8 SHORT COLUMNS

8.1 INTRODUCTION: AXIAL COMPRESSION

Columns are defined as members that carry loads chiefly in compression. Usually columns carry bending moments as well, about one or both axes of the cross section, and the bending action may produce tensile forces over a part of the cross section. Even in such cases, columns are generally referred to as compression members, because the compression forces dominate their behavior. In addition to the most common type of compression member, i.e., vertical elements in structures, compression members include arch ribs, rigid frame members inclined or otherwise, compression elements in trusses, shells, or portions thereof that carry axial compression, and other forms. In this chapter the term *column* will be used interchangeably with the term *compression member*, for brevity and in conformity with general usage.

Three types of reinforced concrete compression members are in use:

1. Members reinforced with longitudinal bars and lateral ties.
2. Members reinforced with longitudinal bars and continuous spirals.
3. Composite compression members reinforced longitudinally with structural steel shapes, pipe, or tubing, with or without additional longitudinal bars, and various types of lateral reinforcement.

Types 1 and 2 are by far the most common, and most of the discussion of this chapter will refer to them.

The main reinforcement in columns is longitudinal, parallel to the direction of the load, and consists of bars arranged in a square, rectangular, or circular pattern, as was shown in Fig. 1.15. Figure 8.1 shows an ironworker tightening splices for the main reinforcing steel during construction of the 60-story Bank of America Corporate Center in Charlotte, North Carolina. The ratio of longitudinal steel area A_{st} to gross concrete cross section A_g is in the range from 0.01 to 0.08, according to ACI Code 10.9.1. The lower limit is necessary to ensure resistance to bending moments not accounted for in the analysis and to reduce the effects of creep and shrinkage of the concrete under sustained compression. Ratios higher than 0.08 not only are uneconomical, but also would cause difficulty owing to congestion of the reinforcement, particularly where the steel must be spliced. Most columns are designed with ratios below 0.04. Larger-diameter bars are used to reduce placement costs and to avoid unnecessary congestion. The special large-diameter No. 14 and No. 18 (No. 43 and No. 57) bars are produced mainly for use in columns. According to ACI Code 10.9.2, a minimum of four longitudinal bars is required when the bars are enclosed by spaced rectangular or circular ties, and a minimum of six bars must be used when the longitudinal bars are enclosed by a continuous spiral.

FIGURE 8.1
Reinforcement for primary column of 60-story Bank of America Corporate Center in Charlotte, North Carolina. *(Courtesy of Walter P. Moore and Associates.)*

Columns may be divided into two broad categories: *short columns*, for which the strength is governed by the strength of the materials and the geometry of the cross section, and *slender columns*, for which the strength may be significantly reduced by lateral deflections. A number of years ago, an ACI-ASCE survey indicated that 90 percent of columns braced against sidesway and 40 percent of unbraced columns could be designed as short columns. Effective lateral bracing, which prevents relative lateral movement of the two ends of a column, is commonly provided by shear walls, elevator and stairwell shafts, diagonal bracing, or a combination of these. Although slender columns are more common now because of the wider use of high-strength materials and improved methods of dimensioning members, it is still true that most columns in ordinary practice can be considered short columns. Only short columns will be discussed in this chapter; the effects of slenderness in reducing column strength will be covered in Chapter 9.

The behavior of short, axially loaded compression members was discussed in Section 1.9 in introducing the basic aspects of reinforced concrete. It is suggested that the earlier material be reviewed at this point. In Section 1.9, it was demonstrated that, for lower loads for which both materials remain elastic, the steel carries a relatively small portion of the total load. The steel stress f_s is equal to n times the concrete stress:

$$f_s = nf_c \tag{8.1}$$

where $n = E_s/E_c$ is the modular ratio. In this range the axial load P is given by

$$P = f_c[A_g + (n - 1)A_{st}] \tag{8.2}$$

where the term in brackets is the area of the transformed section (see Fig. 1.17). Equations (8.2) and (8.1) can be used to find concrete and steel stresses respectively, for given loads, provided both materials remain elastic. Example 1.1 demonstrated the use of these equations.

In Section 1.9, it was further shown that the nominal strength of an axially loaded column can be found, recognizing the nonlinear response of both materials, by

$$P_n = 0.85f_c'A_c + A_{st}f_y \tag{8.3a}$$

or

$$P_n = 0.85f_c'(A_g - A_{st}) + A_{st}f_y \tag{8.3b}$$

i.e., by summing the strength contributions of the two components of the column. At this stage, the steel carries a significantly larger fraction of the load than was the case at lower total load.

The calculation of the nominal strength of an axially loaded column was demonstrated in Section 1.9.

According to ACI Code 10.3.6, the *design strength* of an axially loaded column is to be found based on Eq. (8.3*b*) with the introduction of certain strength reduction factors. The ACI factors are lower for columns than for beams, reflecting their greater importance in a structure. A beam failure would normally affect only a local region, whereas a column failure could result in the collapse of the entire structure. In addition, these factors reflect differences in the behavior of tied columns and spirally reinforced columns that will be discussed in Section 8.2. A basic ϕ factor of 0.70 is used for spirally reinforced columns, and 0.65 for tied columns, vs. $\phi = 0.90$ for most beams.

A further limitation on column strength is imposed by ACI Code 10.3.6 to allow for accidental eccentricities of loading not considered in the analysis. This is done by imposing an upper limit on the axial load that is less than the calculated design strength. This upper limit is taken as 0.85 times the design strength for spirally reinforced columns, and 0.80 times the calculated strength for tied columns. Thus, according to ACI Code 10.3.6, for spirally reinforced columns

$$\phi P_{n(max)} = 0.85\phi[0.85f_c'(A_g - A_{st}) + f_yA_{st}] \tag{8.4a}$$

with $\phi = 0.70$. For tied columns

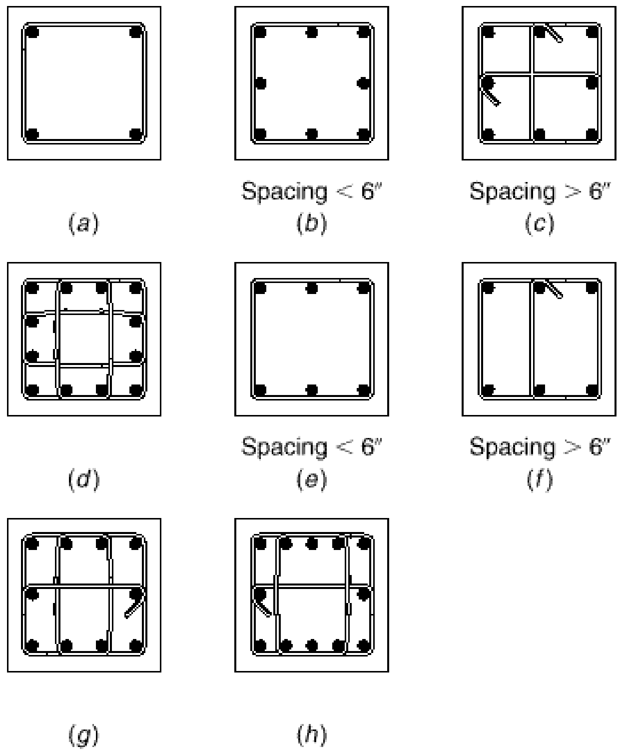
$$\phi P_{n(max)} = 0.80\phi[0.85f_c'(A_g - A_{st}) + f_yA_{st}] \tag{8.4b}$$

with $\phi = 0.65$.

8.2 Lateral Ties and Spirals

Figure 1.15 shows cross sections of the simplest types of columns, spirally reinforced or provided with lateral ties. Other cross sections frequently found in buildings and bridges are shown in Fig. 8.2. In general, in members with large axial forces and small moments, longitudinal bars are spaced more or less uniformly around the perimeter (Fig. 8.2*a* to *d*). When bending moments are large, much of the longitudinal steel is

FIGURE 8.2
Tie arrangements for square and rectangular columns.



concentrated at the faces of highest compression or tension, i.e., at maximum distances from the axis of bending (Fig. 8.2*e* to *h*). Specific recommended patterns for many combinations and arrangements of bars are found in Refs. 8.1 and 8.2. In heavily loaded columns with large steel percentages, the result of a large number of bars, each of them positioned and held individually by ties, is steel congestion in the forms and difficulties in placing the concrete. In such cases, bundled bars are frequently employed. Bundles consist of three or four bars tied in direct contact, wired, or otherwise fastened together. These are usually placed in the corners. Tests have shown that adequately bundled bars act as one unit; i.e., they are detailed as if a bundle constituted a single round bar of area equal to the sum of the bundled bars.

Lateral reinforcement, in the form of individual relatively widely spaced ties or a continuous closely spaced spiral, serves several functions. For one, such reinforcement is needed to hold the longitudinal bars in position in the forms while the concrete is being placed. For this purpose, longitudinal and transverse steel is wired together to form cages, which are then moved into the forms and properly positioned before placing the concrete. For another, transverse reinforcement is needed to prevent the highly stressed, slender longitudinal bars from buckling outward by bursting the thin concrete cover.

Closely spaced spirals serve these two functions. Ties, which can be arranged and spaced in various ways, must be so designed that these two requirements are met. This means that the spacing must be sufficiently small to prevent buckling between ties and that, in any tie plane, a sufficient number of ties must be provided to position and hold all bars. On the other hand, in columns with many longitudinal bars, if the column section is crisscrossed by too many ties, they interfere with the placement of concrete in the forms. To achieve adequate tying yet hold the number of ties to a minimum, ACI Code 7.10.5 gives the following rules for tie arrangement:

> All bars of tied columns shall be enclosed by *lateral ties*, at least No. 3 (No. 10) in size for longitudinal bars up to No. 10 (No. 32), and at least No. 4 (No. 13) in size for Nos. 11, 14, and 18 (Nos. 36, 43, and 57) and bundled longitudinal bars. The spacing of the ties shall not exceed 16 diameters of longitudinal bars, 48 diameters of tie bars, nor the least dimension of the column. The ties shall be so arranged that every corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie having an included angle of not more than 135°, and no bar shall be farther than 6 in. clear on either side from such a laterally supported bar. Deformed wire or welded wire fabric of equivalent area may be used instead of ties. Where the bars are located around the periphery of a circle, complete circular ties may be used.

For spirally reinforced columns ACI Code 7.10.4 requirements for lateral reinforcement may be summarized as follows:

> Spirals shall consist of a continuous bar or wire not less than $\frac{3}{8}$ in. in diameter, and the clear spacing between turns of the spiral must not exceed 3 in. nor be less than 1 in.

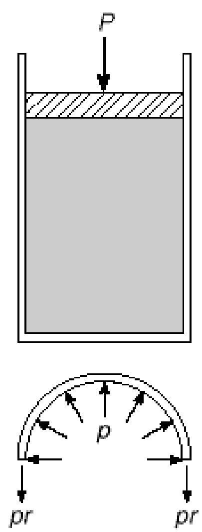


FIGURE 8.3
Model for action of a spiral.

In addition, a minimum ratio of spiral steel is imposed such that the structural performance of the column is significantly improved, with respect to both ultimate load and the type of failure, compared with an otherwise identical tied column.

The structural effect of a spiral is easily visualized by considering as a model a steel drum filled with sand (Fig. 8.3). When a load is placed on the sand, a lateral pressure is exerted by the sand on the drum, which causes hoop tension in the steel wall. The load on the sand can be increased until the hoop tension becomes large enough to burst the drum. The sand pile alone, if not confined in the drum, would have been able to support hardly any load. A cylindrical concrete column, to be sure, does have a definite strength without any lateral confinement. As it is being loaded, it shortens longitudinally and expands laterally, depending on Poisson's ratio. A closely spaced spiral confining the column counteracts the expansion, as did the steel drum in the model. This causes hoop tension in the spiral, while the carrying capacity of the confined concrete in the core is greatly increased. Failure occurs only when the spiral steel yields, which greatly reduces its confining effect, or when it fractures.

A tied column fails at the load given by Eq. (8.3*a* or *b*). At this load the concrete fails by crushing and shearing outward along inclined planes, and the longitudinal steel by buckling outward between ties (Fig. 8.4). In a spirally reinforced column, when the same load is reached, the longitudinal steel and the concrete within the core are prevented from moving outward by the spiral. The concrete in the outer shell, however, not being so confined, does fail; i.e., the outer shell spalls off when the load P_n is reached. It is at this stage that the confining action of the spiral has a significant effect, and if sizable spiral steel is provided, the load that will ultimately fail the column by causing the spiral steel to yield or fracture can be much larger than that at which the shell spalled off. Furthermore, the axial strain limit when the column fails will be much greater than otherwise; the toughness of the column has been much increased.

In contrast to the practice in some foreign countries, it is reasoned in the United States that any excess capacity beyond the spalling load of the shell is wasted because the member, although not actually failed, would no longer be considered serviceable. For this reason, the ACI Code provides a minimum spiral reinforcement of such an

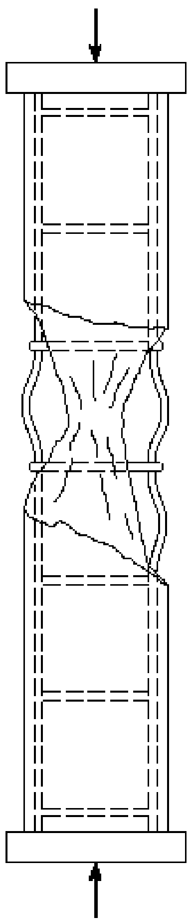


FIGURE 8.4
Failure of a tied column.

amount that its contribution to the carrying capacity is just slightly larger than that of the concrete in the shell. The situation is best understood from Fig. 8.5, which compares the performance of a tied column with that of a spiral column whose spalling load is equal to the ultimate load of the tied column. The failure of the tied column is abrupt and complete. This is true, to almost the same degree, of a spiral column with a spiral so light that its strength contribution is considerably less than the strength lost in the spalled shell. With a heavy spiral the reverse is true, and with considerable prior deformation the spalled column would fail at a higher load. The "ACI spiral," its strength contribution about compensating for that lost in the spalled shell, hardly increases the ultimate load. However, by preventing instantaneous crushing of concrete and buckling of steel, it produces a more gradual and ductile failure, i.e., a tougher column.

It has been found experimentally (Refs. 8.3 to 8.5) that the increase in compressive strength of the core concrete in a column provided through the confining effect of spiral steel is closely represented by the equation

$$f_c^* - 0.85 f_c' = 4.0 f_2' \qquad (a)$$

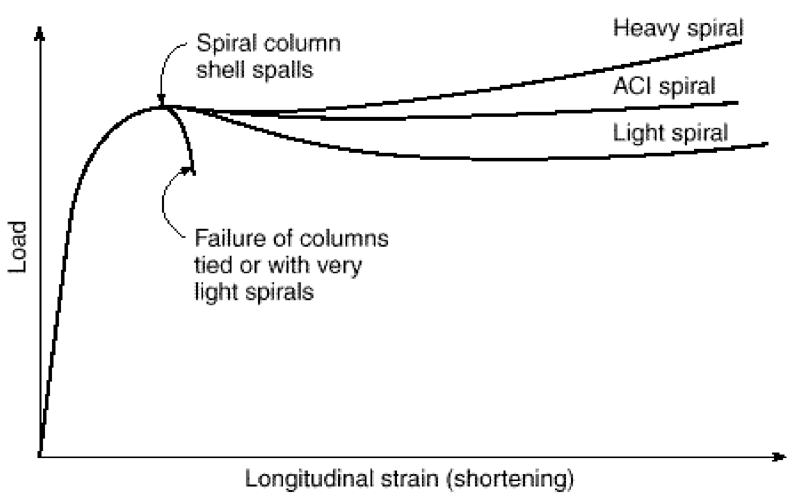
where f_c^* = compressive strength of spirally confined core concrete
$0.85 f_c'$ = compressive strength of concrete if unconfined
f_2' = lateral confinement stress in core concrete produced by spiral

The confinement stress f_2' is calculated assuming that the spiral steel reaches its yield stress f_y when the column eventually fails. With reference to Fig. 8.6, a hoop tension analysis of an idealized model of a short segment of column confined by one turn of lateral steel shows that

$$f_2' = \frac{2A_{sp}f_y}{d_c s} \qquad (b)$$

where A_{sp} = cross-sectional area of spiral wire
f_y = yield strength of spiral steel
d_c = outside diameter of spiral
s = spacing or pitch of spiral wire

FIGURE 8.5
Behavior of spirally reinforced and tied columns.



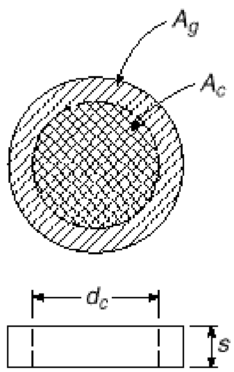


FIGURE 8.6
Confinement of core concrete due to hoop tension.

A volumetric ratio is defined as the ratio of the volume of spiral steel to the volume of core concrete:

$$\rho_s = \frac{2\pi d_c A_{sp}}{2} \frac{4}{\pi d_c^2 s}$$

from which

$$A_{sp} = \frac{\rho_s d_c s}{4} \tag{c}$$

Substituting the value of A_{sp} from Eq. (*c*) into Eq. (*b*) results in

$$f_2 = \frac{\rho_s f_y}{2} \tag{d}$$

To find the right amount of spiral steel one calculates

$$\text{Strength contribution of the shell} = 0.85 f'_c (A_g - A_c) \tag{e}$$

where A_g and A_c are, respectively, the gross and core concrete areas. Then substituting the confinement stress from Eq. (*d*) into Eq. (*a*) and multiplying by the core concrete area,

$$\text{Strength provided by the spiral} = 2\rho_s f_y A_c \tag{f}$$

The basis for the design of the spiral is that the strength gain provided by the spiral should be at least equal to that lost when the shell spalls, so combining Eqs. (*e*) and (*f*),

$$0.85 f'_c (A_g - A_c) = 2\rho_s f_y A_c$$

from which

$$\rho_s = 0.425 \left(\frac{A_g}{A_c} - 1 \right) \frac{f'_c}{f_y} \tag{g}$$

According to the ACI Code, this result is rounded upward slightly, and ACI Code 10.9.3 states that the ratio of spiral reinforcement shall not be less than

$$\rho_s = 0.45 \left(\frac{A_g}{A_c} - 1 \right) \frac{f'_c}{f_y} \tag{8.5}$$

It is further stipulated in the ACI Code that f_y must not be taken greater than 60,000 psi.

It follows from this development that two concentrically loaded columns designed to the ACI Code, one tied and one with spiral but otherwise identical, will fail at about the same load, the former in a sudden and brittle manner, the latter gradually with prior spalling of the shell and with more ductile behavior. This advantage of the spiral column is much less pronounced if the load is applied with significant eccentricity or when bending from other sources is present simultaneously with axial load. For this reason, while the ACI Code permits somewhat larger design loads on spiral than on tied columns when the moments are small or zero ($\phi = 0.70$ for spirally reinforced columns vs. $\phi = 0.65$ for tied), the difference is not large, and it is even further reduced for large eccentricities, for which ϕ approaches 0.90 for both.

The design of spiral reinforcement according to the ACI Code provisions is easily reduced to tabular form, as in Table A.14 of Appendix A.

8.3 Compression Plus Bending of Rectangular Columns

Members that are axially, i.e., concentrically, compressed occur rarely, if ever, in buildings and other structures. Components such as columns and arches chiefly carry loads in compression, but simultaneous bending is almost always present. Bending moments are caused by continuity, i.e., by the fact that building columns are parts of monolithic frames in which the support moments of the girders are partly resisted by the abutting columns, by transverse loads such as wind forces, by loads carried eccentrically on column brackets, or in arches when the arch axis does not coincide with the pressure line. Even when design calculations show a member to be loaded purely axially, inevitable imperfections of construction will introduce eccentricities and consequent bending in the member as built. For this reason members that must be designed for simultaneous compression and bending are very frequent in almost all types of concrete structures.

When a member is subjected to combined axial compression P and moment M, such as in Fig. 8.7*a*, it is usually convenient to replace the axial load and moment with an equal load P applied at eccentricity $e = M \cdot P$, as in Fig. 8.7*b*. The two loadings are statically equivalent. All columns may then be classified in terms of the equivalent eccentricity. Those having relatively small e are generally characterized by compression over the entire concrete section, and if overloaded will fail by crushing of the concrete accompanied by yielding of the steel in compression on the more heavily loaded side. Columns with large eccentricity are subject to tension over at least a part of the section, and if overloaded may fail due to tensile yielding of the steel on the side farthest from the load.

For columns, load stages below the ultimate are generally not important. Cracking of concrete, even for columns with large eccentricity, is usually not a serious problem, and lateral deflections at service load levels are seldom, if ever, a factor. Design of columns is therefore based on the factored load, which must not exceed the design strength, as usual, i.e.,

$$\cdot M_n \geq M_u \tag{8.6a}$$

$$\cdot P_n \geq P_u \tag{8.6b}$$

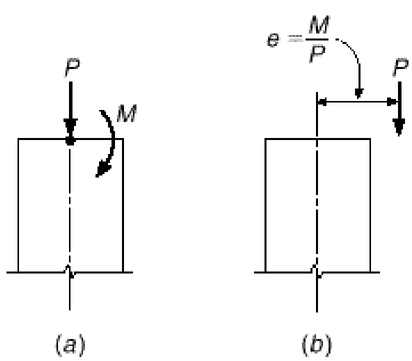


FIGURE 8.7
Equivalent eccentricity of column load.

8.4 Strain Compatibility Analysis and Interaction Diagrams

Figure 8.8*a* shows a member loaded parallel to its axis by a compressive force P_n at an eccentricity *e* measured from the centerline. The distribution of strains at a section *a*-*a* along its length, at incipient failure, is shown in Fig. 8.8*b*. With plane sections assumed to remain plane, concrete strains vary linearly with distance from the neutral axis, which is located a distance *c* from the more heavily loaded side of the member. With full compatibility of deformations, the steel strains at any location are the same as the strains in the adjacent concrete; thus, if the ultimate concrete strain is ϵ_u, the strain in the bars nearest the load is ϵ'_s, while that in the tension bars at the far side is ϵ_s. Compression steel with area A'_s and tension steel with area A_s are located at distances d' and d, respectively, from the compression face.

The corresponding stresses and forces are shown in Fig. 8.8*c*. Just as for simple bending, the actual concrete compressive stress distribution is replaced by an equivalent rectangular distribution having depth $a = \beta_1 c$. A large number of tests on columns with a variety of shapes has shown that the strengths computed on this basis are in satisfactory agreement with test results (Ref. 8.6).

Equilibrium between external and internal axial forces shown in Fig. 8.8*c* requires that

$$P_n = 0.85 f'_c ab + A'_s f'_s - A_s f_s \tag{8.7}$$

Also, the moment about the centerline of the section of the internal stresses and forces must be equal and opposite to the moment of the external force P_n, so that

$$M_n = P_n e = 0.85 f'_c ab \left(\frac{h}{2} - \frac{a}{2}\right) + A'_s f'_s \left(\frac{h}{2} - d'\right) + A_s f_s \left(d - \frac{h}{2}\right) \tag{8.8}$$

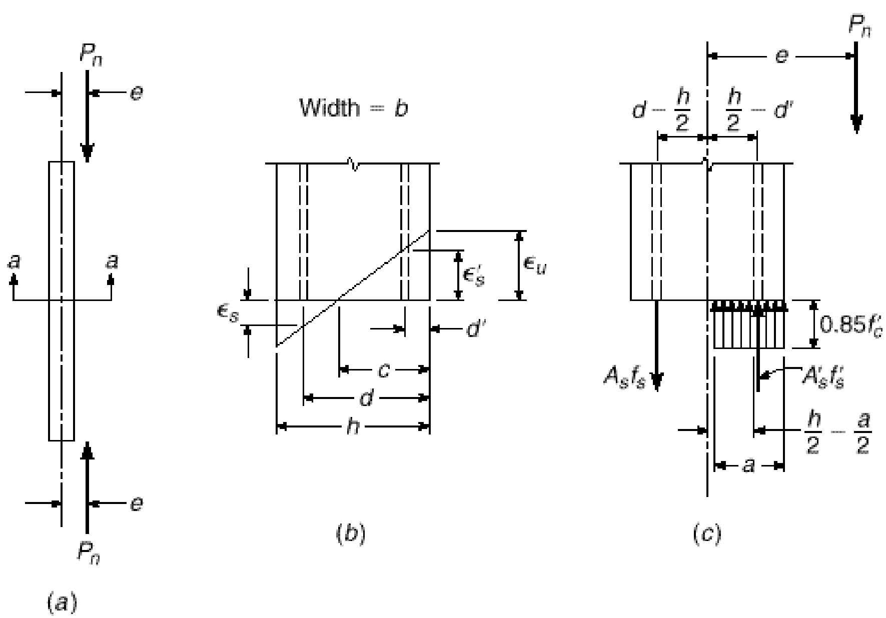


FIGURE 8.8
Column subject to eccentric compression: (*a*) loaded column; (*b*) strain distribution at section *a*-*a*; (*c*) stresses and forces at nominal strength.

These are the two basic equilibrium relations for rectangular eccentrically compressed members.

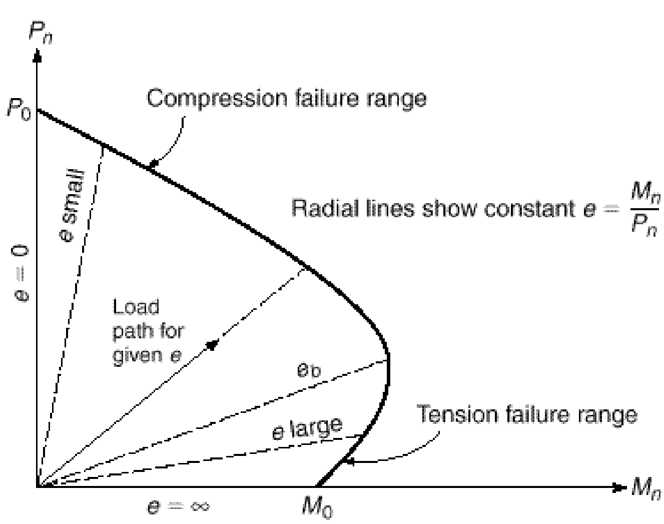
The fact that the presence of the compression reinforcement A'_s has displaced a corresponding amount of concrete of area A'_s is neglected in writing these equations. If necessary, particularly for large reinforcement ratios, one can account for this very simply. Evidently, in the above equations a nonexistent concrete compression force of amount $A'_s \cdot 0.85f'_c$ has been included as acting in the displaced concrete at the level of the compression steel. This excess force can be removed in both equations by multiplying A'_s by $f'_s - 0.85f'_c$ rather than by f'_s.

For large eccentricities, failure is initiated by yielding of the tension steel A_s. Hence, for this case, $f_s = f_y$. When the concrete reaches its ultimate strain ϵ_u, the compression steel may or may not have yielded; this must be determined based on compatibility of strains. For small eccentricities the concrete will reach its limit strain ϵ_u before the tension steel starts yielding; in fact, the bars on the side of the column farther from the load may be in compression, not tension. For small eccentricities, too, the analysis must be based on compatibility of strains between the steel and the adjacent concrete.

For a given eccentricity determined from the frame analysis (i.e., $e = M_u/P_u$) it is possible to solve Eqs. (8.7) and (8.8) for the load P_n and moment M_n that would result in failure as follows. In both equations, f'_s, f_s, and a can be expressed in terms of a single unknown c, the distance to the neutral axis. This is easily done based on the geometry of the strain diagram, with ϵ_u taken equal to 0.003 as usual, and using the stress-strain curve of the reinforcement. The result is that the two equations contain only two unknowns, P_n and c, and can be solved for those values simultaneously. However, to do so in practice would be complicated algebraically, particularly because of the need to incorporate the limit f_y on both f'_s and f_s.

A better approach, providing the basis for practical design, is to construct a *strength interaction diagram* defining the failure load and failure moment for a given column for the full range of eccentricities from zero to infinity. For any eccentricity, there is a unique pair of values of P_n and M_n that will produce the state of incipient failure. That pair of values can be plotted as a point on a graph relating P_n and M_n, such as shown in Fig. 8.9. A series of such calculations, each corresponding to a dif-

FIGURE 8.9
Interaction diagram for nominal column strength in combined bending and axial load.



ferent eccentricity, will result in a curve having a shape typically as shown in Fig. 8.9. On such a diagram, any radial line represents a particular eccentricity $e = M/P$. For that eccentricity, gradually increasing the load will define a load path as shown, and when that load path reaches the limit curve, failure will result. Note that the vertical axis corresponds to $e = 0$, and P_0 is the capacity of the column if concentrically loaded, as given by Eq. (8.3*b*). The horizontal axis corresponds to an infinite value of e, i.e., pure bending at moment capacity M_0. Small eccentricities will produce failure governed by concrete compression, while large eccentricities give a failure triggered by yielding of the tension steel.

For a given column, selected for trial, the interaction diagram is most easily constructed by selecting successive choices of neutral axis distance c, from infinity (axial load with eccentricity 0) to a very small value found by trial to give $P_n = 0$ (pure bending). For each selected value of c, the steel strains and stresses and the concrete force are easily calculated as follows. For the tension steel,

$$\epsilon_s = \epsilon_u \frac{d - c}{c} \tag{8.9}$$

$$f_s = \epsilon_u E_s \frac{d - c}{c} \leq f_y \tag{8.10}$$

while for the compression steel,

$$\epsilon'_s = \epsilon_u \frac{c - d'}{c} \tag{8.11}$$

$$f'_s = \epsilon_u E_s \frac{c - d'}{c} \leq f_y \tag{8.12}$$

The concrete stress block has depth

$$a = \beta_1 c \leq h \tag{8.13}$$

and consequently the concrete compressive resultant is

$$C = 0.85 f'_c ab \tag{8.14}$$

The nominal axial force P_n and nominal moment M_n corresponding to the selected neutral axis location can then be calculated from Eqs. (8.7) and (8.8), respectively, and thus a single point on the strength interaction diagram is established. The calculations are then repeated for successive choices of neutral axis to establish the curve defining the strength limits, such as Fig. 8.9. The calculations, of a repetitive nature, are easily programmed for the computer or performed using a spreadsheet.

8.5 Balanced Failure

As already noted, the interaction curve is divided into a compression failure range and a tension failure range.† It is useful to define what is termed a *balanced failure mode* and corresponding eccentricity e_b with the load P_b and moment M_b acting in combination to produce failure, with the concrete reaching its limit strain ϵ_u at precisely the

† The terms *compression failure range* and *tension failure range* are used for the purpose of general description and are distinct from *tension-controlled* and *compression-controlled* failures, as described in Chapter 3 and Section 8.9.

same instant that the tensile steel on the far side of the column reaches yield strain. This point on the interaction diagram is the dividing point between compression failure (small eccentricities) and tension failure (large eccentricities).

The values of P_b and M_b are easily computed with reference to Fig. 8.8. For balanced failure,

$$c = c_b = d \frac{\epsilon_u}{\epsilon_u + \epsilon_y} \tag{8.15}$$

and

$$a = a_b = \beta_1 c_b \tag{8.16}$$

Equations (8.9) through (8.14) are then used to obtain the steel stresses and the compressive resultant, after which P_b and M_b are found from Eqs. (8.7) and (8.8).

It is to be noted that, in contrast to beam design, one cannot restrict column designs such that yielding failure rather than crushing failure would always be the result of overloading. The type of failure for a column depends on the value of eccentricity e, which in turn is defined by the load analysis of the building or other structure.

It is important to observe, in Fig. 8.9, that in the region of compression failure the larger the axial load P_n, the smaller the moment M_n that the section is able to sustain before failing. However, in the region of tension failure the reverse is true; the larger the axial load, the larger the simultaneous moment capacity. This is easily understood. In the compression failure region, failure occurs through overstraining of the concrete. The larger the concrete compressive strain caused by the axial load alone, the smaller the margin of additional strain available for the added compression caused by bending. On the other hand, in the tension failure region, yielding of the steel initiates failure. If the member is loaded in simple bending to the point at which yielding begins in the tension steel, and if an axial compression load is then added, the steel compressive stresses caused by this load will superimpose on the previous tensile stresses. This reduces the total steel stress to a value below its yield strength. Consequently, an additional moment can now be sustained of such magnitude that the combination of the steel stress from the axial load and the increased moment again reaches the yield strength.

The typical shape of a column interaction diagram shown in Fig. 8.9 has important design implications. In the range of tension failure, a *reduction in axial load* may produce failure for a given moment. In carrying out a frame analysis, the designer must consider all combinations of loading that may occur, including that which would produce minimum axial load paired with a given moment (the specific load combinations are specified in ACI Code 8.8 and described in Section 12.3). Only that amount of compression that is certain to be present should be used in calculating the capacity of a column subject to a given moment.

EXAMPLE 8.1

Column strength interaction diagram. A 12 × 20 in. column is reinforced with four No. 9 (No. 29) bars of area 1.0 in^2 each, one in each corner as shown in Fig. 8.10a. The concrete cylinder strength is f'_c = 4000 psi and the steel yield strength is 60 ksi. Determine (a) the load P_b, moment M_b, and corresponding eccentricity e_b for balanced failure; (b) the load and moment for a representative point in the tension failure region of the interaction curve; (c) the load and moment for a representative point in the compression failure region; (d) the axial load strength for zero eccentricity. Then (e) sketch the strength interaction diagram for this column. Finally, (f) design the transverse reinforcement, based on ACI Code provisions.

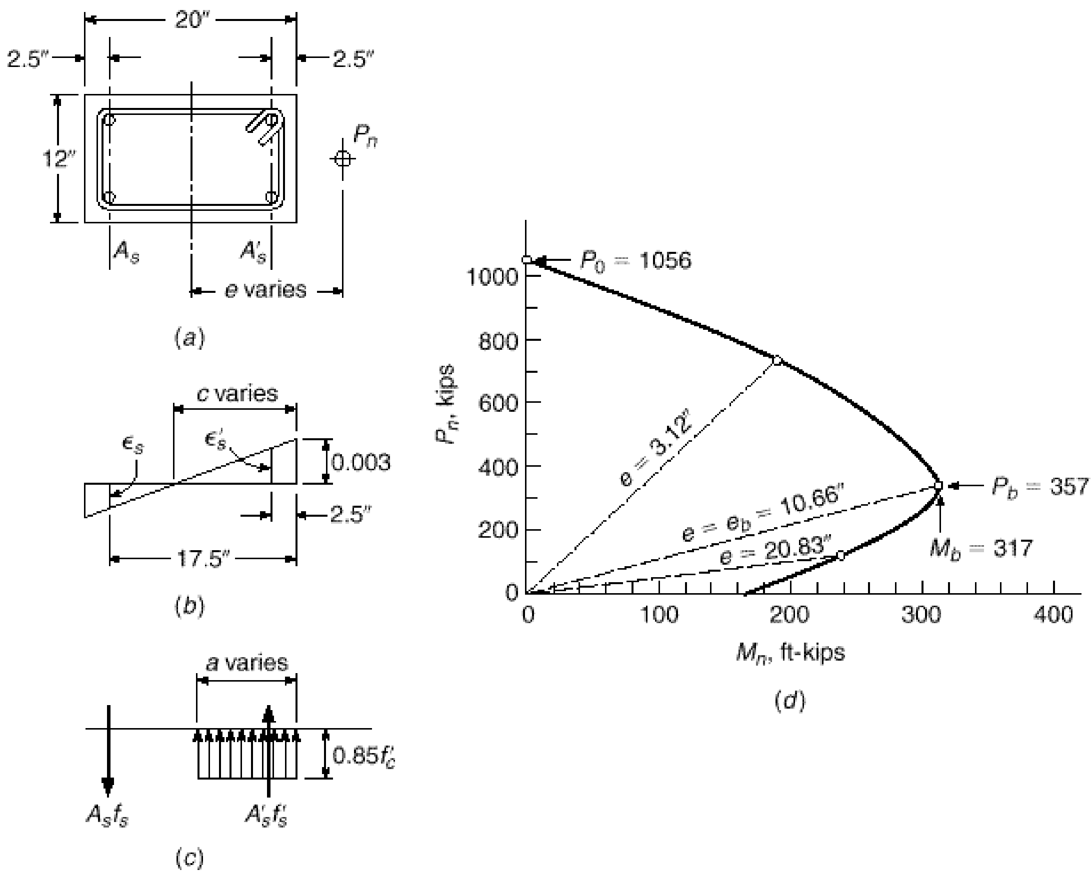


FIGURE 8.10

Column interaction diagram for Example 8.1: (*a*) cross section; (*b*) strain distribution; (*c*) stresses and forces; (*d*) strength interaction diagram.

Solution.

(*a*) The neutral axis for the balanced failure condition is easily found from Eq. (8.15) with $\epsilon_u = 0.003$ and $\epsilon_y = 60/29{,}000 = 0.0021$:

$$c_b = 17.5 \times \frac{0.003}{0.0051} = 10.3 \text{ in.}$$

giving a stress-block depth $a = 0.85 \times 10.3 = 8.76$ in. For the balanced failure condition, by definition, $f_s = f_y$. The compressive steel stress is found from Eq. (8.12):

$$f'_s = 0.003 \times 29{,}000 \frac{10.3 - 2.5}{10.3} = 65.9 \text{ ksi} \quad \text{but} \quad \leq 60 \text{ ksi}$$

confirming that the compression steel, too, is at the yield. The concrete compressive resultant is

$$C = 0.85 \times 4 \times 8.76 \times 12 = 357 \text{ kips}$$

The balanced load P_b is then found from Eq. (8.7) to be

$$P_b = 357 + 2.0 \times 60 - 2.0 \times 60 = 357 \text{ kips}$$

and the balanced moment from Eq. (8.8) is

$$M_b = 357(10 - 4.38) + 2.0 \times 60(10 - 2.5) + 2.0 \times 60(17.5 - 10)$$
$$= 3806 \text{ in-kips} = 317 \text{ ft-kips}$$

The corresponding eccentricity of load is $e_b = 10.66$ in.

(*b*) Any choice of *c smaller than* $c_b = 10.3$ in. will give a point in the tension failure region of the interaction curve, with eccentricity larger than e_b. For example, choose $c = 5.0$ in. By definition, $f_s = f_y$. The compressive steel stress is found to be

$$f'_s = 0.003 \times 29{,}000 \frac{5.0 - 2.5}{5.0} = 43.5 \text{ ksi}$$

With the stress-block depth $a = 0.85 \times 5.0 \times 4.25$, the compressive resultant is $C = 0.85 \times 4 \times 4.25 \times 12 = 173$ kips. Then from Eq. (8.7), the thrust is

$$P_n = 173 + 2.0 \times 43.5 - 2.0 \times 60 = 140 \text{ kips}$$

and the moment capacity from Eq. (8.8) is

$$M_n = 173(10 - 2.12) + 2.0 \times 43.5(10 - 2.5) + 2.0 \times 60(17.5 - 10)$$
$$= 2916 \text{ in-kips} = 243 \text{ ft-kips}$$

giving eccentricity $e = 2916/140 = 20.83$ in., well above the balanced value.

(*c*) Now selecting a c value *larger than* c_b to demonstrate a compression failure point on the interaction curve, choose $c = 18.0$ in., for which $a = 0.85 \times 18.0 = 15.3$ in. The compressive concrete resultant is $C = 0.85 \times 4 \times 15.3 \times 12 = 624$ kips. From Eq. (8.10) the stress in the steel at the left side of the column is

$$f_s = 0.003 \times 29{,}000 \frac{17.5 - 18.0}{18.0} = -2 \text{ ksi}$$

Note that the negative value of f_s indicates correctly that A_s is in compression if c is greater than d, as in the present case. The compressive steel stress is found from Eq. (8.12) to be

$$f'_s = 0.003 \times 29{,}000 \frac{18.0 - 2.5}{18.0} = 75 \text{ ksi} \quad \text{but} \quad \le 60 \text{ ksi}$$

Then the column capacity is

$$P_n = 624 + 2.0 \times 60 + 2.0 \times 2 = 748 \text{ kips}$$
$$M_n = 624(10 - 7.65) + 2.0 \times 60(10 - 2.5) - 2.0 \times 2(17.5 - 10)$$
$$= 2336 \text{ in-kips} = 195 \text{ ft-kips}$$

giving eccentricity $e = 2336/748 = 3.12$ in.

(*d*) The axial strength of the column if concentrically loaded corresponds to $c = \infty$ and $e = 0$. For this case,

$$P_n = 0.85 \times 4 \times 12 \times 20 + 4.0 \times 60 = 1056 \text{ kips}$$

Note that, for this as well as the preceding calculations, subtraction of the concrete displaced by the steel has been neglected. For comparison, if the deduction were made in the last calculation:

$$P_n = 0.85 \times 4(12 \times 20 - 4) + (4.0 \times 60) = 1042 \text{ kips}$$

The error in neglecting this deduction is only 1 percent in this case; the difference generally can be neglected, except perhaps for columns with reinforcement ratios close to the maximum of 8 percent. In the case of design aids, however, such as those presented in Refs. 8.2 and 8.7 and discussed in Section 8.10, the deduction is usually included for all reinforcement ratios.

(*e*) From the calculations just completed, plus similar repetitive calculations that will not be given here, the strength interaction curve of Fig. 8.10*d* is constructed. Note the characteristic shape, described earlier, the location of the balanced failure point as well as the "small eccentricity" and "large eccentricity" points just found, and the axial load capacity.

(*f*) In the process of developing a strength interaction curve, it is possible to select the values of steel strain ϵ_s, as done in step (*a*), for use in steps (*b*) and (*c*). Selecting ϵ_s uniquely establishes the neutral axis depth *c*, as shown by Eqs. (8.9) and (8.15), and is useful in determining M_n and P_n for values of steel strain that correspond to changes in the strength reduction factor ϕ, as will be discussed in Section 8.9.

(*g*) The design of the column ties will be carried out following the ACI Code restrictions. For the minimum permitted tie diameter of $\frac{3}{8}$ in., used with No. 9 (No. 29) longitudinal bars having a diameter of 1.128 in a column the least dimension of which is 12 in., the tie spacing is not to exceed:

$$48 \times \frac{3}{8} = 18 \text{ in.}$$

$$16 \times 1.128 = 18.05 \text{ in.}$$

$$b = 12 \text{ in.}$$

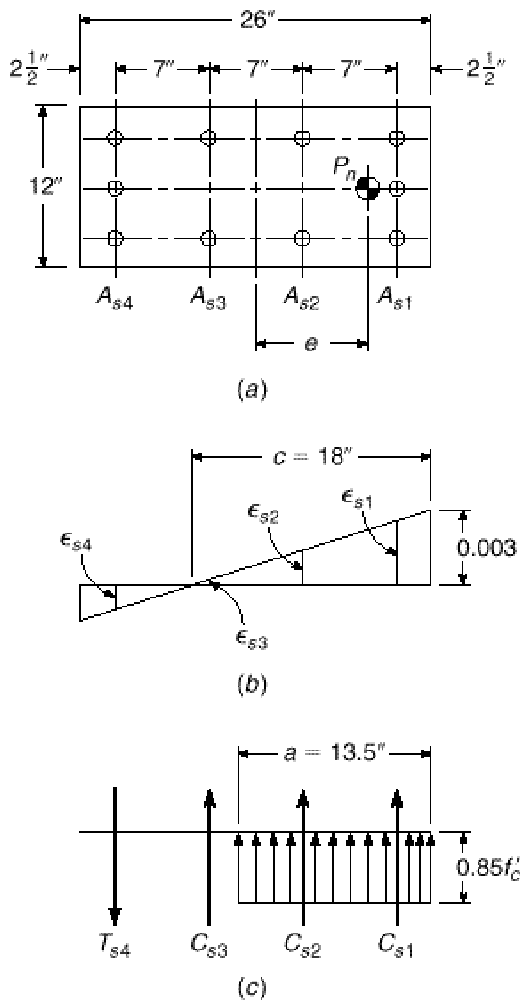
The last restriction controls in this case, and No. 3 (No. 10) ties will be used at 12 in. spacing, detailed as shown in Fig. 8.10*a*. Note that the permitted spacing as controlled by the first and second criteria, 18 in., must be reduced because of the 12 in. column dimension, indicating that a saving in tie steel could be realized using a smaller tie diameter; however, this would not meet the ACI Code restriction on the minimum tie diameter in this case.

8.6 Distributed Reinforcement

When large bending moments are present, it is most economical to concentrate all or most of the steel along the outer faces parallel to the axis of bending. Such arrangements are shown in Fig. 8.2*e* to *h*. On the other hand, with small eccentricities so that axial compression is predominant, and when a small cross section is desired, it is often advantageous to place the steel more uniformly around the perimeter, as in Fig. 8.2*a* to *d*. In this case, special attention must be paid to the intermediate bars, i.e., those that are not placed along the two faces that are most highly stressed. This is so because when the ultimate load is reached, the stresses in these intermediate bars are usually below the yield point, even though the bars along one or both extreme faces may be yielding. This situation can be analyzed by a simple and obvious extension of the previous analysis based on compatibility of strains. A strength interaction diagram may be constructed just as before. A sequence of choices of neutral axis location results in a set of paired values of P_n and M_n, each corresponding to a particular eccentricity of load.

EXAMPLE 8.2 **Analysis of eccentrically loaded column with distributed reinforcement.** The column in Fig. 8.11*a* is reinforced with ten No. 11 (No. 36) bars distributed around the perimeter as shown. Load P_n will be applied with eccentricity *e* about the strong axis. Material strengths are $f'_c = 6000$ psi and $f_y = 75$ ksi. Find the load and moment corresponding to a failure point with neutral axis $c = 18$ in. from the right face.

FIGURE 8.11
Column in Example 8.2: (*a*) cross section; (*b*) strain distribution; (*c*) stresses and forces.



Solution. When the concrete reaches its limit strain of 0.003, the strain distribution is that shown in Fig. 8.11*b*, the strains at the locations of the four bar groups are found from similar triangles, after which the stresses are found by multiplying strains by $E_s = 29{,}000$ ksi applying the limit value f_y:

$$\epsilon_{s1} = 0.00258 \qquad f_{s1} = 75.0 \text{ ksi compression}$$
$$\epsilon_{s2} = 0.00142 \qquad f_{s2} = 41.2 \text{ ksi compression}$$
$$\epsilon_{s3} = 0.00025 \qquad f_{s3} = 7.3 \text{ ksi compression}$$
$$\epsilon_{s4} = 0.00091 \qquad f_{s4} = 26.4 \text{ ksi tension}$$

For $f_c' = 6000$ psi, $\beta_1 = 0.75$ and the depth of the equivalent rectangular stress block is $a = 0.75 \times 18 = 13.5$ in. The concrete compressive resultant is $C = 0.85 \times 6 \times 13.5 \times 12 = 826$ kips, and the respective steel forces in Fig. 8.11*c* are:

$$C_{s1} = 4.68 \times 75.0 = 351 \text{ kips}$$
$$C_{s2} = 3.12 \times 41.2 = 129 \text{ kips}$$
$$C_{s3} = 3.12 \times 7.3 = 23 \text{ kips}$$
$$T_{s4} = 4.68 \times 26.4 = 124 \text{ kips}$$

The axial load and moment that would produce failure for a neutral axis 18 in. from the right face are found by the obvious extensions of Eqs. (8.7) and (8.8):

$$P_n = 826 + 351 + 129 + 23 - 124 = 1205 \text{ kips}$$

$$M_n = 826 \cdot 13 - 6.75 \cdot + 351 \cdot 13 - 2.5 \cdot + 129 \cdot 13 - 9.5 \cdot - 23 \cdot 13 - 9.5 \cdot$$

$$+ 124 \cdot 13 - 2.5 \cdot$$

$$= 10{,}520 \text{ in-kips}$$

$$= 877 \text{ ft-kips}$$

The corresponding eccentricity is $e = 10{,}520 \cdot 1205 = 8.73$ in. Other points on the interaction diagram can be computed in a similar way.

Two general conclusions can be made from this example:

1. Even with the relatively small eccentricity of about one-third of the depth of the section, only the bars of group 1 just barely reached their yield strain, and consequently their yield stress. All other bar groups of the relatively high-strength steel that was used are stressed far below their yield strength, which would also have been true for group 1 for a slightly larger eccentricity. It follows that the use of the more expensive high-strength steel is economical in symmetrically reinforced columns only for very small eccentricities, e.g., in the lower stories of tall buildings.
2. The contribution of the intermediate bars of groups 2 and 3 to both P_n and M_n is quite small because of their low stresses. Again, intermediate bars, except as they are needed to hold ties in place, are economical only for columns with very small eccentricities.

8.7 Unsymmetrical Reinforcement

Most reinforced concrete columns are symmetrically reinforced about the axis of bending. However, for some cases, such as the columns of rigid portal frames in which the moments are uniaxial and the eccentricity large, it is more economical to use an unsymmetrical pattern of bars, with most of the bars on the tension side such as shown in Fig. 8.12. Such columns can be analyzed by the same strain compatibility approach as described above. However, for an unsymmetrically reinforced column to be loaded concentrically, the load must pass through a point known as the *plastic centroid*. The plastic centroid is defined as the point of application of the resultant force for the column cross section (including concrete and steel forces) if the column is compressed uniformly to the failure strain $\cdot_u = 0.003$ over its entire cross section. Eccentricity of the applied load must be measured with respect to the plastic centroid, because only then will $e = 0$ correspond to an axial load with no moment. The location of the plastic centroid for the column of Fig. 8.12 is the resultant of the three internal forces to be accounted for. Its distance from the left face is

$$x = \frac{0.85 f'_c bh^2 \cdot 2 + A_s f_y d + A'_s f_y d \cdot}{0.85 f'_c bh + A_s f_y + A'_s f_y} \qquad (8.17)$$

Clearly, in a symmetrically reinforced cross section, the plastic centroid and the geometric center coincide.

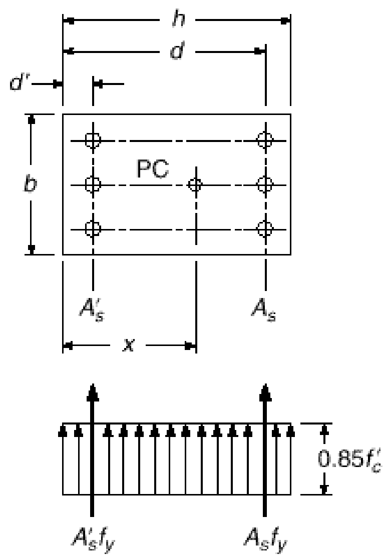


FIGURE 8.12
Plastic centroid of an unsymmetrically reinforced column.

8.8 CIRCULAR COLUMNS

It was mentioned in Section 8.2 that when load eccentricities are small, spirally reinforced columns show greater toughness, i.e., greater ductility, than tied columns, although this difference fades out as the eccentricity is increased. For this reason, as discussed in Section 8.2, the ACI Code provides a more favorable reduction factor ϕ = 0.70 for spiral columns, compared with ϕ = 0.65 for tied columns. Also, the maximum stipulated design load for entirely or nearly axially loaded members is larger for spirally reinforced members than for comparable tied members (see Section 8.9). It follows that spirally reinforced columns permit a somewhat more economical utilization of the materials, particularly for small calculated eccentricities. A further advantage lies in the fact that the circular shape is frequently desired by the architect.

Figure 8.13 shows the cross section of a spirally reinforced column. Six or more longitudinal bars of equal size are provided for longitudinal reinforcement, depending on column diameter. The strain distribution at the instant at which the ultimate load is reached is shown in Fig. 8.13*b*. Bar groups 2 and 3 are seen to be strained to much smaller values than groups 1 and 4. The stresses in the four bar groups are easily found. For any of the bars with strains in excess of yield strain $\epsilon_y = f_y/E_s$, the stress at failure is evidently the yield stress of the bar. For bars with smaller strains, the stress is found from $f_s = \epsilon_s E_s$.

One then has the internal forces shown in Fig. 8.13*c*. They must be in force and moment equilibrium with the nominal strength P_n. It will be noted that the situation is analogous to that discussed in Sections 8.4 to 8.6 for rectangular columns. Calculations can be carried out exactly as in Example 8.1, except that for circular columns the concrete compression zone subject to the equivalent rectangular stress distribution has the shape of a segment of a circle, shown shaded in Fig. 8.13*a*.

Although the shape of the compression zone and the strain variation in the different groups of bars make longhand calculations awkward, no new principles are involved and computer solutions are easily developed.

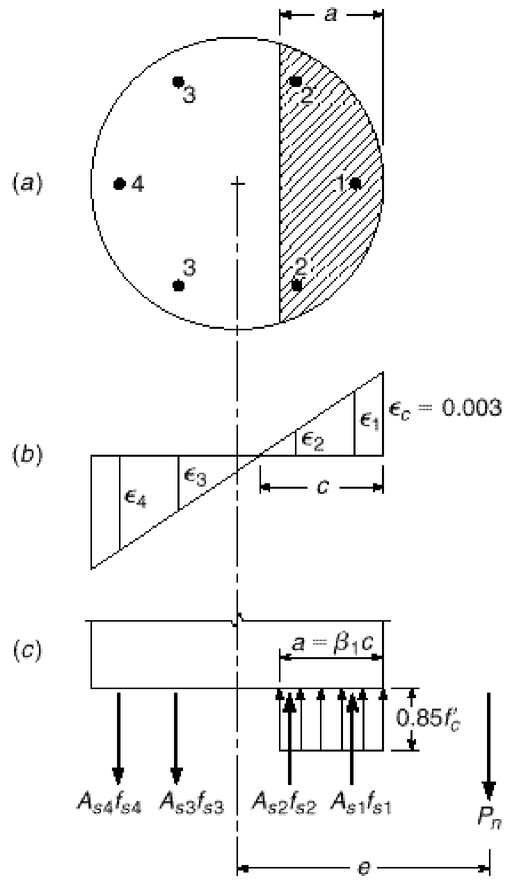


FIGURE 8.13
Circular column with compression plus bending.

Design or analysis of spirally reinforced columns is usually carried out by means of design aids, such as Graphs A.13 to A.16 of Appendix A. Additional tables and graphs are available, e.g., in Ref. 8.7. In developing such design aids, the entire steel area is often assumed to be arranged in a uniform, concentric ring, rather than being concentrated in the actual bar locations; this simplifies calculations without noticeably affecting results if the column contains at least eight longitudinal bars. When fewer bars are used, the interaction curve should be calculated based on the weakest orientation in bending.

It should be noted that, to qualify for the more favorable safety provisions for spiral columns, the reinforcement ratio of the spiral must be at least equal to that given by Eq. (8.5) for reasons discussed in Section 8.2.

8.9 ACI Code Provisions for Column Design

For columns, as for all members designed according to the ACI Code, adequate safety margins are established by applying load factors to the service loads and strength reduction factors to the nominal strengths. Thus, for columns, $\phi P_n \geq P_u$ and $\phi M_n \geq M_u$ are the basic safety criteria. For most members subject to axial compression or compression plus flexure (compression-controlled members, as described in Chapter 3), the ACI Code provides basic reduction factors:

$$\phi = 0.65 \text{ for tied columns}$$

$$\phi = 0.70 \text{ for spirally reinforced columns}$$

The spread between these two values reflects the added safety furnished by the greater toughness of spirally reinforced columns.

There are various reasons why the ϕ values for columns are lower than those for flexure or shear (0.90 and 0.75, respectively). One is that the strength of underreinforced flexural members is not much affected by variations in concrete strength, since it depends primarily on the yield strength of the steel, while the strength of axially loaded members depends strongly on the concrete compressive strength. Because the cylinder strength of concrete under site conditions is less closely controlled than the yield strength of mill-produced steel, a larger occasional strength deficiency must be allowed for. This is particularly true for columns, in which concrete, being placed from the top down in a long, narrow form, is more subject to segregation than in horizontally cast beams. Moreover, electrical and other conduits are frequently located in building columns; this reduces their effective cross sections, often to an extent unknown to the designer, even though this is poor practice and restricted by the ACI Code. Finally, the consequences of a column failure, say in a lower story, would be more catastrophic than that of a single beam in a floor system in the same building.

For high eccentricities, as the eccentricity increases from e_b to infinity (pure bending), the ACI Code recognizes that the member behaves progressively more like a flexural member and less like a column. As described in Chapter 3, this is acknowledged in ACI Code 9.3.2 by providing a linear transition in ϕ from values of 0.65 and 0.70 to 0.90 as the net tensile strain in the extreme tensile steel ϵ_t increases from f_y/E_s (which may be taken as 0.002 for Grade 60 reinforcement) to 0.005.

At the other extreme, for columns with very small or zero calculated eccentricities, the ACI Code recognizes that accidental construction misalignments and other unforeseen factors may produce actual eccentricities in excess of these small design values. Also, the concrete strength under high, sustained axial loads may be somewhat smaller than the short-term cylinder strength. Therefore, regardless of the magnitude of the calculated eccentricity, ACI Code 10.3.5 limits the maximum design strength to $0.80\phi P_0$ for tied columns (with $\phi = 0.65$) and to $0.85\phi P_0$ for spirally reinforced columns (with $\phi = 0.70$), where P_0 is the nominal strength of the axially loaded column with zero eccentricity [see Eq. (8.4)].

The effects of the safety provisions of the ACI Code are shown in Fig. 8.14. The solid curve labeled "nominal strength" is the same as Fig. 8.9 and represents the actual carrying capacity, as nearly as can be predicted. The smooth curve shown partially dashed, then solid, then dashed, represents the basic design strength obtained by reducing the nominal strengths P_n and M_n, for each eccentricity, by $\phi = 0.65$ for tied columns and $\phi = 0.70$ for spiral columns. The horizontal cutoff at $\alpha\phi P_0$ represents the maximum design load stipulated in the ACI Code for small eccentricities, i.e., large axial loads, as just discussed. At the other end, for large eccentricities, i.e., small axial loads, the ACI Code permits a linear transition of ϕ from 0.65 or 0.70, applicable for $\epsilon_t \leq f_y/E_s$ (or 0.002 for Grade 60 reinforcement) to 0.90 at $\epsilon_t = 0.005$. By definition,

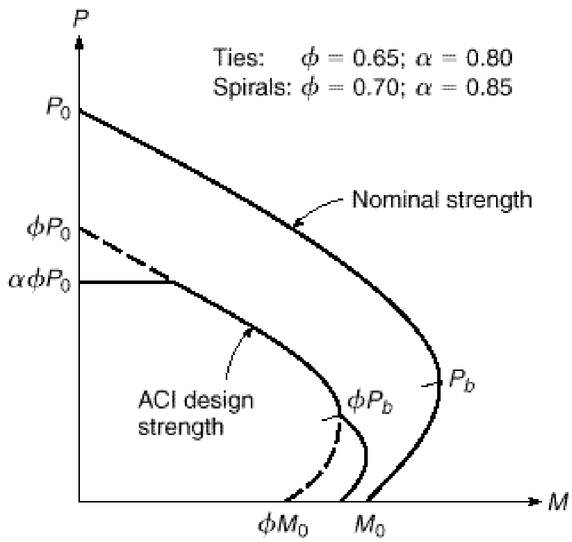


FIGURE 8.14
ACI safety provisions superimposed on column strength interaction diagram.

$\epsilon_t = f_y/E_s$ at the balanced condition. The effect of the transition in ϕ is shown at the lower right end of the design strength curve.†

8.10 Design Aids

The design of eccentrically loaded columns using the strain compatibility method of analysis described requires that a trial column be selected. The trial column is then investigated to determine if it is adequate to carry any combination of P_u and M_u that may act on it should the structure be overloaded, i.e., to see if P_u and M_u from the analysis of the structure, when plotted on a strength interaction diagram such as Fig. 8.14, fall within the region bounded by the curve labeled "ACI design strength." Furthermore, economical design requires that the controlling combination of P_u and M_u be close to the limit curve. If these conditions are not met, a new column must be selected for trial.

While a simple computer program or spreadsheet can be developed, based on the strain compatibility analysis, to calculate points on the design strength curve, and even to plot the curve, for any trial column, in practice design aids are used such as are available in handbooks and special volumes published by the American Concrete Institute (Ref. 8.7) and the Concrete Reinforcing Steel Institute (Ref. 8.2). They cover the most frequent practical cases, such as symmetrically reinforced rectangular and square columns and circular spirally reinforced columns. There are also a number of commercially available computer programs (e.g., PCACOLUMN, Portland Cement Association, Skokie, Illinois, and HBCOLUMN, Concrete Reinforcing Steel Institute, Schaumburg, Illinois).

† While the general intent of the ACI Code safety provisions relating to eccentric columns is clear and fundamentally sound, the end result is a set of strangely shaped column design curves following no discernible physical law, as is demonstrated in Fig. 8.14. Improved column safety provisions, resulting in a smooth design curve appropriately related to the strength curve, would be simpler to use and more rational as well.

Graphs A.5 through A.16 of Appendix A are representative of column design charts (such as found in Ref. 8.7), in this case for concrete with $f'_c = 4000$ psi and steel with yield strength $f_y = 60$ ksi, for varying cover distances.[†] Reference 8.7 includes charts for a broad range of material strengths. Graphs A.5 through A.8 are drawn for rectangular columns with reinforcement distributed around the column perimeter; Graphs A.9 through A.12 are for rectangular columns with reinforcement along two opposite faces. Circular columns with bars in a circular pattern are shown in Graphs A.13 through A.16.

The graphs are seen to consist of nominal strength interaction curves of the type shown in Fig. 8.14. However, instead of plotting P_n versus M_n, corresponding parameters have been used to make the charts more generally applicable, i.e., load is plotted as $K_n = P_n/(f'_c A_g)$, while moment is expressed as $R_n = P_n e/(f'_c A_g h)$. Families of curves are drawn for various values of $\rho_g = A_{st}/A_g$ between 0.01 and 0.08. The graphs also include radial lines representing different eccentricity ratios e/h, as well as lines representing different ratios of stress f_s/f_y or values of strain $\epsilon_t = 0.002$ and 0.005 in the extreme tension steel.

Charts such as these permit the direct design of eccentrically loaded columns throughout the common range of strength and geometric variables. They may be used in one of two ways as follows. For a given factored load P_u and equivalent eccentricity $e = M_u/P_u$:

1. **(a)** Select trial cross section dimensions b and h (refer to Fig. 8.8).
   (b) Calculate the ratio γ based on required cover distances to the bar centroids, and select the corresponding column design chart.
   (c) Calculate $K_n = P_u/(\phi f'_c A_g)$ and $R_n = P_u e/(\phi f'_c A_g h)$, where $A_g = bh$.
   (d) From the graph, for the values found in **(c)**, read the required reinforcement ratio ρ_g.
   (e) Calculate the total steel area $A_{st} = \rho_g bh$.
2. **(a)** Select the reinforcement ratio ρ_g.
   (b) Choose a trial value of h and calculate e/h and γ.
   (c) From the corresponding graph, read $K_n = P_u/(\phi f'_c A_g)$ and calculate the required A_g.
   (d) Calculate $b = A_g/h$.
   (e) Revise the trial value of h if necessary to obtain a well-proportioned section.
   (f) Calculate the total steel area $A_{st} = \rho_g bh$.

Use of the column design charts will be illustrated in Examples 8.3 and 8.4.

Other design aids pertaining to ties and spirals, as well as recommendations for standard practice, will be found in Refs. 8.2 and 8.7.

EXAMPLE 8.3 **Selection of reinforcement for column of given size.** In a three-story structure, an exterior column is to be designed for a service dead load of 222 kips, maximum live load of 333 kips, dead load moment of 162 ft-kips, and live load moment of 232 ft-kips. The minimum live load compatible with the full live load moment is 166 kips, obtained when no live load is placed on the roof but a full live load is placed on the second floor. Architectural considerations require that a rectangular column be used, with dimensions $b = 20$ in. and $h = 25$ in.

[†] Graphs A.5 through A.16 were developed for the specific bar configurations shown on the graphs. The curves exhibit changes in curvature, especially apparent near the balanced load, that result when bars within the cross section yield. The values provided in the graphs, however, are largely insensitive to the exact number of bars in the cross section and may be used for columns with similar bar configurations, but with smaller or larger numbers of bars.

(*a*) Find the required column reinforcement for the condition that the full live load acts.
(*b*) Check to ensure that the column is adequate for the condition of no live load on the roof.

Material strengths are $f_c' = 4000$ psi and $f_y = 60{,}000$ psi.

Solution.

(*a*) The column will be designed initially for full load, then checked for adequacy when live load is partially removed. According to the ACI safety provisions, the column must be designed for a factored load $P_u = 1.2 \times 222 + 1.6 \times 333 = 799$ kips and a factored moment $M_u = 1.2 \times 162 + 1.6 \times 232 = 566$ ft-kips. A column 20 × 25 in. is specified, and reinforcement distributed around the column perimeter will be used. Bar cover is estimated to be 2.5 in. from the column face to the steel centerline for each bar. The column parameters (assuming bending about the strong axis) are

$$K_n = \frac{P_u}{\cdot f_c A_g} = \frac{799}{0.65 \times 4 \times 500} = 0.615$$

$$R_n = \frac{M_u}{\cdot f_c A_g h} = \frac{566 \times 12}{0.65 \times 4 \times 500 \times 25} = 0.209$$

With 2.5 in. cover, the parameter · $= (25 - 5) \cdot 25 = 0.80$. For this column geometry and material strengths, Graph A.7 of Appendix A applies. From that figure, with the calculated values of K_n and R_n, $\cdot_g = 0.024$. Thus, the required reinforcement is $A_{st} = 0.024 \times 500 = 12.00$ in². Twelve No. 9 (No. 29) bars will be used, one at each corner and two evenly spaced along each face of the column, providing $A_{st} = 12.00$ in².

(*b*) With the roof live load absent, the column will carry a factored load $P_u = 1.2 \times 222 + 1.6 \times 166 = 532$ kips and factored moment $M_u = 566$ ft-kips, as before. Thus, the column parameters for this condition are

$$K_n = \frac{P_u}{\cdot f_c A_g} = \frac{532}{0.65 \times 4 \times 500} = 0.409$$

$$R_n = \frac{M_u}{\cdot f_c A_g h} = \frac{566 \times 12}{0.65 \times 4 \times 500 \times 25} = 0.209$$

and · $= 0.80$ as before. From Graph A.7 it is found that a reinforcement ratio of $\cdot_g = 0.016$ is sufficient for this condition, less than that required in part (*a*), so no modification is required.

Selecting No. 3 (No. 10) ties for trial, the maximum tie spacing must not exceed $48 \times 0.375 = 18$ in., $16 \times 1.128 = 18.05$ in., or 20 in. Spacing is controlled by the diameter of the ties, and No. 3 (No. 10) ties will be used at 18 in. spacing, in the pattern shown in Fig. 8.2*d*.

EXAMPLE 8.4 **Selection of column size for a given reinforcement ratio.** A column is to be designed to carry a factored load $P_u = 481$ kips and factored moment $M_u = 492$ ft-kips. Material strengths $f_y = 60{,}000$ psi and $f_c' = 4000$ psi are specified. Cost studies for the particular location indicate that a reinforcement ratio $\cdot_g$ of about 0.03 is optimum. Find the required dimensions *b* and *h* of the column. Bending will be about the strong axis, and an arrangement of steel with bars concentrated in two layers, adjacent to the outer faces of the column and parallel to the axis of bending, will be used.

Solution. It is convenient to select a trial column dimension *h*, perpendicular to the axis of bending; a value of $h = 25$ in. will be selected, and assuming a concrete cover of 2.5 in. to the bar centers, the parameter · $= 0.80$. Graph A.11 of Appendix A applies. For the stated loads the eccentricity is $e = 492 \times 12/481 = 12.3$ in., and $e/h = 12.3/25 = 0.49$. From Graph A.11 with $e/h = 0.49$ and $\rho_g = 0.03$, $K_n = P_u/\phi f'_c A_g = 0.51$. For the trial dimension $h = 25$ in., the required column width is

$$b = \frac{P_u}{\phi f'_c K_n h} = \frac{481}{0.65 \times 4 \times 0.51 \times 25} = 14.5 \text{ in.}$$

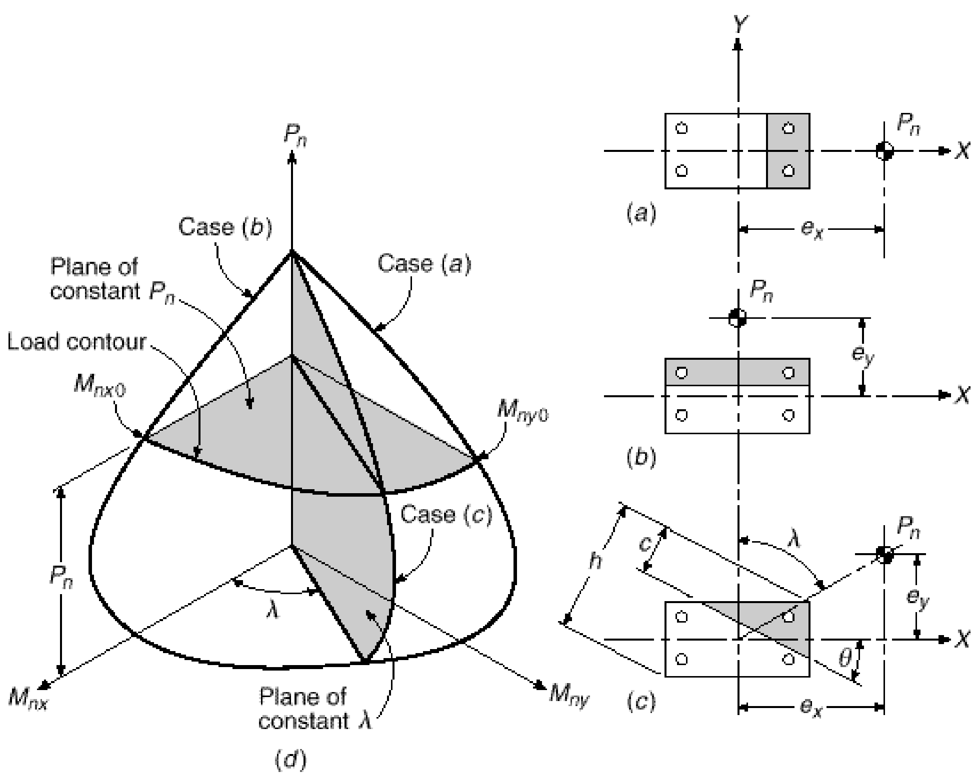
A column 15 × 25 in. will be used, for which the required steel area is $A_{st} = 0.03 \times 15 \times 25 = 11.25$ in^2. Eight No. 11 (No. 36) bars will be used, providing $A_{st} = 12.48$ in^2, arranged in two layers of four bars each, similar to the sketch shown in Graph A.11.

8.11 Biaxial Bending

The methods discussed in the preceding sections permit rectangular or square columns to be designed if bending is present about only one of the principal axes. There are situations, by no means exceptional, in which axial compression is accompanied by simultaneous bending about both principal axes of the section. Such is the case, for instance, in corner columns of buildings where beams and girders frame into the columns in the directions of both walls and transfer their end moments into the columns in two perpendicular planes. Similar loading may occur at interior columns, particularly if the column layout is irregular.

The situation with respect to strength of biaxially loaded columns is shown in Fig. 8.15. Let X and Y denote the directions of the principal axes of the cross section.

FIGURE 8.15
Interaction diagram for compression plus biaxial bending: (*a*) uniaxial bending about *Y* axis; (*b*) uniaxial bending about *X* axis; (*c*) biaxial bending about diagonal axis; (*d*) interaction surface.



In Fig. 8.15*a*, the section is shown subject to bending about the *Y* axis only, with load eccentricity e_x measured in the *X* direction. The corresponding strength interaction curve is shown as Case (*a*) in the three-dimensional sketch in Fig. 8.15*d* and is drawn in the plane defined by the axes P_n and M_{ny}. Such a curve can be established by the usual methods for uniaxial bending. Similarly, Fig. 8.15*b* shows bending about the *X* axis only, with eccentricity e_y measured in the *Y* direction. The corresponding interaction curve is shown as Case (*b*) in the plane of P_n and M_{nx} in Fig. 8.15*d*. For Case (*c*), which combines *X* and *Y* axis bending, the orientation of the resultant eccentricity is defined by the angle · :

$$\cdot = \arctan \frac{e_x}{e_y} = \arctan \frac{M_{ny}}{M_{nx}}$$

Bending for this case is about an axis defined by the angle · with respect to the *X* axis. The angle · in Fig. 8.15*c* establishes a plane in Fig. 8.15*d*, passing through the vertical P_n axis and making an angle · with the M_{nx} axis, as shown. In that plane, column strength is defined by the interaction curve labeled Case (*c*). For other values of · , similar curves are obtained to define a *failure surface* for axial load plus biaxial bending, such as shown in Fig. 8.15*d*. The surface is exactly analogous to the *interaction curve* for axial load plus uniaxial bending. Any combination of P_u, M_{ux}, and M_{uy} falling inside the surface can be applied safely, but any point falling outside the surface would represent failure. Note that the failure surface can be described either by a set of curves defined by radial planes passing through the P_n axis, such as shown by Case (*c*), or by a set of curves defined by horizontal plane intersections, each for a constant P_n, defining load contours.

Constructing such an interaction surface for a given column would appear to be an obvious extension of uniaxial bending analysis. In Fig. 8.15*c*, for a selected value of · , successive choices of neutral axis distance *c* could be taken. For each, using strain compatibility and stress-strain relations to establish bar forces and the concrete compressive resultant, then using the equilibrium equations to find P_n, M_{nx}, and M_{ny}, one can determine a single point on the interaction surface. Repetitive calculations, easily done by computer, then establish sufficient points to define the surface. The triangular or trapezoidal compression zone, such as shown in Fig. 8.15*c*, is a complication, and in general the strain in each reinforcing bar will be different, but these features can be incorporated.

The main difficulty, however, is that the neutral axis will not, in general, be perpendicular to the resultant eccentricity, drawn from the column center to the load P_n. For each successive choice of neutral axis, there are unique values of P_n, M_{nx}, and M_{ny}, and only for special cases will the ratio of $M_{ny} \cdot M_{nx}$ be such that the eccentricity is perpendicular to the neutral axis chosen for the calculation. The result is that, for successive choices of *c* for any given · , the value of · in Fig. 8.15*c* and *d* will vary. Points on the failure surface established in this way will wander up the failure surface for increasing P_n, not representing a plane intersection, as shown for Case (*c*) in Fig. 8.15*d*.

In practice, the factored load P_u and the factored moments M_{ux} and M_{uy} to be resisted are known from the frame analysis of the structure. Therefore, the actual value of · = arctan($M_{uy} \cdot M_{ux}$) is established, and one needs only the curve of Case (*c*), Fig. 8.15*d*, to test the adequacy of the trial column. An iterative computer method to establish the interaction line for the particular value of · that applies will be described in Section 8.14.

Alternatively, simple approximate methods are widely used. These will be described in Sections 8.12 and 8.13.

8.12 LOAD CONTOUR METHOD

The load contour method is based on representing the failure surface of Fig. 8.15*d* by a family of curves corresponding to constant values of P_n (Ref. 8.8). The general form of these curves can be approximated by a nondimensional interaction equation:

$$\left(\frac{M_{nx}}{M_{nx0}}\right)^{\alpha_1} + \left(\frac{M_{ny}}{M_{ny0}}\right)^{\alpha_2} = 1.0 \qquad (8.18)$$

where

$$M_{nx} = P_n e_y$$
$$M_{nx0} = M_{nx} \qquad \text{when } M_{ny} = 0$$
$$M_{ny} = P_n e_x$$
$$M_{ny0} = M_{ny} \qquad \text{when } M_{nx} = 0$$

and α_1 and α_2 are exponents depending on column dimensions, amount and distribution of steel reinforcement, stress-strain characteristics of steel and concrete, amount of concrete cover, and size of lateral ties or spiral. When $\alpha_1 = \alpha_2 = \alpha$, the shapes of such interaction contours are as shown in Fig. 8.16 for specific α values.

Calculations reported by Bresler in Ref. 8.9 indicate that α falls in the range from 1.15 to 1.55 for square and rectangular columns. Values near the lower end of that range are the more conservative. Methods and design aids permitting a more defined estimation of α are found in Ref. 8.7.

In practice, the values of P_u, M_{ux}, and M_{uy} are known from the analysis of the structure. For a trial column section, the values of M_{nx0} and M_{ny0} corresponding to the load P_u/ϕ can easily be found by the usual methods for uniaxial bending. Then replacing M_{nx} with M_{ux}/ϕ and M_{ny} with M_{uy}/ϕ and using $\alpha_1 = \alpha_2 = \alpha$ in Eq. (8.18), or alternatively by plotting $(M_{nx}/\phi)/M_{nx0}$ and $(M_{ny}/\phi)/M_{ny0}$ in Fig. 8.16, it can be confirmed

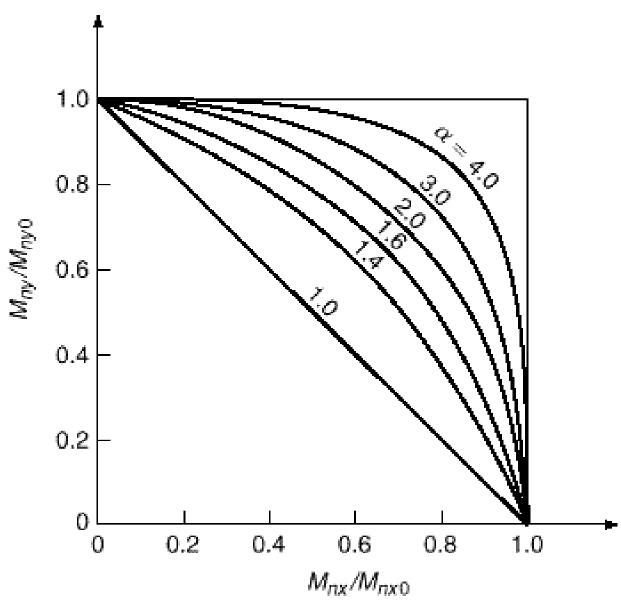


FIGURE 8.16
Interaction contours at constant P_n for varying α. *(Adapted from Ref. 8.8.)*

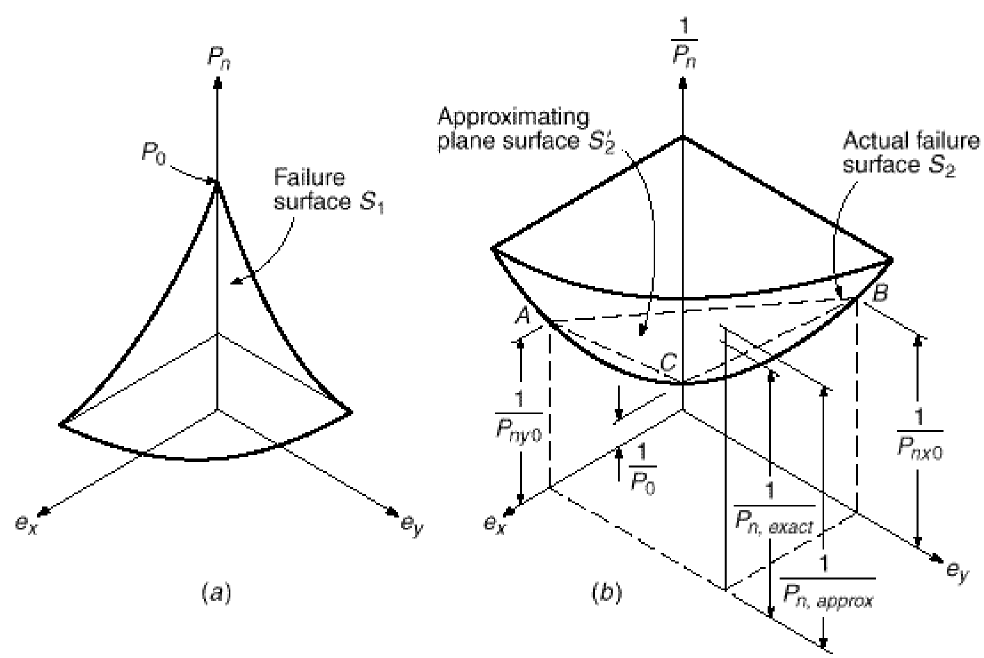
that a particular combination of factored moments falls within the load contour (safe design) or outside the contour (failure), and the design modified if necessary.

An approximate approach to the load contour method, in which the curved load contour is represented by a bilinear approximation, will be found in Ref. 8.10. It leads to a method of *trial design* in which the biaxial bending moments are represented by an equivalent uniaxial bending moment. Design charts based on this approximate approach will be found in the *ACI Design Handbook* (Ref. 8.7). Trial designs arrived at in this way should be checked for adequacy by the load contour method, described above, or by the method of reciprocal loads that follows.

8.13 Reciprocal Load Method

A simple, approximate design method developed by Bresler (Ref. 8.9) has been satisfactorily verified by comparison with results of extensive tests and accurate calculations (Ref. 8.11). It is noted that the column interaction surface in Fig. 8.15*d* can, alternatively, be plotted as a function of the axial load P_n and eccentricities $e_x = M_{ny} \cdot P_n$ and $e_y = M_{nx} \cdot P_n$, as is shown in Fig. 8.17*a*. The surface S_1 of Fig. 8.17*a* can be transformed into an equivalent failure surface S_2, as shown in Fig. 8.17*b*, where e_x and e_y are plotted against $1 \cdot P_n$ rather than P_n. Thus, $e_x = e_y = 0$ corresponds to the inverse of the capacity of the column if it were concentrically loaded, P_0, and this is plotted as point *C*. For $e_y = 0$ and any given value of e_x, there is a load P_{ny0} (corresponding to moment M_{ny0}) that would result in failure. The reciprocal of this load is plotted as point *A*. Similarly, for $e_x = 0$ and any given value of e_y, there is a certain load P_{nx0} (corresponding to moment M_{nx0}) that would cause failure, the reciprocal of which is point *B*. The values of P_{nx0} and P_{ny0} are easily established, for known eccentricities of loading applied to a given column, using the methods already established for uniaxial bending, or using design charts for uniaxial bending.

FIGURE 8.17
Interaction surfaces for the reciprocal load method.



An oblique plane S_2' is defined by the three points: A, B, and C. This plane is used as an approximation of the actual failure surface S_2. Note that, for any point on the surface S_2 (i.e., for any given combination of e_x and e_y), there is a corresponding plane S_2'. Thus, the approximation of the true failure surface S_2 involves an infinite number of planes S_2' determined by particular pairs of values of e_x and e_y, i.e., by particular points A, B, and C.

The vertical ordinate $1/P_{n,exact}$ to the true failure surface will always be conservatively estimated by the distance $1/P_{n,approx}$ to the oblique plane ABC (extended), because of the concave upward eggshell shape of the true failure surface. In other words, $1/P_{n,approx}$ is always greater than $1/P_{n,exact}$, which means that $P_{n,approx}$ is always less than $P_{n,exact}$.

Bresler's reciprocal load equation derives from the geometry of the approximating plane. It can be shown that

$$\frac{1}{P_n} = \frac{1}{P_{nx0}} + \frac{1}{P_{ny0}} - \frac{1}{P_0} \tag{8.19}$$

where P_n = approximate value of nominal load in biaxial bending with eccentricities e_x and e_y

P_{ny0} = nominal load when only eccentricity e_x is present ($e_y = 0$)

P_{nx0} = nominal load when only eccentricity e_y is present ($e_x = 0$)

P_0 = nominal load for concentrically loaded column

Equation (8.19) has been found to be acceptably accurate for design purposes provided $P_n \geq 0.10P_0$. It is not reliable where biaxial bending is prevalent and accompanied by an axial force smaller than $P_0/10$. In the case of such strongly prevalent bending, failure is initiated by yielding of the steel in tension, and the situation corresponds to the lowest tenth of the interaction diagram of Fig. 8.15*d*. In this range, it is conservative and accurate enough to neglect the axial force entirely and to calculate the section for biaxial bending only.

Over most of the range for which the Bresler method is applicable, above $0.10P_0$, is constant, although for very small eccentricities the ACI Code imposes an upper limit on the maximum design strength that has the effect of flattening the upper part of the column strength interaction curve (see Section 8.9 and Graphs A.5 through A.16 of Appendix A). When using the Bresler method for biaxial bending, it is necessary to use the uniaxial strength curve *without* the horizontal cutoff (as shown by the lighter lines in the graphs of Appendix A) in obtaining values for use in Eq. (8.19). The value of P_n obtained in this way should then be subject to the restriction, as for uniaxial bending, that it must not exceed $0.80 P_0$ for tied columns and $0.85 P_0$ for spirally reinforced columns.

In a typical design situation, given the size and reinforcement of the trial column and the load eccentricities e_y and e_x, one finds by computation or from design charts the nominal loads P_{nx0} and P_{ny0} for uniaxial bending around the X and Y axes respectively, and the nominal load P_0 for concentric loading. Then $1/P_n$ is computed from Eq. (8.19) and, from that, P_n is calculated. The design requirement is that the factored load P_u must not exceed P_n, as modified by the horizontal cutoff mentioned above, if applicable.

EXAMPLE 8.5 **Design of column for biaxial bending.** The 12 × 20 in. column shown in Fig. 8.18 is reinforced with eight No. 9 (No. 29) bars arranged around the column perimeter, providing an area A_{st} = 8.00 in^2. A factored load P_u of 255 kips is to be applied with eccentricities e_y = 3 in. and e_x = 6 in., as shown. Material strengths are f_c' = 4 ksi and f_y = 60 ksi. Check the

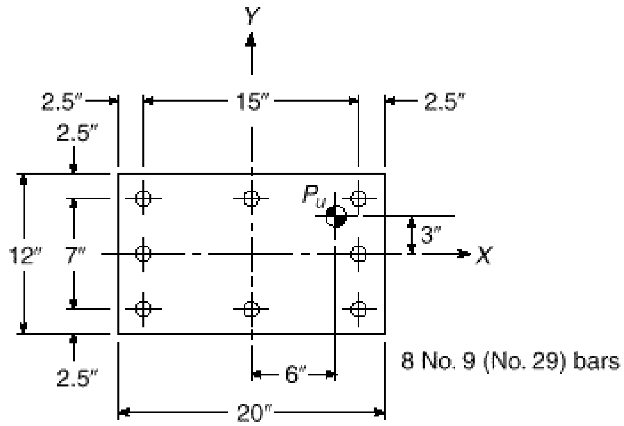


FIGURE 8.18
Column cross section for Example 8.5.

adequacy of the trial design (*a*) using the reciprocal load method and (*b*) using the load contour method.

Solution.

(*a*) By the reciprocal load method, first considering bending about the *Y* axis, · = 15· 20 = 0.75, and $e\cdot h$ = 6· 20 = 0.30. With the reinforcement ratio of $A_{st}\cdot bh$ = 8.00· 240 = 0.033, using the average of Graphs A.6 (· = 0.70) and A.7 (· = 0.80)

$$\frac{P_{ny0}}{f_c A_g}\cdot\text{avg}\cdot = \frac{0.62 + 0.66}{2} = 0.64 \qquad P_{ny0} = 0.64 \times 4 \times 240 = 614 \text{ kips}$$

$$\frac{P_0}{f_c A_g} = 1.31 \qquad P_0 = 1.31 \times 4 \times 240 = 1258 \text{ kips}$$

Then for bending about the *X* axis, · = 7· 12 = 0.58 (say 0.60), and $e\cdot h$ = 3· 12 = 0.25. Graph A.5 of Appendix A gives

$$\frac{P_{nx0}}{f_c A_g} = 0.65 \qquad P_{nx0} = 0.65 \times 4 \times 240 = 624 \text{ kips}$$

$$\frac{P_0}{f_c A_g} = 1.31 \qquad P_0 = 1.31 \times 4 \times 240 = 1258 \text{ kips}$$

Substituting these values in Eq. (8.19) results in

$$\frac{1}{P_n} = \frac{1}{624} + \frac{1}{614} - \frac{1}{1258} = 0.00244$$

from which P_n = 410 kips. Thus, according to the Bresler method, the design load of P_u = 0.65 × 410 = 267 kips can be applied safely.

(*b*) By the load contour method, for *Y* axis bending with $P_u\cdot(\cdot f_c'A_g)$ = 255· (0.65 × 4 × 240) = 0.41. The average from Graphs A.6 and A.7 of Appendix A is

$$\frac{M_{ny0}}{f_c A_g h}\cdot\text{avg}\cdot = \frac{0.212 + 0.235}{2} = 0.224$$

Hence, M_{ny0} = 0.224 × 4 × 240 × 20 = 4300 in-kips. Then for *X* axis bending, with $P_u\cdot(\cdot f_c'A_g)$ = 0.41, as before, from Graph A.5,

$$\frac{M_{nx0}}{f_c A_g h} = 0.186$$

So $M_{nx0} = 0.186 \times 4 \times 240 \times 12 = 2140$ in-kips. The factored load moments about the Y and X axes respectively are

$$M_{uy} = 255 \times 6 = 1530 \text{ in-kips}$$

$$M_{ux} = 255 \times 3 = 765 \text{ in-kips}$$

Adequacy of the trial design will now be checked using Eq. (8.18) with an exponent α conservatively taken equal to 1.15. Then with $M_{nx} = M_{ux}/\phi$ and $M_{ny} = M_{uy}/\phi$, that equation indicates

$$\left(\frac{765/0.65}{2140}\right)^{1.15} + \left(\frac{1530/0.65}{4300}\right)^{1.15} = 0.502 + 0.500 = 1.002$$

This is close enough to 1.0 that the design would be considered safe by the load contour method also.

In actual practice, the values of α used in Eq. (8.18) should be checked, for the specific column, because predictions of that equation are quite sensitive to changes in α. In Ref. 8.10, it is shown that $\alpha = \log 0.5/\log \beta$, where values of β can be tabulated for specific column geometries, material strengths, and load ranges (see Ref. 8.7). For the present example, it can be confirmed from Ref. 8.7 that $\beta = 0.56$ and hence $\alpha = 1.19$, approximately as chosen.

One observes that, in Example 8.5*a*, an eccentricity in the Y direction equal to 50 percent of that in the X direction causes a reduction in nominal capacity of 33 percent, i.e., from 614 to 410 kips. For cases in which the ratio of eccentricities is smaller, there is some justification for the frequent practice in framed structures of neglecting the bending moments in the direction of the smaller eccentricity. *In general, biaxial bending should be taken into account when the estimated eccentricity ratio approaches or exceeds 0.2.*

8.14 Computer Analysis for Biaxial Bending of Columns

Although the load contour method and the reciprocal load method are widely used in practice, each has serious shortcomings. With the load contour method, selection of the appropriate value of the exponent α is made difficult by a number of factors relating to column shape and bar distribution. For many cases, the usual assumption that $\alpha_1 = \alpha_2$ is a poor approximation. Design aids are available, but they introduce further approximations, e.g., the use of a bilinear representation of the load contour. The reciprocal load method is very simple to use, but the representation of the curved failure surface by an approximating plane is not reliable in the range of large eccentricities, where failure is initiated by steel yielding.

With the general availability of desktop computers, it is better to use simpler methods to obtain faster, and more exact, solutions to the biaxial column problem. Such a method is that developed by Ehsani (Ref. 8.12). A column strength interaction curve is established for a trial column, exactly analogous to the curve for axial load plus uniaxial bending, as described in Sections 8.3 to 8.7. However, the curve is generated for the particular value of the eccentricity angle that applies, as determined by the ratio of M_{uy}/M_{ux} from the structural frame analysis [see Case (*c*) of Fig. 8.15*d*]. This is done by taking successive choices of neutral axis distance, measured in this case along one face of the column from the most heavily compressed corner, from very small (large eccentricity) to very large (small eccentricity), then calculating the axial force P_n and moments M_{nx} and M_{ny}. For each neutral axis distance, iteration is performed with successive values of the orientation angle · , Fig. 8.15*c*, until · = arctan $M_{ny} \cdot M_{nx}$ is in agreement with the value of · = arctan $M_{uy} \cdot M_{ux}$ from the structural frame analysis. Thus, one point on the curve (*c*) of Fig. 8.15*d* is established. The sequence of calculations is repeated: another choice of neutral axis distance is made, a value of · is selected, the axial force and moments are calculated, · is found, and the value of · is iterated until · is correct. Thus, the next point is established, and so on, until the complete strength interaction curve for that particular value of · is complete. ACI Code safety provisions may then be imposed in the usual way, and the adequacy of the proposed design tested, for the known load and moments, against the design strength curve for the trial column.

The method is obviously impractical for manual calculation, but the iterative steps are easily and quickly performed by desktop computers, which can also provide a graphical presentation of results. Full details will be found in Ref. 8.12.

A number of computer programs for biaxial bending are available commercially, such as PCACOLUMN (Portland Cement Association, Skokie, Illinois) and HBCOLUMN (Concrete Reinforcing Steel Institute, Schaumburg, Illinois).

8.15 Bar Splicing in Columns

The main vertical reinforcement in columns is usually spliced just above each floor, or sometimes at alternate floors. This permits the column steel area to be reduced progressively at the higher levels in a building, where loads are smaller, and in addition avoids handling and supporting very long column bars. Column steel may be spliced by lapping, by butt welding, by various types of mechanical connections, or by direct end bearing, using special devices to ensure proper alignment of bars.

Special attention must be given to the problem of bar congestion at splices. Lapping the bars, for example, effectively doubles the steel area in the column cross section at the level of the splice and can result in problems placing concrete and meeting the ACI Code requirement for minimum lateral spacing of bars ($1.5d_b$ or 1.5 in.). To avoid difficulty, column steel percentages are often limited in practice to not more than about 4 percent, or the bars are extended two stories and staggered splices are used.

The most common method of splicing column steel is the simple lapped bar splice, with the bars in contact throughout the lapped length. It is standard practice to offset the lower bars, as shown in Fig. 8.19, to permit the proper positioning of the upper bars. To prevent outward buckling of the bars at the bottom bend point of such an offset, with spalling of the concrete cover, it is necessary to provide special lateral reinforcement in the form of extra ties. According to ACI Code 7.8.1, the slope of the inclined part of an offset bar must not exceed 1 in 6, and lateral steel must be provided to resist $1\frac{1}{2}$ times the horizontal component of the computed force in the inclined part of the offset bar. This special reinforcement must be placed not more than 6 in. from the point of bend, as shown in Fig. 8.19. Elsewhere in the column, above and below the floor, the usual spacing requirements described in Section 8.2 apply, except that ties must be located not more than one-half the normal spacing s above the floor. Where beams frame from four directions into a joint, as shown in Fig. 8.19, ties may be terminated not more than 3 in. below the lowest reinforcement in the shallowest of such beams, according to ACI Code 7.10.5. If beams are not present on four sides, such as for exterior columns, ties must be placed vertically at the usual spacing

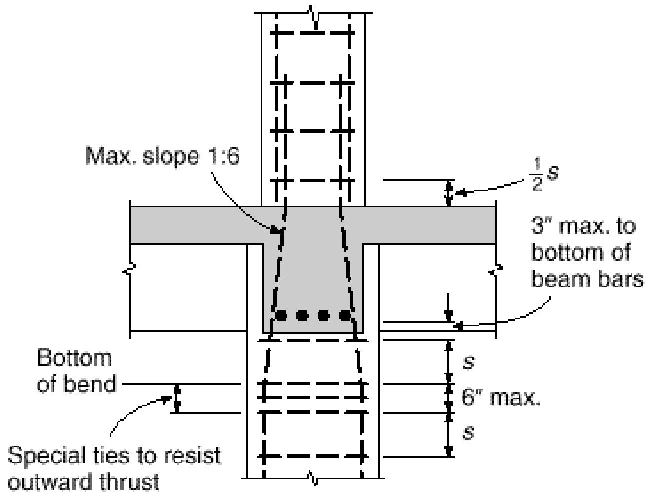


FIGURE 8.19
Splice details at typical interior column.

through the depth of the joint to a level not more than one-half the usual spacing s below the lowest reinforcement in the slab.

Analogous requirements are found in ACI Code 7.10.4 and are illustrated in Ref. 8.1 for spirally reinforced columns.

As discussed in Section 5.11, in frames subjected to lateral loading, a viable alternative to splicing bars just above the floor is to splice them in the center half of the column height, where the moment due to lateral loading is much lower than at floor level. Splicing near midheight is mandatory in "special moment frames" designed for seismic loading (Chapter 20). The use of midheight splices removes the requirement for the special ties shown in Fig. 8.19 because bent bars are not used.

Column splices are mainly compression splices, although load combinations producing moderate to large eccentricity require that splices transmit tension as well. ACI Code 12.17 permits splicing by lapping, butt welding, mechanical connectors, or end bearing. As discussed in Section 5.11, the length of compression lap splices may be reduced in cases where ties or spiral reinforcement throughout the lap length meet specific requirements. If the column bars are in tension, Class A tension lap splices are permitted if the tensile stress does not exceed $0.5f_y$ and less than one-half of the bars are spliced at any section. Class B tension splices are required if the tensile stresses are higher than $0.5f_y$ under factored loads or where more than one-half of the reinforcement is spliced at one location. When end bearing splices are used, they must be staggered or additional reinforcement must be added so that the continuing bars on each column face possess a tensile strength not less than $0.25f_y$ times the area of the vertical reinforcement on that face.

Full requirements for both compression and tension lap splices are discussed in Section 5.11, and the design of a compression splice in a typical column is illustrated in Example 5.4.

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8.8. F. N. Pannell, "Failure Surfaces for Members in Compression and Biaxial Bending," *J. ACI*, vol. 60, no. 1, 1963, pp. 129–140.
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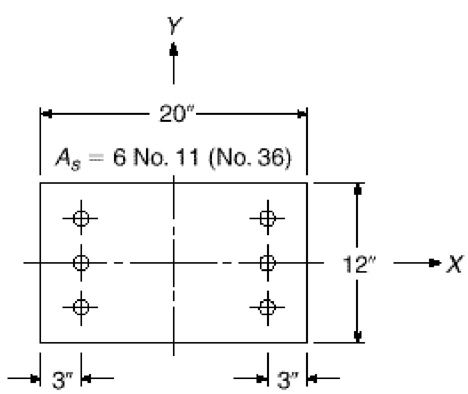
8.11. L. N. Ramamurthy, "Investigation of the Ultimate Strength of Square and Rectangular Columns under Biaxially Eccentric Loads," in *Symp. Reinforced Concrete Columns*, SP-13, American Concrete Institute, Detroit, MI, 1966, pp. 263–298.
8.12. M. R. Ehsani, "CAD for Columns," *Concr. Intl.*, vol. 8, no. 9, 1986, pp. 43–47.

8.1. A 16 in. square column is reinforced with four No. 14 (No. 43) bars, one in each corner, with cover distances 3 in. to the steel center in each direction. Material strengths are f'_c = 5000 psi and f_y = 60,000 psi. Construct the interaction diagram relating axial strength P_n and flexural strength M_n. Bending will be about an axis parallel to one face. Calculate the coordinates for P_o, P_b, and at least three other representative points on the curve.

8.2. Plot the design strength curve relating · P_n and · M_n for the column of Problem 8.1. Design and detail the tie steel required by the ACI Code. Is the column a good choice to resist a load P_u = 540 kips applied with an eccentricity e = 4.44 in?

8.3. The short column shown in Fig. P8.3 will be subjected to an eccentric load causing uniaxial bending about the Y axis. Material strengths are f_y = 60 ksi and f'_c = 4 ksi.

FIGURE P8.3



(*a*) Construct the nominal strength interaction curve for this column, calculating no fewer than five points, including those corresponding to pure bending, pure axial thrust, and balanced failure.

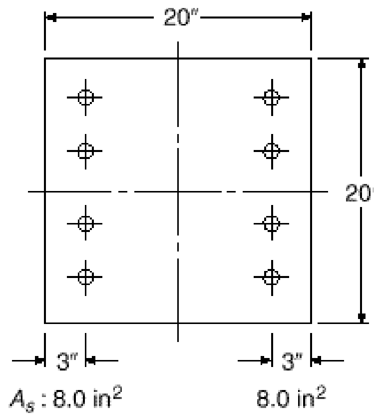
(*b*) Compare the calculated values with those obtained using Graph A.10 in Appendix A.

(*c*) Show on the same drawing the design strength curve obtained through introduction of the ACI ϕ factors.

(*d*) Design the lateral reinforcement for the column, giving key dimensions for ties.

8.4. The column shown in Fig. P8.4 is subjected to axial load and bending moment causing bending about an axis parallel to that of the rows of bars. What moment would cause the column to fail if the axial load applied simultaneously was 500 kips? Material strengths are $f_c' = 4000$ psi and $f_y = 60$ ksi.

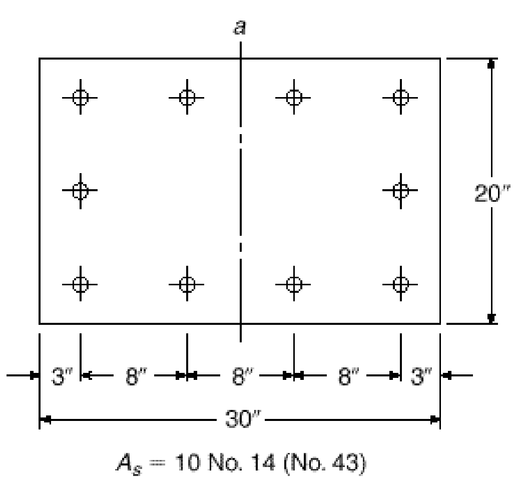
FIGURE P8.4



8.5. What is the strength M_n of the column of Problem 8.4 if it were loaded in pure bending (axial force = 0) about one principal axis?

8.6. Construct the interaction diagram relating P_n to M_n for the building column shown in Fig. P8.6. Bending will be about the axis *a-a*. Calculate specific coordinates for concentric loading ($e = 0$), for P_b, and at least three other points, well chosen, on the curve. Material strengths are $f_c' = 8000$ psi and $f_y = 60{,}000$ psi.

FIGURE P8.6



8.7. A short rectangular reinforced concrete column shown in Fig. P8.7 is to be a part of a long-span rigid frame and will be subjected to high bending moments combined with relatively low axial loads, causing bending about the strong axis. Because of the high eccentricity, steel is placed unsymmetrically as shown, with three No. 14 (No. 43) bars near the tension face and two No. 11 (No. 36) bars near the compression face. Material strengths are $f'_c = 6$ ksi and $f_y = 75$ ksi. Construct the complete strength interaction diagram plotting P_n vs. M_n, relating eccentricities to the plastic centroid of the column (not the geometric center).

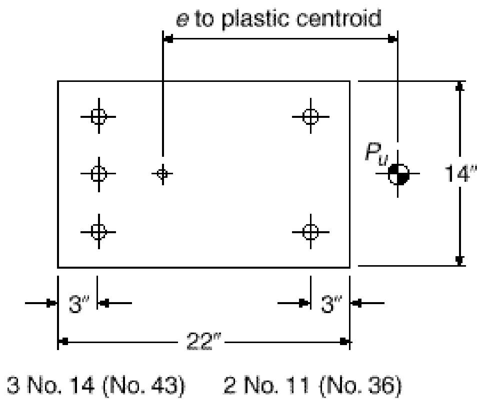


FIGURE P8.7

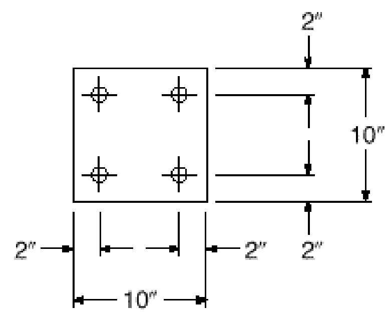


FIGURE P8.8

8.8. The square column shown in Fig. P8.8 must be designed for a factored axial load of 130 kips. Material strengths are $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

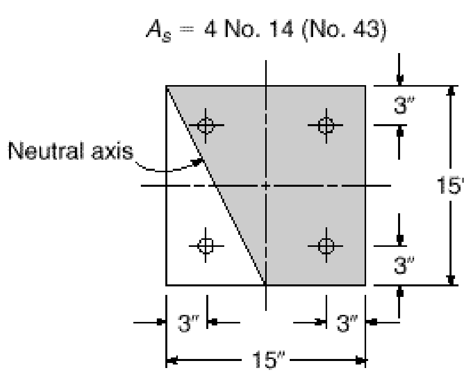
(*a*) Select the longitudinal and transverse reinforcement for an eccentricity $e_y = 2.7$ in.

(*b*) Select the longitudinal and transverse reinforcement for the same axial load with $e_x = e_y = 2.7$ in.

(*c*) Construct the strength interaction diagram and design strength curves for the column designed in part (*b*), given that the column will be subjected to biaxial bending with equal eccentricities about both principal axes.

8.9. The square column shown in Fig. P8.9 is a corner column subject to axial load and biaxial bending. Material strengths are $f_y = 60{,}000$ psi and $f'_c = 4000$ psi.

FIGURE P8.9



(*a*) Find the unique combination of P_n, M_{nx}, and M_{ny} that will produce incipient failure with the neutral axis located as in the figure. The compressive zone is shown shaded. Note that the actual neutral axis is shown, not the equivalent rectangular stress block limit; however, the rectangular stress block may be used as the basis of calculations.

(*b*) Find the angle between the neutral axis and the eccentricity axis, the latter defined as the line from the column center to the point of load.

8.10. For the axial load P_n found in Problem 8.9, and for the same column, with the same eccentricity ratio $e_y \cdot e_x$, find the values of M_{nx} and M_{ny} that would produce incipient failure using the load contour method. Compare with the results of Problem 8.9. Take · = 1.30, and use the graphs in Appendix A, as appropriate.

8.11. For the eccentricities e_x and e_y found in Problem 8.9, find the value of axial load P_n that would produce incipient failure using the reciprocal load (Bresler) method. Use the graphs in Appendix A, as appropriate. Compare with the results of Problems 8.9 and 8.10.