Experiment 1-B – Hydrostatic Force and Center of Pressure

This experiment is designed to help you understand how to locate the center of pressure and compute the hydrostatic force acting on a submerged surface.

Objectives:

- To determine experimentally the resultant hydrostatic force (total force) applied on a submerged surface.
- To determine the experimental and the theoretical center of pressure.

Description of the apparatus:

The apparatus consists of a transparent rectangular water tank that supports a counter balance arm. Attached to the counter balance arm is a torroidal quadrant. The tank has a drain at one end and a leveling screw at each corner of its base. On the top edge of the tank are two knife-edge supports that hold the counter balance arm. The counter-balance arm has an adjustable weight at one end and a weight pan at the other end. The arm is balanced by the use of a level mounted in the middle of the arm. Finally, a hook gauge mounted at one end of tank is used to measure water level.

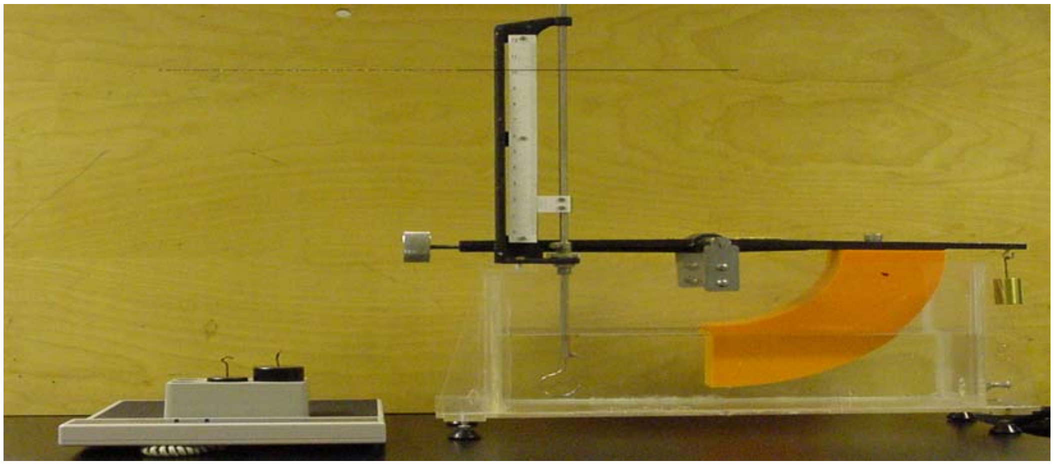

Figure 1.1: Hydrostatic Force Apparatus

Experimental Procedure:

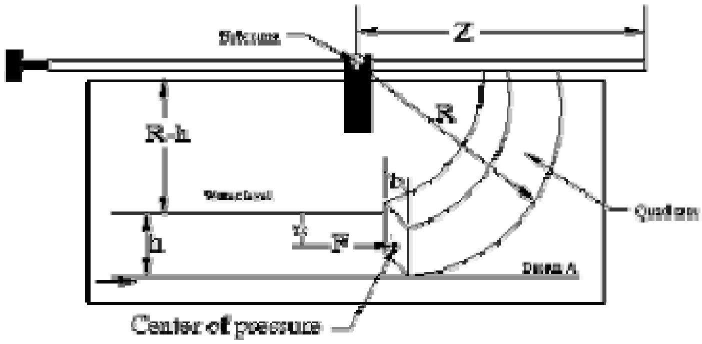

Figure 1.2: Sketch of the Apparatus

1. Mount one of the quadrants and measure Z, R and b as shown in the sketch.

2. Use the hose attached to the nearby faucet to add water to the tank until it reaches a height of approximately one inch. Adjust the leveling screws on the base of the tank to level it by visual inspection.

3. Use the levels located on top of the counter-balance arm to level it along its length and perpendicular to its length. Add masses as necessary to the left of the beam to balance it along its length; use the tank leveling screws to level it in the perpendicular direction.

4. Slowly add additional water to the tank until the water surface touches the lowest point of the surface “S” shown in Figure 1.3 below. Raise the point of the hook gauge until it just touches the water surface and tighten the venire against the rod at zero on the gauge. This establishes the datum “A,” or zero level.

5. Raise the hook gauge until the venire is at two inches on the gauge and secure the rod in place. Add water until it just reaches the point of the gauge. If you add too much water, simply raise the point of the hook gauge to the actual water surface. Record the actual height difference.

6. Add mass to the right side of the counter-balance arm to return the quadrant to the datum position. Record this weight.

7. Release the hook gauge and raise the rod an additional inch. Repeat the procedure outlined in steps 5 and 6 above.

8. Repeat steps 1 through 8 for two additional quadrants, using the same water depths.

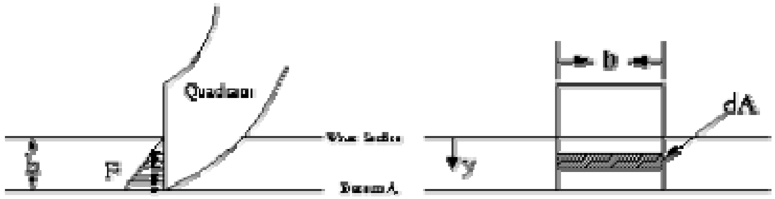

Figure 1.3: Hydrostatic Force Acting on Surface "S"

Assumptions and Formulation:

Assumptions:

1- Standard atmospheric pressure acts equally on all sides
2- Incompressible fluid
3- Acceleration of gravity is constant

Formulation:

1- Total force on submerged portion of the surface "S:"

The basic equation of the resultant hydrostatic force is as follows:

$$F_r = \int_A -pdA \qquad (1.1)$$

Equation 1.1 provides an expression for the resultant force F_r as a function of the pressure acting over the differential area element $dA = bdh$.

Assume the localized acceleration of gravity is a constant 9.81 m/s^2 and that the water is incompressible with constant density 996 kg/m^3. From the basic pressure-height relation for static fluid:

$$\frac{dp}{dh} = \rho \cdot g \qquad (1.2)$$

$$dp = \rho \cdot g \cdot dh \qquad (1.3)$$

$$p - p_0 = \int_0^h \rho \cdot g \cdot dh \qquad (1.4)$$

$$p = p_0 + \rho \cdot g \cdot h \qquad (1.5)$$

In equations 1.2 through 1.5 above, *p* is the pressure at any depth *h* measured positive downward from the water free surface and b is width of the surface “S.” Knowing that p_0 is zero, the resultant force equation becomes:

$$F_r = \int_0^h \rho \cdot g \cdot b \cdot h \cdot dh \qquad (1.6)$$

Integrating equation 1.6 yields:

$$F_r = \rho \cdot g \cdot b \cdot \frac{h^2}{2} \qquad (1.7)$$

2- Moment *M* of the total force F_r at the water surface:

Calculation of the moment of the hydrostatic force is analogous to similar calculations in solid mechanics:

$$M = \int_0^h h \cdot F_r \qquad (1.8)$$

$$M = \int_0^h \rho \cdot g \cdot b \cdot h^2 \cdot dh \qquad (1.9)$$

$$M = \rho \cdot g \cdot b \cdot \frac{h^3}{3} \qquad (1.10)$$

Note that the point of application of the resultant force (the center of pressure) must be such that the moment of the resultant force about any axis is equal to the moment of the distributed force about the same axis.

3-Theoretical center of pressure Y_c :

The theoretical center of pressure is given as:

$$Y_c = \frac{M}{Fr} \qquad (1.11)$$

With the above results:

$$Y_c = \frac{2}{3}h \qquad (1.12)$$

4-Experimental center of pressure Y_c :

A static equilibrium is reached once the quadrant is brought back to the datum position by adding weight to the right side of the counter-balance arm. Once the quadrant is in equilibrium, the sum of the moments about any point on the surface “S” is equal to zero.

Summing moments about the fulcrum axis:

$$\sum M_f = 0 = F_r \cdot (Y_c - (R-h)) - W \cdot Z \tag{1.13}$$

Where $(R-h)$ is the vertical distance between the water surface and the fulcrum axis, W is the weight on the weight pan (m*g), and Z is the horizontal distance from the fulcrum axis to the weight pan.

$$F_r \cdot (Y_c + (R-h)) = W \cdot Z \tag{1.14}$$

$$Y_c = \left(\frac{W \cdot Z}{F_r}\right) - (R-h) \tag{1.15}$$

5- Percentage of error:

$$\text{Percentage of error} = \left[\frac{(Y_c)_{theoretical} - (Y_c)_{experimental}}{(Y_c)_{theoretical}}\right] \cdot 100 \tag{1.16}$$

Report requirement:

1-Compute the following:

- The total force (F_r) acting on the submerged portion of the surface
- The moment (M) of the total force (F_r) at the water surface
- The theoretical center of pressure (Y_c)
- The experimental center of pressure (Y_c), by summing moments about the fulcrum axis
- The percentage of error between the theoretical and experimental values of Y_c

2- In the results section; discuss results, sources of error, and possible discrepancies with theoretical data.