9 Slender Columns

9.1 Introduction

The material presented in Chapter 8 pertained to concentrically or eccentrically loaded *short columns*, for which the strength is governed entirely by the strength of the materials and the geometry of the cross section. Most columns in present-day practice fall in that category. However, with the increasing use of high-strength materials and improved methods of dimensioning members, it is now possible, for a given value of axial load, with or without simultaneous bending, to design a much smaller cross section than in the past. This clearly makes for more slender members. It is because of this, together with the use of more innovative structural concepts, that rational and reliable design procedures for slender columns have become increasingly important.

A column is said to be *slender* if its cross-sectional dimensions are small compared with its length. The degree of slenderness is generally expressed in terms of the slenderness ratio l/r, where l is the unsupported length of the member and r is the radius of gyration of its cross section, equal to $\sqrt{I/A}$. For square or circular members, the value of r is the same about either axis; for other shapes r is smallest about the minor principal axis, and it is generally this value that must be used in determining the slenderness ratio of a free-standing column.

It has long been known that a member of great slenderness will collapse under a smaller compression load than a stocky member with the same cross-sectional dimensions. When a stocky member, say with $l/r = 10$ (e.g., a square column of length equal to about 3 times its cross-sectional dimension h), is loaded in axial compression, it will fail at the load given by Eq. (8.3), because at that load both concrete and steel are stressed to their maximum carrying capacity and give way, respectively, by crushing and by yielding. If a member with the same cross section has a slenderness ratio $l/r = 100$ (e.g., a square column hinged at both ends and of length equal to about 30 times its section dimension), it may fail under an axial load equal to one-half or less of that given by Eq. (8.3). In this case, collapse is caused by buckling, i.e., by sudden lateral displacement of the member between its ends, with consequent overstressing of steel and concrete by the bending stresses that are superimposed on the axial compressive stresses.

Most columns in practice are subjected to bending moments as well as axial loads, as was made clear in Chapter 8. These moments produce lateral deflection of a member between its ends and may also result in relative lateral displacement of joints. Associated with these lateral displacements are *secondary moments* that add to the primary moments and that may become very large for slender columns, leading to failure. A practical definition of a slender column is one for which there is a significant reduction in axial load capacity because of these secondary moments. In the development of ACI Code column provisions, for example, any reduction greater than about 5 percent is considered significant, requiring consideration of slenderness effects.

The ACI Code and Commentary contain detailed provisions governing the design of slender columns. ACI Code 10.11, 10.12, and 10.13 present approximate methods for accounting for slenderness through the use of *moment magnification factors*. The provisions are quite similar to those used for steel columns designed under the American Institute of Steel Construction (AISC) Specification. Alternatively, in ACI Code 10.10, a more fundamental approach is endorsed, in which the effect of lateral displacements is accounted for directly in the frame analysis. Because of the increasing complexity of the moment magnification approach, as it has been refined in recent years, with its many detailed requirements, and because of the universal availability of computers in the design office, there is increasing interest in "second-order analysis" as suggested in ACI Code 10.10, in which the effect of lateral displacements is computed directly.

As noted, most columns in practice continue to be short columns. Simple expressions are included in the ACI Code to determine whether slenderness effects must be considered. These will be presented in Section 9.4 following the development of background information in Sections 9.2 and 9.3 relating to column buckling and slenderness effects.

9.2 Concentrically Loaded Columns

The basic information on the behavior of straight, concentrically loaded slender columns was developed by Euler more than 200 years ago. In generalized form, it states that such a member will fail by buckling at the critical load

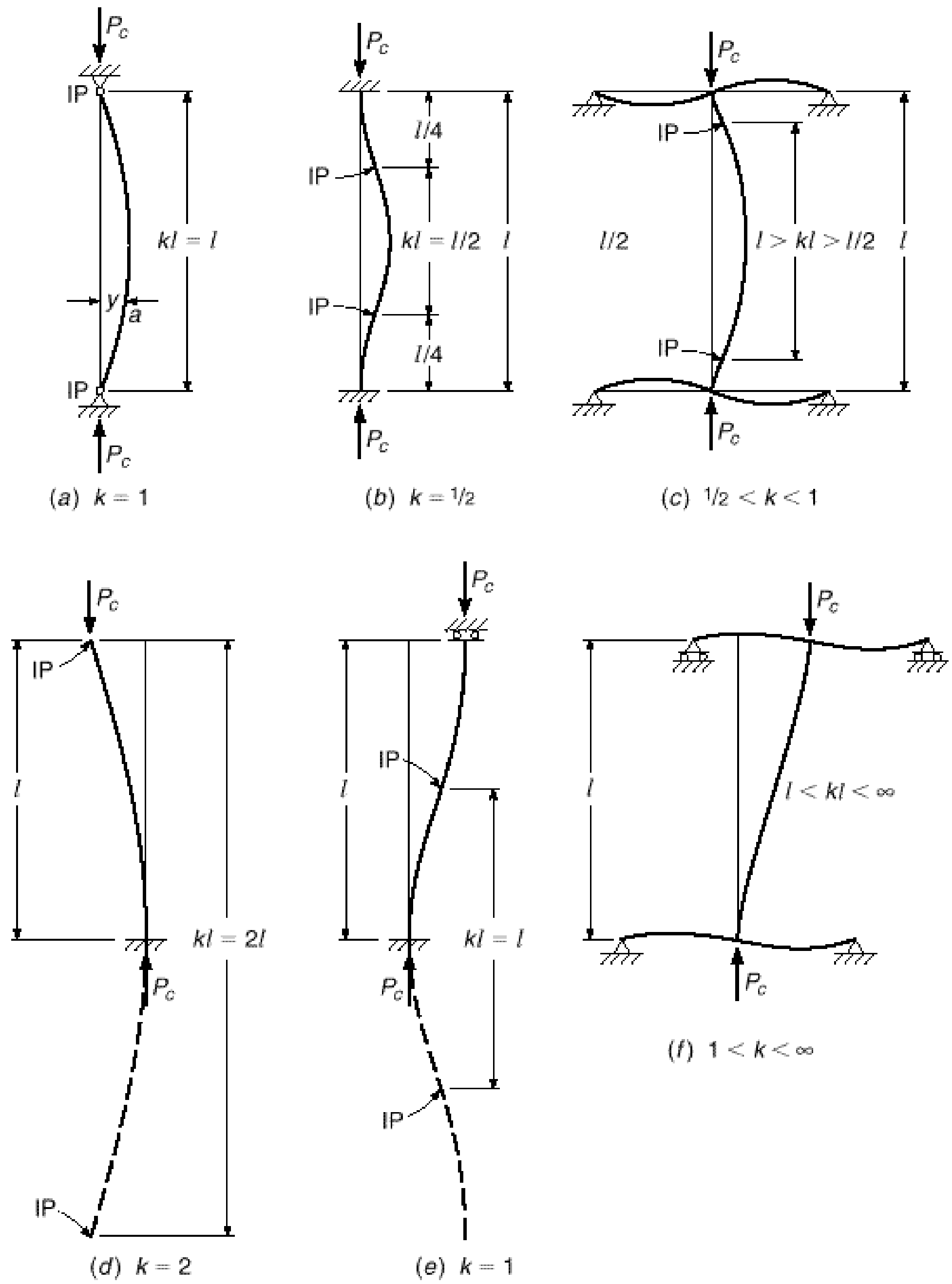
$$P_c = \frac{\pi^2 E_t I}{(kl)^2} \tag{9.1}$$

It is seen that the buckling load decreases rapidly with increasing *slenderness ratio* kl/r (Ref. 9.1).

For the simplest case of a column hinged at both ends and made of elastic material, E_t simply becomes Young's modulus and kl is equal to the actual length l of the column. At the load given by Eq. (9.1), the originally straight member buckles into a half sine wave, as shown in Fig. 9.1*a*. In this bent configuration, bending moments Py act at any section such as a; y is the deflection at that section. These deflections continue to increase until the bending stress caused by the increasing moment, together with the original compression stress, overstresses and fails the member.

If the stress-strain curve of a short piece of the given member has the shape shown in Fig. 9.2*a*, as it would be for reinforced concrete columns, E_t is equal to Young's modulus, provided that the buckling stress P_c/A is below the proportional limit f_p. If the strain is larger than f_p, buckling occurs in the inelastic range. In this case, in Eq. (9.1), E_t is the tangent modulus, i.e., the slope of the tangent to the stress-strain curve. As the stress increases, E_t decreases. A plot of the buckling load vs. the slenderness ratio, the so-called column curve, therefore has the shape given in Fig. 9.2*b*, which shows the reduction in buckling strength with increasing slenderness. For very stocky columns, the value of the buckling load, calculated from Eq. (9.1), exceeds the direct crushing strength of the stocky column P_n, given by Eq. (8.3). This is also shown in Fig. 9.2*b*. Correspondingly, there is a limiting slenderness ratio

FIGURE 9.1
Buckling and effective length of axially loaded columns.



$(kl \cdot r)_{lim}$. For values smaller than this, failure occurs by simple crushing, regardless of $kl \cdot r$; for values larger than $(kl \cdot r)_{lim}$, failure occurs by buckling, the buckling load or stress decreasing for greater slenderness.

If a member is fixed against rotation at both ends, it buckles in the shape of Fig. 9.1*b*, with inflection points (IP) as shown. The portion between the inflection points is in precisely the same situation as the hinge-ended column of Fig. 9.1*a*, and thus, the *effective length kl* of the fixed-fixed column, i.e., the distance between inflection

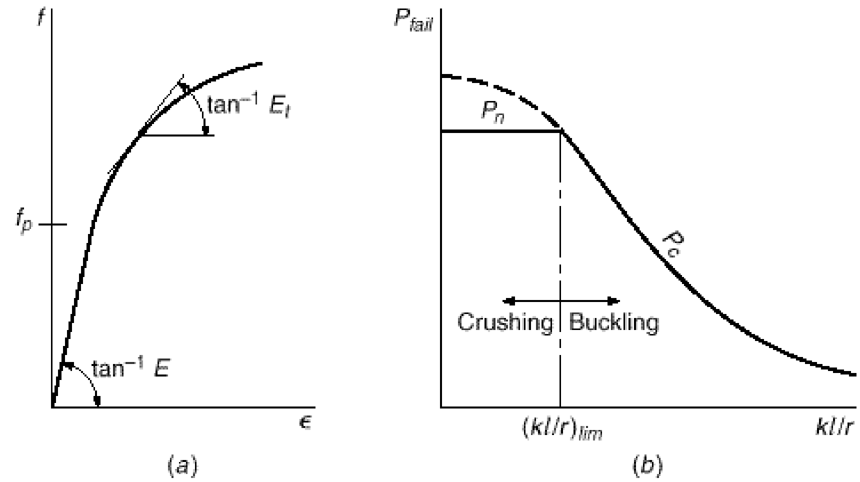


FIGURE 9.2
Effect of slenderness on strength of axially loaded columns.

points, is seen to be $kl = l/2$. Equation (9.1) shows that an elastic column fixed at both ends will carry 4 times as much load as when hinged.

Columns in real structures are rarely either hinged or fixed but have ends partially restrained against rotation by abutting members. This is shown schematically in Fig. 9.1*c*, from which it is seen that for such members the effective length kl, i.e., the distance between inflection points, has a value between l and $l/2$. The precise value depends on the degree of end restraint, i.e., on the ratio of the stiffness EI/l of the column to the sum of stiffnesses EI/l of the restraining members at both ends.

In the columns of Fig. 9.1*a* to *c*, it was assumed that one end was prevented from moving laterally relative to the other end, by horizontal bracing or otherwise. In this case, it is seen that the effective length kl is always smaller than (or at most it is equal to) the real length l.

If a column is fixed at one end and entirely free at the other (cantilever column or flagpole), it buckles as shown in Fig. 9.1*d*. That is, the upper end moves laterally with respect to the lower, a kind of deformation known as *sidesway*. It buckles into a quarter of a sine wave and is therefore analogous to the upper half of the hinged column in Fig. 9.1*a*. The inflection points, one at the end of the actual column and the other at the imaginary extension of the sine wave, are a distance $2l$ apart, so that the effective length is $kl = 2l$.

If the column is rotationally fixed at both ends but one end can move laterally with respect to the other, it buckles as shown in Fig. 9.1*e*, with an effective length $kl = l$. If one compares this column, fixed at both ends but free to sidesway, with a fixed-fixed column that is braced against sidesway (Fig. 9.1*b*), one sees that the effective length of the former is twice that of the latter. By Eq. (9.1), this means that the buckling strength of an elastic fixed-fixed column that is free to sidesway is only one-quarter that of the same column when braced against sidesway. This is an illustration of the general fact that *compression members free to buckle in a sidesway mode are always considerably weaker than when braced against sidesway.*

Again, the ends of columns in actual structures are rarely either hinged, fixed, or entirely free but are usually restrained by abutting members. If sidesway is not prevented, buckling occurs as shown in Fig. 9.1*f*, and the effective length, as before, depends on the degree of restraint. If the cross beams are very rigid compared with the

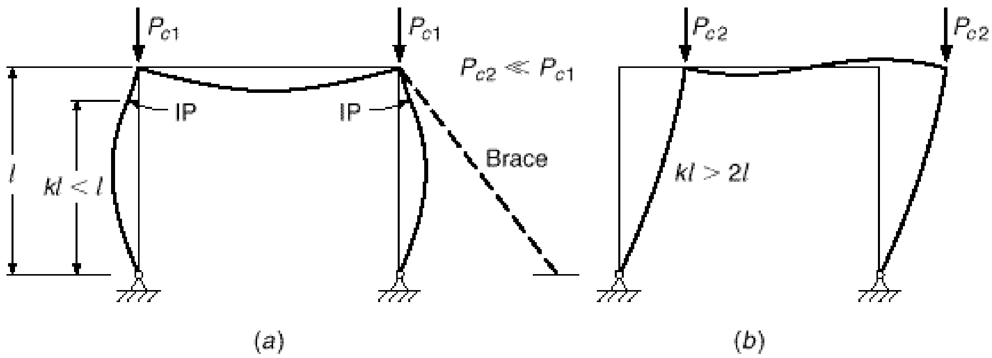


FIGURE 9.3
Rigid-frame buckling: (*a*) laterally braced; (*b*) unbraced.

column, the case of Fig. 9.1*e* is approached and kl is only slightly larger than l. On the other hand, if the restraining members are extremely flexible, a hinged condition is approached at both ends. Evidently, a column hinged at both ends and free to sidesway is unstable. It will simply topple, being unable to carry any load whatever.

In reinforced concrete structures, one is rarely concerned with single members but rather with rigid frames of various configurations. The manner in which the relationships just described affect the buckling behavior of frames is illustrated by the simple portal frame shown in Fig. 9.3, with loads applied concentrically to the columns. If sidesway is prevented, as indicated schematically by the brace in Fig. 9.3*a*, the buckling configuration will be as shown. The buckled shape of the column corresponds to that in Fig. 9.1*c*, except that the lower end is hinged. It is seen that the effective length kl is smaller than l. On the other hand, if no sidesway bracing is provided to an otherwise identical frame, buckling occurs as shown in Fig. 9.3*b*. The column is in a situation similar to that shown in Fig. 9.1*d*, upside down, except that the upper end is not fixed but only partially restrained by the girder. It is seen that the effective length kl exceeds $2l$ by an amount depending on the degree of restraint. The buckling strength depends on kl/r in the manner shown in Fig. 9.2*b*. As a consequence, even though they are dimensionally identical, the unbraced frame will buckle at a radically smaller load than the braced frame.

In summary, the following can be noted:

1. The strength of concentrically loaded columns decreases with increasing slenderness ratio kl/r.
2. In columns that are *braced against sidesway* or that are parts of frames braced against sidesway, the effective length kl, i.e., the distance between inflection points, falls between $l/2$ and l, depending on the degree of end restraint.
3. The effective lengths of columns that are *not braced against sidesway* or that are parts of frames not so braced are always larger than l, the more so the smaller the end restraint. In consequence, the buckling load of a frame not braced against sidesway is always substantially smaller than that of the same frame when braced.

9.3 Compression Plus Bending

Most reinforced concrete compression members are also subject to simultaneous flexure, caused by transverse loads or by end moments owing to continuity. The behavior of members subject to such combined loading also depends greatly on their slenderness.

FIGURE 9.4
Moments in slender members with compression plus bending, bent in single curvature.

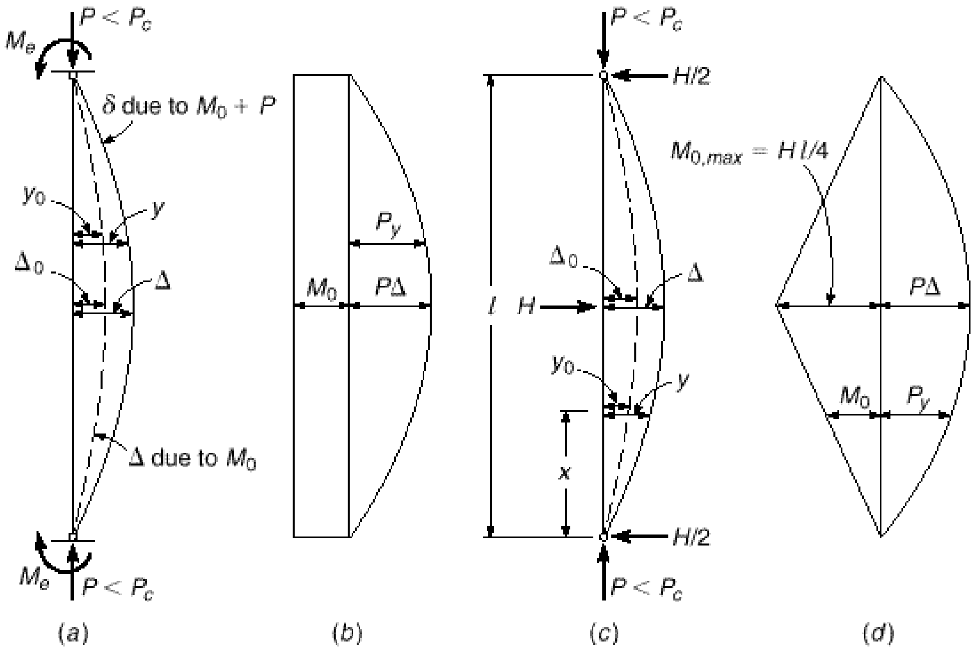


Figure 9.4*a* shows such a member, axially loaded by P and bent by equal end moments M_e. If no axial load were present, the moment M_0 in the member would be constant throughout and equal to the end moments M_e. This is shown in Fig. 9.4*b*. In this situation, i.e., in simple bending without axial compression, the member deflects as shown by the dashed curve of Fig. 9.4*a*, where y_0 represents the deflection at any point caused by bending only. When P is applied, the moment at any point increases by an amount equal to P times its lever arm. The increased moments cause additional deflections, so that the deflection curve under the simultaneous action of P and M_0 is the solid curve of Fig. 9.4*a*. At any point, then, the total moment is now

$$M = M_0 + Py \tag{9.2}$$

i.e., the total moment consists of the moment M_0 that acts in the presence of P and the additional moment caused by P, equal to P times the deflection. This is one illustration of the so-called P-Δ effect.

A similar situation is shown in Fig. 9.4*c*, where bending is caused by the transverse load H. When P is absent, the moment at any point x is $M_0 = Hx \cdot 2$, with a maximum value at midspan equal to $Hl \cdot 4$. The corresponding M_0 diagram is shown in Fig. 9.4*d*. When P is applied, additional moments Py are caused again, distributed as shown, and the total moment at any point in the member consists of the same two parts as in Eq. (9.2).

The deflections y of elastic columns of the type shown in Fig. 9.4 can be calculated from the deflections y_0, that is, from the deflections of the corresponding beam without axial load, using the following expression (see, for example, Ref. 9.1).

$$y = y_0 \frac{1}{1 - P \cdot P_c} \tag{9.3}$$

If Δ is the deflection at the point of maximum moment M_{max}, as shown in Fig. 9.4, M_{max} can be calculated using Eqs. (9.2) and (9.3).

$$M_{max} = M_0 + P\ \ = M_0 + P\ \ _0 \frac{1}{1 - P/P_c} \tag{9.4}$$

It can be shown (Ref. 9.2) that Eq. (9.4) can be written

$$M_{max} = M_0 \frac{1 + \ \ P/P_c}{1 - P/P_c} \tag{9.5}$$

where   is a coefficient that depends on the type of loading and varies between about ±0.20 for most practical cases. Because P/P_c is always significantly smaller than 1, the second term in the numerator of Eq. (9.5) is small enough to be neglected. Doing so, one obtains the simplified design equation

$$M_{max} = M_0 \frac{1}{1 - P/P_c} \tag{9.6}$$

where $1/(1 - P/P_c)$ is known as the *moment magnification factor*, which reflects the amount by which the moment M_0 is magnified by the presence of a simultaneous axial force P.

Since P_c decreases with increasing slenderness ratio, it is seen from Eq. (9.6) that the moment M in the member increases with the slenderness ratio kl/r. The situation is shown schematically in Fig. 9.5. It indicates that, for a given transverse loading (i.e., a given value of M_0), an axial force P causes a larger additional moment in a slender member than in a stocky member.

In the two members in Fig. 9.4, the largest moment caused by P, namely $P\Delta$, adds directly to the maximum value of M_0; for example,

$$M_0 = \frac{Hl}{4}$$

in Fig. 9.4*d*. As P increases, the maximum moment at midspan increases at a rate faster than that of P in the manner given by Eqs. (9.2) and (9.6) and shown in Fig. 9.6. The member will fail when the simultaneous values of P and M become equal to P_n and M_n, the nominal strength of the cross section at the location of maximum moment.

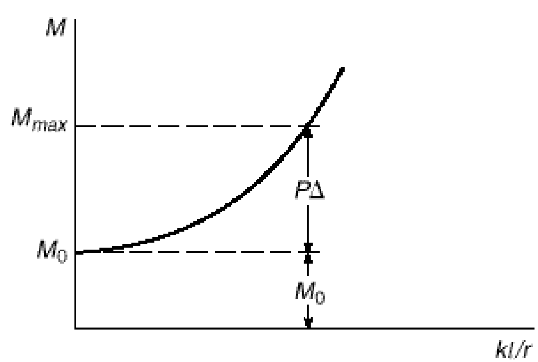


FIGURE 9.5
Effect of slenderness on column moments.

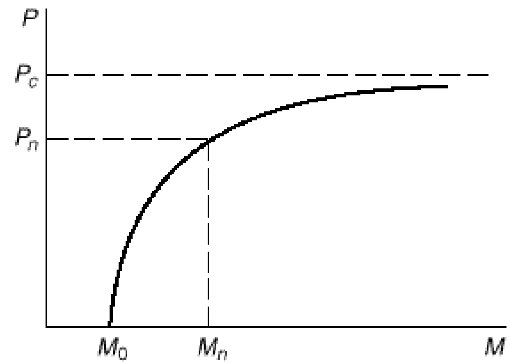


FIGURE 9.6
Effect of axial load on column moments.

This direct addition of the maximum moment caused by P to the maximum moment caused by the transverse load, clearly the most unfavorable situation, does not result for all types of deformations. For instance, the member in Fig. 9.7*a*, with equal and opposite end moments, has the M_0 diagram shown in Fig. 9.7*b*. The deflections caused by M_0 alone are again magnified when an axial load P is applied. In this case, these deflections under simultaneous bending and compression can be approximated by (Ref. 9.1)

$$y = y_0 \frac{1}{1 - P/4P_c} \tag{9.7}$$

By comparison with Eq. (9.3) it is seen that the deflection magnification here is much smaller.

The additional moments Py caused by the axial load are distributed as shown in Fig. 9.7*c*. Although the M_0 moments are largest at the ends, the Py moments are seen

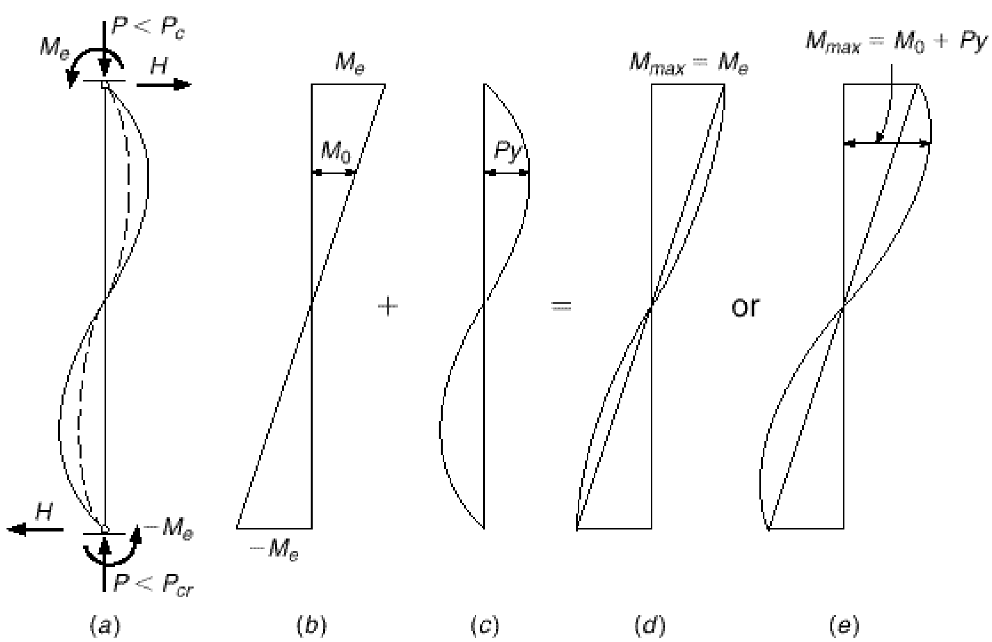


FIGURE 9.7
Moments in slender members with compression plus bending, bent in double curvature.

to be largest at some distance from the ends. Depending on their relative magnitudes, the total moments $M = M_0 + Py$ are distributed as shown in either Fig. 9.7*d* or *e*. In the former case, the maximum moment continues to act at the end and to be equal to M_e; the presence of the axial force, then, does not result in any increase in the maximum moment. Alternatively, in the case of Fig. 9.7*e*, the maximum moment is located at some distance from the end; at that location M_0 is significantly smaller than its maximum value M_e, and for this reason the added moment Py increases the maximum moment to a value only moderately greater than M_e.

Comparing Figs. 9.4 and 9.7, one can generalize as follows. The moment M_0 will be magnified most strongly when the location where M_0 is largest coincides with that where the deflection y_0 is largest. This occurs in members bent into single curvature by symmetrical loads or equal end moments. If the two end moments of Fig. 9.4*a* are unequal but of the same sign, i.e., producing single curvature, M_0 will still be strongly magnified, though not quite so much as for equal end moments. On the other hand, as evident from Fig. 9.7, there will be little or possibly no magnification if the end moments are of opposite sign and produce an inflection point along the member.

It can be shown (Ref. 9.2) that the way in which moment magnification depends on the relative magnitude of the two end moments (as in Figs. 9.4*a* and 9.7*a*) can be expressed by a modification of Eq. (9.6):

$$M_{max} = M_0 \frac{C_m}{1 - P{\cdot}P_c} \tag{9.8}$$

where

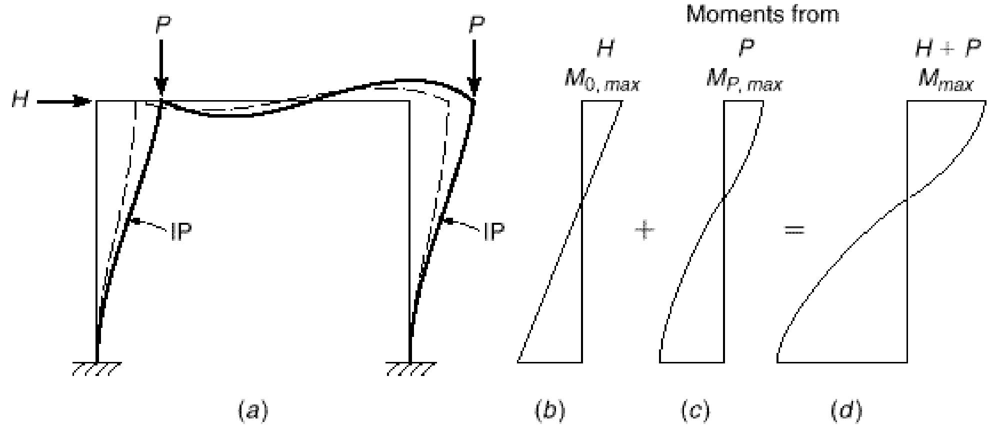
$$C_m = 0.6 + 0.4 \frac{M_1}{M_2} \geq 0.4 \tag{9.9}$$

Here M_1 is the numerically smaller and M_2 the numerically larger of the two end moments; hence, by definition, $M_0 = M_2$. The fraction $M_1{\cdot}M_2$ is defined as positive if the end moments produce single curvature and negative if they produce double curvature. It is seen that when $M_1 = M_2$, as in Fig. 9.4*a*, $C_m = 1$, so that Eq. (9.8) becomes Eq. (9.6), as it should. It is to be noted that Eq. (9.9) *applies only to members braced against sidesway*. As will become apparent from the discussion that follows, for members not braced against sidesway, maximum moment magnification usually occurs, that is, $C_m = 1$.

Members that are braced against sidesway include columns that are parts of structures in which sidesway is prevented in one of various ways: by walls sufficiently strong and rigid in their own planes to effectively prevent horizontal displacement; by special bracing in vertical planes; in buildings by designing the utility core to resist horizontal loads and furnish bracing to the frames; or by bracing the frame against some other essentially immovable support.

If no such bracing is provided, *sidesway can occur only for the entire frame simultaneously*, not for individual columns in the frame. If this is the case, the combined effect of bending and axial load is somewhat different from that in braced columns. As an illustration, consider the simple portal frame of Fig. 9.8*a* subject to a horizontal load H, such as a wind load, and compression forces P, such as from gravity loads. The moments M_0 caused by H alone, in the absence of P, are shown in Fig. 9.8*b*; the corresponding deformation of the frame is given in dashed curves. When P is added, horizontal moments are caused that result in the magnified deformations shown in solid curves and in the moment diagram of Fig. 9.8*d*. It is seen that the maximum values of M_0, both positive and negative, and the maximum values of the additional

FIGURE 9.8
Fixed portal frame, laterally unbraced.



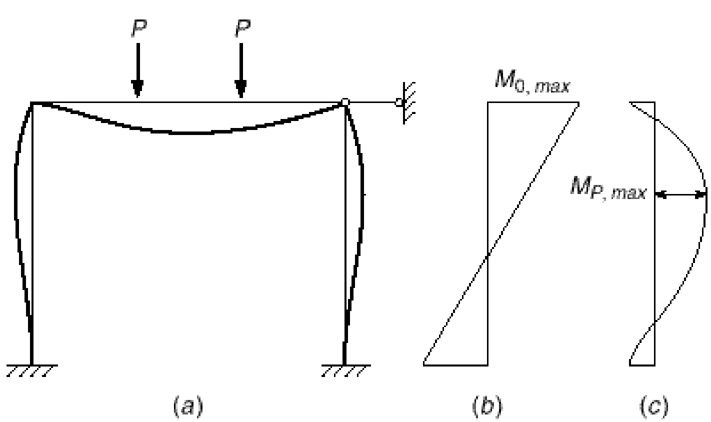
moments M_P of the same sign occur at the same locations, namely, at the ends of the columns. They are therefore fully additive, leading to a large moment magnification. In contrast, if the frame in Fig. 9.8 is laterally braced and vertically loaded, Fig. 9.9 shows that the maximum values of the two different moments occur in different locations; the moment magnification, if any, is therefore much smaller, as correctly expressed by C_m.

It should be noted that the moments that cause a frame to sidesway need not be caused by horizontal loads as in Fig. 9.8. Asymmetries, either of frame configuration or vertical loading or both, also result in sidesway displacements. In this case, the presence of axial column loads again results in the increased deflection and moment magnification.

In summary, it can be stated as follows:

1. In flexural members, the presence of axial compression causes additional deflections and additional moments Py. Other things being equal, the additional moments increase with increasing slenderness ratio kl/r.
2. In *members braced against sidesway* and bent in single curvature, the maxima of both types of moments, M_0 and Py, occur at the same or at nearby locations and are fully additive; this leads to large moment magnifications. If the M_0 moments result in double curvature (i.e., in the occurrence of an inflection point), the opposite is true and less or no moment magnification occurs.

FIGURE 9.9
Fixed portal frame, laterally braced.



3. In *members in frames not braced against sidesway*, the maximum moments of both kinds, M_0 and Py, almost always occur at the same locations, the ends of the columns; they are fully additive, regardless of the presence or absence of an inflection point. Here, too, other things being equal, the additional deflections and the corresponding moments increase with increasing kl/r.

This discussion is a simplified presentation of a fairly complex subject. The provisions of the ACI Code regarding slender columns are based on the behavior and the corresponding equations that have just been presented. They take account, in an approximate manner, of the additional complexities that arise from the fact that concrete is not an elastic material, that tension cracking changes the moment of inertia of a member, and that under sustained load, creep increases the short-term deflections and, thereby, the moments caused by these deflections.

9.4 ACI Criteria for Neglecting of Slenderness Effects

The procedure of designing slender columns is inevitably lengthy, particularly because it involves a trial-and-error process. At the same time, studies have shown that most columns in existing buildings are sufficiently stocky that slenderness effects reduce their capacity only a few percent. As stated in Chapter 8, an ACI-ASCE survey indicated that 90 percent of columns braced against sway, and 40 percent of unbraced columns, could be designed as short columns; i.e., they could develop essentially the full cross-sectional strength with little or no reduction from slenderness (Ref. 9.3). Furthermore, lateral bracing is usually provided by shear walls, elevator shafts, stairwells, or other elements for which resistance to lateral deflection is much greater than for the columns of the building frame. It can be concluded that in most cases in reinforced concrete buildings, slenderness effects may be neglected.

To permit the designer to dispense with the complicated analysis required for slender column design for these ordinary cases, ACI Code 10.12.2 and 10.13.2 provide limits below which the effects of slenderness are insignificant and may be neglected. These limits are adjusted to result in a maximum unaccounted reduction in column capacity of no more than 5 percent. Separate limits are applied to braced and unbraced frames, alternately described in the ACI Code as *nonsway* and *sway* frames, respectively. The Code provisions are as follows:

1. For compression members in nonsway frames, the effects of slenderness may be neglected when $kl_u/r \leq 34 - 12M_1/M_2$, where $(34 - 12M_1/M_2)$ is not taken greater than 40.
2. For compression members in sway frames, the effects of slenderness may be neglected when kl_u/r is less than 22.

In these provisions, k is the effective length factor (see Section 9.2); l_u is the unsupported length, taken as the clear distance between floor slabs, beams, or other members providing lateral support; M_1 is the smaller factored end moment on the compression member, positive if the member is bent in single curvature and negative if bent in double curvature; and M_2 is the larger factored end moment on the compression member, always positive.

The radius of gyration r for rectangular columns may be taken as $0.30h$, where h is the overall cross-sectional dimension in the direction in which stability is being considered. For circular members, it may be taken as 0.25 times the diameter. For other shapes, r may be computed for the gross concrete section.

Nilson–Darwin–Dolan: Design of Concrete Structures, Thirteenth Edition | 9. Slender Columns | Text

In accordance with ACI Code 10.12.1, k must be taken as 1.0 for nonsway frames, unless a lower value is supported by analysis. For sway frames, k must be determined by analysis in all cases, in accordance with ACI Code 10.13.1. The ACI criteria for determining k for both braced and unbraced columns are discussed in Section 9.6.

9.5 ACI Criteria for Nonsway Versus Sway Frames

The discussion of Section 9.3 clearly shows important differences in the behavior of slender columns in nonsway (braced) frames and corresponding columns in sway (unbraced) frames. ACI Code provisions and Commentary guidelines for the approximate design of slender columns reflect this, and there are separate provisions in each relating to the important parameters in nonsway vs. sway frames, including moment magnification factors and effective length factors.

In actual structures, a frame is seldom either completely braced or completely unbraced. It is necessary, therefore, to determine in advance if bracing provided by shear walls, elevator and utility shafts, stairwells, or other elements is adequate to restrain the frame against significant sway effects. Both the ACI Code and Commentary provide guidance.

As suggested in ACI Commentary 10.11.4, a compression member can be assumed braced if it is located in a story in which the bracing elements (shear walls, etc.) have a stiffness substantial enough to limit lateral deflection to the extent that the column strength is not substantially affected. Such a determination can often be made by inspection. If not, ACI Code 10.11.4 provides two alternate criteria for determining if columns and stories are treated as nonsway or sway.

To be considered as a nonsway or braced column, the first criterion requires that the increase in column end moment due to second-order effects must not exceed 5 percent of the first-order end moments. The designer is free to select the method for such a determination.

As an alternative, the Code allows a story to be considered nonsway when the *stability index*

$$Q = \frac{\Sigma P_u \Delta_o}{V_u l_c} \tag{9.10}$$

for a story is not greater than 0.05, where ΣP_u and V_u are the total factored vertical load and story shear, respectively, for the story; Δ_o is the first-order relative deflection between the top and the bottom of the story due to V_u; and l_c is the length of the compressive member measured center-to-center of the joints in the frame. ACI Commentary 10.11.4 provides the guidance that ΣP_u should be based on the lateral loading that maximizes the value of ΣP_u; the case of $V_u = 0$ is not included. In most cases, this calculation involves the combinations of load factors in Table 1.2 for wind, earthquake, or soil pressure (e.g., $1.2D + 1.6W + 1.0L + 0.5L_r$).

As shown in Refs. 9.3 and 9.4, for Q not greater than 0.6, the stability index closely approximates the ratio P/P_c used in the calculation of the moment magnification factor, so that $1/(1 - P/P_c)$ can be replaced by $1/(1 - Q)$. Thus, for $Q = 0.05$, $M_{max} \approx 1.05M_0$.†

† The near equivalence of Q to P/P_{cr} for reinforced concrete columns can be demonstrated using a single sway column with ends fixed against rotation, as shown in Fig. 9.1*e*. For this column, $Q = P_u\Delta_o/V_u l_c$. Since V_u/Δ_o = the lateral stiffness of the column $= 12EI/l_c^3$, the stability index can be expressed as $Q = P_u/(12EI/l_c^2)$. For an unsupported length of the column (the length used to calculate P_c) $l_u = 0.9l_c$ and $P = P_u$, $Q = P_u/(9.72EI/l_u^2)$ compared to $P/P_c = P_u/(\pi^2EI/l_u^2) = P_u/(9.87EI/l_u^2)$.

In accordance with ACI Code 10.11.1, the section properties of the frame members used to calculate Q must take into account the effects of axial loads, cracked regions along the length of the member, and the duration of the loads. Alternately, the section properties may be represented using the modulus of elasticity E_c given in Eq. (2.3) and the following section properties:

Moments of inertia	
Beams	$0.35I_g$
Columns	$0.70I_g$
Walls—uncracked	$0.70I_g$
—cracked	$0.35I_g$
Flat plates and flat slabs	$0.25I_g$
Area	$1.0A_g$

The moments of inertia must be divided by $(1 + \beta_d)$ when sustained lateral loads act or for stability checks (under ACI Code 10.13.6—described in Section 9.7).

where I_g and A_g are based on the gross concrete cross section, neglecting reinforcement, and β_d for calculating Δ_o in Eq. (9.10) is the ratio of the maximum factored sustained shear within a story to the maximum factored shear in that story to account for the effects of creep. As discussed in Section 12.5, I_g for T beams can be closely approximated as 2 times I_g for the web. The reduced values of I given above take into account the effect of nonlinear material behavior on the effective stiffness of the members. Reference 9.3 shows that the Code values for moments of inertia underestimate the true moments of inertia and conservatively overestimate second-order effects by 20 to 25 percent for reinforced concrete frames.

9.6 ACI Moment Magnifier Method for Nonsway Frames

A slender reinforced concrete column reaches the limit of its strength when the combination of P and M at the most highly stressed section causes that section to fail. In general, P is essentially constant along the length of the member. This means that the column approaches failure when, at the most highly stressed section, the axial force P combines with a moment $M = M_{max}$, as given by Eq. (9.8), so that this combination becomes equal to P_n and M_n, which will cause the section to fail. This is easily visualized by means of Fig. 9.10.

For a column of given cross section, Fig. 9.10 presents a typical interaction diagram. For simplicity, suppose that the column is bent in single curvature with equal eccentricities at both ends. For this eccentricity, the strength of the cross section is given by point A on the interaction curve. If the column is stocky enough for the moment magnification to be negligibly small, then $P_{n,stocky}$ at point A represents the member strength of the column under the simultaneous moment $M_{n,stocky} = e_0 P_{n,stocky}$.

On the other hand, if the same column is sufficiently slender, significant moment magnification will occur with increasing P. Then the moment at the most highly stressed section is M_{max}, as given by Eq. (9.8), with $C_m = 1$ because of equal end eccentricities. The solid curve in Fig. 9.10 shows the nonlinear increase of M_{max} as P increases. The point where this curve intersects the interaction curve, i.e., point B, defines the member strength $P_{n,slender}$ of the slender column, combined with the simultaneously applied end moments $M_0 = e_0 P_{n,slender}$. If end moments are unequal, the factor C_m will be less than 1, as discussed in Section 9.3.

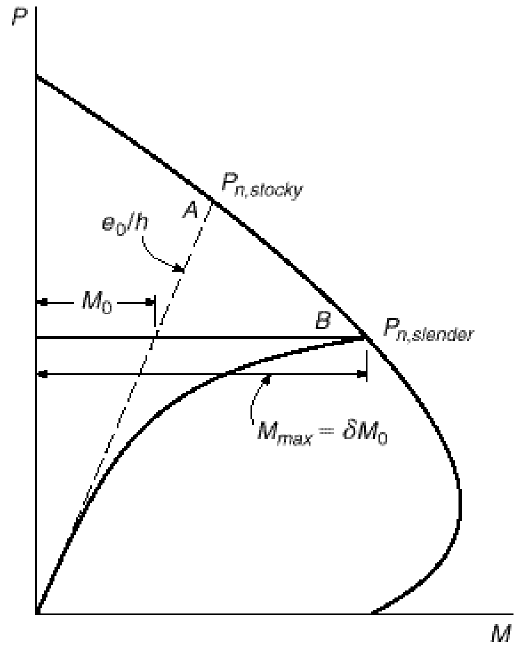


FIGURE 9.10
Effect of slenderness on carrying capacity.

ACI Code 10.11.1 specifies that axial loads and end moments in columns must be determined by a conventional elastic frame analysis (see Chapter 12) using the section properties given in Section 9.5. The member is then designed for that axial load and a simultaneous magnified column moment.

For a nonsway frame, the ACI Code equation for magnified moment, acting with the factored axial load P_u, is written as follows:

$$M_c = \delta_{ns} M_2 \tag{9.11}$$

where the moment magnification factor is

$$\delta_{ns} = \frac{C_m}{1 - P_u/0.75P_c} \geq 1 \tag{9.12}$$

In Eqs. (9.11) and (9.12), the subscript *ns* denotes a nonsway frame. The 0.75 term in Eq. (9.12) is a *stiffness reduction factor*, designed to provide a conservative estimate of P_c. The critical load P_c, in accordance with Eq. (9.1), is given as

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} \tag{9.13}$$

where l_u is defined as the unsupported length of the compression member. The value of k in Eq. (9.13) should be set equal to 1.0, unless calculated using the values of E_c and I given in Section 9.5 and procedures described later in this section.

In Eq. (9.12), the value of C_m is as previously given in Eq. (9.9):

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4 \tag{9.9}$$

for members braced against sidesway and without transverse loads between supports. Here M_2 is the larger of the two end moments, and M_1/M_2 is positive when the end

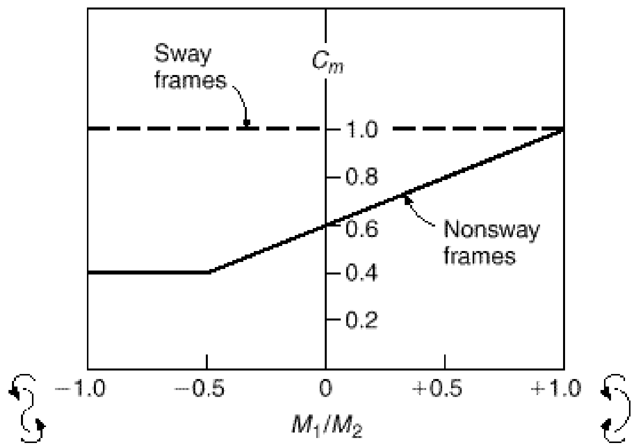


FIGURE 9.11
Values of C_m for slender columns in sway and nonsway frames.

moments produce single curvature and negative when they produce double curvature. The variation of C_m with $M_1 \cdot M_2$ is shown in Fig. 9.11. In Eq. (9.12), when the calculated value of $\cdot_{ns}$ is smaller than 1, it indicates that the larger of the two end moments, M_2, is the largest moment in the column, a situation depicted in Fig. 9.7*d*.

In this way the ACI Code provides for the capacity-reducing effects of slenderness in nonsway frames by means of the moment magnification factor $\cdot_{ns}$. However, it is well known that for columns with no or very small applied moments, i.e., axially or nearly axially loaded columns, increasing slenderness also reduces the column strength. For this situation, ACI Code 10.12.3.2 provides that the factored moment M_2 in Eq. (9.11) shall not be taken less than:

$$M_{2,min} = P_u(0.6 + 0.03h) \tag{9.14}$$

about each axis separately, where 0.6 and h are in inches. For members in which $M_{2,min}$ exceeds M_2, the value of C_m in Eq. (9.9) is taken equal to 1.0 or is based on the ratio of the computed end moments M_1 and M_2.

The value of EI used in Eq. (9.13) to calculate P_c for an individual member must be both accurate and reasonably conservative to account for the greater variability inherent in the properties of individual columns, as compared to the properties of the reinforced concrete frame, as a whole. The values of EI provided in Section 9.5 are adequate for general frame analysis but not for establishing P_c for individual columns.

In homogeneous elastic members, such as steel columns, EI is easily obtained from Young's modulus and the usual moment of inertia. Reinforced concrete columns, however, are nonhomogeneous, since they consist of both steel and concrete. Whereas steel is substantially elastic, concrete is not and is, in addition, subject to creep and to cracking if tension occurs on the convex side of the column. All of these factors affect the effective value of EI for a reinforced concrete member. It is possible by computer methods to calculate fairly realistic effective section properties, taking account of these factors. Even these calculations are no more accurate than the assumptions on which they are based. On the basis of elaborate studies, both analytical and experimental (Ref. 9.5), the ACI Code requires that EI be determined by either

$$EI = \frac{0.2E_cI_g + E_sI_{se}}{1 + \cdot_d} \tag{9.15}$$

or by the simpler expression

$$EI = \frac{0.4E_cI_g}{1 + \beta_d} \tag{9.16}$$

where E_c = modulus of elasticity of concrete, psi
I_g = moment of inertia of gross section of column, in^4
E_s = modulus of elasticity of steel = 29,000,000 psi
I_{se} = moment of inertia of reinforcement about centroidal axis of member cross section, in^4
β_d = ratio of maximum factored axial sustained load to maximum factored axial load associated with the same load combination (this definition differs from that used in Section 9.5 to calculate Δ_o)

The factor β_d accounts approximately for the effects of creep. That is, the larger the sustained loads, the larger are the creep deformations and corresponding curvatures. Consequently, the larger the sustained loads relative to the temporary loads, the smaller the effective rigidity, as correctly reflected in Eqs. (9.15) and (9.16). Because, of the two materials, only concrete is subject to creep, and reinforcing steel as ordinarily used is not, the argument can be made that the creep parameter $1 + \beta_d$ should be applied only to the term $0.2E_cI_g$ in Eq. (9.15). However, as explained in ACI Commentary 10.12.3, the creep parameter is applied to both terms because of the potential for premature yielding of steel in columns under sustained loading.

Both Eqs. (9.15) and (9.16) are conservative lower limits for large numbers of investigated actual members (Ref. 9.3). The simpler but more conservative Eq. (9.16) is not unreasonable for lightly reinforced members, but it greatly underestimates the effect of reinforcement for more heavily reinforced members, i.e., for the range of higher ρ values. Equation (9.15) is more reliable for the entire range of ρ and definitely preferable for medium and high ρ values (Ref. 9.6).

An accurate determination of the effective length factor k is essential in connection with Eqs. (9.11) and (9.13). In Section 9.2, it was shown that, for frames braced against sidesway (nonsway frames), k varies from $\frac{1}{2}$ to 1, whereas for laterally unbraced frames (sway frames), it varies from 1 to ∞, depending on the degree of rotational restraint at both ends. This was illustrated in Fig. 9.1. For frames, it is seen that this degree of rotational restraint depends on whether the stiffnesses of the beams framing into the column at top and bottom are large or small compared with the stiffness of the column itself. An approximate but generally satisfactory way of determining k is by means of *alignment charts* based on isolating the given column plus all members framing into it at top and bottom, as shown in Fig. 9.12. The *degree of end restraint* at each end is $\psi = \Sigma(EI/l_c$ of columns$) \div \Sigma(EI/l$ of floor members). Only floor members that are in a plane at either end of the column are to be included. The value of k can be read directly from the chart of Fig. 9.13, as illustrated by the dashed lines.†

It is seen that k must be known before a column in a frame can be dimensioned. Yet k depends on the stiffness EI/l of the members to be dimensioned, as well as on that of the abutting members. Thus, the dimensioning process necessarily involves iteration; i.e., one assumes member sizes, calculates member stiffnesses and corresponding k values, and then calculates the critical buckling load and more accurate

† Alternative to the use of charts are equations for the determination of effective length factors k, developed in Refs. 9.7 through 9.9 and given in ACI Commentary 10.12.1. Even more accurate expressions for k are given in Ref. 9.10. The equations are more convenient in developing computer solutions.

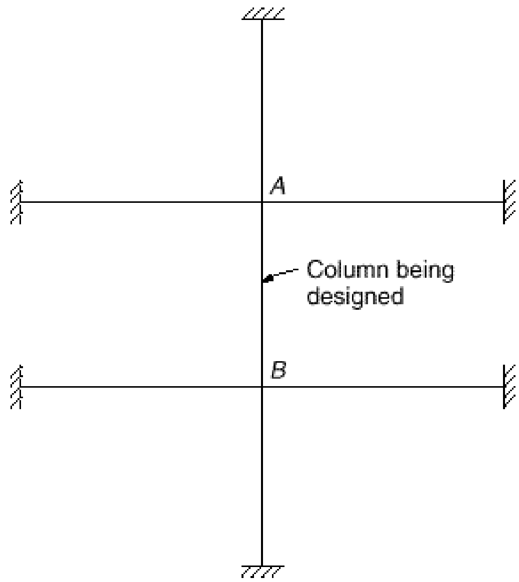
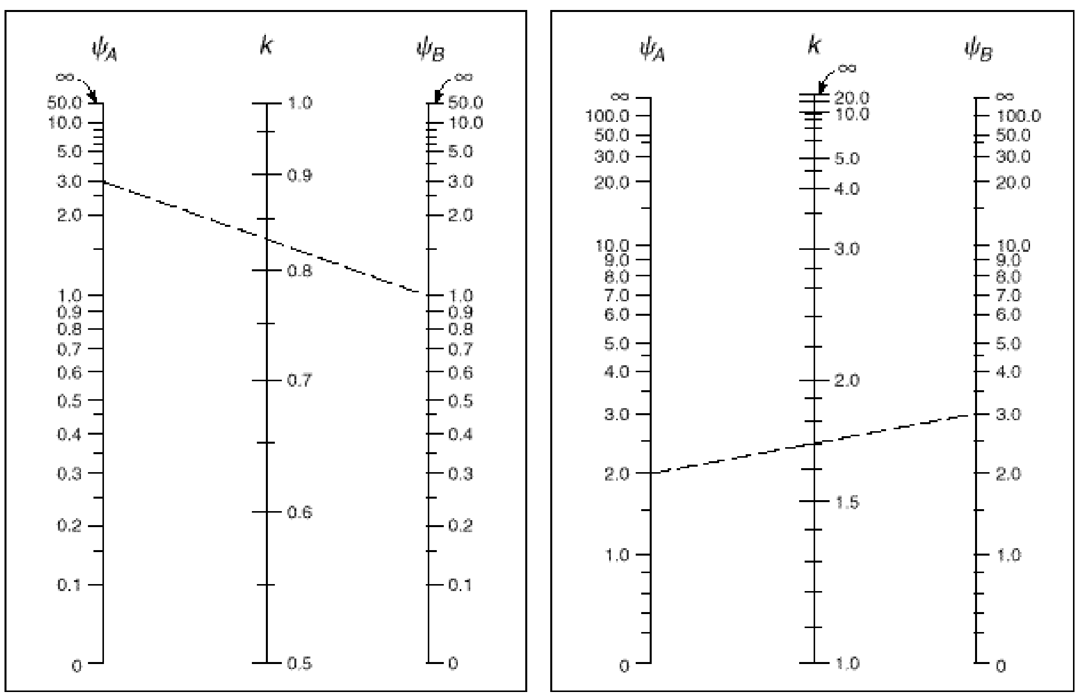


FIGURE 9.12
Section of rigid frame including column to be designed.



(*a*) Nonsway frames (*b*) Sway frames

FIGURE 9.13
Alignment charts for effective length factors *k*.

member sizes on the basis of these k values until assumed and final member sizes coincide or are satisfactorily close. The stiffness EI/l should be calculated based on the values of E_c and I given in Section 9.5, and the span lengths of the members l_c and l should be measured center-to-center of the joints.

An outline of the separate steps in the analysis/design procedure for nonsway frames follows along these lines:

1. Select a trial column section to carry the factored axial load P_u and moment $M_u = M_2$ from the elastic first-order frame analysis, assuming short column behavior and following the procedures of Chapter 8.
2. Determine if the frame should be considered as nonsway or sway, using the criteria of Section 9.5.
3. Find the unsupported length l_u.
4. For the trial column, check for consideration of slenderness effects using the criteria of Section 9.4 with $k = 1.0$.
5. If slenderness is tentatively found to be important, refine the calculation of k based on the alignment chart in Fig. 9.13*a*, with member stiffnesses $EI \cdot l$ (Section 9.5) and rotational restraint factors · based on trial member sizes. Recheck against the slenderness criteria.
6. If moments from the frame analysis are small, check to determine if the minimum moment from Eq. (9.14) controls.
7. Calculate the equivalent uniform moment factor C_m from Eq. (9.9).
8. Calculate $\cdot_d$, EI from Eq. (9.15) or (9.16), and P_c from Eq. (9.13) for the trial column.
9. Calculate the moment magnification factor $\cdot_{ns}$ from Eq. (9.12) and magnified moment M_c from Eq. (9.11).
10. Check the adequacy of the column to resist axial load and magnified moment, using the column design charts of Appendix A in the usual way. Revise the column section and reinforcement if necessary.
11. If column dimensions are altered, repeat the calculations for k, · , and P_c based on the new cross section. Determine the revised moment magnification factor and check the adequacy of the new design.

EXAMPLE 9.1

Design of a slender column in a nonsway frame. Figure 9.14 shows an elevation view of a multistory concrete frame building, with 48 in. wide × 12 in. deep beams on all column lines, carrying two-way slab floors and roof. The clear height of the columns is 13 ft. Interior columns are tentatively dimensioned at 18 × 18 in., and exterior columns at 16 × 16 in. The frame is effectively braced against sway by stair and elevator shafts having concrete walls that are monolithic with the floors, located in the building corners (not shown in the figure). The structure will be subjected to vertical dead and live loads. Trial calculations by first-order analysis indicate that the pattern of live loading shown in Fig. 9.14, with full load distribution on roof and upper floors and a checkerboard pattern adjacent to column *C*3, produces maximum moments with single curvature in that column, at nearly maximum axial load. Dead loads act on all spans. Service load values of dead and live load axial force and moments for the typical interior column *C*3 are as follows:

Dead load	*Live load*
$P = 230$ kips	$P = 173$ kips
$M_2 = 2$ ft-kips	$M_2 = 108$ ft-kips
$M_1 = -2$ ft-kips	$M_1 = 100$ ft-kips

The column is subjected to double curvature under dead load alone and single curvature under live load.

Design column *C*3, using the ACI moment magnifier method. Use $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

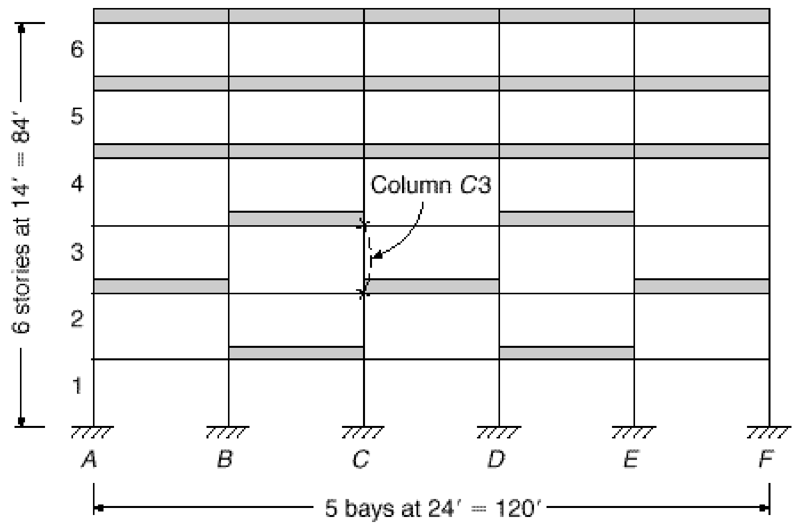


FIGURE 9.14
Concrete building frame for Example 9.1.

Solution. The column will first be designed as a short column, assuming no slenderness effect. With the application of the usual load factors,

$$P_u = 1.2 \times 230 + 1.6 \times 173 = 553 \text{ kips}$$

$$M_u = 1.2 \times 2 + 1.6 \times 108 = 175 \text{ ft-kips}$$

For an 18 × 18 in. column, with the 1.5 in. clear to the outside steel, No. 3 (No. 10) stirrups, and (assumed) No. 10 (No. 32) longitudinal steel:

$$\cdot = \cdot 18.00 - 2 \times 1.50 - 2 \times 0.38 - 1.27 \cdot\cdot 18 = 0.72$$

Graph A.6 for $\cdot = 0.70$, with bars arranged around the column perimeter, will be used. Then

$$\frac{P_u}{\cdot f_c' A_g} = \frac{553}{0.65 \times 4 \times 324} = 0.656$$

$$\frac{M_u}{\cdot f_c' A_g h} = \frac{175 \times 12}{0.65 \times 4 \times 324 \times 18} = 0.138$$

and from the graph $\cdot_g = 0.02$. This is low enough that an increase in steel area could be made, if necessary, to allow for slenderness, and the 18 × 18 in. concrete dimensions will be retained.

For an initial check on slenderness, an effective length factor $k = 1.0$ will be used. Then

$$\frac{kl_u}{r} = \frac{1.0 \times 13 \times 12}{0.3 \times 18} = 28.9$$

For a braced frame, the upper limit for short column behavior is

$$34 - 12\frac{M_1}{M_2} = 34 - 12\frac{1.2 \times \cdot -2 \cdot + 1.6 \times 100}{1.2 \times 2 + 1.6 \times 108} = 23.2$$

The calculated value of 28.9 exceeds this, so slenderness must be considered in the design. A more refined calculation of the effective length factor k is thus called for.

Because E_c is the same for column and beams, it will be canceled in the stiffness calculations. For this step, the column moment of inertia is $0.7I_g = 0.7 \times 18 \times 18^3 \cdot 12 = 6124$ in^4, giving $I \cdot l_c = 6124 \cdot (14 \times 12) = 36.5$ in^3. For the beams, the moment of inertia will be taken as $0.35I_g$, where I_g is taken as 2 times the gross moment of inertia of the web.

Thus, $0.35I_g = 0.35 \times 2 \times 48 \times 12^3/12 = 4838$ in^4, and $I/l = 4838/(24 \times 12) = 16.8$ in^3. Rotational restraint factors at the top and bottom of column $C3$ are the same and are

$$\psi_a = \psi_b = \frac{36.5 + 36.5}{16.8 + 16.8} = 2.17$$

From Fig. 9.13a for the braced frame, the value of k is 0.87, rather than 1.0 as used previously. Consequently,

$$\frac{kl_u}{r} = \frac{0.87 \times 13 \times 12}{0.3 \times 18} = 25.1$$

This is still above the limit value of 23.3, confirming that slenderness must be considered.

A check will now be made of minimum moment. According to Eq. (9.14), $M_{2,min} = 553 \times (0.6 + 0.03 \times 18)/12 = 53$ ft-kips. It is seen that this does not control.

The coefficient C_m can now be found from Eq. (9.9) with $M_1 = 1.2 \times (-2) + 1.6 \times 100 = 158$ ft-kips and $M_2 = 1.2 \times 2 + 1.6 \times 108 = 175$ ft-kips:

$$C_m = 0.6 + 0.4\frac{158}{175} = 0.96$$

Next the factor β_d will be found based on the ratio of the maximum factored sustained axial load (the factored dead load in this case) to the maximum factored axial load:

$$\beta_d = \frac{1.2 \times 230}{1.2 \times 230 + 1.6 \times 173} = 0.50$$

For a relatively low reinforcement ratio, one estimated to be in the range of 0.02 to 0.03, the more approximate Eq. (9.16) for EI will be used, and

$$EI = \frac{0.4 \times 3.60 \times 10^6 \times 18 \times 18^3/12}{(1 + 0.50)} = 8.40 \times 10^9 \text{ in}^2\text{-lb}$$

The critical buckling load is found from Eq. (9.13) to be

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2 \times 8.40 \times 10^9}{(0.87 \times 13 \times 12)^2} = 4.50 \times 10^6 \text{ lb}$$

The moment magnification factor can now be found from Eq. (9.12).

$$\delta_{ns} = \frac{C_m}{1 - P_u/0.75P_c} = \frac{0.96}{1 - 553/(0.75 \times 4500)} = 1.15$$

Thus the required axial strength of the column is $P_u = 553$ kips (as before), while the magnified design moment is $M_c = \delta_{ns}M_2 = 1.15 \times 175 = 201$ ft-kips. With reference again to the column design chart A.6 with

$$\frac{P_u}{\phi f_c' A_g} = \frac{553}{0.65 \times 4 \times 324} = 0.657$$

$$\frac{M_u}{\phi f_c' A_g h} = \frac{201 \times 12}{0.65 \times 4 \times 324 \times 18} = 0.159$$

it is seen that the required reinforcement ratio is increased from 0.020 to 0.026 because of slenderness. The steel area now required is

$$A_{st} = 0.026 \times 324 = 8.42 \text{ in}^2$$

which can be provided using four No. 10 (No. 32) and four No. 9 (No. 29) bars ($A_{st} = 9.08$ in^2), arranged as shown in Fig. 9.15. Number 3 (No. 10) ties will be used at a spacing not to exceed the least dimension of the column (18 in.), 48 tie diameters (18 in.), or 16 bar

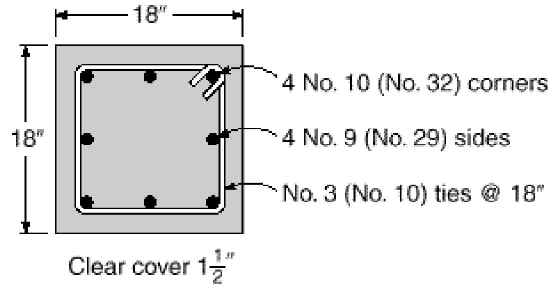


FIGURE 9.15
Cross section of column $C3$, Example 9.1.

diameters (18 in.). Single ties at 18 in. spacing, as shown in the figure, will meet requirements of the ACI Code.

Further refinements in the design could, of course, be made by recalculating the critical buckling load using Eq. (9.15). This extra step is not justified here because the column slenderness is barely above the upper limit for short column behavior and the moment magnification is not great.

9.7 ACI Moment Magnifier Method for Sway Frames

The important differences in behavior between columns braced against sidesway and columns for which sidesway is possible were discussed in Sections 9.2 and 9.3. The critical load for a column P_c depends on the effective length kl_u, and although the effective length factor k falls between 0.5 and 1.0 for braced columns, it is between 1.0 and ∞ for columns that are unbraced (see Figs. 9.1 and 9.13). Consequently, an unbraced column will buckle at a much smaller load than will a braced column that is otherwise identical.

Columns subject to sidesway do not normally stand alone but are part of a structural system including floors and roof. A floor or roof is normally very stiff in its own plane. Consequently, all columns at a given story level in a structure are subject to essentially identical sway displacements; i.e., sidesway of a particular story can occur only by simultaneous lateral motion of all columns of that story. Clearly, all columns at a given level must be considered together in evaluating slenderness effects relating to sidesway.

On the other hand, it is also possible for a single column in a sway frame to buckle individually under gravity loads, the ends of the column being held against relative lateral movement by other, stiffer columns at the same floor level. This possibility, resulting in magnification of nonsway moments due to gravity loads, must also be considered in the analysis and design of slender columns in unbraced frames.

The ACI moment magnifier approach can still be used, but in frames subject to sidesway, it is necessary, according to ACI Code 10.13.3, to separate the loads acting on a structure into two categories: loads that result in no appreciable sidesway and loads that result in appreciable sidesway. Clearly two separate frame analyses are required, one for loads of each type. In general, gravity loads acting on reasonably symmetrical frames produce little sway, and the effects of gravity load may therefore be placed in the first category. This is confirmed by tests and analyses in Ref. 9.11 that show that the sway magnification of gravity moments by the sway multiplier is unwarranted.

The maximum magnified moments caused by sway loading occur at the ends of the column, but those due to gravity loads may occur somewhere in the midheight of the column, the exact location of the latter varying depending on the end moments. Because magnified gravity moments and magnified sway moments do not occur at the same location, the argument can be made that, in most cases, no magnification should be applied to the nonsway moments when sway moments are considered; that is, it is unlikely that the actual maximum moment will exceed the sum of the nonmagnified gravity moment and the magnified sway moment. Consequently, for cases involving sidesway, Eq. (9.11) is replaced by

$$M_1 = M_{1ns} + \delta_s M_{1s} \tag{9.17}$$

$$M_2 = M_{2ns} + \delta_s M_{2s} \tag{9.18}$$

where M_1 = smaller factored end moment on compression member
M_2 = larger factored end moment on compression member
M_{1ns} = factored end moment on compression member at the end at which M_1 acts, due to loads that cause no appreciable sidesway, calculated using a first-order elastic frame analysis
M_{2ns} = factored end moment on compression member at the end at which M_2 acts, due to loads that cause no appreciable sidesway, calculated using a first-order elastic frame analysis
M_{1s} = factored end moment on compression member at the end at which M_1 acts, due to loads that cause appreciable sidesway, calculated using a first-order elastic frame analysis
M_{2s} = factored end moment on compression member at the end at which M_2 acts, due to loads that cause appreciable sidesway, calculated using a first-order elastic frame analysis
δ_s = moment magnification factor for frames not braced against sidesway, to reflect lateral drift resulting from lateral (and sometimes gravity) loads

The need to calculate M_1 as well as M_2 will be explained shortly.

ACI Code 10.13.4 provides three alternate methods for calculating the magnified sway moments, $\delta_s M_s$.

With the first alternative, the column end moments are calculated using a second-order analysis based on the member stiffnesses given in Section 9.5.

With the second alternative, the magnified sway moments are calculated as

$$\delta_s M_s = \frac{M_s}{1 - Q} \geq M_s \tag{9.19}$$

where Q is the stability index calculated using Eq. (9.10). The ACI Code limits application of Eq. (9.19) to values of $\delta_s = 1/(1 - Q) \leq 1.5$. One of the other two alternate methods must be used for higher values of δ_s.

For the third alternative, the ACI Code permits the magnified sway moment to be calculated as

$$\delta_s M_s = \frac{M_s}{1 - \Sigma P_u / 0.75 \Sigma P_c} \geq M_s \tag{9.20}$$

in which ΣP_u is the total axial load on all columns and ΣP_c is the total critical buckling load for all columns in the story under consideration. As with Eq. (9.12), the 0.75 factor in Eq. (9.20) is a stiffness reduction factor to provide a conservative estimate of the critical buckling loads P_c. The individual values of P_c are calculated using Eq.

(9.13) with effective length factors k for unbraced frames (Fig. 9.13b) and values of EI from Eq. (9.15) or (9.16).

For the three alternate methods used to calculate $\cdot_s M_s$, the factor $\cdot_d$ is defined differently than it is for nonsway frames. As described earlier, in Section 9.5, for sway frames $\cdot_d$ is the ratio of the maximum factored sustained shear within a story to the maximum factored shear in that story. Thus, for most applications, $\cdot_d = 0$ for the purpose of calculating $\cdot_s M_s$. In unusual situations, $\cdot_d \neq 0$ will occur, such as a building located on a sloping site that is subjected to soil pressure on a single side (Refs. 9.12 and 9.13).

It is noted in ACI Commentary 10.13.4 that the moment magnifier procedure may underestimate the magnified moment for columns in structures that undergo significant torsional displacement under lateral load. The discrepancy is greatest for columns farthest from the center of twist. The Commentary suggests that three-dimensional second-order analysis should be considered in such cases.

Equations (19.17) and (19.18) are based on the assumption that the maximum moments in columns in sway frames are the end moments. However, as shown in Fig. 9.7e, the maximum moment may occur at any point along a length of a column. As described in Ref. 9.14, for values of

$$\frac{l_u}{r} \cdot \frac{35}{\sqrt{\dfrac{P_u}{f'_c A_g}}} \qquad (9.21)$$

the maximum moment is likely to exceed the value calculated in Eq. (9.18) by more than 5 percent. To account for the additional moment magnification, ACI Code 10.13.5 requires that under the conditions represented by the inequality in Eq. (9.21), columns must be designed as *nonsway columns* based on Eqs. (9.11), (9.13), and (9.9), with M_1 and M_2 calculated using Eqs. (9.17) and (9.18), respectively. The smaller moment, M_1, is used in Eq. (9.9) to calculate C_m. $\cdot_d$ is defined for the load combination under consideration, and k is defined for a *nonsway frame*.

To protect against sidesway buckling of an entire story under gravity loads alone, ACI Code 10.13.6 places additional restrictions on sway frames. The form of the restriction depends on the method used to calculate $\cdot_s M_s$:

1. When $\cdot_s M_s$ is computed based on a second-order elastic analysis, the ratio of the second-order lateral deflections to the first-order lateral deflections under $1.4D + 1.7L$ plus lateral load shall not exceed 2.5.
2. When $\cdot_s M_s$ is computed using Eq. (9.19), the value of Q using ΣP_u for $1.4D + 1.7L$ shall not exceed 0.60. [This corresponds to $\cdot_s = 1 \cdot (1 - Q) = 2.5$.]
3. When $\cdot_s M_s$ is computed using Eq. (9.20), $\cdot_s$ computed using ΣP_u and ΣP_c corresponding to $1.4D + 1.7L$ shall be positive and shall not exceed 2.5.

For all three methods of checking sidesway instability under gravity loads, $\cdot_d$ is calculated for the full story as the ratio of the maximum factored sustained axial load to the maximum factored axial load for the story, rather than as the ratio of maximum factored sustained shear within the story to maximum factored shear in the story, and the gravity load ΣP_u is based on $1.4D + 1.7L$, rather than $1.2D + 1.6L$ or $1.2D + 1.0L$.[†] Method (1) involves two analyses, one first-order and one second-order, for the

[†] The load factors, 1.4 and 1.7, which were used for dead and live load, respectively, in ACI Codes prior to 2002, have been retained in the 2002 ACI Code for this portion of the slenderness calculations.

structure under factored gravity loads plus lateral load. Any reasonable lateral load distribution can be used for the analyses. The ratio of the deflection from the second-order analysis to the deflection from the first-order analysis is limited to a value of 2.5.

For method (2), the value of Q calculated in Eq. (9.10) can be conservatively modified for use in the stability check by multiplying by

$$\frac{(1.4D + 1.7L)(1 + \beta_{dA})}{(1.2D + 1.0L)(1 + \beta_{dV})} \tag{a}$$

where the subscripts A and V represent β_d based on total story axial load and total story shear, respectively. ACI Commentary 10.13.6 points out that if Q from Eq. (9.10) is less than or equal to 0.2, the stability check is satisfied. This conservative limit on Q from Eq. (9.10) is based on assumed values of $\beta_{dV} = 0$ and $\beta_{dA} = 1$ (i.e., all axial load is sustained) and an L/D ratio of 2. Assuming that the live load is twice the dead load,

$$0.20\frac{(1.4 \times 1 + 1.7 \times 2)(1 + 1)}{(1.2 \times 1 + 1.0 \times 2)(1 + 0)} = 0.60$$

exactly satisfying the upper limit, $Q = 0.6$, in ACI Code 10.13.6. If Q from Eq. (9.10) is somewhat greater than 0.2, the actual values of D, L, β_{dA}, β_{dV} should be used to determine Q for the stability check.

For method (3), δ_s can be calculated from δ_s in Eq. (9.20) by multiplying the original $\Sigma P_u/\Sigma P_c$ term by the same term as used to modify Q [see Eq. (*a*)]. ACI Commentary 10.13.6 points out that, although $\delta_s = 2.5$ is very high, the value is selected to offset the conservatism inherent in the moment magnifier procedure. In any case, if the appropriate restriction is violated, the structure must be stiffened.

The sequence of design steps for slender columns in sway frames is similar to that outlined in Section 9.6 for nonsway frames, except for the requirement that loads be separated into gravity loads, which are assumed to produce no sway, and horizontal loads producing sway. Separate frame analyses are required, and different equivalent length factors k and creep coefficients β_d, along with extra checks specified for Eq. (9.21) and for the possibility of sidesway instability under gravity loads, must be applied. It will be noted that according to ACI Code 9.2 (see also Table 1.2 of Chapter 1), if wind effects W are included in the design, four possible factored load combinations are to be applied:

$$U = 1.2D + 1.6L$$

$$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + 0.8W$$

$$U = 1.2D + 1.6W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$$

$$U = 0.9D + 1.6W$$

Similar provisions are included for cases where earthquake loads are to be considered. This represents a significant complication in the sway frame analysis; however, the factored loads can be separated into gravity effects and sway effects, as required, and a separate analysis can be performed for each.

It is important to realize that, for sway frames, *the beams must be designed for the total magnified end moments of the compression members at the joint.* Even though the columns may be very rigid, if plastic hinges were to form in the restraining beams adjacent to the joints, the effective column length would be greatly increased and the critical column load much reduced.

The choice of which of the three alternate methods to use for calculating $\delta_s M_s$ depends upon the desired level of accuracy and the available analytical tools.

Second-order analysis (discussed in more detail in Section 9.8) provides the most accurate estimate of the magnified sway moments but requires more sophisticated techniques. The extra effort required for second-order analysis, however, usually produces a superior design. The second alternative, Eq. (9.19), will in most cases be the easiest to apply, since matrix analysis is used for virtually all frames to determine member forces under gravity and lateral loading. Such an analysis automatically generates the value of Δ_o, the first-order relative deflection within a story, allowing Q to be calculated for each story within a structure. The third alternative, Eq. (9.20), is retained with minor modifications from previous versions of the ACI Code. As will be demonstrated in the following example, calculations using Eq. (9.20) are more tedious than those needed for Eq. (9.19) but do not require knowledge of Δ_o. Application of Eq. (9.19) is limited by the Code to values of $\delta_s \leq 1.5$. For $\delta_s > 1.5$, application of Eq. (9.20) is mandatory if a second-order analysis is not used.

EXAMPLE 9.2

Design of a slender column in a sway frame. Consider now that the concrete building frame of Example 9.1 acts as a *sway frame*, without the stairwells or elevator shafts described earlier. An initial evaluation is carried out using the member dimensions and reinforcement given in Example 9.1. The reinforcement for the interior 18 × 18 in. columns, shown in Fig. 9.15, consists of four No. 10 (No. 32) bars at the corners and four No. 9 (No. 29) bars at the center of each side. Reinforcement for the exterior 16 × 16 in. columns consists of eight No. 8 (No. 25) bars distributed in a manner similar to that shown for the longitudinal reinforcement in Fig. 9.15. The building will be subjected to gravity dead and live loads and horizontal wind loads. Elastic first-order analysis of the frame at service loads (all load factors = 1.0) using the values of E and I defined in Section 9.5 gives the following results at the third story:

	Cols. A*3 and* F*3*	*Cols.* B*3 and* E*3*	*Cols.* C*3 and* D*3*
P_{dead}	115 kips	230 kips	230 kips
P_{live}	90 kips	173 kips	173 kips
P_{wind}	±30 kips	±18 kips	±6 kips
V_{wind}	5.5 kips	11 kips	11 kips
$M_{2,dead}$			2 ft-kips
$M_{2,live}$			108 ft-kips
$M_{2,wind}$			±84 ft-kips
$M_{1,dead}$			−2 ft-kips
$M_{1,live}$			100 ft-kips
$M_{1,wind}$			±70 ft-kips

To simplify the analysis, roof loads will not be considered. The relative lateral deflection for the third story under total wind shear $V_{wind} = 55$ kips is 0.76 in.

Column $C3$ is to be designed for the critical loading condition, using $f'_c = 4000$ psi and $f_y = 60{,}000$ psi as before.

Solution. The column size and reinforcement must satisfy requirements for each of the four load conditions noted above.

Initially, a check is made to see if a sway frame analysis is required. The factored shear $V_u = 1.6 \times V_{wind} = 1.6 \times 55 = 88$ kips. The corresponding deflection $\Delta_o = 1.6 \times 0.76 = 1.22$ in. The total factored axial force on the story is obtained using the load table.

Columns $A3$ and $F3$: $\quad P_u = 1.2 \times 115 + 1.0 \times 90 = 228$ kips

Columns $B3$, $C3$, $D3$, and $E3$: $\quad P_u = 1.2 \times 230 + 1.0 \times 173 = 449$ kips

Note that in this case the values of P_{wind} in the columns are not considered since they cancel out for the floor as a whole, i.e., $\Sigma P_{wind} = 0$. Thus, $\Sigma P_u = 2 \times 228 + 4 \times 449 = 2252$ kips, and the stability index is

$$Q = \frac{\Sigma P_u \Delta_o}{V_u l_c} = \frac{2252 \times 1.22}{88 \times 14 \times 12} = 0.19$$

Since $Q > 0.05$, sway frame analysis is required for this story.

(a) Gravity loads only. All columns in sway frames must first be considered as braced columns under gravity loads acting alone, i.e., for $U = 1.2D + 1.6L$. This check has already been made for column $C3$ in Example 9.1.

(b) Gravity plus wind loads. Three additional load combinations must be considered when wind effects are included: $U = 1.2D + 1.6(L_r$ or S or $R) + 0.8W$, $U = 1.2D + 1.6W + 1.0L + 0.5(L_r$ or S or $R)$, and $U = 0.9D + 1.6W$. By inspection, the second of these will control for this case, and the others will not be considered further. From Example 9.1, $\psi_a = \psi_b = 2.17$. With reference to the alignment chart in Fig. 9.13*b*, the effective length factor for an unbraced frame $k = 1.64$ and

$$\frac{kl_u}{r} = \frac{1.64 \times 13 \times 12}{0.3 \times 18} = 47.4$$

This is much above the limit value of 22 for short column behavior in an unbraced frame. (This should be no surprise since $kl_u/r = 25.1$ for column $C3$ in the braced condition.) For sway frame analysis, the loads must be separated into gravity loads and sway loads, and the appropriate magnification factor must be computed and applied to the sway moments. The factored end moments resulting from the nonsway loads on column $C3$ are

$$M_{1ns} = 1.2 \times (-2) + 1.0 \times 100 = 98 \text{ ft-kips}$$

$$M_{2ns} = 1.2 \times 2 + 1.0 \times 108 = 110 \text{ ft-kips}$$

The sway effects will amplify the moments

$$M_{1s} = 1.6(-70) = -112 \text{ ft-kips}$$

$$M_{2s} = 1.6 \times 84 = 134 \text{ ft-kips}$$

For the purposes of comparison, the magnified sway moments will be calculated based on both Q [Eq. (9.19)] and $\Sigma P_u/\Sigma P_c$ [Eq. (9.20)].

Using Eq. (9.19),

$$\delta_s M_{1s} = \frac{M_{1s}}{1 - Q} = \frac{-112}{1 - 0.19} = -138 \text{ ft-kips}$$

$$\delta_s M_{2s} = \frac{M_{2s}}{1 - Q} = \frac{134}{0.81} = 165 \text{ ft-kips}$$

To use Eq. (9.20), the critical loads must be calculated for each of the columns as follows. For columns $A3$ and $F3$,

Columns: $I = 0.7I_g = 0.7 \times 16 \times 16^3/12 = 3823 \text{ in}^4$ and $I/l_c = 3823/(14 \times 12) = 22.8 \text{ in}^3$

Beams: $I = 4838 \text{ in}^4$ and $I/l_c = 16.8 \text{ in}^3$

Rotational restraint factors for this case, with two columns and one beam framing into the joint, are

$$\psi_a = \psi_b = \frac{22.8 + 22.8}{16.8} = 2.71$$

which, with reference to the alignment chart for unbraced frames, gives $k = 1.77$. For wind load, $\beta_d = 0$. Since reinforcement has been initially selected for one column, EI will be calculated using Eq. (9.15).

$$EI = 0.2E_cI_g + E_sI_{se} = 0.2 \times 3.6 \times 10^6 \times 16 \times 16^3/12 + 29 \times 10^6 \times 6 \times 0.79 \times 6.6^2$$

$$= 9.92 \times 10^9 \text{ in}^2\text{-lb}$$

Then the critical load is

$$P_c = \frac{\pi^2 \times 9.92 \times 10^9}{(1.77 \times 13 \times 12)^2} = 1.51 \times 10^6 \text{ lb}$$

For columns $B3$, $C3$, $D3$, and $E3$, from earlier calculations for column $C3$, $k = 1.64$ for the sway loading case. For these columns,

$$EI = 0.2 \times 3.6 \times 10^6 \times 18 \times 18^3/12 + 29 \times 10^6(4 \times 1.27 \times 6.4^2 + 2 \times 1.0 \times 6.5^2)$$

$$= 14.8 \times 10^9 \text{ in}^2\text{-lb}$$

$$P_c = \frac{\pi^2 \times 14.8 \times 10^9}{(1.64 \times 13 \times 12)^2} = 2.62 \times 10^6 \text{ lb}$$

Thus, for all of the columns at this level of the structure,

$$\Sigma P_c = 2 \times 1510 + 4 \times 2620 = 13{,}500 \text{ kips}$$

and finally, the magnified sway moments for both the top and the bottom of column $C3$ are

$$\delta_sM_{1s} = \frac{M_{1s}}{1 - \Sigma P_u/0.75\Sigma P_c} = \frac{-112}{1 - 2252/(0.75 \times 13{,}500)} = -144 \text{ ft-kips}$$

$$\delta_sM_{2s} = \frac{M_{2s}}{1 - \Sigma P_u/0.75\Sigma P_c} = \frac{134}{1 - 0.222} = 172 \text{ ft-kips}$$

The values of δ_sM_s are higher based on $\Sigma P_u/\Sigma P_c$ than they are based on Q (172 ft-kips vs. 165 ft-kips for δ_sM_{2s}), emphasizing the conservative nature of the moment magnifier approach based on Eq. (9.20). The design will proceed using the less conservative value of δ_sM_s.

The total magnified moments are

$$M_1 = 98 - 138 = -40 \text{ ft-kips}$$

$$M_2 = 110 + 165 = 275 \text{ ft-kips}$$

combined with factored axial load $P_u = 459$ kips (now including $1.6P_{\text{wind}}$). In reference to Graph A.6 with column parameters

$$\frac{P_u}{\phi f_c'A_g} = \frac{459}{0.65 \times 4 \times 324} = 0.545 \text{ ksi}$$

$$\frac{M_u}{\phi f_c'A_gh} = \frac{275 \times 12}{0.65 \times 4 \times 324 \times 18} = 0.218 \text{ ksi}$$

it is seen that $\rho_g = 0.030$. This is slightly higher than the value of 0.026 required for column $C3$ in a braced frame. The required steel area of

$$A_{st} = 0.03 \times 324 = 9.72 \text{ in}^2$$

will be provided using eight No. 10 (No. 32) bars, arranged as shown in Fig. 9.16. Spacing of No. 3 (No. 10) ties must not exceed the least dimension of the column, 48 tie diameters, or 16 main bar diameters. The second criterion controls, and No. 3 (No. 10) ties at 18 in. spacing will be used in the pattern shown in Fig. 9.16.

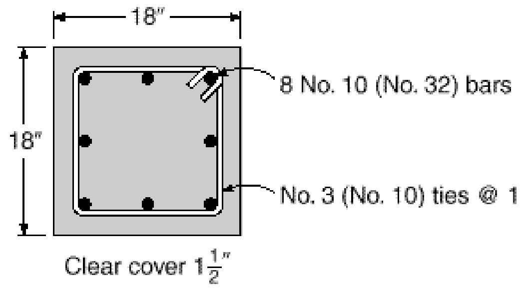


FIGURE 9.16
Cross section of column $C3$, Example 9.2.

Two more checks are required to complete the design. First, in accordance with Eq. (9.21), a higher magnified moment must be calculated using the values of M_1 and M_2 and Eqs. (9.11), (9.13), and (9.9) if $l_u/r > 35/\sqrt{P_u/f'_c A_g}$. In this case, $l_u/r = 13 \cdot 12/(0.3 \times 18) = 29$ compared to $35/\sqrt{459/(4 \times 324)} = 59$, indicating that the current analysis and design are satisfactory.

A second check is needed to protect against sidesway instability of the entire story under gravity loads. When Q is used to compute $\delta_s M_s$, Q may not exceed 0.60. For this check, the moments of inertia used to calculate Δ_o for use in Eq. (9.10) must be divided by $1 + \beta_d$, with β_d equal to the ratio of the sustained factored axial load to the maximum factored axial load and ΣP_u must be based on $1.4D + 1.7L$. This check is satisfied without further calculation, however, if the value of Q from Eq. (9.10) does not exceed 0.20. Since Q based on Eq. (9.10) is 0.19, the stability check is satisfied for the third story.

9.8 Second-Order Analysis for Slenderness Effects

It may be evident from the preceding examples that, although the ACI moment magnifier method works well enough for nonsway frames, its application to sway frames is complicated, with many opportunities for error, especially when Eq. (9.20) is used to calculate $\delta_s M_s$.

With the universal availability of computers in design offices, and because of the complexity of the moment magnifier method, it is advantageous to apply rational second-order frame analysis, or P-Δ analysis, in which the effects of lateral deflection on moments, axial forces, and, in turn, lateral deflections are computed directly. The resulting moments and deflections include the effects of slenderness, and so the problem is strictly nonlinear.

Second-order analysis is encouraged in general by ACI Code 10.10.1 and in particular for sway frames by ACI Code 10.13.4.1. A second-order analysis is *required* by ACI Code 10.11.5 for all compression members with kl_u/r greater than 100. According to the Code, such analyses "shall be based on the factored forces and moments from a second-order analysis considering material nonlinearity and cracking, as well as the effects of member curvature and lateral drift, duration of the loads, shrinkage and creep, and interaction with the supporting foundation." Member dimensions used in the second-order analysis must be within 10 percent of the final dimensions. Otherwise, the frame must be reanalyzed. ACI Code 10.10.1 requires that the second-order analysis procedure be one that provides a strength prediction that is in "substantial agreement" with test results for reinforced concrete columns in statically indeterminate frames. ACI Commentary 10.10.1 suggests that a prediction within 15 percent of the test results is satisfactory. It also suggests that a stiffness reduction factor ϕ_K of 0.80 be used to provide consistency with the second-order analysis for sway frames described in ACI Code 10.13.4.1.

Satisfying all of the requirements of ACI Code 10.10.1 on a member-by-member basis would be highly inefficient. As pointed out in Ref. 9.14, the key requirement for EI values for second-order frame analysis is that they be representative of member stiffness just prior to failure. The values of E and I in Section 9.5 (ACI Code 10.11.1) meet that requirement and include a stiffness reduction factor of 0.875 (Ref. 9.14). The value of the stiffness reduction factor and the moments of inertia in Section 9.5 are higher than the factor 0.75 in Eqs. (9.12) and (9.19) and the effective values of I in Eqs. (9.15) and (9.16), respectively, because of the inherently lower variability in the total stiffness of a frame compared to that of an individual member. ACI Code 10.13.4 authorizes the use of E and I from Section 9.5 in second-order analyses to determine magnified sway moments.

A rational second-order analysis gives a better approximation of actual moments and forces than the moment magnifier method. Differences are particularly significant for irregular frames, for frames subject to significant sway forces, and for lightly braced frames. There may be important economies in the resulting design.

Practical methods for performing a full second-order analysis are described in the literature (Refs. 9.3, 9.15, 9.16, 9.17, and 9.18 to name a few), and general-purpose programs that perform a full nonlinear analysis including sway effects are commercially available. Existing linear first-order analysis programs, however, can be modified to produce acceptable results. This requires an iterative approach, which can be summarized as follows.

Figure 9.17*a* shows a simple frame subject to lateral loads H and vertical loads P. The lateral deflection Δ is calculated by ordinary first-order analysis. As the frame is displaced laterally, the column end moments must equilibrate the lateral loads and a moment equal to $(\Sigma P)\Delta$:

$$\Sigma(M_{top} + M_{bot}) = Hl_c + (\Sigma P)\Delta \tag{9.22}$$

where Δ is the lateral deflection of the top of the frame with respect to the bottom, and ΣP is the sum of the vertical forces acting. The moment $\Sigma P\Delta$ in a given story can be represented by equivalent shear forces $(\Sigma P)\Delta/l_c$, where l_c is the story height, as shown in Fig. 9.17*b*. These shears give an overturning moment equal to that of the loads P acting at a displacement Δ.

Fig. 9.17*c* shows the story shears acting in a three-story frame. The algebraic sum of the story shears from the columns above and below a given floor correspond in effect to a sway force dH acting on that floor. For example, at the second floor the sway force is

$$dH_2 = \frac{(\Sigma P_1)\Delta_1}{l_1} - \frac{(\Sigma P_2)\Delta_2}{l_2} \tag{9.23}$$

The sway forces must be added to the applied lateral force H at any story level, and the structure is then reanalyzed, giving new deflections and increased moments. If the lateral deflections increase significantly (say more than 5 percent), new dH sway forces are computed, and the structure is reanalyzed for the sum of the applied lateral forces and the new sway forces. Iteration is continued until changes are insignificant. Generally one or two cycles of iteration are adequate for structures of reasonable lateral stiffness (Ref. 9.3).

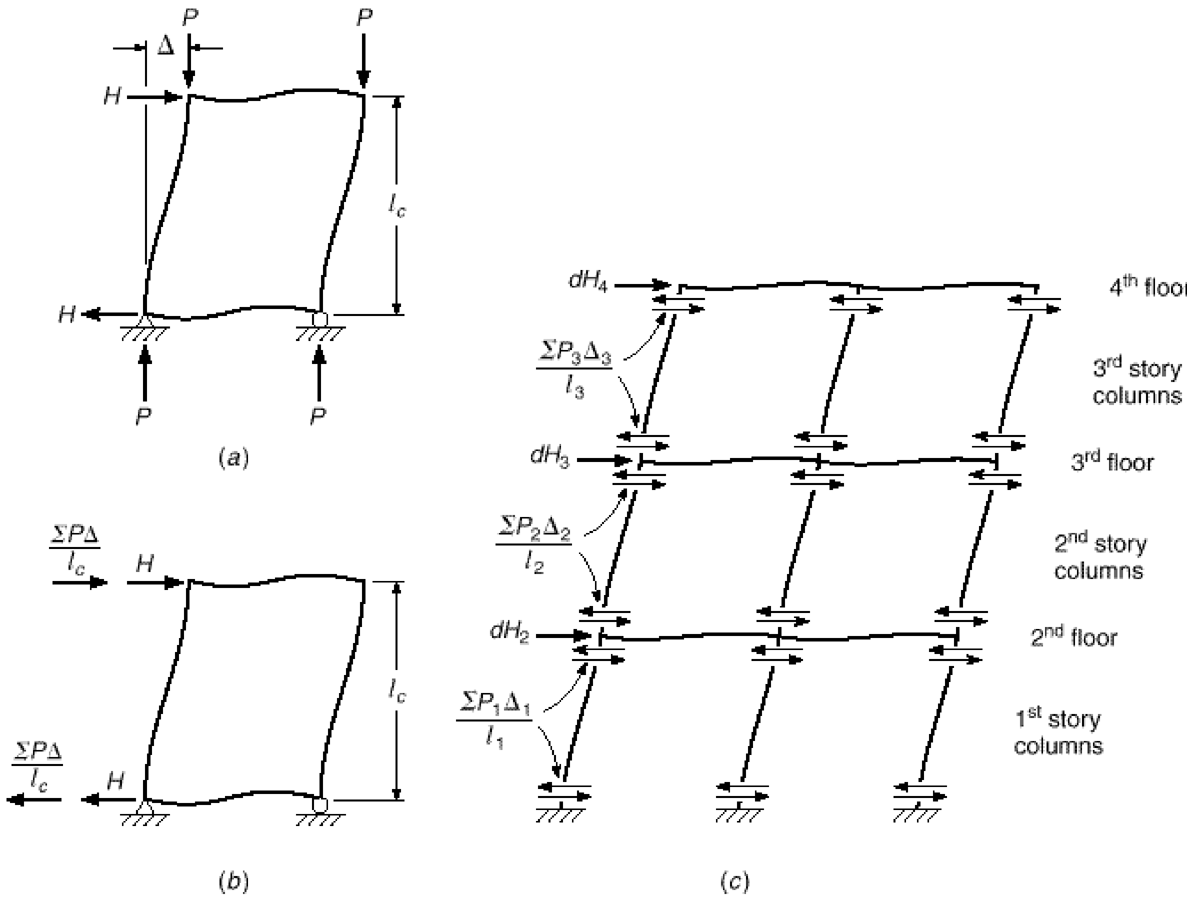


FIGURE 9.17
Basis for iterative P-Δ analysis: (*a*) vertical and lateral loads on rectangular frame; (*b*) real lateral forces H and fictitious sway forces dH; (*c*) three-story frame subject to sway forces. (*Adapted from Ref. 9.15.*)

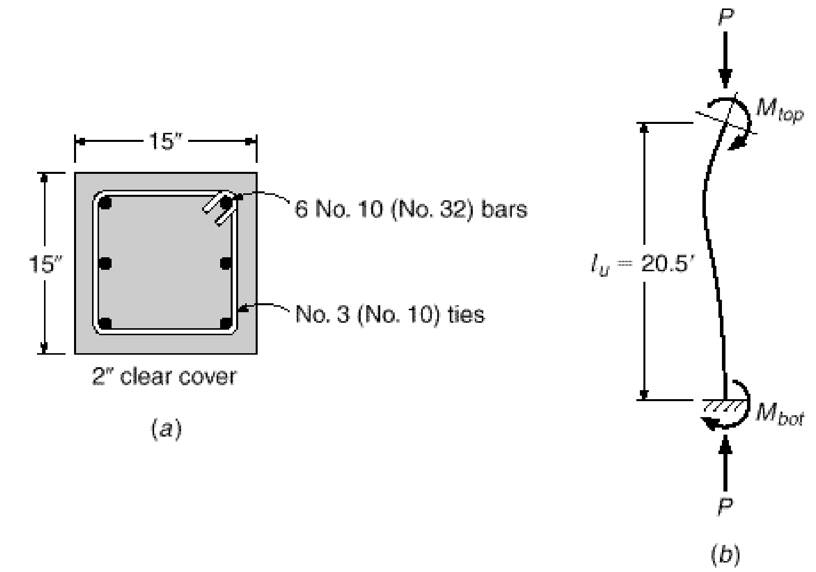
It is noted in Ref. 9.15 that a correction must be made in the analysis to account for the differences in shape between the $P\Delta$ moment diagram that has the same shape as the deflected column, and the moment diagram associated with the $P\Delta \cdot l$ forces, which is linear between the joints at the column ends. The area of the actual $P\Delta$ moment diagram is larger than the linear equivalent representation, and consequently lateral deflections will be larger. The difference will vary depending on the relative stiffnesses of the column and the beams framing into the joints. In Ref. 9.15, it is suggested that the increased deflection can be accounted for by taking the sway forces dH as 15 percent greater than the calculated value for each iteration. Iteration and the 15 percent increase in deflection are not required if the program performs a full nonlinear geometric analysis, since the $P\Delta$ moments are calculated in full.

The accuracy of the results of a P-Δ analysis will be strongly influenced by the values of member stiffness used, by foundation rotations, if any, and by the effects of concrete creep. In connection with creep effects, lateral loads causing significant sway are usually wind or earthquake loads of short duration, so creep effects are minimal. In general, the use of sway frames to resist *sustained* lateral loads, e.g., from earth or liquid pressures, is not recommended, and it would be preferable to include shear walls or other elements to resist these loads.

9.1. S. P. Timoshenko and J. M. Gere, *Theory of Elastic Stability*, 3d ed., McGraw-Hill, New York, 1969.
9.2. T. V. Galambos (ed.), *Guide to Stability Design Criteria for Metal Structures*, 5th ed., John Wiley & Sons, New York, 1998.
9.3. J. G. MacGregor and S. E. Hage, "Stability Analysis and Design of Concrete Frames," *J. Struct. Div.*, ASCE, vol. 103, no. ST 10, 1977, pp. 1953–1977.
9.4. S.-M. A. Lai and J. G. MacGregor, "Geometric Nonlinearities in Unbraced Multistory Frames," *J. Struct. Eng.*, ASCE, vol. 109, no. 11, 1983, pp. 2528–2545.
9.5. J. G. MacGregor, J. E. Breen, and E. O. Pfrang, "Design of Slender Concrete Columns," *J. ACI*, vol. 67, no. 1, 1970, pp. 6–28.
9.6. J. G. MacGregor, V. H. Oelhafen, and S. E. Hage, "A Reexamination of the *EI* Value for Slender Columns," *Reinforced Concrete Columns*, American Concrete Institute, Detroit, 1975, pp. 1–40.
9.7. *Code of Practice for the Structural Use of Concrete*, Part 1, "Design Materials and Workmanship," (CP110: Part 1, 1972), British Standards Institution, London, 1972.
9.8. W. B. Cranston, "Analysis and Design of Reinforced Concrete Columns," *Research Report* No. 20, Paper 41.020, Cement and Concrete Association, London, 1972.
9.9. R. W. Furlong, "Column Slenderness and Charts for Design," *J. ACI*, vol. 68, no. 1, 1971, pp. 9–18.
9.10. M. Valley and P. Dumonteil, Disc. of "*K*-Factor Equation to Alignment Charts for Column Design," by L. Duan, W.-S. King, and W.-F. Chen, *ACI Struct. J.*, vol. 91, no. 2, Mar.-Apr. 1994, pp. 229–230.
9.11. J. S. Ford, D. C. Chang, and J. E. Breen, "Design Indications from Tests of Unbraced Multipanel Concrete Frames," *Conc. Intl.*, vol. 3, no. 3, 1981, pp. 37–47.
9.12. *Building Code Requirements for Structural Concrete*, ACI 318-02, American Concrete Institute, Farmington Hills, MI, 2002.
9.13. *Commentary on Building Code Requirements for Structural Concrete*, ACI 318R-02, American Concrete Institute, Farmington Hills, MI, 2002 (published as part of Ref. 9.12).
9.14. J. G. MacGregor, "Design of Slender Concrete Columns —Revisited," *ACI Struct. J.*, vol. 90, no. 3, 1993, pp. 302–309.
9.15. J. G. MacGregor, *Reinforced Concrete*, 3d ed., Prentice-Hall, Upper Saddle River, NJ, 1997.
9.16. B. R. Wood, D. Beaulieu, and P. F. Adams, "Column Design by *P*-Delta Model," *Proc. ASCE*, vol. 102, no. ST2, 1976, pp. 487–500.
9.17. B. R. Wood, D. Beaulieu, and P. F. Adams, "Further Aspects of Design by *P*-Delta Model," *J. Struct. Div.*, ASCE, vol. 102, no. ST3, 1976, pp. 487–500.
9.18. R. W. Furlong, "Rational Analysis of Multistory Concrete Structures," *Conc. Intl.*, vol. 3, no. 6, 1981, pp. 29–35.

9.1. The 15 × 15 in. column shown in Fig. P9.1 must extend from footing level to the second floor of a braced frame structure with an unsupported length of 20.5 ft. Exterior exposure requires 2 in. clear cover for the outermost steel. Analysis indicates the critical loading corresponds with the following service loads: (*a*) from dead loads, $P = 170$ kips, $M_{top} = 29$ ft-kips, $M_{bot} = 14.5$ ft-kips; (*b*) from live loads, $P = 100$ kips, $M_{top} = 50$ ft-kips, $M_{bot} = 25$ ft-kips, with the column bent in double curvature as shown. The effective length factor k determined using Fig. 9.13*a* is 0.90. Material strengths are $f'_c = 4000$ psi and $f_y = 60{,}000$ psi. Using the ACI moment-magnifier method, determine whether the column is adequate to resist these loads.

FIGURE P9.1



9.2. The structure shown in Fig. P9.2*a* requires tall slender columns at the left side. It is fully braced by shear walls on the right. All columns are 16 × 16 in., as shown in Fig. P9.2*b*, and all beams are 24 × 18 in. with 6 in. monolithic floor slab, as in Fig. P9.2*c*. Trial calculations call for column reinforcement as shown. Alternate load analysis indicates the critical condition with column *AB*

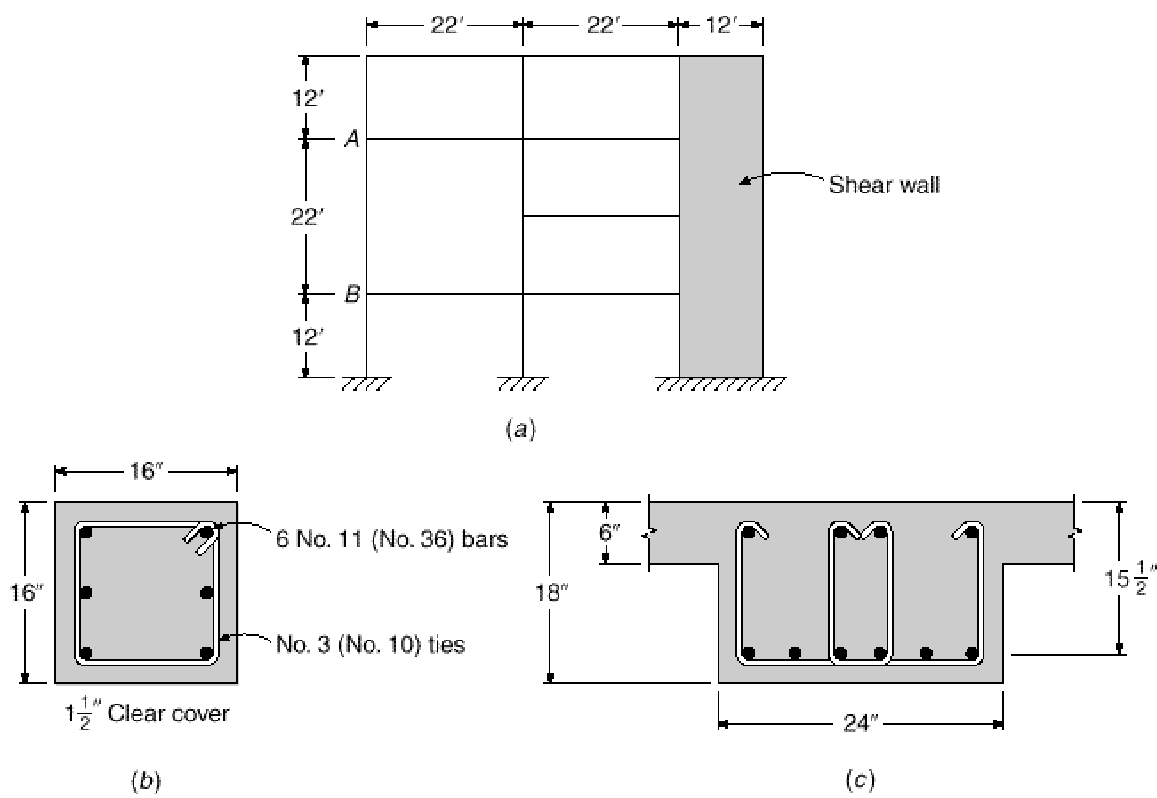


FIGURE P9.2

bent in single curvature, and service loads and moments as follows: from dead loads, P = 139 kips, M_{top} = 61 ft-kips, M_{bot} = 41 ft-kips; from live load, P = 93 kips, M_{top} = 41 ft-kips, M_{bot} = 27 ft-kips. Material strengths are f'_c = 4000 psi and f_y = 60,000 psi. Is the proposed column, reinforced as shown, satisfactory for this load condition? Use Eq. (9.16) to calculate EI for the column.

9.3. Refine the calculations of Problem 9.2, using Eq. (9.15) to calculate EI for the column. The reinforcement will be as given in Problem 9.2. Comment on your results.

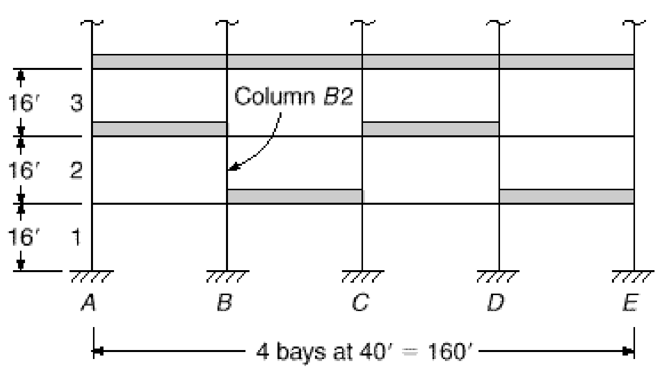
9.4. An interior column in a braced frame has an unsupported length of 20 ft and carries the following service load forces and moments: (*a*) from dead loads, P = 180 kips, M_{top} = 28 ft-kips, M_{bot} = −28 ft-kips; (*b*) from live loads, P = 220 kips, M_{top} = 112 ft-kips, M_{bot} = 112 ft-kips, with the signs of the moments representing double curvature under dead load and single curvature under live load. Rotational restraint factors at the top and bottom may be taken equal to 1.0. Design a square tied column to resist these loads, with a reinforcement ratio of about 0.02. Use f'_c = 4000 psi and f_y = 60,000 psi.

9.5. The first three floors of a multistory building are shown in Fig. P9.5. The lateral load resisting frame consists of 20 × 20 in. exterior columns, 24 × 24 in. interior columns, and 36 in. wide × 24 in. deep girders. The center-to-center column height is 16 ft. For the second-story columns, the service gravity dead and live loads and the horizontal wind loads based on an elastic first-order analysis of the frame are:

	Cols. A2 *and* E2	*Cols.* B2 *and* D2	*Col.* C2
P_{dead}	348 kips	757 kips	688 kips
P_{live}	137 kips	307 kips	295 kips
P_{wind}	±19 kips	±9 kips	0 kips
V_{wind}	6.5 kips	13.5 kips	13.5 kips
$M_{2,dead}$		31 ft-kips	
$M_{2,live}$		161 ft-kips	
$M_{2,wind}$		105 ft-kips	
$M_{1,dead}$		−34 ft-kips	
$M_{1,live}$		108 ft-kips	
$M_{1,wind}$		−98 ft-kips	

A matrix analysis for the total unfactored wind shear of 53.5 kips, using values of E and I specified in Section 9.5, indicates that the relative lateral deflection of the second story is 0.24 in. Design columns $B2$ and $D2$ using Eq. (9.19) to calculate ·$_sM_s$. Material strengths are f'_c = 4000 psi and f_y = 60,000 psi.

FIGURE P9.5



9.6. Repeat Problem 9.5 using Eq. (9.20) to calculate $\cdot {}_sM_s$. Comment on your results.

9.7. Redesign column *C*3 from Example 9.2 for a story height of 16 ft, a column unsupported length of 15 ft, and a relative lateral displacement of the third story of 1.10 in. Loads and other dimensions remain unchanged.