 bar diameters |
| Nos. 14 and 18 (Nos. 43 and 57) | 10 bar diameters |

## b. Development Length and Modification Factors for Hooked Bars

Hooked bars resist pullout by the combined actions of bond along the straight length of bar leading to the hook and anchorage provided by the hook. Tests indicate that the main cause of failure of hooked bars in tension is splitting of the concrete in the plane of the hook. This splitting is due to the very high stresses in the concrete inside of the hook; these stresses are influenced mainly by the bar diameter $d_b$ for a given tensile force, and the radius of bar bend. Resistance to splitting has been found to depend on the concrete cover for the hooked bar, measured laterally from the edge of the member to the bar perpendicular to the plane of the hook, and measured to the top (or bottom) of the member from the point where the hook starts, parallel to the plane of the hook. If these distances must be small, the strength of the anchorage can be substantially increased by providing confinement steel in the form of closed stirrups or ties.

ACI Code 12.5 provisions for hooked bars in tension are based on research summarized in Refs. 5.8 and 5.9. The Code requirements account for the combined contribution of bond along the straight bar leading to the hook, plus the hooked anchorage. A total development length $l_{dh}$ is defined as shown in Fig. 5.10 and is measured

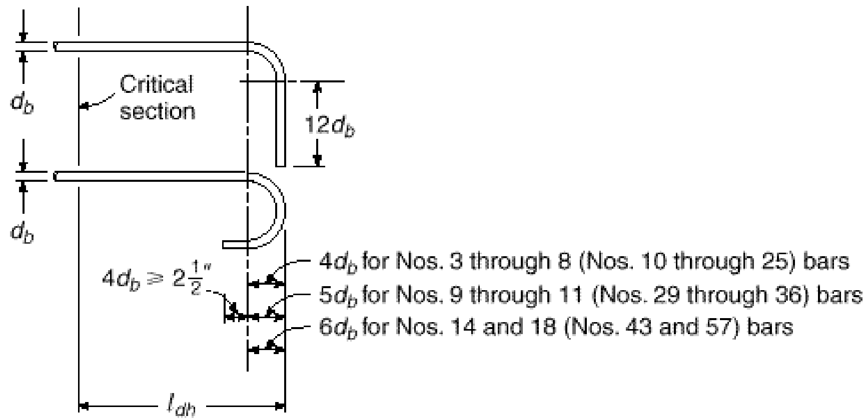



**FIGURE 5.10**
Bar details for development of standard hooks.

from the critical section to the farthest point on the bar, parallel to the straight part of the bar. For standard hooks, as shown in Fig. 5.9, the development length is

$$l_{dh} = \cdot \frac{0.02 \cdot \cdot f_y}{\cdot \overline{f_c'}} \cdot d_b \tag{5.6}$$

with · = 1.2 for epoxy-coated reinforcement and · = 1.3 for lightweight aggregate concrete. For other cases, · and · are taken as 1.0.

The development length $l_{dh}$ should be multiplied by certain applicable modifying factors, summarized in Table 5.3. These factors are combined as appropriate; e.g., if side cover of at least $2\frac{1}{2}$ in. is provided for a 180° hook, and if, in addition, ties are provided, the development length is multiplied by the product of 0.7 and 0.8. In any case, the length $l_{dh}$ is not to be less than 8 bar diameters and not less than 6 in.

Transverse confinement steel is essential if the full bar strength must be developed with minimum concrete confinement, such as when hooks may be required at the ends of a simply supported beam or where a beam in a continuous structure frames into an end column and does not extend past the column or when bars must be anchored in a short cantilever, as shown in Fig. 5.11 (Ref. 5.11). According to ACI Code 12.5.4, for bars hooked at the discontinuous ends of members with both side cover and top or bottom cover less than $2\frac{1}{2}$ in., hooks *must* be enclosed with closed stirrups or ties along the full development length, as shown in Fig. 5.11. The spacing of the confinement steel must not exceed 3 times the diameter of the hooked bar $d_b$, and the first stirrup or tie must enclose the bent portion of the hook within a distance equal to $2d_b$ of the outside of the bend. In such cases, the factor 0.8 of Table 5.3 does not apply.

## c. Mechanical Anchorage

For some special cases, e.g., at the ends of main flexural reinforcement in deep beams, there is not room for hooks or the necessary confinement steel, and special mechanical anchorage devices must be used. These may consist of welded plates, manufactured devices, or T-headed bars, the adequacy of which must be established by tests. Development of reinforcement, when such devices are employed, may consist of the combined contributions of bond along the length of the bar leading to the critical section, plus that of the mechanical anchorage; that is to say, the total resistance is the sum of the parts.

**TABLE 5.3**
**Development lengths for hooked deformed bars in tension**

| | |
|---|---|
| *A.* Development length $l_{dh}$ for hooked bars | $\cdot\ \frac{0.02\ \cdot\ \cdot\ f_y}{\cdot\ \sqrt{f'_c}}\ \cdot\ d_b$ |
| *B.* Modification factors applied to $l_{dh}$ | |
| For No. 11 (No. 36) and smaller bar hooks with side cover (normal to plane of hook) not less than $2\frac{1}{2}$ in., and for 90° hooks with cover on bar extension beyond hook not less than 2 in. | 0.7 |
| For 90° hooks of No. 11 (No. 36) and smaller bars that are either enclosed within ties or stirrups perpendicular to the bar being developed, spaced not greater than $3d_b$ along the development length $l_{dh}$ of the hook; or enclosed within ties or stirrups parallel to the bar being developed, spaced not greater than $3d_b$ along the length of the tail extension of the hook plus bend | 0.8 |
| For 180° hooks of No. 11 (No. 36) and smaller bars that are enclosed within ties or stirrups perpendicular to the bar being developed, spaced not greater than $3d_b$ along the development length $l_{dh}$ of the hook | 0.8 |
| Where anchorage or development for $f_y$ is not specifically required, reinforcement in excess of that required by analysis | $\frac{A_s \text{ required}}{A_s \text{ needed}}$ |
| · : | |
| For epoxy-coated bars | 1.2 |
| For other bars | 1.0 |
| · : | |
| For epoxy-coated bars | 1.3 |
| For normal-weight concrete | 1.0 |

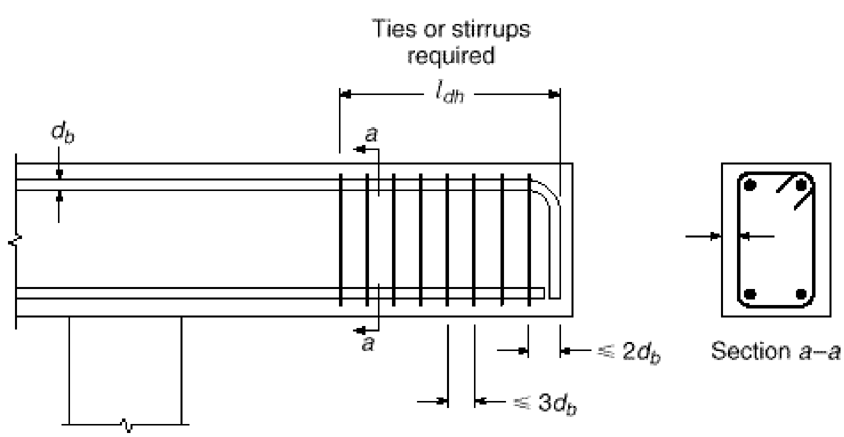



**FIGURE 5.11**
Transverse reinforcement requirements at discontinuous ends of members with small cover distances.

**EXAMPLE 5.2** **Development of hooked bars in tension.** Referring to the beam-column joint shown in Fig. 5.8, the No. 11 (No. 36) negative bars are to be extended into the column and terminated in a standard 90° hook, keeping 2 in. clear to the outside face of the column. The column width in the direction of beam width is 16 in. Find the minimum length of embedment of the hook past the column face, and specify the hook details.

**Solution.** The development length for hooked bars, measured from the critical section along the bar to the far side of the vertical hook, is given by Eq. (5.6):

$$l_{dh} = \frac{0.02 \times 60{,}000}{\sqrt{4000}} 1.41 = 27 \text{ in.}$$

In this case, side cover for the No. 11 (No. 36) bars exceeds 2.5 in. and cover beyond the bent bar is adequate, so a modifying factor of 0.7 can be applied. The only other factor applicable is for excess reinforcement, which is 0.93 as for Example 5.1. Accordingly, the minimum development length for the hooked bars is

$$l_{dh} = 27 \times 0.7 \times 0.93 = 18 \text{ in.}$$

With 21 − 2 = 19 in. available, the required length is contained within the column. The hook will be bent to a minimum diameter of 8 × 1.41 = 11.28 in. The bar will continue for 12 bar diameters, or 17 in. past the end of the bend in the vertical direction.

## 5.5 Anchorage Requirements for Web Reinforcement

Stirrups should be carried as close as possible to the compression and tension faces of a beam, and special attention must be given to proper anchorage. The truss model (see Section 4.8 and Fig. 4.19) for design of shear reinforcement indicates the development of diagonal compressive struts, the thrust from which is equilibrated, near the top and bottom of the beam, by the tension web members (i.e., the stirrups). Thus, at the factored load, the tensile strength of the stirrups must be developed for almost their full height. Clearly, it is impossible to do this by development length. For this reason, stirrups normally are provided with 90° or 135° hooks at their upper end (see Fig. 5.9*b* for standard hook details) and, at their lower end, are bent 90° to pass around the longitudinal reinforcement. In simple spans, or in the positive bending region of continuous spans, where no top bars are required for flexure, stirrup support bars must be used. These are usually about the same diameter as the stirrups themselves, and they not only provide improved anchorage of the hooks but also facilitate fabrication of the reinforcement cage, holding the stirrups in position during placement of the concrete.

ACI Code 12.13 includes special provisions for anchorage of web reinforcement. The ends of single-leg, simple-U, or multiple-U stirrups are to be anchored by one of the following means:

1. For No. 5 (No. 16) bars and smaller, and for Nos. 6, 7, and 8 (Nos. 19, 22, and 25) bars with $f_y$ of 40,000 psi or less, a standard hook around longitudinal reinforcement, as shown in Fig. 5.12*a*.
2. For Nos. 6, 7, and 8 (Nos. 19, 22, and 25) stirrups with $f_y$ greater than 40,000 psi, a standard hook around a longitudinal bar, plus an embedment between midheight of the member and the outside end of the hook equal to or greater than $0.014 d_b f_y / \sqrt{f'_c}$ in., as shown in Fig. 5.12*b*.

ACI Code 12.13 specifies further that, between anchored ends, each bend in the continuous portion of a simple-U or multiple-U stirrup shall enclose a longitudinal bar, as in Fig. 5.12*c*. Longitudinal bars bent to act as shear reinforcement, if extended into a region of tension, shall be continuous with longitudinal reinforcement and, if extended into a region of compression, shall be anchored beyond middepth $d/2$ as

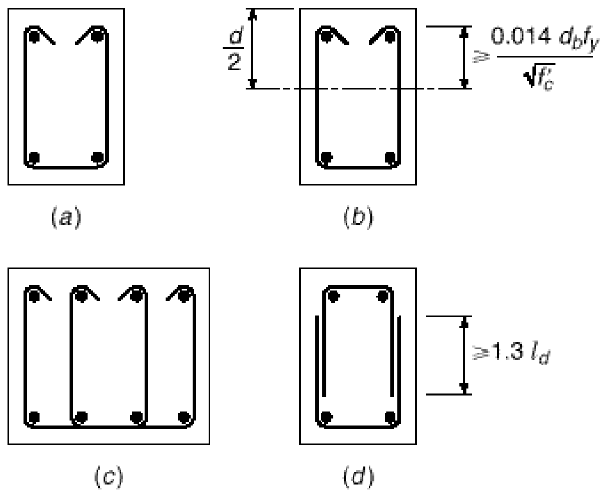



**FIGURE 5.12**
ACI requirements for stirrup anchorage: (*a*) No. 5 (No. 16) stirrups and smaller, and Nos. 6, 7, and 8 (Nos. 19, 22, and 25) stirrups with yield stress not exceeding 40,000 psi; (*b*) Nos. 6, 7, and 8 stirrups (Nos. 19, 22, and 25) with yield stress exceeding 40,000 psi; (*c*) wide beam with multiple-leg U stirrups; (*d*) pairs of U stirrups forming a closed unit. See Fig. 5.9 for alternative standard hook details.

specified for development length. Pairs of U-stirrups or ties so placed as to form a closed unit shall be considered properly spliced when length of laps are $1.3l_d$ as in Fig. 5.12*d*. In members at least 18 in. deep, such splices are considered adequate if the stirrup legs extend the full depth of the member.

Other provisions are contained in the ACI Code relating to the use of welded wire reinforcement, which is sometimes used for web reinforcement in precast and prestressed concrete beams.

## 5.6 Welded Wire Reinforcement

Tensile steel consisting of welded wire reinforcement (often referred to as welded wire fabric), with either deformed or smooth wires, is commonly used in one-way and two-way slabs and certain other types of members (see Section 2.15). For *deformed* wire reinforcement, some of the development is assigned to the welded cross wires and some to the embedded length of the deformed wire. According to ACI Code 12.7, the development length of welded deformed wire reinforcement measured from the point of the critical section to the end of the wire is computed as the product of the development length $l_d$ from Table 5.1 or from the more accurate Eq. (5.4) and the appropriate modification factor or factors related to those equations, except that the epoxy coating factor · is taken as 1.0 and the development length is not to be less than 8 in. Additionally, for welded deformed wire reinforcement with at least one cross wire within the development length and not less than 2 in. from the point of the critical section, a *wire fabric factor* equal to the greater of

$$\frac{f_y - 35{,}000}{f_y} \tag{5.7a}$$

or

$$\frac{5d_b}{s_w} \tag{5.7b}$$

can be applied, where $s_w$ is the lateral spacing of the wire being developed; but this factor need not exceed 1.0. For welded wire deformed reinforcement with no cross wires within the development length or with a single cross wire less than 2 in. from the point of the critical section, the wire fabric factor is taken to be equal to 1.0 and the development length determined as for the deformed wire.

For *smooth* welded wire reinforcement, development is considered to be provided by embedment of two cross wires, with the closer wire not less than 2 in. from the critical section. However, the development length measured from the critical section to the outermost cross wire is not to be less than

$$l_d = 0.27 \frac{A_w f_y}{s_w \sqrt{f_c'}} \tag{5.8}$$

according to ACI Code 12.8, where $A_w$ is the cross-sectional area of an individual wire to be developed or spliced. Modification factors pertaining to excess reinforcement and lightweight concrete may be applied, but $l_d$ is not to be less than 6 in. for the smooth welded wire reinforcement.†

## 5.7 Development of Bars in Compression

Reinforcement may be required to develop its compressive strength by embedment under various circumstances, e.g., where bars transfer their share of column loads to a supporting footing or where lap splices are made of compression bars in column (see Section 5.11). In the case of bars in compression, a part of the total force is transferred by bond along the embedded length, and a part is transferred by end bearing of the bars on the concrete. Because the surrounding concrete is relatively free of cracks and because of the beneficial effect of end bearing, shorter basic development lengths are permissible for compression bars than for tension bars. If transverse confinement steel is present, such as spiral column reinforcement or special spiral steel around an individual bar, the required development length is further reduced. Hooks such as are shown in Fig. 5.9 are *not* effective in transferring compression from bars to concrete, and, if present for other reasons, should be disregarded in determining required embedment length.

According to ACI Code 12.3, the development length in compression is the greater of

$$l_{dc} = \frac{0.02 f_y}{\sqrt{f_c'}} d_b \tag{5.9a}$$

$$l_{dc} = 0.0003 f_y d_b \tag{5.9b}$$

Modification factors summarized in part *B* of Table 5.4, as applicable, are applied to the development length in compression to obtain the value of development length $l_{dc}$ to be used in design. In no case is $l_d$ to be less than 8 in., according to the ACI Code. Basic and modified compressive development lengths are given in Table A.11 of Appendix A.

## 5.8 Bundled Bars

It was pointed out in Section 3.6c that it is sometimes advantageous to "bundle" tensile reinforcement in large beams, with two, three, or four bars in contact, to provide

† The ACI Code offers no explanation as to why $l_{d,min}$ = 6 in. for smooth wire fabric, but 8 in. for deformed welded wire reinforcement.

**TABLE 5.4**
**Development lengths for deformed bars in compression**

| | | |
|---|---|---|
| *A.* | Basic development length $l_{dc}$ | $\geq \dfrac{0.02 f_y}{\sqrt{f_c'}} d_b$ <br> $\geq 0.0003 f_y d_b$ |
| *B.* | Modification factors to be applied to $l_{dc}$ | |
| | Reinforcement in excess of that required by analysis | $\dfrac{A_s \text{ required}}{A_s \text{ provided}}$ |
| | Reinforcement enclosed within spiral reinforcement not less than $\frac{1}{4}$ in. diameter and not more than 4 in. pitch or within No. 4 (No. 13) ties spaced at not more than 4 in. on centers | 0.75 |

for improved placement of concrete around and between bundles of bars. Bar bundles are typically triangular or L shaped for three bars, and square for four. When bars are cut off in a bundled group, the cutoff points must be staggered at least 40 diameters. The development length of individual bars within a bundle, for both tension and compression, is that of the individual bar increased by 20 percent for a three-bar bundle and 33 percent for a four-bar bundle, to account for the probable deficiency of bond at the inside of the bar group.

## 5.9 Bar Cutoff and Bend Points in Beams

Chapter 3 dealt with moments, flexural stresses, concrete dimensions, and longitudinal bar areas at the critical moment sections of beams. These critical moment sections are generally at the face of the supports (negative bending) and near the middle of the span (positive bending). Occasionally, haunched members having variable depth or width are used so that the concrete flexural capacity will agree more closely with the variation of bending moment along a span or series of spans. Usually, however, prismatic beams with constant concrete cross-section dimensions are used to simplify formwork and thus to reduce cost.

The steel requirement, on the other hand, is easily varied in accordance with requirements for flexure, and it is common practice either to cut off bars where they are no longer needed to resist stress or, sometimes in the case of continuous beams, to bend up the bottom steel (usually at 45°) so that it provides tensile reinforcement at the top of the beam over the supports.

### a. Theoretical Points of Cutoff or Bend

The tensile force to be resisted by the reinforcement at any cross section is

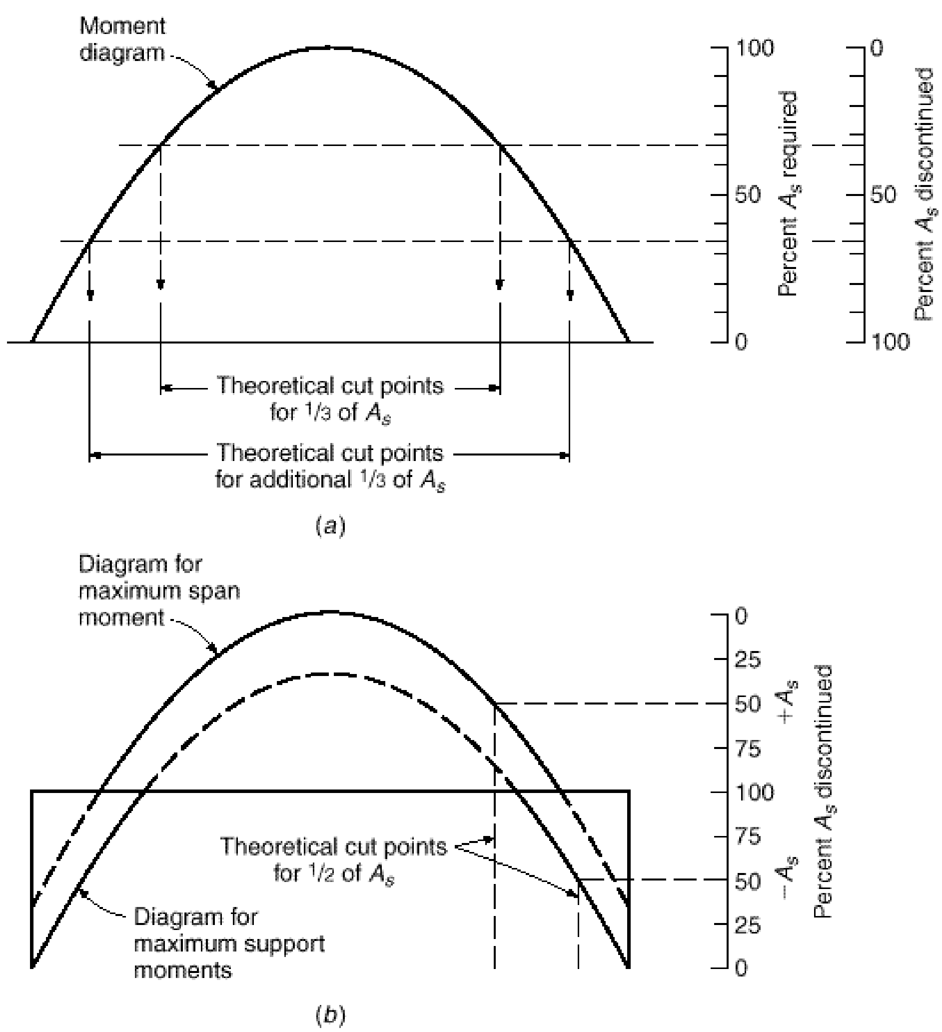
$$T = A_s f_s = \frac{M}{z}$$

where $M$ is the value of bending moment at that section and $z$ is the internal lever arm of the resisting moment. The lever arm $z$ varies only within narrow limits and is never less than the value at the maximum-moment section. Consequently, the tensile force can be taken with good accuracy directly proportional to the bending moment. Since it is desirable to design so that the steel everywhere in the beam is as nearly fully stressed as possible, it follows that the required steel area is very nearly proportional to the bending moment.

To illustrate, the moment diagram for a uniformly loaded simple-span beam shown in Fig. 5.13*a* can be used as a steel-requirement diagram. At the maximum-moment section, 100 percent of the tensile steel is required (0 percent can be discontinued or bent), while at the supports, 0 percent of the steel is theoretically required (100 percent can be discontinued or bent). The percentage of bars that could be discontinued elsewhere along the span is obtainable directly from the moment diagram, drawn to scale. To facilitate the determination of cutoff or bend points for simple spans, Graph A.2 of Appendix A has been prepared. It represents a half-moment diagram for a uniformly loaded simple span.

To determine cutoff or bend points for continuous beams, the moment diagrams resulting from loading for maximum span moment and maximum support moment are

**FIGURE 5.13**
Bar cutoff points from moment diagrams.



drawn. A moment envelope results that defines the range of values of moment at any section. Cutoff or bend points can be found from the appropriate moment curve as for simple spans. Figure 5.13*b* illustrates, for example, a continuous beam with moment envelope resulting from alternate loadings to produce maximum span and maximum support moments. The locations of the points at which 50 percent of the bottom and top steel may theoretically be discontinued are shown.

According to ACI Code 8.3, uniformly loaded, continuous reinforced concrete beams of fairly regular span may be designed using moment coefficients (see Table 12.1). These coefficients, analogous to the numerical constant in the expression $\frac{1}{8}wL^2$ for simple-beam bending moment, give a conservative approximation of span and support moments for continuous beams. When such coefficients are used in design, cutoff and bend points may conveniently be found from Graph A.3 of Appendix A. Moment curves corresponding to the various span and support-moment coefficients are given at the top and bottom of the chart, respectively.

Alternatively, if moments are found by frame analysis rather than from ACI moment coefficients, the location along the span where bending moment reduces to any particular value (e.g., as determined by the bar group after some bars are cut off), or to zero, is easily computed by statics.

## b. Practical Considerations and ACI Code Requirements

Actually, in no case should the tensile steel be discontinued exactly at the theoretically described points. As described in Section 4.4 and shown in Fig. 4.9, when diagonal tension cracks form, an internal redistribution of forces occurs in a beam. Prior to cracking, the steel tensile force at any point is proportional to the moment at a vertical section passing through the point. However, after the crack has formed, the tensile force in the steel at the crack is governed by the moment at a section nearer midspan, which may be much larger. Furthermore, the actual moment diagram may differ from that used as a design basis, due to approximation of the real loads, approximations in the analysis, or the superimposed effect of settlement or lateral loads. In recognition of these facts, ACI Code 12.10 requires that every bar should be continued at least a distance equal to the effective depth of the beam or 12 bar diameters (whichever is larger) beyond the point at which it is theoretically no longer required to resist stress.

In addition, it is necessary that the calculated stress in the steel at each section be developed by adequate embedded length or end anchorage, or a combination of the two. For the usual case, with no special end anchorage, this means that the full development length $l_d$ must be provided beyond critical sections at which peak stress exists in the bars. These critical sections are located at points of maximum moment and at points where adjacent terminated reinforcement is no longer needed to resist bending.[†]

Further reflecting the possible change in peak-stress location, ACI Code 12.11 requires that at least one-third of the positive-moment steel (one-fourth in continuous

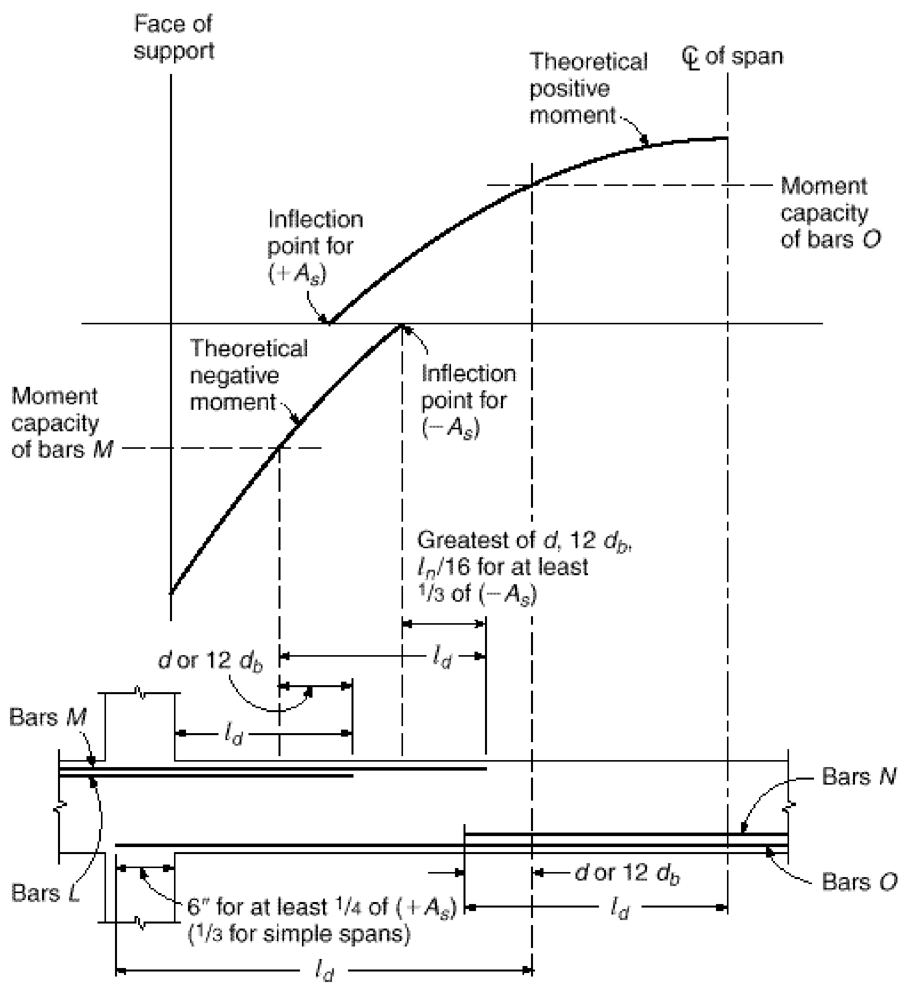
[†] The ACI Code is ambiguous as to whether or not the extension length $d$ or $12d_b$ is to be added to the required development length $l_d$. The Code Commentary presents the view that these requirements need not be superimposed, and Fig. 5.14 has been prepared on that basis. However, the argument just presented regarding possible shifts in moment curves or steel stress distribution curves leads to the conclusion that these requirements should be superimposed. In such cases, each bar should be continued a distance $l_d$ plus the greater of $d$ or $12d_b$ beyond the peak stress location.

spans) must be continued uninterrupted along the same face of the beam a distance at least 6 in. into the support. When a flexural member is a part of a primary lateral load resisting system, positive-moment reinforcement required to be extended into the support must be anchored to develop the yield strength of the bars at the face of support to account for the possibility of reversal of moment at the supports. According to ACI Code 12.12, at least one-third of the total reinforcement provided for negative moment at the support must be extended beyond the extreme position of the point of inflection a distance not less than one-sixteenth the clear span, or $d$, or $12d_b$, whichever is greatest.

Requirements for bar-cutoff or bend-point locations are summarized in Fig. 5.14. If negative bars $L$ are to be cut off, they must extend a full development length $l_d$ beyond the face of the support. In addition, they must extend a distance $d$ or $12d_b$ beyond the theoretical point of cutoff defined by the moment diagram. The remaining negative bars $M$ (at least one-third of the total negative area) must extend at least $l_d$ beyond the theoretical point of cutoff of bars $L$ and in addition must extend $d$, $12d_b$, or $l_n$· 16 (whichever is greatest) past the point of inflection of the negative-moment diagram.

If the positive bars $N$ are to be cut off, they must project $l_d$ past the point of theoretical maximum moment, as well as $d$ or $12d_b$ beyond the cutoff point from the positive-

**FIGURE 5.14**
Bar cutoff requirements of the ACI Code.



moment diagram. The remaining positive bars *O* must extend $l_d$ past the theoretical point of cutoff of bars *N* and must extend at least 6 in. into the face of the support.

When bars are cut off in a tension zone, there is a tendency toward the formation of premature flexural and diagonal tension cracks in the vicinity of the cut end. This may result in a reduction of shear capacity and a loss in overall ductility of the beam. ACI Code 12.10 requires special precautions, specifying that no flexural bar shall be terminated in a tension zone unless *one* of the following conditions is satisfied:

1. The shear is not over two-thirds of the design strength · $V_n$.
2. Stirrups in excess of those normally required are provided over a distance along each terminated bar from the point of cutoff equal to $\frac{3}{4}$ $d$. These "binder" stirrups shall provide an area $A_v \geq 60\ b_w s \cdot f_y$. In addition, the stirrup spacing shall not exceed $d \cdot 8 \cdot {}_b$, where $\cdot_b$ is the ratio of the area of bars cut off to the total area of bars at the section.
3. The continuing bars, if No. 11 (No. 36) or smaller, provide twice the area required for flexure at that point, and the shear does not exceed three-quarters of the design strength · $V_n$.

As an alternative to cutting off the steel, tension bars may be anchored by bending them across the web and making them continuous with the reinforcement on the opposite face. Although this leads to some complication in detailing and placing the steel, thus adding to construction cost, some engineers prefer the arrangement because added insurance is provided against the spread of diagonal tension cracks. In some cases, particularly for relatively deep beams in which a large percentage of the total bottom steel is to be bent, it may be impossible to locate the bend-up point for bottom bars far enough from the support for the same bars to meet the requirements for top steel. The theoretical points of bend should be checked carefully for both bottom and top steel.

Because the determination of cutoff or bend points may be rather tedious, particularly for frames that have been analyzed by elastic methods rather than by moment coefficients, many designers specify that bars be cut off or bent at more or less arbitrarily defined points that experience has proven to be safe. For nearly equal spans, uniformly loaded, in which not more than about one-half the tensile steel is to be cut off or bent, the locations shown in Fig. 5.15 are satisfactory. Note, in Fig. 5.15, that the beam at the exterior support at the left is shown to be simply supported. If the beam is monolithic with exterior columns or with a concrete wall at that end, details for a typical interior span could be used for the end span as well.

## c. Special Requirements near the Point of Zero Moment

While the basic requirement for flexural tensile reinforcement is that a full development length $l_d$ be provided beyond the point where the bar is assumed fully stressed to $f_y$, this requirement may *not* be sufficient to ensure safety against bond distress. Figure 5.16 shows the moment and shear diagram representative of a uniformly loaded continuous beam. Positive bars provided to resist the maximum moment at $c$ are required to have a full development length beyond the point $c$, measured in the direction of decreasing moment. Thus $l_d$ in the limiting case could be exactly equal to the distance from point $c$ to the point of inflection. However, if that requirement were exactly met, then at point $b$, halfway from $c$ to the point of inflection, those bars would

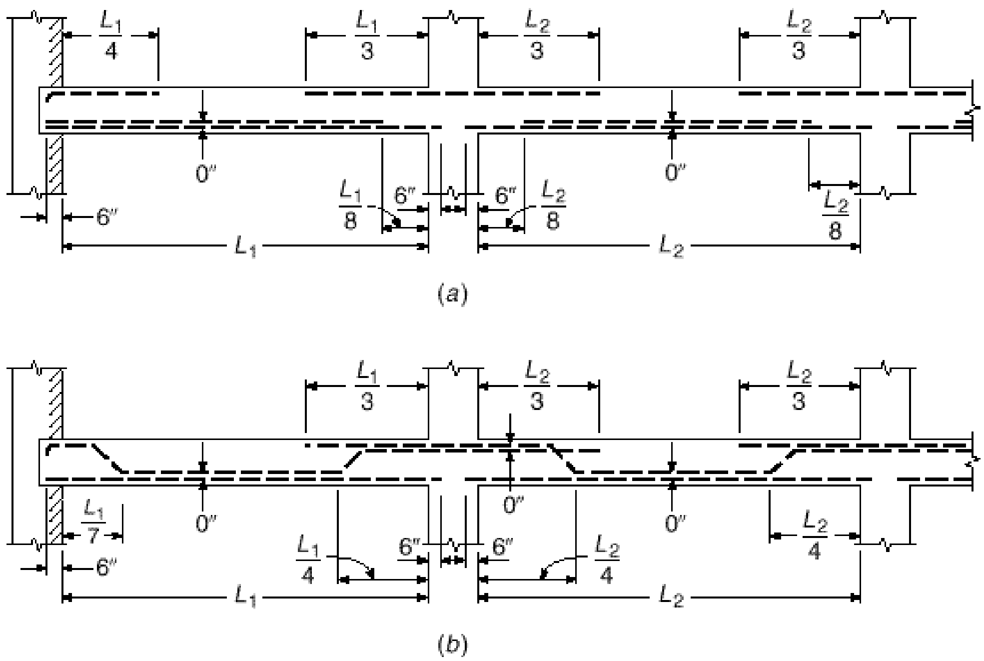



**FIGURE 5.15**
Cutoff or bend points for bars in approximately equal spans with uniformly distributed loads.

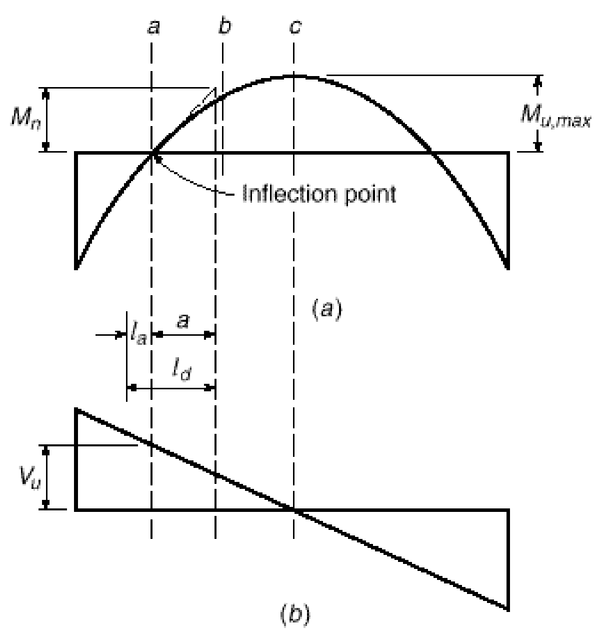



**FIGURE 5.16**
Development length requirement at point of inflection.

have only half their development length remaining, whereas the moment would be three-quarters of that at point $c$, and three-quarters of the bar force must yet be developed. This situation arises whenever the moments over the development length are greater than those corresponding to a linear reduction to zero. Therefore, the problem is a concern in the positive-moment region of continuous uniformly loaded spans, but not in the negative-moment region.

The bond force $U$ per unit length along the tensile reinforcement in a beam is $U = dT/dx$, where $dT$ is the change in bar tension in the length $dx$. Since $dT = dM/z$, this can be written

$$U = \frac{dM}{z\,dx} \qquad (a)$$

that is, the bond force per unit length of bar, generated by bending, is proportional to the slope of the moment diagram. In reference to Fig. 5.16*a*, the maximum bond force $U$ in the positive moment region would therefore be at the point of inflection, and $U$ would gradually diminish along the beam toward point $c$. Clearly, a conservative approach in evaluating adequacy in bond for those bars that are continued as far as the point of inflection (not necessarily the full $A_s$ provided for $M_u$ at point $c$) would be to require that the bond resistance, which is assumed to increase linearly along the bar from its end, would be governed by the maximum rate of moment increase, i.e., the maximum slope $dM/dx$ of the moment diagram, which for positive bending is seen to occur at the inflection point.

From elementary mechanics, it is known that the slope of the moment diagram at any point is equal to the value of the shear force at that point. Therefore, with reference to Fig. 5.16, the slope of the moment diagram at the point of inflection is $V_u$. A dashed line may therefore be drawn tangent to the moment curve at the point of inflection having the slope equal to the value of shear force $V_u$. Then if $M_n$ is the nominal flexural strength provided by those bars that extend to the point of inflection, and if the moment diagram were conservatively assumed to vary linearly along the dashed line tangent to the actual moment curve, from the basic relation that $M_n/a = V_u$, a distance $a$ is established:

$$a = \frac{M_n}{V_u} \qquad (b)$$

If the bars in question were fully stressed at a distance $a$ to the right of the point of inflection, and if the moments diminished linearly to the point of inflection, as suggested by the dashed line, then bond failure would not occur if the development length $l_d$ did not exceed the distance $a$. The actual moments are less than indicated by the dashed line, so the requirement is on the safe side.

If the bars extend past the point of inflection toward the support, as is always required, then the extension can be counted as contributing toward satisfying the requirement for embedded length. Arbitrarily, according to ACI Code 12.11, a length past the point of inflection not greater than the larger of the beam depth $d$ or 12 times the bar diameter $d_b$ may be counted toward satisfying the requirement. Thus, the requirement for tensile bars at the point of inflection is that

$$l_d \le \frac{M_n}{V_u} + l_a \qquad (5.10)$$

where $M_n$ = nominal flexural strength assuming all reinforcement at section to be stressed to $f_y$

$V_u$ = factored shear force at section

$l_a$ = embedded length of bar past point of zero moment, but not to exceed the greater of $d$ or $12d_b$

A corresponding situation occurs near the supports of simple spans carrying uniform loads, and similar requirements must be imposed. However, because of the beneficial effect of vertical compression in the concrete at the end of a simply supported span, which tends to prevent splitting and bond failure along the bars, the value $M_n/V_u$ may be increased 30 percent for such cases, according to ACI Code 12.11. Thus, at the ends of a simply supported span, the requirement for tension reinforcement is

$$l_d \leq 1.3 \frac{M_n}{V_u} + l_a \tag{5.11}$$

The consequence of these special requirements at the point of zero moment is that, in some cases, smaller bar sizes must be used to obtain smaller $l_d$, even though requirements for development past the point of maximum stress are met.

It may be evident from review of Sections 5.9b and 5.9c that the determination of cutoff or bend points in flexural members is complicated and can be extremely time-consuming in design. It is important to keep the matter in perspective and to recognize that the overall cost of construction will be increased very little if some bars are slightly longer than absolutely necessary, according to calculation, or as dictated by ACI Code provisions. In addition, simplicity in construction is a desired goal, and can, in itself, produce compensating cost savings. Accordingly, many engineers in practice continue *all* positive reinforcement into the face of the supports the required 6 in. and extend *all* negative reinforcement the required distance past the points of inflection, rather than using staggered cutoff points.

## d. Structural Integrity Provisions

Experience with structures that have been subjected to damage to a major supporting element, such as a column, owing to accident or abnormal loading has indicated that total collapse can be prevented through relatively minor changes in bar detailing. If some reinforcement, properly confined, is carried continuously through a support, then even if that support is damaged or destroyed, catenary action of the beams can prevent total collapse. In general, if beams have bottom and top steel meeting or exceeding the requirements summarized in Sections 5.9b and 5.9c, and if binding steel is provided in the form of properly detailed stirrups, then that catenary action can usually be ensured.

According to ACI Code 7.13.2, beams at the perimeter of the structure must have continuous reinforcement consisting of at least one-sixth of the tension reinforcement required for negative moment at the support, but not less than two bars, and at least one-quarter of the tension reinforcement required for positive moment at midspan, but not less than two bars. The continuous reinforcement must be enclosed by the corners of U stirrups having not less than 135° hooks around continuous top bars or by one-piece closed stirrups with not less than 135° hooks around one of the continuous top bars. Although spacing of such stirrups is not specified, the requirements for minimum shear steel given in Section 4.5b provide guidance in regions where shear does not require closer spacing. Stirrups need not be extended through the joints. The required continuity of longitudinal steel can be provided with top reinforcement spliced at midspan, and bottom reinforcement spliced at or near the supports (see Section 5.11a).

In other than perimeter beams, when stirrups as described in the preceding paragraph are not provided, at least one-quarter of the positive-moment reinforcement required at midspan, but not less than two bars, must be continuous or spliced over or near the support with a Class A tension splice, and at noncontinuous supports must be terminated with a standard hook.

Note that these provisions require very little additional steel in the structure. At least one-quarter of the bottom bars must be extended 6 in. into the support by other

ACI Code provisions; the structural integrity provisions merely require that these bars be made continuous or spliced. Similarly, other ACI Code provisions require that at least one-third of the negative bars be extended a certain minimum distance past the point of inflection; the structural integrity provisions for perimeter beams require only that half of those bars be further extended and spliced at midspan.

## 5.10 Integrated Beam Design Example

In this and in the preceding chapters, the several aspects of the design of reinforced concrete beams have been studied more or less separately: first the flexural design, then design for shear, and finally for bond and anchorage. The following example is presented to show how the various requirements for beams, which are often in some respects conflicting, are satisfied in the overall design of a representative member.

**EXAMPLE 5.3** **Integrated design of T beam.** A floor system consists of single span T beams 8 ft on centers, supported by 12 in. masonry walls spaced at 25 ft between inside faces. The general arrangement is shown in Fig. 5.17*a*. A 5 in. monolithic slab carries a uniformly distributed service live load of 165 psf. The T beams, in addition to the slab load and their own weight, must carry two 16,000 lb equipment loads applied over the stem of the T beam 3 ft from the span centerline as shown. A complete design is to be provided for the T beams, using concrete of 4000 psi strength and bars with 60,000 psi yield stress.

Solution. According to the ACI Code, the span length is to be taken as the clear span plus the beam depth, but need not exceed the distance between the centers of supports. The latter provision controls in this case, and the effective span is 26 ft. Estimating the beam web dimensions to be 12 × 24 in., the calculated and factored dead loads are

Slab:

$$\frac{5}{12} \times 150 \times 7 = 440 \text{ lb/ft}$$

Beam:

$$\frac{12 \times 24}{144} 150 = 300$$

$$w_d = 740 \text{ lb/ft}$$

$$1.2w_d = 890 \text{ lb/ft}$$

The uniformly distributed live load is

$$w_l = 165 \times 8 = 1320 \text{ lb/ft}$$

$$1.6w_l = 2110 \text{ lb/ft}$$

Live load overload factors are applied to the two concentrated loads to obtain $P_u = 16{,}000 \times 1.6 = 25{,}600$ lb. Factored loads are summarized in Fig. 5.17*b*.

In lieu of other controlling criteria, the beam web dimensions will be selected on the basis of shear. The left and right reactions under factored load are 25.6 + 3.00 × 13 = 64.6 kips. With the effective beam depth estimated to be 20 in., the maximum shear that need be considered in design is 64.6 − 3.00(0.50 + 1.67) = 58.1 kips. Although the ACI Code permits $V_s$ as high as $8\sqrt{f'_c}\, b_w d$, this would require very heavy web reinforcement. A lower limit of $4\sqrt{f'_c}\, b_w d$ will be adopted. With $V_c = 2\sqrt{f'_c}\, b_w d$ this results in a maximum $V_n = 6\sqrt{f'_c}\, b_w d$. Then $b_w d = V_u/(6\phi\sqrt{f'_c}) = 58{,}100/(6 \times 0.75\sqrt{4000}) = 204$ in$^2$. Cross-sectional dimensions $b_w = 12$ in. and $d = 18$ in. are selected, providing a total beam depth of 22 in. The assumed dead load of the beam need not be revised.

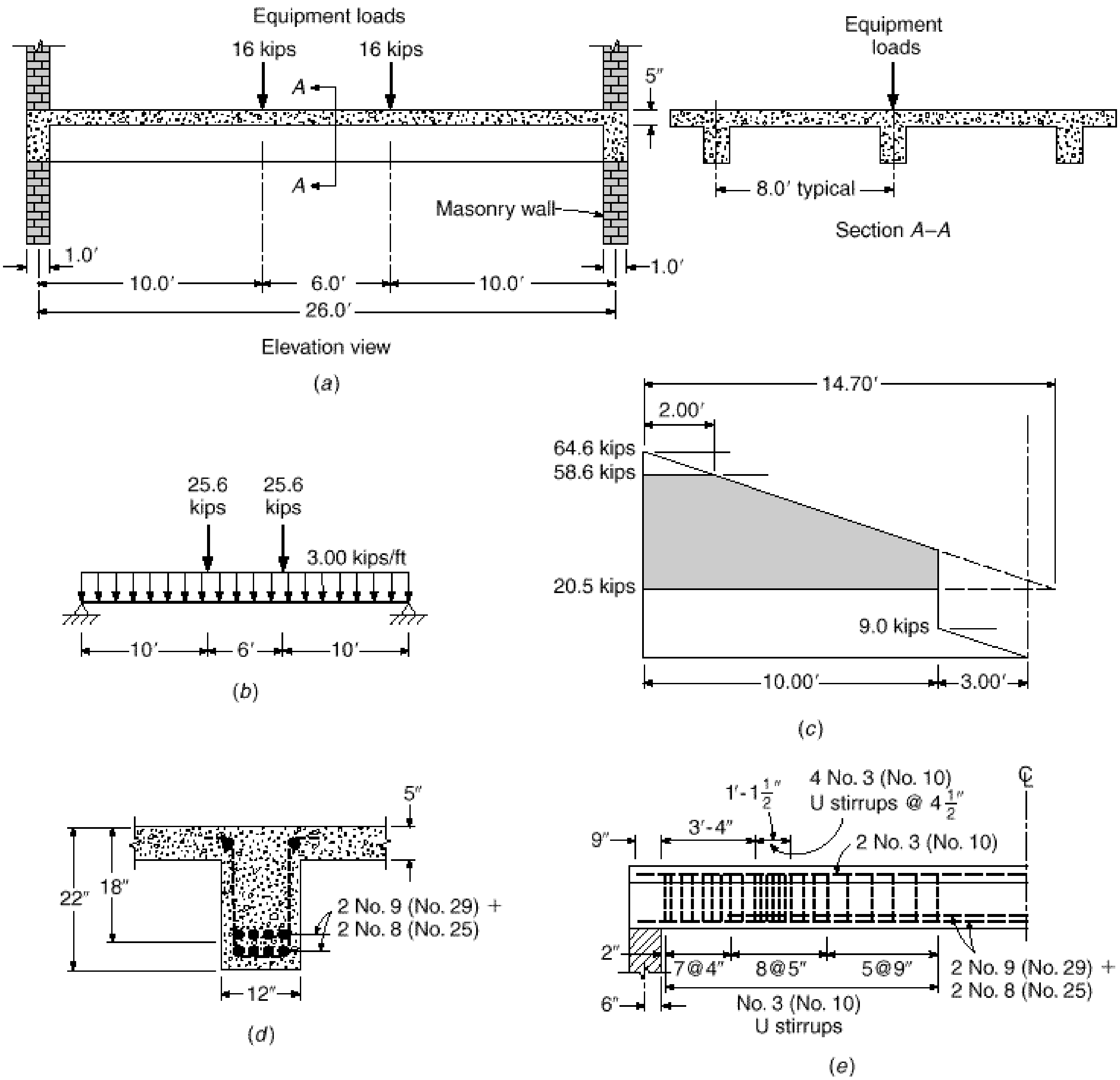



**FIGURE 5.17**
T beam design for Example 5.3.

According to the Code, the effective flange width $b$ is the smallest of the three quantities

$$\frac{L}{4} = \frac{26 \times 12}{4} = 78 \text{ in.}$$

$$16h_f + b_w = 80 + 12 = 92 \text{ in.}$$

$$\text{Centerline spacing} = 96 \text{ in.}$$

The first controls in this case. The maximum moment is at midspan, where

$$M_u = \frac{1}{8} \times 3.00 \times 26^2 + 25.6 \times 10 = 510 \text{ ft-kips}$$

Assuming for trial that the stress-block depth will equal the slab thickness leads to

$$A_s = \frac{M_u}{\phi f_y (d - a/2)} = \frac{510 \times 12}{0.90 \times 60 \times 15.5} = 7.31 \text{ in}^2$$

Then

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{7.31 \times 60}{0.85 \times 4 \times 78} = 1.65 \text{ in.}$$

The stress-block depth is seen to be less than the slab depth; rectangular beam equations are valid. An improved determination of $A_s$ is

$$A_s = \frac{510 \times 12}{0.90 \times 60 \times 17.11} = 6.60 \text{ in}^2$$

A check confirms that this is well below the maximum permitted reinforcement ratio. Four No. 9 (No. 29) plus four No. 8 (No. 25) bars will be used, providing a total area of 7.14 in$^2$. They will be arranged in two rows, as shown in Fig. 5.17*d*, with No. 9 (No. 29) bars at the outer end of each row. Beam width $b_w$ is adequate for this bar arrangement.

While the ACI Code permits discontinuation of two-thirds of the longitudinal reinforcement for simple spans, in the present case it is convenient to discontinue only the upper layer of steel, consisting of one-half of the total area. The moment capacity of the member after the upper layer of bars has been discontinued is then found:

$$a = \frac{3.57 \times 60}{0.85 \times 4 \times 78} = 0.81 \text{ in.}$$

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right) = 0.90 \times 3.57 \times 60 \times 18.66 \times \frac{1}{12} = 300 \text{ ft-kips}$$

For the present case, with a moment diagram resulting from combined distributed and concentrated loads, the point at which the applied moment is equal to this amount must be calculated. (In the case of uniformly loaded beams, Graphs A.2 and A.3 in Appendix A are helpful.) If $x$ is the distance from the support centerline to the point at which the moment is 300 ft-kips, then

$$64.6x - \frac{3.00x^2}{2} = 300$$

$$x = 5.30$$

The upper bars must be continued at least $d = 1.50$ ft or $12d_b = 1.13$ ft beyond this theoretical point of cutoff. In addition, the full development length $l_d$ must be provided past the maximum-moment section at which the stress in the bars to be cut is assumed to be $f_y$. Because of the heavy concentrated loads near the midspan, the point of peak stress will be assumed to be at the concentrated load rather than at midspan. For the four upper bars, assuming 1.50 in. clear cover to the outside of the No. 3 (No. 10) stirrups, the clear side cover is $1.50 + 0.38 = 1.88$ in., or $1.66d_b$. Assuming equal clear spacing between all four bars, that clear spacing is $[12.00 - 2 \times (1.50 + 0.38 + 1.13 + 1.00)]/3 = 1.33$ in., or $1.18d_b$. Noting that the ACI Code requirements for minimum stirrups are met, it is clear that all restrictions for the use of the simplified equation for development length are met. From Table 5.1 (Section 5.3), the required development length is

$$l_d = \frac{60{,}000}{20\sqrt{4000}} 1.13 = 47 \times 1.13 = 53 \text{ in.}$$

or 4.42 ft. Thus, the bars must be continued at least 3.00 + 4.42 = 7.42 ft past the midspan point, but in addition they must continue to a point 5.30 − 1.50 = 3.80 ft from the support centerline. The second requirement controls and the upper layer of the bars will be terminated, as shown in Fig. 5.17*e*, 3.30 ft from the support face. The bottom layer of bars will be extended to a point 3 in. from the end of the beam, providing 5.55 ft embedment past the critical section for cutoff of the upper bars. This exceeds the development length of the lower set of bars, confirming that cutoff and extension requirements are met.

Note that a simpler design, using very little extra steel, would result from extending all eight positive bars into the support. Whether or not the more elaborate calculations and more complicated placement are justified would depend largely on the number of repetitions of the design in the total structure.

Checking by Eq. (5.12) to ensure that the continued steel is of sufficiently small diameter determines that

$$l_d \leq 1.3\frac{333 \times 12}{64.6} + 3 = 83 \text{ in.}$$

The actual $l_d$ of 53 in. meets this restriction.

Since the cut bars are located in the tension zone, special binding stirrups will be used to control cracking; these will be selected after the normal shear reinforcement has been determined.

The shear diagram resulting from application of factored loads is shown in Fig. 5.17*c*. The shear contribution of the concrete is

$$\cdot V_c = 0.75 \times 2 \cdot \sqrt{4000} \times 12 \times 18 = 20{,}500 \text{ lb}$$

Thus web reinforcement must be provided for that part of the shear diagram shown shaded.

No. 3 (No. 10) stirrups will be selected. The maximum spacings must not exceed $d \cdot 2$ = 9 in., 24 in., or $A_v f_y \cdot (0.75 \cdot \sqrt{f_c'}\, b_w) = 0.22 \times 60{,}000 \cdot (0.75 \cdot \sqrt{4000} \times 12) = 23$ in. $\leq A_v f_y \cdot 50 b_w = 0.22 \times 60{,}000 \cdot 50 \times 12 = 22$ in. The first criterion controls here. For reference, from Eq. (4.14*a*) the hypothetical stirrup spacing at the support is

$$s_0 = \frac{0.75 \times 0.22 \times 60 \times 18}{64.6 - 20.5} = 4.04 \text{ in.}$$

and at 2 ft intervals along the span,

$$s_2 = 4.68 \text{ in.}$$
$$s_4 = 5.55 \text{ in.}$$
$$s_6 = 6.83 \text{ in.}$$
$$s_8 = 8.87 \text{ in.}$$
$$s_{10} = 12.64 \text{ in.}$$

The spacing need not be closer than that required 2.00 ft from the support centerline. In addition, stirrups are not required past the point of application of concentrated load, since beyond that point the shear is less than half of $\cdot V_c$. The final spacing of vertical stirrups selected is

1 space at 2 in. = 2 in.
7 spaces at 4 in. = 28 in.
8 spaces at 5 in. = 40 in.
5 spaces at 9 in. = 45 in.
Total = 115 in. = 9 ft 7 in. from the face of the support (121 in. = 10 ft 1 in. from the support centerline)

Two No. 3 (No. 10) longitudinal bars will be added to meet anchorage requirements and fix the top of the stirrups.

In addition to the shear reinforcement just specified, it is necessary to provide extra web reinforcement over a distance equal to $\frac{3}{4}\,d$, or 13.5 in., from the cut ends of the discontinued steel. The spacing of this extra web reinforcement must not exceed $d/8\beta_b = 18/(8 \times \frac{1}{2}) = 4.5$ in. In addition, the area of added steel within the distance $s$ must not be less than $60b_w s/f_y = 60 \times 12 \times 4.5/60{,}000 = 0.054$ in$^2$. For convenience, No. 3 (No. 10) stirrups will be used for this purpose also, providing an area of 0.22 in$^2$ in the distance $s$. The placement of the four extra stirrups is shown in Fig. 5.17*e*.

## 5.11 Bar Splices

In general, reinforcing bars are stocked by suppliers in lengths of 60 ft for bars from No. 5 to No. 18 (No. 16 to No. 57), and in 20 or 40 ft lengths for smaller sizes. For this reason, and because it is often more convenient to work with shorter bar lengths, it is frequently necessary to splice bars in the field. Splices in reinforcement at points of maximum stress should be avoided, and when splices are used they should be staggered, although neither condition is practical, for example, in compression splices in columns.

Splices for No. 11 (No. 36) bars and smaller are usually made simply by lapping the bars a sufficient distance to transfer stress by bond from one bar to the other. The lapped bars are usually placed in contact and lightly wired so that they stay in position as the concrete is placed. Alternatively, splicing may be accomplished by welding or by sleeves or mechanical devices. ACI Code 12.14.2 prohibits use of lapped splices for bars larger than No. 11 (No. 36), except that No. 14 and No. 18 (No. 43 and No. 57) bars may be lapped in compression with No. 11 (No. 36) and smaller bars per ACI Code 12.16.2 and 15.8.2.3. For bars that will carry only compression, it is possible to transfer load by end bearing of square cut ends, if the bars are accurately held in position by a sleeve or other device.

Lap splices of bars in bundles are based on the lap splice length required for individual bars within the bundle but must be increased in length by 20 percent for three-bar bundles and by 33 percent for four-bar bundles because of the reduced effective perimeter. Individual bar splices within a bundle should not overlap, and entire bundles must not be lap spliced.

According to ACI Code 12.14.3, welded splices must develop at least 125 percent of the specified yield strength of the bar. The same requirement applies to full mechanical connections. This ensures that an overloaded spliced bar would fail by ductile yielding in the region away from the splice, rather than at the splice where brittle failure is likely. Mechanical connections of No. 5 (No. 16) and smaller bars not meeting this requirement may be used at points of less than maximum stress, in accordance with ACI Code 12.15.4.

### a. Lap Splices in Tension

The required length of lap for tension splices is stated in terms of the development length $l_d$. In the process of calculating $l_d$, the usual modification factors are applied except that the reduction factor for excess reinforcement should not be applied because that factor is already accounted for in the splice specification.

Two different classifications of lap splices are established, corresponding to the minimum length of lap required: a Class A splice requires a lap of $1.0l_d$, and a class B splice requires a lap of $1.3l_d$. In either case, a minimum length of 12 in. applies. Lap splices, in general, must be class B splices, according to ACI Code 12.15.2, except that class A splices are allowed when the area of reinforcement provided is at least twice that required by analysis over the entire length of the splice *and* when one-half or less of the total reinforcement is spliced within the required lap length. The effect of these requirements is to encourage designers to locate splices away from regions of maximum stress, to a location where the actual steel area is at least twice that required by analysis, and to stagger splices.

Spiral reinforcement is spliced with a lap of $48d_b$ for uncoated bars and $72d_b$ for epoxy-coated bars, in accordance with ACI Code 7.10.4.5. The lap for epoxy-coated bars is reduced to $48d_b$ if the bars are anchored with a standard stirrup or tie hook.

## b. Compression Splices

Reinforcing bars in compression are spliced mainly in columns, where bars are most often terminated just above each floor or every other floor. This is done partly for construction convenience, to avoid handling and supporting very long column bars, but it is also done to permit column steel area to be reduced in steps, as loads become lighter at higher floors.

Compression bars may be spliced by lapping, by direct end bearing, or by welding or mechanical devices that provide positive connection. The minimum length of lap for compression splices is set according to ACI Code 12.16:

| | |
|---|---|
| For bars with $f_y \leq 60{,}000$ psi | $0.0005 f_y d_b$ |
| For bars with $f_y \cdot$ 60,000 psi | $\cdot 0.0009 f_y - 24 \cdot d_b$. |

but not less than 12 in. For $f'_c$ less than 3000 psi, the required lap is increased by one-third. When bars of different size are lap spliced in compression, the splice length is to be the larger of the development length of the larger bar and the splice length of the smaller bar. In exception to the usual restriction on lap splices for large diameter bars, No. 14 and No. 18 bars *may* be lap spliced to No. 11 and smaller bars.

Direct end bearing of the bars has been found by test and experience to be an effective means for transmitting compression. In such a case, the bars must be held in proper alignment by a suitable device. The bar ends must terminate in flat surfaces within 1.5° of a right angle, and the bars must be fitted within 3° of full bearing after assembly, according to ACI Code 12.16.4. Ties, closed stirrups, or spirals must be used.

## c. Column Splices

Lap splices, butt-welded splices, mechanical connections, or end-bearing splices may be used in columns, with certain restrictions. Reinforcing bars in columns may be subjected to compression or tension, or, for different load combinations, both tension and compression. Accordingly, column splices must conform in some cases to the requirements for compression splices only or tension splices only or to requirements for both. ACI Code 12.17 requires that a minimum tension capacity be provided in each face of all columns, even where analysis indicates compression only. Ordinary compressive lap splices provide sufficient tensile resistance, but end-bearing splices may require additional bars for tension, unless the splices are staggered.

For lap splices, where the bar stress due to factored loads is compression, column lap splices must conform to the requirements presented in Section 5.11b for compression splices. Where the stress is tension and does not exceed $0.5f_y$, lap splices must be Class B if more than half the bars are spliced at any section, or Class A if half or fewer are spliced and alternate lap splices are staggered by $l_d$. If the stress is tension and exceeds $0.5f_y$, then lap splices must be Class B, according to ACI Code.

If lateral ties are used throughout the splice length having an area of at least $0.0015hs$, where $s$ is the spacing of ties and $h$ is the overall thickness of the member, the required splice length may be multiplied by 0.83 but must not be less than 12 in. If spiral reinforcement confines the splice, the length required may be multiplied by 0.75 but again must not be less than 12 in.

End-bearing splices, as described above, may be used for column bars stressed in compression, if the splices are staggered or additional bars are provided at splice locations. The continuing bars in each face must have a tensile strength of not less than $0.25f_y$ times the area of reinforcement in that face.

As mentioned in Section 5.11b, column splices are commonly made just above a floor. However, for frames subjected to lateral loads, a better location is within the center half of the column height, where the moments due to lateral loads are much lower than at floor level. Such placement is mandatory for columns in "special moment frames" designed for seismic loads, as will be discussed in Chapter 20.

**EXAMPLE 5.4**

**Compression splice of column reinforcement.** In reference to Fig. 5.8, four No. 11 (No. 36) column bars from the floor below are to be lap spliced with four No. 10 (No. 32) column bars from above, and the splice is to be made just above a construction joint at floor level. The column, measuring 12 in. × 21 in. in cross section, will be subject to compression only for all load combinations. Transverse reinforcement consists of No. 4 (No. 13) ties at 16 in. spacing. All vertical bars may be assumed to be fully stressed. Calculate the required splice length. Material strengths are $f_y = 60{,}000$ psi and $f'_c = 4000$ psi.

**Solution.** The length of the splice must be the larger of the development length of the No. 11 (No. 36) bars and the splice length of the No. 10 (No. 32) bars. For the No. 11 (No. 36) bars, the development length is equal to the larger of the values obtained with Eqs. (5.9*a*) and (5.9*b*):

$$l_{dc} = \frac{0.02 \times 60{,}000}{\sqrt{4000}} 1.41 = 27 \text{ in.}$$

$$l_{dc} = 0.0003 \times 60{,}000 \times 1.41 = 25 \text{ in.}$$

The first criterion controls. No modification factors apply. For the No. 10 (No. 32) bars, the compression splice length is $0.0005 \times 60{,}000 \times 1.27 = 38$ in. In the check for use of the modification factor for tied columns, the critical column dimension is 21 in., and the required effective tie area is thus $0.0015 \times 21 \times 16 = 0.50$ in$^2$. The No. 4 (No. 13) ties provide an area of only $0.20 \times 2 = 0.40$ in$^2$, so the reduction factor of 0.83 cannot be applied to the splice length. Thus the compression splice length of 38 in., which exceeds the development length of 27 in. for the No. 11 (No. 36) bars, controls here, and a lap splice of 38 in. is required. Note that if the spacing of the ties at the splice were reduced to 12.8 in. or less (say 12 in.), the required lap would be reduced to $38 \times 0.83 = 32$ in. This would save steel, and, although placement cost would increase slightly, would probably represent the more economical design.

## References


5.1. R. M. Mains, "Measurement of the Distribution of Tensile and Bond Stresses along Reinforcing Bars," *J. ACI*, vol. 23, no. 3, 1951, pp. 225–252.

5.2. A. H. Nilson, "Internal Measurement of Bond Slip," *J. ACI*, vol. 69, no. 7, 1972, pp. 439–441.

5.3. Y. Goto, "Cracks Formed in Concrete around Deformed Tension Bars," *J. ACI*, vol. 68, no. 4, 1971, pp. 244–251.

5.4. L. A. Lutz and P. Gergely, "Mechanics of Bond and Slip of Deformed Bars in Concrete," *J. ACI*, vol. 64, no. 11, 1967, pp. 711–721.

5.5. P. M. Ferguson and J. N. Thompson, "Development Length of High Strength Reinforcing Bars in Bond," *J. ACI*, vol. 59, no. 7, 1962, pp. 887–922.

5.6. R. G. Mathey and D. Watstein, "Investigation of Bond in Beam and Pullout Specimens with High-Strength Reinforcing Bars," *J. ACI*, vol. 32, no. 9, 1961, pp. 1071–1090.

5.7. ACI Committee 408, "Bond Stress—The State of the Art," *J. ACI*, vol. 63, no. 11, 1966, pp. 1161–1190.

5.8. ACI Committee 408, "Suggested Development, Splice, and Standard Hook Provisions for Deformed Bars in Tension," *Concr. Intl.*, vol. 1, no. 7, 1979, pp. 44–46.

5.9. J. O. Jirsa, L. A. Lutz, and P. Gergely, "Rationale for Suggested Development, Splice, and Standard Hook Provisions for Deformed Bars in Tension," *Concr. Intl.*, vol. 1, no. 7, 1979, pp. 47–61.

5.10. C. O. Orangun, J. O. Jirsa, and J. E. Breen, "A Reevaluation of the Test Data on Development Length and Splices," *J. ACI*, vol. 74, no. 3, 1977, pp. 114–122.

5.11. L. A. Lutz, S. A. Mirza, and N. K. Gosain, "Changes to and Applications of Development and Lap Splice Length Provisions for Bars in Tension," *ACI Struct. J.*, vol. 90, no. 4, 1993, pp. 393–406.

5.12. D. Darwin, M. L. Tholen, E. K. Idun, and J. Zuo, "Splice Strength of High Relative Rib Area Reinforcing Bars," *ACI Struct. J.*, vol. 93, no. 1, 1996, pp. 95–107.

5.13. D. Darwin, J. Zuo, M. L. Tholen, and E. K. Idun, "Development Length Criteria for Conventional and High Relative Rib Area Reinforcing Bars," *ACI Struct. J.*, vol. 93, no. 3, 1996, pp. 347–359.

5.14. J. Zuo and D. Darwin, "Splice Strength of Conventional and High Relative Rib Area Bars in Normal and High Strength Concrete," *ACI Struct. J.*, vol. 97, no. 4, 2000, pp. 630–641.

5.15. ACI Committee 408, *Bond and Development of Straight Reinforcement in Tension*, ACI 408R-03, American Concrete Institute, Farmington Hills, MI, 2003.

5.16. P. M. Ferguson, "Small Bar Spacing or Cover—A Bond Problem for the Designer," *J. ACI*, vol. 74, no. 9, 1977, pp. 435–439.

5.17. P. R. Jeanty, D. Mitchell, and M. S. Mirza, "Investigation of Top Bar Effects in Beams," *ACI Struct. J.*, vol. 85, no. 3, 1988, pp. 251–257.

5.18. R. G. Mathey and J. R. Clifton, "Bond of Coated Reinforcing Bars in Concrete," *J. Struct. Div.*, ASCE, vol. 102, no. ST1, 1976, pp. 215–228.

5.19. R. A. Treece and J. O. Jirsa, "Bond Strength of Epoxy-Coated Reinforcing Bars," *ACI Matls. J.*, vol. 86, no. 2, 1989, pp. 167–174.

5.20. B. S. Hamad, J. O. Jirsa, and N. I. dePaulo, "Anchorage Strength of Epoxy-Coated Hooked Bars," *ACI Struct. J.*, vol. 90, no. 2, 1993, pp. 210–217.

5.21. H. H. Ghaffari, O. C. Choi, D. Darwin, and S. L. McCabe, "Bond of Epoxy-Coated Reinforcement: Cover, Casting Position, Slump, and Consolidation," *ACI Struct. J.*, vol. 91, no. 1, 1994, pp. 59–68.

5.22. C. J. Hester, S. Salamizavaregh, D. Darwin, and S. L. McCabe, "Bond of Epoxy-Coated Reinforcement: Splices," *ACI Struct. J.*, vol. 90, no. 1, 1993, pp. 89–102.

5.23. D. Darwin and J. Zuo, "Discussion of Proposed Changes to ACI 318 in *ACI 318-02 Discussion and Closure*," *Concr. Intl.*, vol. 24, no. 1, 2002, pp. 91, 93, 97–101.


## Problems

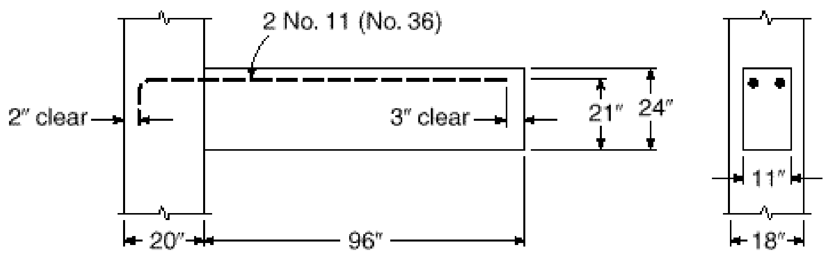
**5.1.** The short beam shown in Fig. P5.1 cantilevers from a supporting column at the left. It must carry a calculated dead load of 2.0 kips/ft including its own weight and a service live load of 3.0 kips/ft. Tensile flexural reinforcement consists of two No. 11 (No. 36) bars at a 21 in. effective depth. Transverse No. 3 (No. 10) U stirrups with 1.5 in. cover are provided at the following spacings from the face of the column: 4 in., 3 at 8 in., 5 at 10.5 in.

(*a*) If the flexural and shear steel use $f_y = 60{,}000$ psi and if the beam uses concrete having $f_c' = 3000$ psi, check to see if proper development length can be provided for the No. 11 (No. 36) bars. Use the simplified development length equations.

(*b*) Recalculate the required development length for the beam bars using the basic Eq. (5.4). Comment on your results.

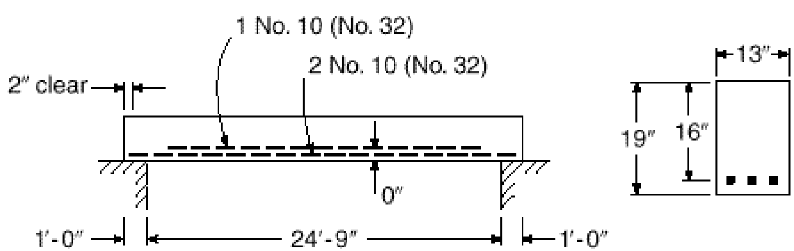
(*c*) If the column material strengths are $f_y$ = 60,000 psi and $f'_c$ = 5000 psi, check to see if adequate embedment can be provided within the column for the No. 11 (No. 36) bars. If hooks are required, specify detailed dimensions.

**FIGURE P5.1**




**5.2.** The beam shown in Fig. P5.2 is simply supported with a clear span of 24.75 ft and is to carry a distributed dead load of 0.72 kips/ft including its own weight, and live load of 1.08 kips/ft, unfactored, in service. The reinforcement consists of three No. 10 (No. 32) bars at a 16 in. effective depth, one of which is to be discontinued where no longer needed. Material strengths specified are $f_y$ = 60,000 psi and $f'_c$ = 4000 psi. No. 3 (No. 10) stirrups are used with a cover of 1.5 in. at spacing less than ACI Code maximum.

**FIGURE P5.2**




(*a*) Calculate the point where the center bar can be discontinued.

(*b*) Check to be sure that adequate embedded length is provided for continued and discontinued bars.

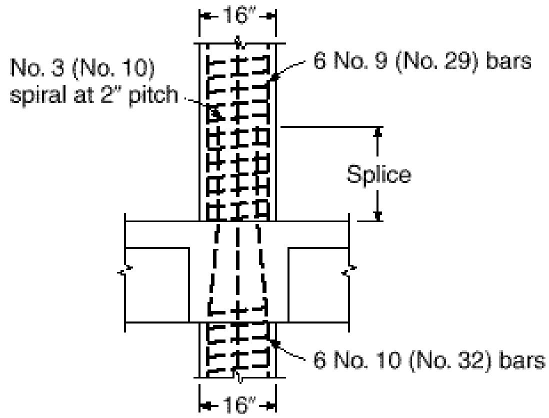
(*c*) Check special requirements at the support, where $M_u = 0$.

(*d*) If No. 3 (No. 10) bars are used for transverse reinforcement, specify special reinforcing details in the vicinity where the No. 10 (No. 32) bar is cut off.

(*e*) Comment on the practical aspects of the proposed design. Would you recommend cutting of the steel as suggested? Could two bars be discontinued rather than one?

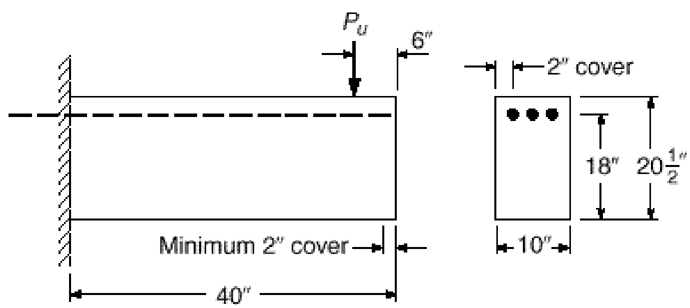
**5.3.** Figure P5.3 shows the column reinforcement for a 16 in. diameter concrete column, with $f_y$ = 60,000 psi and $f'_c$ = 5000 psi. Analysis of the building frame indicates a required $A_s$ = 7.10 in$^2$ in the lower column and 5.60 in$^2$ in the upper column. Spiral reinforcement consists of a $\frac{3}{8}$ in. diameter rod with a 2 in. pitch. Column bars are to be spliced just above the construction joint at the floor level, as shown in the sketch. Calculate the minimum permitted length of splice.

FIGURE P5.3




**5.4.** The short cantilever shown in Fig. P5.4 carries a heavy concentrated load 6 in. from its outer end. Flexural analysis indicates that three No. 8 (No. 25) bars are required, suitably anchored in the supporting wall and extending to a point no closer than 2 in. from the free end. The bars will be fully stressed to $f_y$ at the fixed support. Investigate the need for hooks and transverse confinement steel at the right end of the member. Material strengths are $f_y = 60{,}000$ psi and $f'_c = 4000$ psi. If hooks and transverse steel are required, show details in a sketch.

FIGURE P5.4




**5.5.** A continuous-strip wall footing is shown in cross section in Fig. P5.5. It is proposed that tensile reinforcement be provided using No. 8 (No. 25) bars at 16 in. spacing along the length of the wall, to provide a bar area of 0.59 in$^2$/ft. The bars have strength $f_y = 60{,}000$ psi and the footing concrete has $f'_c = 3000$ psi. The critical section for bending is assumed to be at the face of the supported wall, and the effective depth to the tensile steel is 12 in. Check to ensure that sufficient development length is available for the No. 8 (No. 25) bars, and if hooks are required, sketch details of the hooks giving dimensions.

*Note:* If hooks are required for the No. 8 (No. 25) bars, prepare an alternate design using bars having the same area per foot but of smaller diameter such that hooks could be eliminated; use the largest bar size possible to minimize the cost of steel placement.

Design and detail all splices, following ACI Code provisions. Splices will be staggered, with no more than four bars spliced at any section. Also, investigate the need for special anchorage at the outer ends of main reinforcement, and specify details of special anchorage if required. Material strengths are $f_y$ = 60,000 psi and $f_c'$ = 5000 psi.

FIGURE P5.7

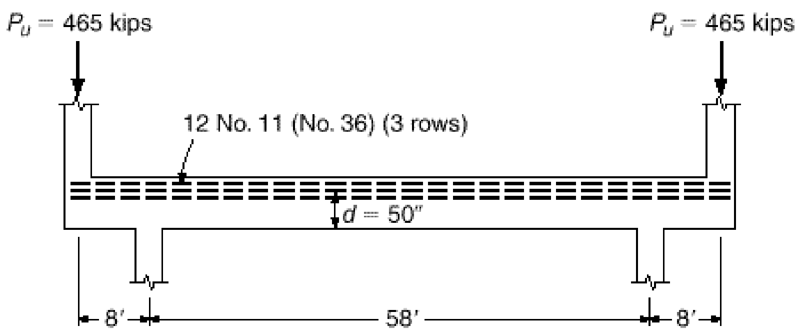
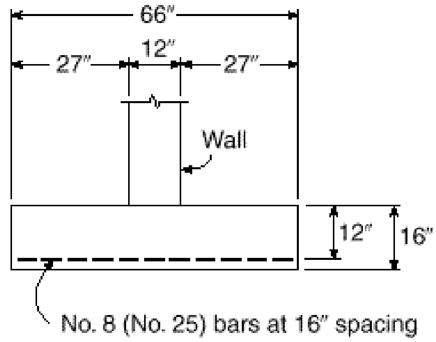
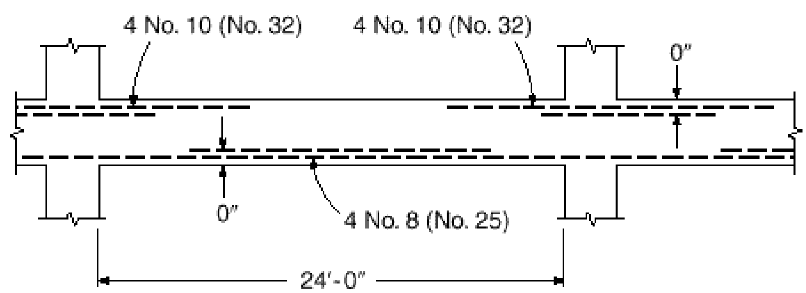


**FIGURE P5.5**




**5.6.** The continuous beam shown in Fig. P5.6 has been designed to carry a service dead load of 2.25 kips/ft including self-weight, and service live load of 3.25 kips/ft. Flexural design has been based on ACI moment coefficients of $\frac{1}{11}$ and $\frac{1}{16}$ at the face of support and midspan respectively, resulting in a concrete section with $b = 14$ in. and $d = 22$ in. Negative reinforcement at the support face is provided by four No. 10 (No. 32) bars, which will be cut off in pairs where no longer required by the ACI Code. Positive bars consist of four No. 8 (No. 25) bars, which will also be cut off in pairs. Specify the exact point of cutoff for all negative and positive steel. Specify also any supplementary web reinforcement that may be required. Check for satisfaction of ACI Code requirements at the point of inflection and suggest modifications of reinforcement if appropriate. Material strengths are $f_y = 60{,}000$ psi and $f_c' = 4000$ psi.

**FIGURE P5.6**




**5.7.** Figure P5.7 shows a deep transfer girder that carries two heavy column loads at its outer ends from a high-rise concrete building. Ground-floor columns must be offset 8 ft as shown. The loading produces an essentially constant moment (neglect self-weight of girder) calling for a concrete section with $b = 22$ in. and $b = 50$ in., with main tensile reinforcement at the top of the girder comprised of 12 No. 11 (No. 36) bars in three layers of four bars each. The maximum available bar length is 60 ft, so tensile splices must be provided.