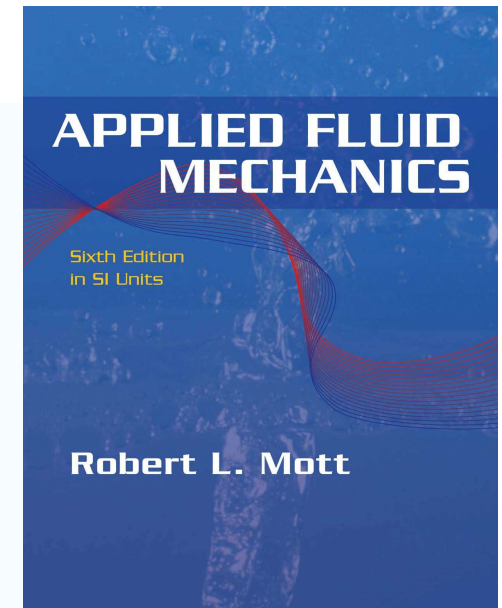


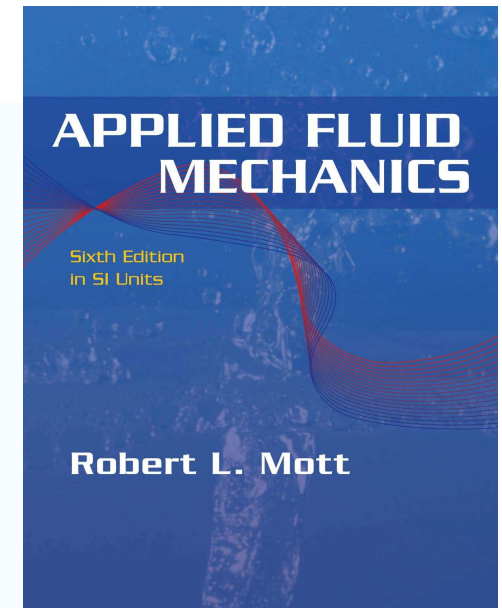
Applied Fluid Mechanics

1. The Nature of Fluid and the Study of Fluid Mechanics
2. Viscosity of Fluid
3. Pressure Measurement
4. Forces Due to Static Fluid
5. Buoyancy and Stability
6. Flow of Fluid and Bernoulli's Equation
7. General Energy Equation
8. Reynolds Number, Laminar Flow, Turbulent Flow and Energy Losses Due to Friction



Applied Fluid Mechanics

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections
10. Minor Losses
11. Series Pipeline Systems
12. Parallel Pipeline Systems
13. Pump Selection and Application
14. Open-Channel Flow
15. Flow Measurement
16. Forces Due to Fluids in Motion

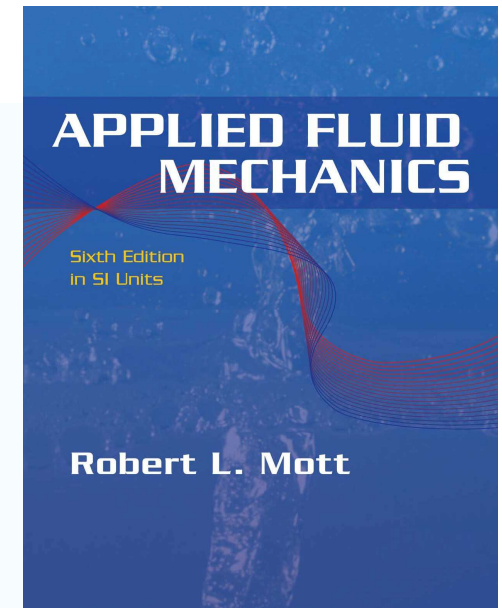


Applied Fluid Mechanics

17. Drag and Lift

18. Fans, Blowers, Compressors
and the Flow of Gases

19. Flow of Air in Ducts



4. Forces due to Static Fluid

Chapter Objectives

1. Compute the force exerted on a plane area by a pressurized gas.
2. Compute the force exerted by any static fluid acting on a horizontal plane area.
3. Compute the resultant force exerted on a rectangular wall by a static liquid.
4. Define the term *center of pressure*.
5. Compute the resultant force exerted on any submerged plane area by a static liquid.
6. Show the vector representing the resultant force on any submerged plane area in its proper location and direction.

4. Forces due to Static Fluid

Chapter Objectives

7. Visualize the distribution of force on a submerged curved surface.
8. Compute the total resultant force on the curved surface.
9. Compute the direction in which the resultant force acts and show its line of action on a sketch of the surface.
10. Include the effect of a pressure head over the liquid on the force on a plane or curved surface.

4. Forces due to Static Fluid

Chapter Outline

1. Introductory Concepts
2. Gases Under Pressure
3. Horizontal Flat Surfaces Under Liquids
4. Rectangular Walls
5. Submerged Plane Areas-General
6. Development of the General Procedure for Forces on Submerged Plane Areas
7. Piezometric Head
8. Distribution of Force on a Submerged Curved Surface
9. Effect of a Pressure Above the Fluid Surface

4. Forces due to Static Fluid

Chapter Outline

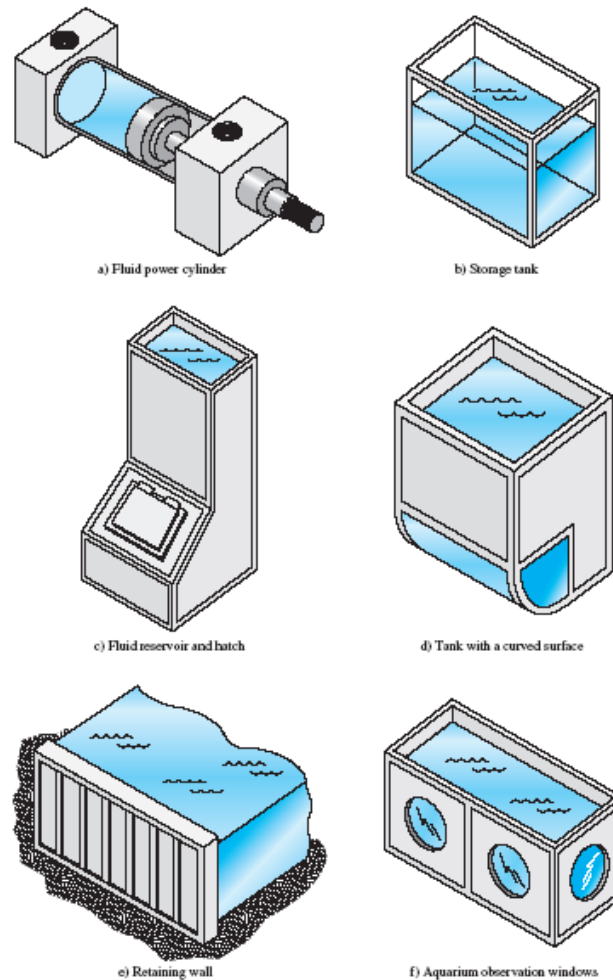
10. Forces on a Curved Surface with Fluid Below It

11. Forces on Curved Surfaces with Fluid Above and Below

4. Forces due to Static Fluid

4.1 Introductory Concepts

- Fig 4.1 shows the examples of cases where forces on submerged areas must be computed.



4. Forces due to Static Fluid

4.1 Introductory Concepts

- In each case, the fluid exerts a force on the surface of interest that acts perpendicular to the surface, considering the basic definition of pressure, $p = F/A$ and the corresponding form, $F = pA$.
- We apply these equations directly only when the pressure is uniform over the entire area of interest.
- An example is when the fluid is a gas for which we consider the pressure to be equal throughout the gas because of its low specific weight.

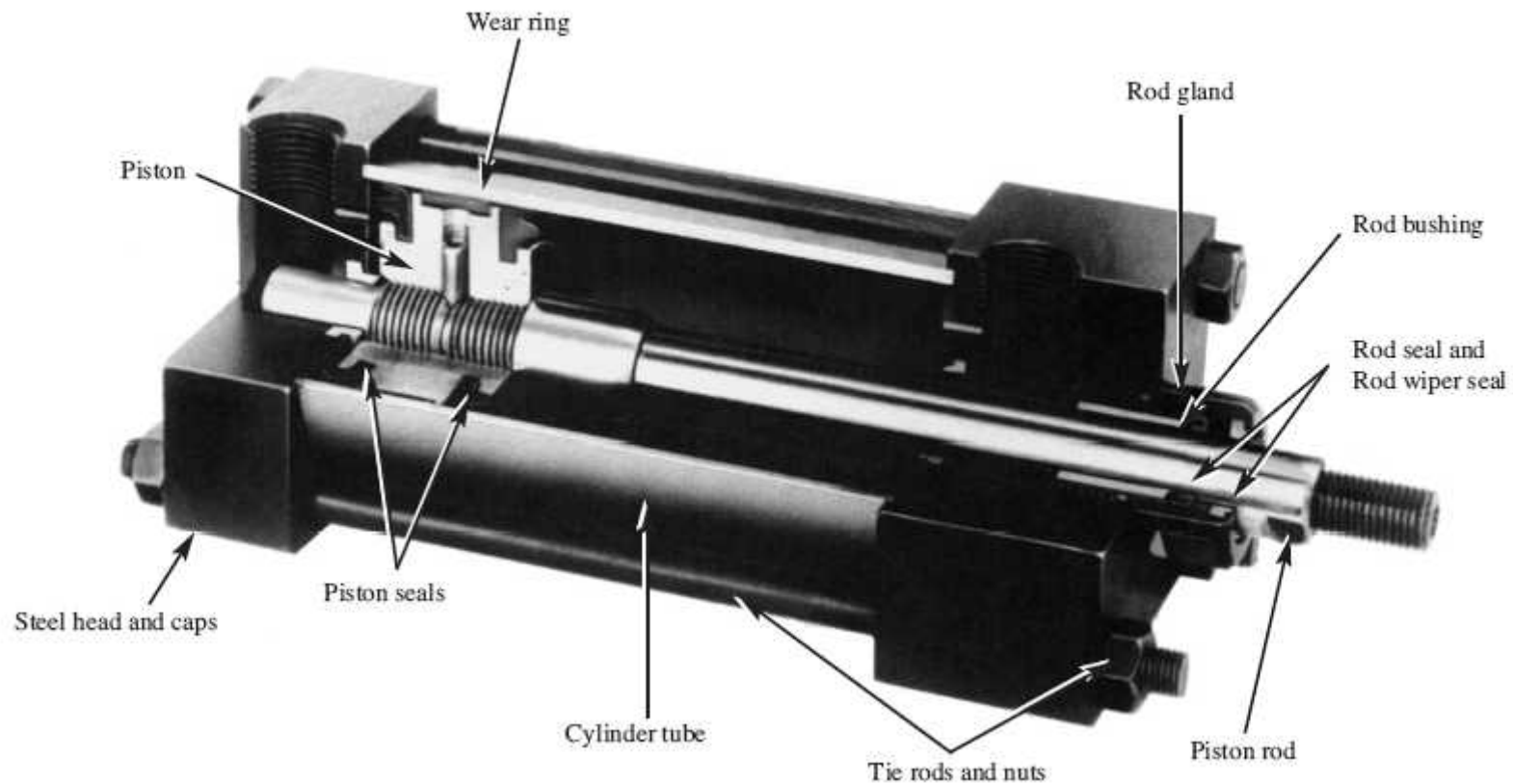
4. Forces due to Static Fluid

4.2 Gases Under Pressure

- Fig 4.2 shows the fluid power cylinder.
- The air pressure acts on the piston face, producing a force that causes the linear movement of the rod.
- The pressure also acts on the end of the cylinder, tending to pull it apart.
- Therefore, we can calculate the force on the piston and the cylinder ends directly from $F = pA$.

4. Forces due to Static Fluid

4.2 Gases Under Pressure



4. Forces due to Static Fluid

Example 4.1

If the cylinder in Fig. 4.2 has an internal diameter of 51 mm and operates at a pressure of 2070 kPa (gage), calculate the force on the ends of the cylinder.

$$F = pA$$

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.051m)^2}{4} = 0.002m^2$$

$$F = \frac{2070 \times 10^3 N / m^2}{m^2} \times 0.002m^2 = 4.41kN$$

4. Forces due to Static Fluid

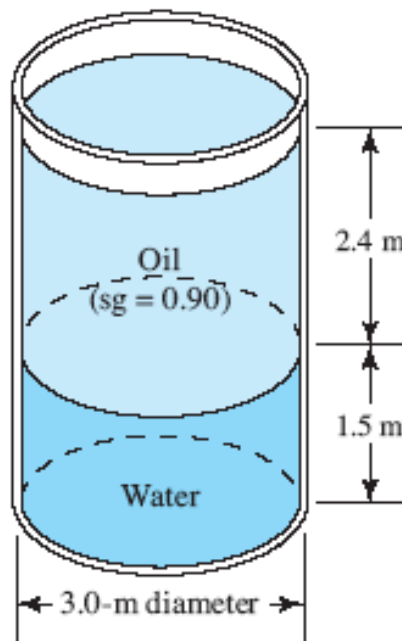
Example 4.1

Notice that gage pressure was used in the calculation of force instead of absolute pressure. The additional force due to atmospheric pressure acts on both sides of the area and is thus balanced. If the pressure on the outside surface is not atmospheric, then all external forces must be considered to determine a net force on the area.

4. Forces due to Static Fluid

4.3 Horizontal Flat Surfaces Under Liquids

- Fig 4.3 shows the cylindrical drum.
- The pressure in the water at the bottom of the drum is uniform across the entire area because it is a horizontal plane in a fluid at rest.
- We can simply use $F = pA$ to calculate the force on the bottom



4. Forces due to Static Fluid

Example 4.2

If the drum in Fig. 4.3 is open to the atmosphere at the top, calculate the force on the bottom.

To use we must first calculate the pressure at the bottom of the drum and the area of the bottom:

$$p_B = p_{\text{atm}} + \gamma_o(2.4 \text{ m}) + \gamma_w(1.5 \text{ m})$$

$$\gamma_o = (\text{sg})_o(9.81 \text{ kN/m}^3) = (0.90)(9.81 \text{ kN/m}^3) = 8.83 \text{ kN/m}^3$$

$$\begin{aligned} p_B &= 0 \text{ Pa(gage)} + (8.83 \text{ kN/m}^3)(2.4 \text{ m}) + (9.81 \text{ kN/m}^3)(1.5 \text{ m}) \\ &= (0 + 21.2 + 14.7) \text{ kPa} = 35.9 \text{ kPa(gage)} \end{aligned}$$

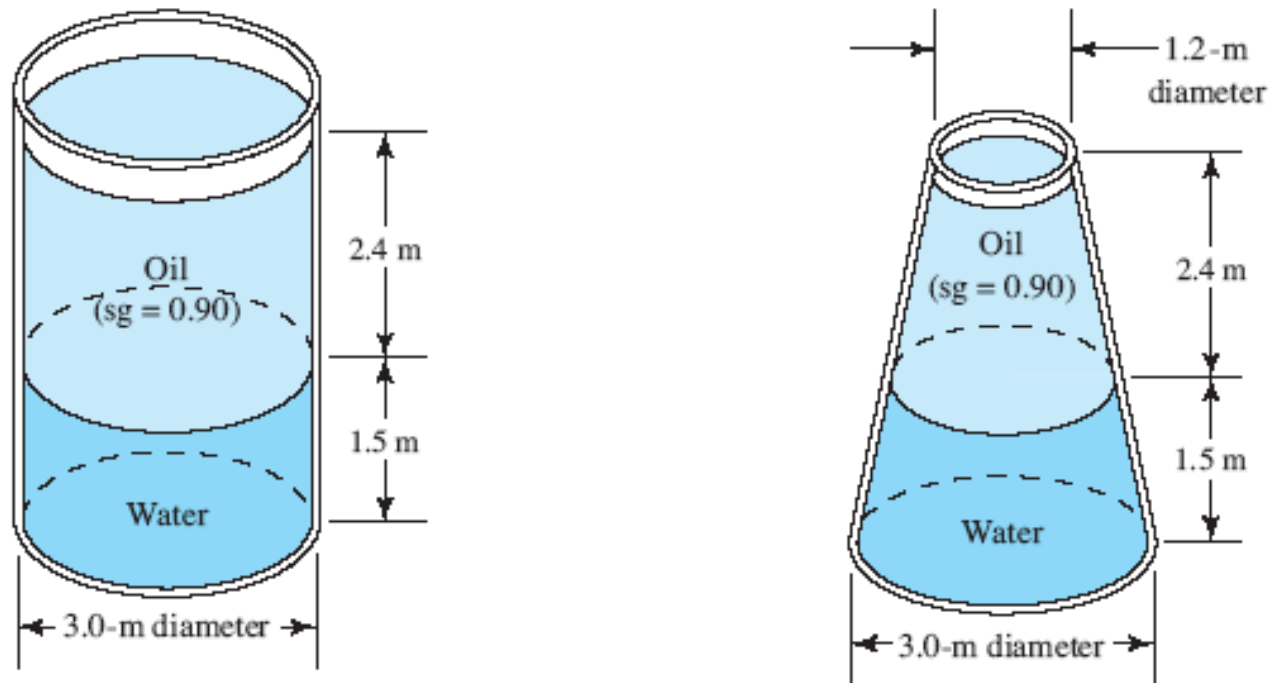
$$A = \pi D^2/4 = \pi(3.0 \text{ m})^2/4 = 7.07 \text{ m}^2$$

$$F = p_B A = (35.9 \text{ kN/m}^2)(7.07 \text{ m}^2) = 253.8 \text{ kN}$$

4. Forces due to Static Fluid

Example 4.2

Would there be any difference between the force on the bottom of the drum in Fig. 4.3 and that on the bottom of the cone-shaped container in Fig. 4.4?



4. Forces due to Static Fluid

Example 4.2

The force would be the same because the pressure at the bottom is dependent only on the depth and specific weight of the fluid in the container. The total weight of fluid is not the controlling factor.

Comment: The force computed in these two example problems is the force exerted by the fluid on the inside bottom of the container. Of course, when designing the support structure for the container, the total weight of the container and the fluids must be considered. For the structural design, the cone-shaped container will be lighter than the cylindrical drum.

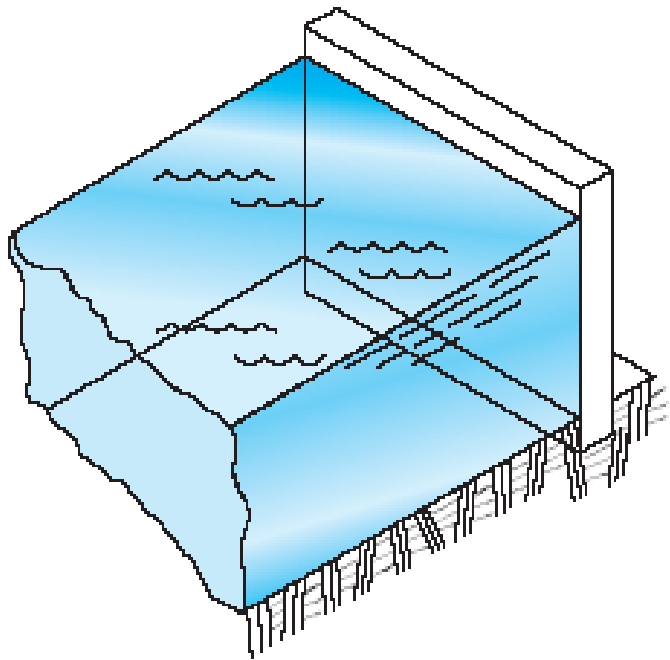
4. Forces due to Static Fluid

4.4 Rectangular Walls

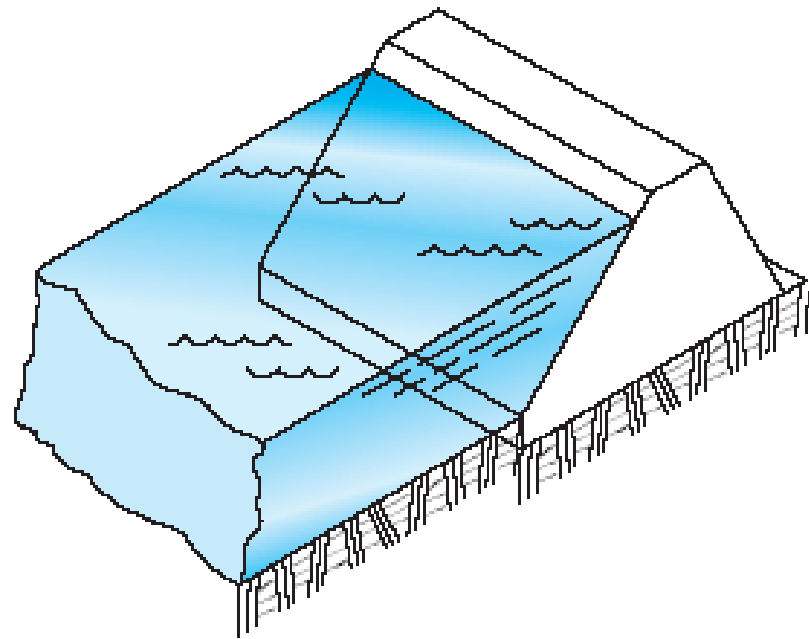
- Fig 4.5 shows the rectangular walls.
- They are walls which exposed to a pressure varying from zero on the surface of the fluid to a maximum at the bottom of the wall.
- The force due to the fluid pressure tends to overturn the wall or break it at the place where it is fixed to the bottom.
- The actual force is distributed over the entire wall, but for the purpose of analysis it is desirable to determine the resultant force and the place where it acts, called the *center of pressure*.

4. Forces due to Static Fluid

4.4 Rectangular Walls



(a) Vertical retaining wall

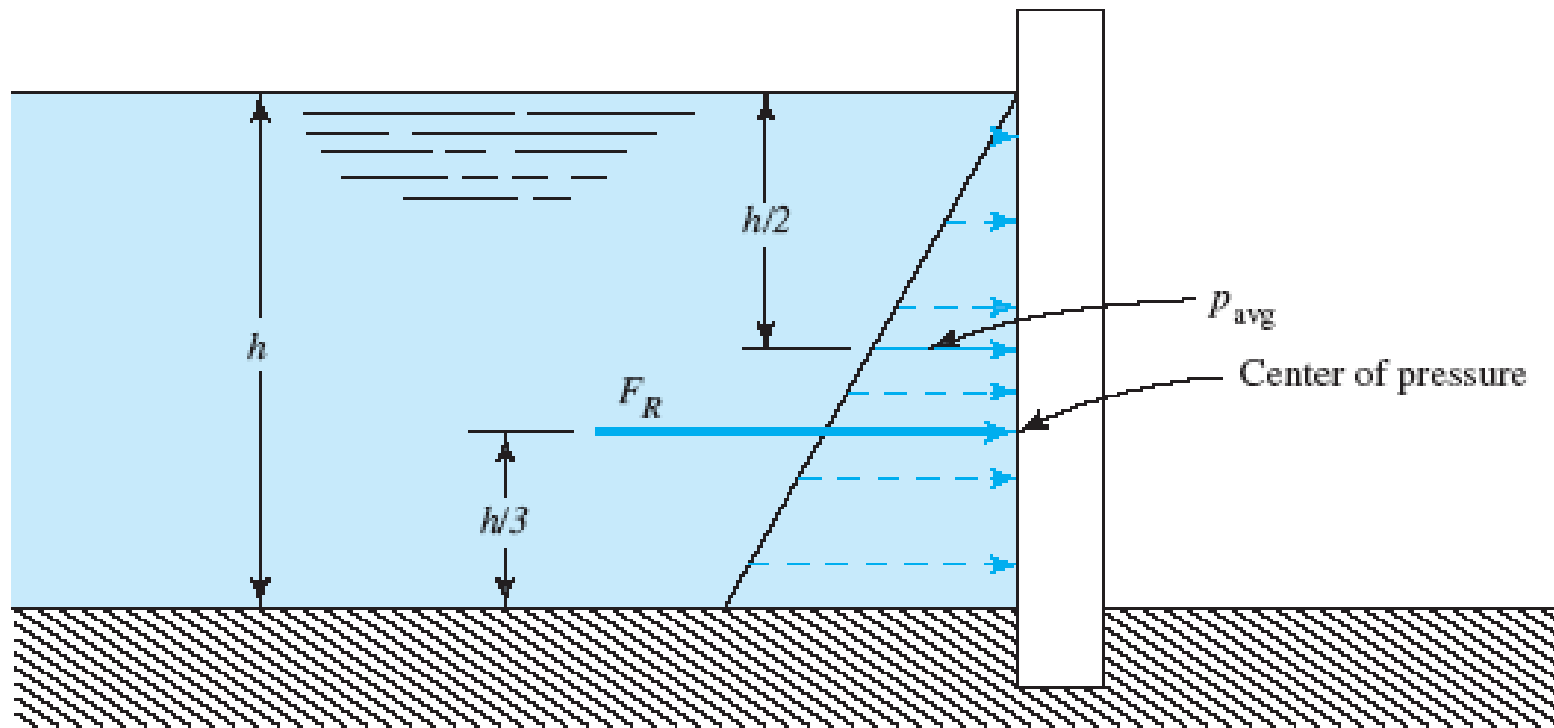


(b) Inclined wall (dam)

4. Forces due to Static Fluid

4.4 Rectangular Walls

- Fig 4.6 shows the vertical rectangular wall.



4. Forces due to Static Fluid

4.4 Rectangular Walls

- The total resultant force can be calculated from the equation

$$F_R = p_{avg} \times A \quad (4-1)$$

where p_{avg} is the average pressure and A is the total area of the wall.

- But the average pressure is that at the middle of the wall and can be calculated from the equation

$$p_{avg} = \gamma(h/2) \quad (4-2)$$

where h is the total depth of the fluid.

4. Forces due to Static Fluid

4.4 Rectangular Walls

- Therefore, we have

$$F_R = \gamma(h/2)A \quad (4-3)$$

- The center of pressure is at the centroid of the pressure distribution triangle, one third of the distance from the bottom of the wall.
- The resultant force acts perpendicular to the wall at this point.

4. Forces due to Static Fluid

4.4 Rectangular Walls

- Below are the procedures for computing the force on a rectangular wall:
 1. Calculate the magnitude of the resultant force from

$$F_R = \gamma(h/2)A$$

γ = Specific weight of the fluid

h = Total depth of the fluid

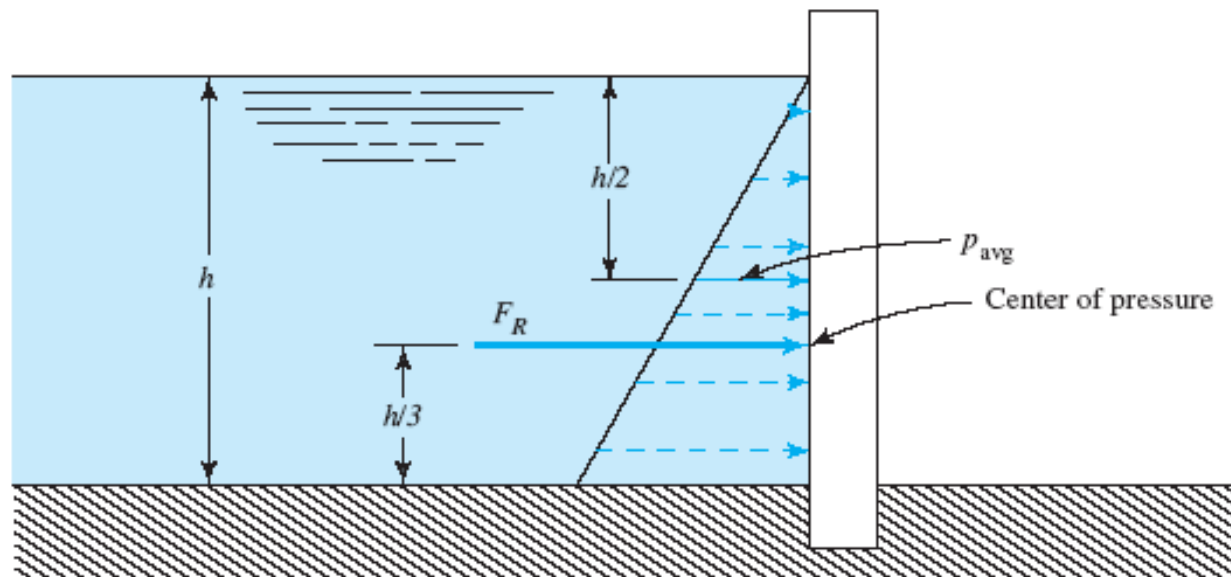
A = Total area of the wall

2. Locate the center of pressure at a vertical distance of from the bottom of the wall.
3. Show the resultant force acting at the center of pressure perpendicular to the wall.

4. Forces due to Static Fluid

Example 4.4

In Fig. 4.6, the fluid is gasoline and the total depth is 3.7 m. The wall is 12.2 m long. Calculate the magnitude of the resultant force on the wall and the location of the center of pressure.



4. Forces due to Static Fluid

Example 4.4

Step 1

$$F_R = \gamma (h / 2) A$$

$$\gamma = (0.68)(9.81 \text{ kN} / \text{m}^3) = 6.67 \text{ kN} / \text{m}^3$$

$$A = (3.7 \text{ m})(12.2 \text{ m}) = 45.14 \text{ m}^2$$

$$F_R = \frac{6.67 \text{ kN} / \text{m}^3}{\text{m}^3} \times \frac{3.7 \text{ m}}{2} \times 45.14 \text{ m}^2 = 557.0 \text{ kN}$$

4. Forces due to Static Fluid

Example 4.4

Step 2

$$h/3 = 3.7 \text{ m}/3 = 1.23 \text{ m}$$

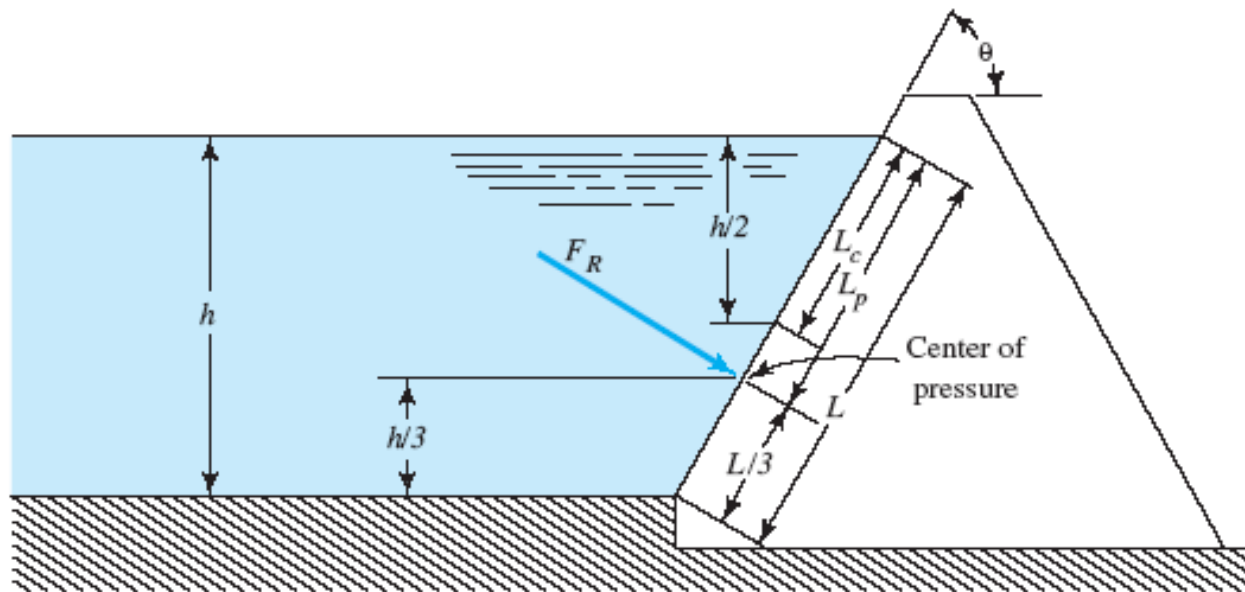
Step 3

The force acts perpendicular to the wall at the center of pressure as shown in Fig. 4.6.

4. Forces due to Static Fluid

Example 4.5

Figure 4.7 shows a dam 30.5 m long that retains 8 m of fresh water and is inclined at an angle of 60° . Calculate the magnitude of the resultant force on the dam and the location of the center of pressure.



4. Forces due to Static Fluid

Example 4.5

Step 1

$$F_R = \gamma(h/2)A$$

To calculate the area of the dam we need the length of its face, called L in Fig. 4.7:

$$\sin \theta = h/L$$

$$L = h/\sin \theta = 8 \text{ m}/\sin 60^\circ = 9.24 \text{ m}$$

Then, the area of the dam is

$$A = (9.24 \text{ m})(30.5 \text{ m}) = 281.8 \text{ m}^2$$

4. Forces due to Static Fluid

Example 4.5

Step 1

Now we can calculate the resultant force:

$$\begin{aligned} F_R &= \gamma(h/2)A = \frac{9.81 \text{ kN}}{\text{m}^3} \times \frac{8 \text{ m}}{2} \times 281.8 \text{ m}^2 \\ &= 11\,060 \text{ kN} = 11.06 \text{ MN} \end{aligned}$$

4. Forces due to Static Fluid

Example 4.5

Step 2

The center of pressure is at a vertical distance of

$$h/3 = 8 \text{ m}/3 = 2.67 \text{ m}$$

from the bottom of the dam, or, measured from the bottom of the dam along the face of the dam, the center of pressure is at

$$L/3 = 9.24 \text{ m}/3 = 3.08 \text{ m}$$

4. Forces due to Static Fluid

Example 4.5

Step 2

Measured along the face of the dam we define

L_p = Distance from the free surface of the fluid to the center of pressure

$$L_p = L - L/3$$

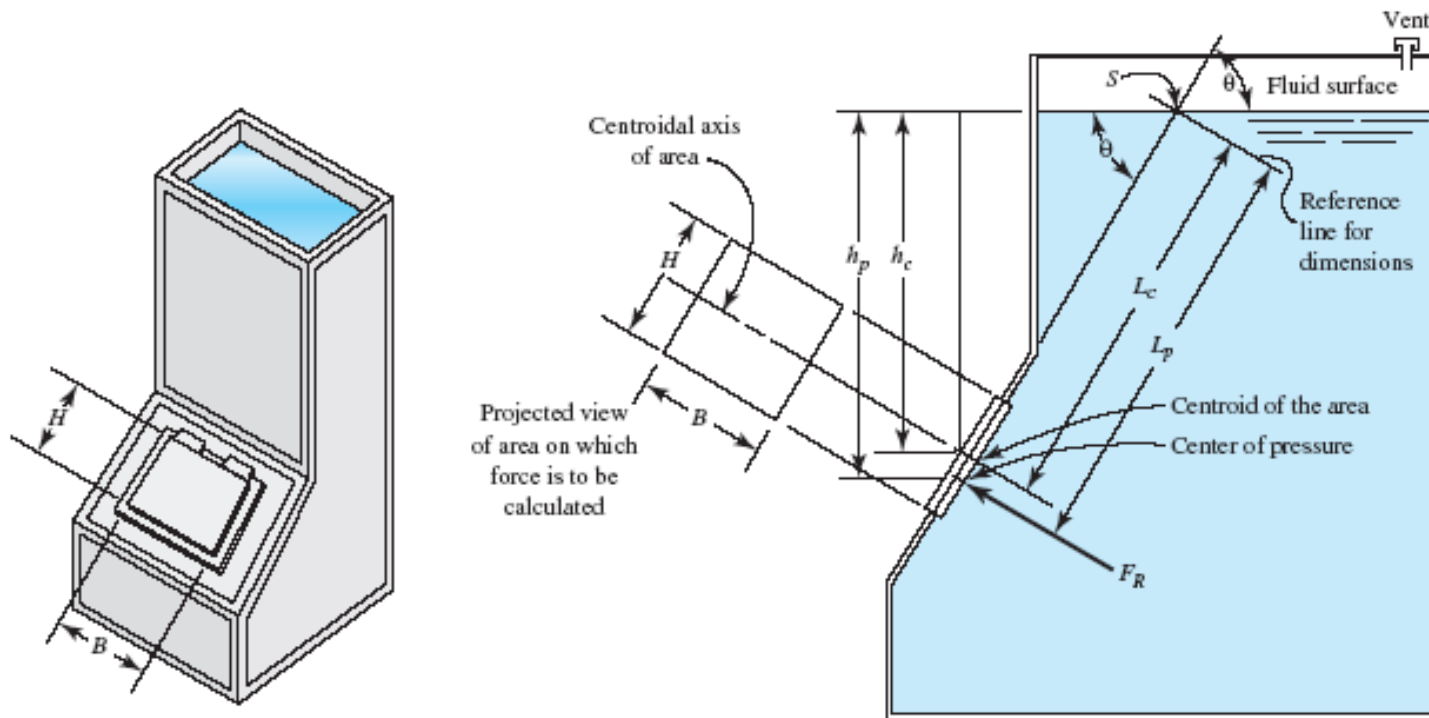
$$L_p = 9.24 \text{ m} - 3.08 \text{ m} = 6.16 \text{ m}$$

We show F_R acting at the center of pressure perpendicular to the wall.

4. Forces due to Static Fluid

4.5 Submerged Plane Areas - General

- Fig 4.85 shows the force on a submerged plane area.



4. Forces due to Static Fluid

4.5 Submerged Plane Areas - General

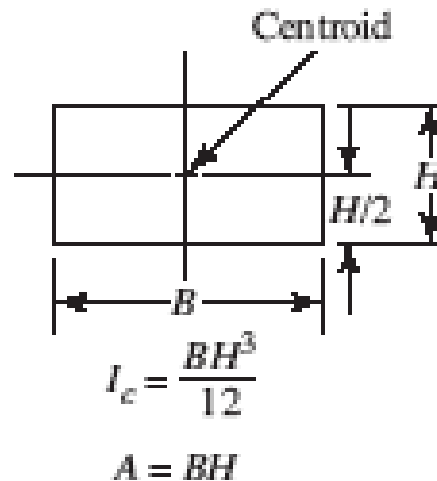
- The standard dimensions and symbols used in the procedure described later are shown in the figure and defined as follows:

F_R	Resultant force on the area due to the fluid pressure
—	The <i>center of pressure</i> of the area is the point at which the resultant force can be considered to act
—	The <i>centroid</i> of the area is the point at which the area would be balanced if suspended from that point; it is equivalent to the center of gravity of a solid body
θ	Angle of inclination of the area
h_c	Depth of fluid from the free surface to the centroid of the area
L_c	Distance from the level of the free surface of the fluid to the centroid of the area, measured along the angle of inclination of the area
L_p	Distance from the level of the free surface of the fluid to the center of pressure of the area, measured along the angle of inclination of the area
h_p	Distance from the free surface to the center of pressure of the area
B, H	Dimensions of the area

4. Forces due to Static Fluid

4.5 Submerged Plane Areas - General

- Fig 4.9 shows the properties of a rectangle.



4. Forces due to Static Fluid

4.5 Submerged Plane Areas - General

- Below are the procedure for computing the force on a submerged plane area:
 1. Identify the point where the angle of inclination of the area of interest intersects the level of the free surface of the fluid. This may require the extension of the angled surface or the fluid surface line. Call this point S .
 2. Locate the centroid of the area from its geometry.
 3. Determine h_c as the *vertical* distance from the level of the free surface down to the centroid of the area.

4. Forces due to Static Fluid

4.5 Submerged Plane Areas - General

4. Determine L_c as the *inclined* distance from the level of the free surface down to the centroid of the area. This is the distance from S to the centroid. Note that h_c and L_c are related by

$$h_c = L_c \sin \theta$$

5. Calculate the total area A on which the force is to be determined.
6. Calculate the resultant force from

$$F_R = \gamma h_c A \quad (4-4)$$

4. Forces due to Static Fluid

4.5 Submerged Plane Areas - General

where γ is the specific weight of the fluid. This equation states that the resultant force is the product of the pressure at the centroid of the area and the total area.

7. Calculate I_c the moment of inertia of the area about its centroidal axis.
8. Calculate the location of the center of pressure from

$$L_p = L_c + \frac{I_c}{L_c A} \quad (4-5)$$

4. Forces due to Static Fluid

4.5 Submerged Plane Areas - General

Notice that the center of pressure is always below the centroid of an area that is inclined with the horizontal. In some cases it may be of interest to calculate only the difference between L_p and L_c from

$$L_p - L_c = \frac{I_c}{L_c A} \quad (4-6)$$

9. Sketch the resultant force F_R acting at the center of pressure, perpendicular to the area.
10. Show the dimension L_p on the sketch in a manner similar to that used in Fig. 4.8.
11. Draw the dimension lines for L_p and L_c from a reference line drawn through point S and perpendicular to the angle of inclination of the area.

4. Forces due to Static Fluid

4.5 Submerged Plane Areas - General

12. If it is desired to compute the vertical depth to the center of pressure, h_p either of two methods can be used. If the distance L_p has already been computed, use

$$h_p = L_p \sin \theta$$

Alternatively, Step 8 could be avoided and h_p can be computed directly from

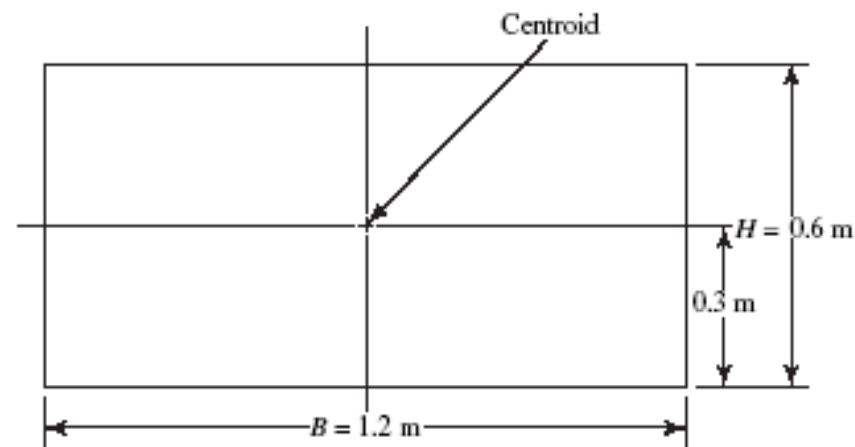
$$h_p = h_c + \frac{I_c \sin^2 \theta}{h_c A}$$

We will now use the programmed instruction approach to illustrate the application of this procedure.

4. Forces due to Static Fluid

Example 4.6

The tank shown in Fig. 4.8 contains a lubricating oil with a specific gravity of 0.91. A rectangular gate with the dimensions $B=1.2$ m and $H=0.6$ m is placed in the inclined wall of the tank ($\theta = 60^\circ$). The centroid of the gate is at a depth of 1.5 m from the surface of the oil. Calculate (a) the magnitude of the resultant force F_R on the gate and (b) the location of the center of pressure.



4. Forces due to Static Fluid

Example 4.6

The terms L_c and h_c are related in this case by

$$h_c = L_c \sin \theta$$

Therefore, we have

$$L_c = h_c / \sin \theta = 1.5 \text{ m} / \sin 60^\circ = 1.73 \text{ m}$$

Because the area of the rectangle is BH ,

$$A = BH = (1.2 \text{ m})(0.6 \text{ m}) = 0.72 \text{ m}^2$$

We need the specific weight of the oil:

$$\begin{aligned} \gamma_o &= (\text{sg})_o (9.81 \text{ kN/m}^3) = (0.91)(9.81 \text{ kN/m}^3) \\ &= 8.92 \text{ kN/m}^3 \end{aligned}$$

4. Forces due to Static Fluid

Example 4.6

Then we have

$$F_R = \gamma_0 h_c A = \frac{8.92 \text{ kN}}{\text{m}^3} \times 1.5 \text{ m} \times 0.72 \text{ m}^2 = 9.63 \text{ kN}$$

We find that for a rectangle,

$$I_c = BH^3/12 = (1.2 \text{ m})(0.6 \text{ m})^3 = 0.02 \text{ m}^4$$

Because $I_c=0.02 \text{ m}^4$, $L_c=1.73 \text{ m}$, and $A=0.72 \text{ m}^2$,

$$L_p = L_c + \frac{I_c}{L_c A} = 1.73 \text{ m} + \frac{0.02 \text{ m}^4}{(1.73 \text{ m})(0.72 \text{ m}^2)}$$

$$L_p = 1.73 \text{ m} + 0.016 \text{ m} = 1.75 \text{ m}$$

4. Forces due to Static Fluid

Example 4.6

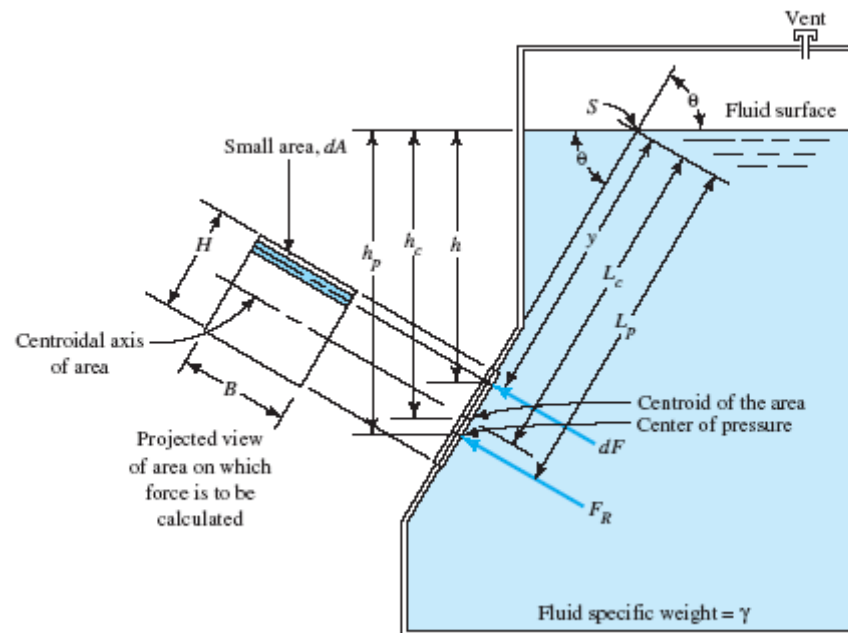
This means that the center of pressure is 0.016 m (or 16 mm) below the centroid of the gate.

Be sure you understand how the dimension L_p is drawn from the reference line.

4. Forces due to Static Fluid

4.6 Development of the General Procedure for Forces on Submerged Plane Areas

- The *resultant force* is defined as the summation of the forces on small elements of interest.
- Fig 4.11 shows the development of the general procedure for forces on submerged plane areas.



4. Forces due to Static Fluid

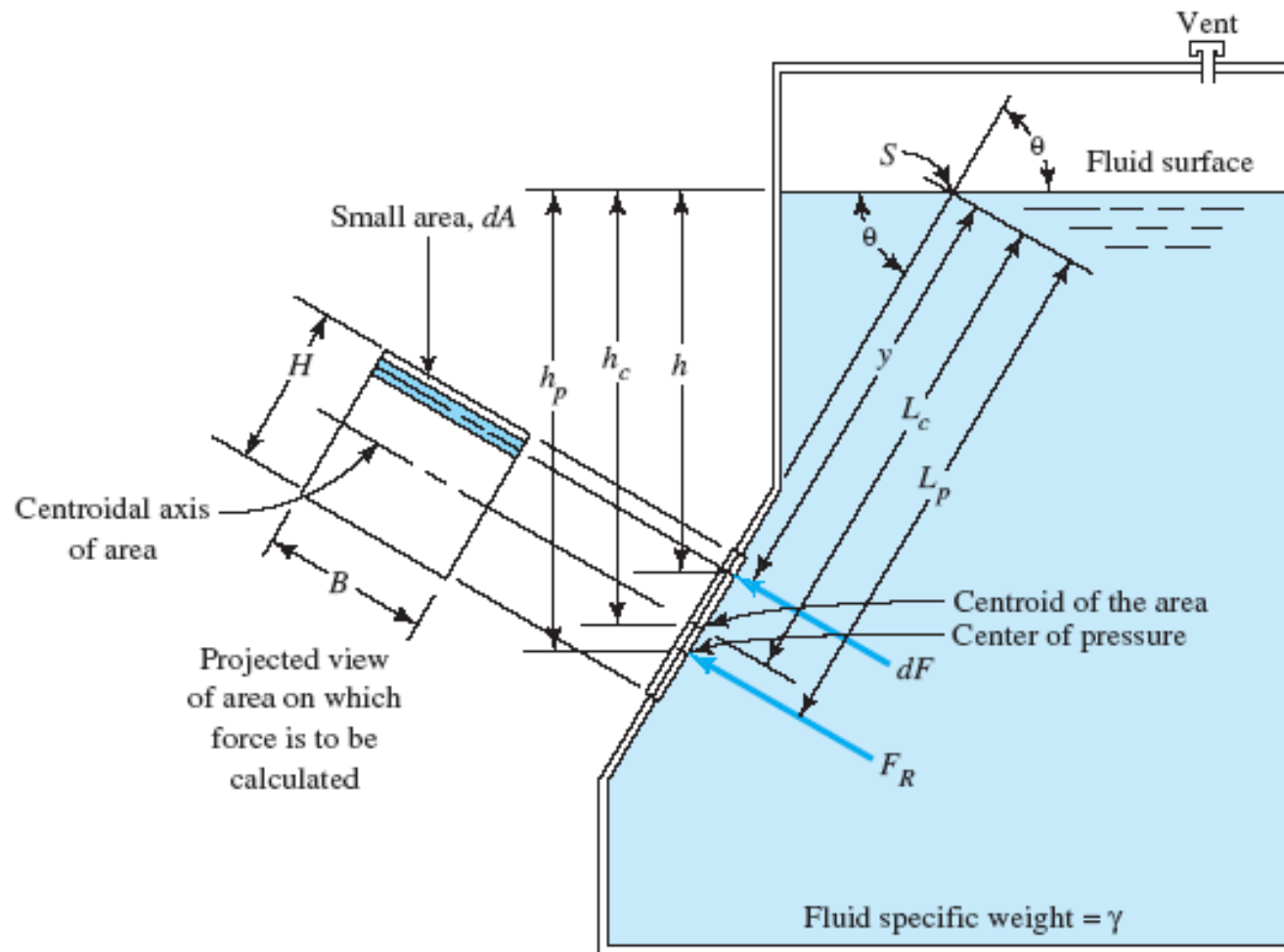
4.6 Development of the General Procedure for Forces on Submerged Plane Areas

- The *resultant force* is defined as the summation of the forces on small elements of interest.
- Fig 4.11 shows the development of the general procedure for forces on submerged plane areas.
- On any small area dA , there exists a force dF acting perpendicular to the area owing to the fluid pressure p . But the magnitude of the pressure at any depth h in a static liquid of specific weight is $p = \gamma h$.
- Then, the force is

$$dF = p(dA) = \gamma h(dA) \quad (4-7)$$

4. Forces due to Static Fluid

4.6 Development of the General Procedure for Forces on Submerged Plane Areas



4. Forces due to Static Fluid

4.6 Development of the General Procedure for Forces on Submerged Plane Areas

- Because the area is inclined at an angle it is convenient to work in the plane of the area, using y to denote the position on the area at any depth h . Note that

$$h = y \sin \theta \quad (4-8)$$

where y is measured from the level of the free surface of the fluid along the angle of inclination of the area. Then,

$$dF = \gamma(y \sin \theta)(dA) \quad (4-9)$$

4. Forces due to Static Fluid

4.6 Development of the General Procedure for Forces on Submerged Plane Areas

- The summation of forces over the entire area is accomplished by the mathematical process of integration,

$$F_R = \int_A dF = \int_A \gamma(y \sin \theta)(dA) = \gamma \sin \theta \int_A y(dA)$$

- From mechanics we learn that $\int y(dA)$ is equal to the product of the total area times the distance to the centroid of the area from the reference axis. That is,

$$\int_A y(dA) = L_c A$$

4. Forces due to Static Fluid

4.6 Development of the General Procedure for Forces on Submerged Plane Areas

- Then, the resultant force is

$$F_R = \gamma \sin \theta (L_c A) \quad (4-10)$$

$$F_R = \gamma h_c A \quad (4-11)$$

- This is the same form as Eq. (4-4). Because each of the small forces dF acted perpendicular to the area, the resultant force also acts perpendicular to the area.

4. Forces due to Static Fluid

4.6.1 Center of Pressure

- The *center of pressure* is that point on an area where the resultant force can be assumed to act so as to have the same effect as the distributed force over the entire area due to fluid pressure.
- The moment of each small force dF is

$$dM = dF \cdot y$$

- But

$$dM = y[\gamma(y \sin \theta)(dA)] = \gamma \sin \theta (y^2 dA)$$

4. Forces due to Static Fluid

4.6.1 Center of Pressure

- Now, if we assume that the resultant force acts at the center of pressure, its moment with respect to the axis through S is $F_R L_p$. Then

$$F_R L_p = \int \gamma \sin \theta (y^2 dA) = \gamma \sin \theta \int (y^2 dA)$$

- Substituting F_R for from Eq. (4–10) gives

$$L_p = \frac{\gamma \sin \theta (I)}{\gamma \sin \theta (L_c A)} = \frac{I}{L_c A} \quad (4-12)$$

4. Forces due to Static Fluid

4.6.1 Center of Pressure

- A more convenient expression can be developed by using the transfer theorem for moment of inertia from mechanics.

$$I = I_c + AL_c^2$$

- Equation (4–12) then becomes

$$L_p = \frac{I}{L_c A} = \frac{I_c + AL_c^2}{L_c A} = \frac{I_c}{L_c A} + L_c \quad (4-13)$$

4. Forces due to Static Fluid

4.6.1 Center of Pressure

- Rearranging gives the same form as Eq. (4–6):

$$L_p - L_c = \frac{I_c}{L_c A}$$

- We now continue the development by creating an expression for the vertical depth to the center of pressure h_p . Starting from Eq. (4–13), note the following relationships:

$$h_p = L_p \sin \theta$$

$$L_c = h_c / \sin \theta$$

4. Forces due to Static Fluid

4.6.1 Center of Pressure

- Then

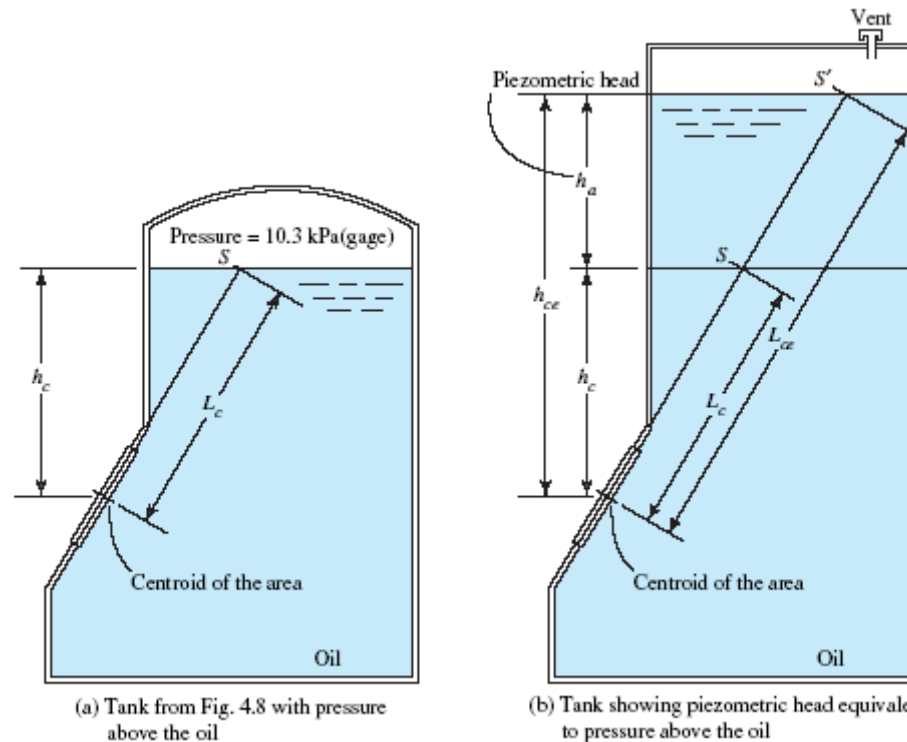
$$h_p = L_p \sin \theta = \sin \theta \left[\frac{h_c}{\sin \theta} + \frac{I_c}{(h_c/\sin \theta)A} \right]$$

$$h_p = h_c + \frac{I_c \sin^2 \theta}{h_c A}$$

4. Forces due to Static Fluid

4.7 Piezometric Head

- In all the problems demonstrated so far, the free surface of the fluid was exposed to the ambient pressure where $p = 0$ (gage).
- Fig 4.12 shows the illustration of piezometric head.



4. Forces due to Static Fluid

4.7 Piezometric Head

- A convenient method uses the concept of *piezometric head*, in which the actual pressure above the fluid, is converted into an equivalent depth of the fluid, that would create the same pressure (Fig. 4.12):

$$h_a = p_a/\gamma \quad (4-14)$$

- This depth is added to any depth h below the free surface to obtain an *equivalent depth*, That is,

$$h_e = h + h_a \quad (4-15)$$

- The equivalent depth to the centroid is

$$h_{ce} = h_c + h_a$$

4. Forces due to Static Fluid

Example 4.7

Repeat Example Problem 4.6, except consider that the tank shown in Fig. 4.8 is sealed at its top and that there is a pressure of 10.3 kPa(gage) above the oil.

Several calculations in the solution to Example Problem 4.6 used the depth to the centroid, given to be 1.5 m below the surface of the oil. With the pressure above the oil, we must add the piezometric head from Eq. (4–14). Using $\gamma = 8.92 \text{ kN/m}^3$, we get

$$h_a = \frac{p_a}{\gamma} = \frac{10.3 \text{ kN} / \text{m}^2}{8.92 \text{ kN} / \text{m}^3} = 1.15 \text{ m}$$

4. Forces due to Static Fluid

Example 4.7

Then, the equivalent depth to the centroid is

$$h_{ce} = h_c + h_a = 1.5 \text{ m} + 1.15 \text{ m} = 2.65 \text{ m}$$

The resultant force is then

$$F_R = \gamma h_{ce} A = (8.92 \text{ kN/m}^3)(2.65 \text{ m})(0.72 \text{ m}^2) = 17.02 \text{ kN}$$

Compare this with the value of 9.63 kN found before for the open tank. The center of pressure also changes because the distance L_c changes L_{ce} to as follows:

$$L_{cce} = h_{ce} / \sin \theta = 2.65 \text{ m} / \sin 60^\circ = 3.0 \text{ m}$$

$$L_{pe} - L_{ce} = \frac{I_c}{L_{ce} A} = \frac{0.02 \text{ m}^4}{(3.0 \text{ m})(0.72 \text{ m}^2)} = 0.009 \text{ m} = 9 \text{ mm}$$

4. Forces due to Static Fluid

Example 4.7

Then, the equivalent depth to the centroid is

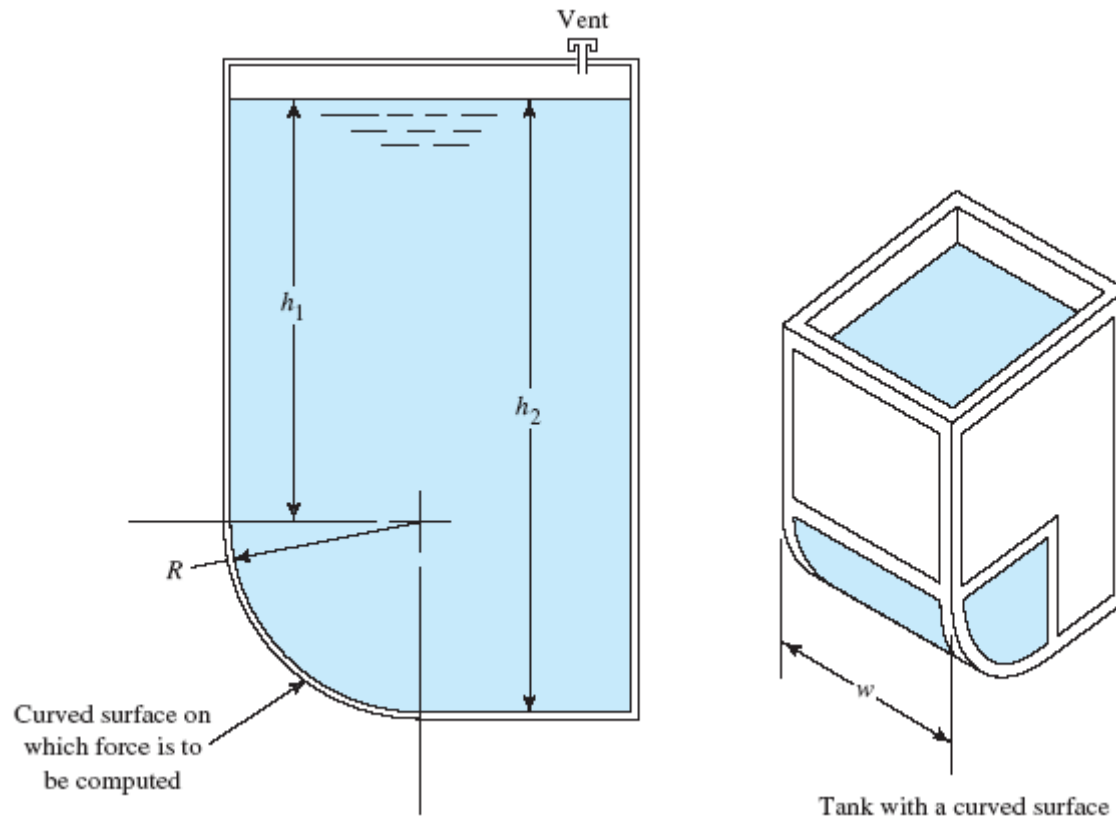
The resultant force is then

Compare this with the value of 9.63 kN found before for the open tank. The center of pressure also changes because the distance L_c changes L_{ce} to as follows:

4. Forces due to Static Fluid

4.8 Distribution of Force on a Submerged Curved Surface

- Fig 4.13 shows the tank with a curved surface containing a static fluid.



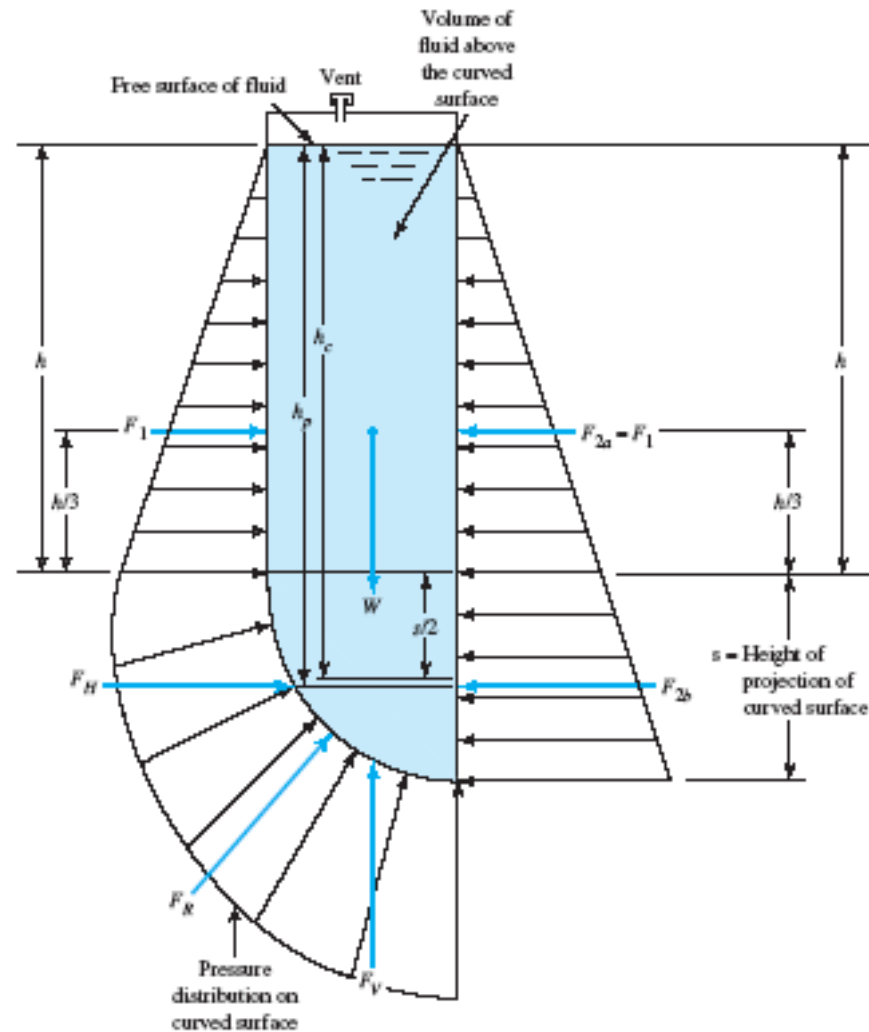
4. Forces due to Static Fluid

4.8 Distribution of Force on a Submerged Curved Surface

- One way to visualize the total force system involved is to isolate the volume of fluid directly above the surface of interest as a free body and show all the forces acting on it, as shown in Fig. 4.14.
- Fig 4.14 shows the free-body diagram of a volume of fluid above the curved surface.

4. Forces due to Static Fluid

4.8 Distribution of Force on a Submerged Curved Surface



4. Forces due to Static Fluid

4.8.1 Horizontal Force

- The vertical solid wall at the left exerts horizontal forces on the fluid in contact with it in reaction to the forces due to the fluid pressure.
- The magnitude of F_{2b} and its location can be found using the procedures developed for plane surfaces. That is,

$$F_{2b} = \gamma h_c A \quad (4-16)$$

where h_c is the depth to the centroid of the projected area.

4. Forces due to Static Fluid

4.8.1 Horizontal Force

- Calling the height of the rectangle s , you can see that $h_c = h + s/2$. Also, the area is sw , where w is the width of the curved surface. Then,

$$F_{2b} = F_H = \gamma sw(h + s/2) \quad (4-17)$$

- Again using the principles developed earlier, we get

$$h_p - h_c = I_c / (h_c A)$$

- For the rectangular projected area, however,

$$I_c = ws^3/12$$

$$A = sw$$

4. Forces due to Static Fluid

4.8.1 Horizontal Force

- Then,

$$h_p - h_c = \frac{ws^3}{12(h_c)(sw)} = \frac{s^2}{12h_c} \quad (4-18)$$

4. Forces due to Static Fluid

4.8.2 Vertical Force

- The vertical component of the force exerted by the curved surface on the fluid can be found by summing forces in the vertical direction.
- Only the weight of the fluid acts downward, and only the vertical component acts upward.

$$F_V = \gamma(\text{volume}) = \gamma Aw \quad (4-19)$$

4. Forces due to Static Fluid

4.8.3 Resultant Force

- The total resultant force is

$$F_R = \sqrt{F_H^2 + F_V^2} \quad (4-20)$$

- The resultant force acts at an angle relative to the horizontal found from

$$\phi = \tan^{-1}(F_V/F_H) \quad (4-21)$$

4. Forces due to Static Fluid

4.8.4 summary of the Procedure for Computing the Force on a Submerged Curved Surface

1. Isolate the volume of fluid above the surface.
2. Compute the weight of the isolated volume.
3. The magnitude of the vertical component of the resultant force is equal to the weight of the isolated volume. It acts in line with the centroid of the isolated volume.
4. Draw a projection of the curved surface onto a vertical plane and determine its height, called s .
5. Compute the depth to the centroid of the projected area from the below, where h is the depth to the top of the projected area.

$$h_c = h + s/2$$

4. Forces due to Static Fluid

4.8.4 summary of the Procedure for Computing the Force on a Submerged Curved Surface

6. Compute the magnitude of the horizontal component of the resultant force from

$$F_H = \gamma s w (h + s/2) = \gamma s w h_c$$

7. Compute the depth to the line of action of the horizontal component from

$$h_p = h_c + s^2 / (12 h_c)$$

8. Compute the resultant force from

$$F_R = \sqrt{F_V^2 + F_H^2}$$

4. Forces due to Static Fluid

4.8.4 summary of the Procedure for Computing the Force on a Submerged Curved Surface

9. Compute the angle of inclination of the resultant force relative to the horizontal from

$$\phi = \tan^{-1}(F_V/F_H)$$

10. Show the resultant force acting on the curved surface in such a direction that its line of action passes through the center of curvature of the surface.

4. Forces due to Static Fluid

Example 4.8

For the tank shown in Fig. 4.13, the following dimensions apply:

$$h_1 = 3.00 \text{ m}$$

$$h_2 = 4.50 \text{ m}$$

$$w = 2.50 \text{ m}$$

$$\gamma = 9.81 \text{ kN/m}^3 \quad (\text{water})$$

Compute the horizontal and vertical components of the resultant force on the curved surface and the resultant force itself. Show these force vectors on a sketch.

4. Forces due to Static Fluid

Example 4.8

Using the steps outlined above:

1. The volume above the curved surface is shown in Fig. 4.15.
2. The weight of the isolated volume is the product of the specific weight of the water times the volume. The volume is the product of the area times the length w .

$$\text{Area} = A_1 + A_2 = h_1 \cdot R + \frac{1}{4} (\pi R^2)$$

$$\text{Area} = (3.00 \text{ m})(1.50 \text{ m}) + \frac{1}{4} [\pi(1.50 \text{ m})^2] = 4.50 \text{ m}^2 + 1.767 \text{ m}^2$$

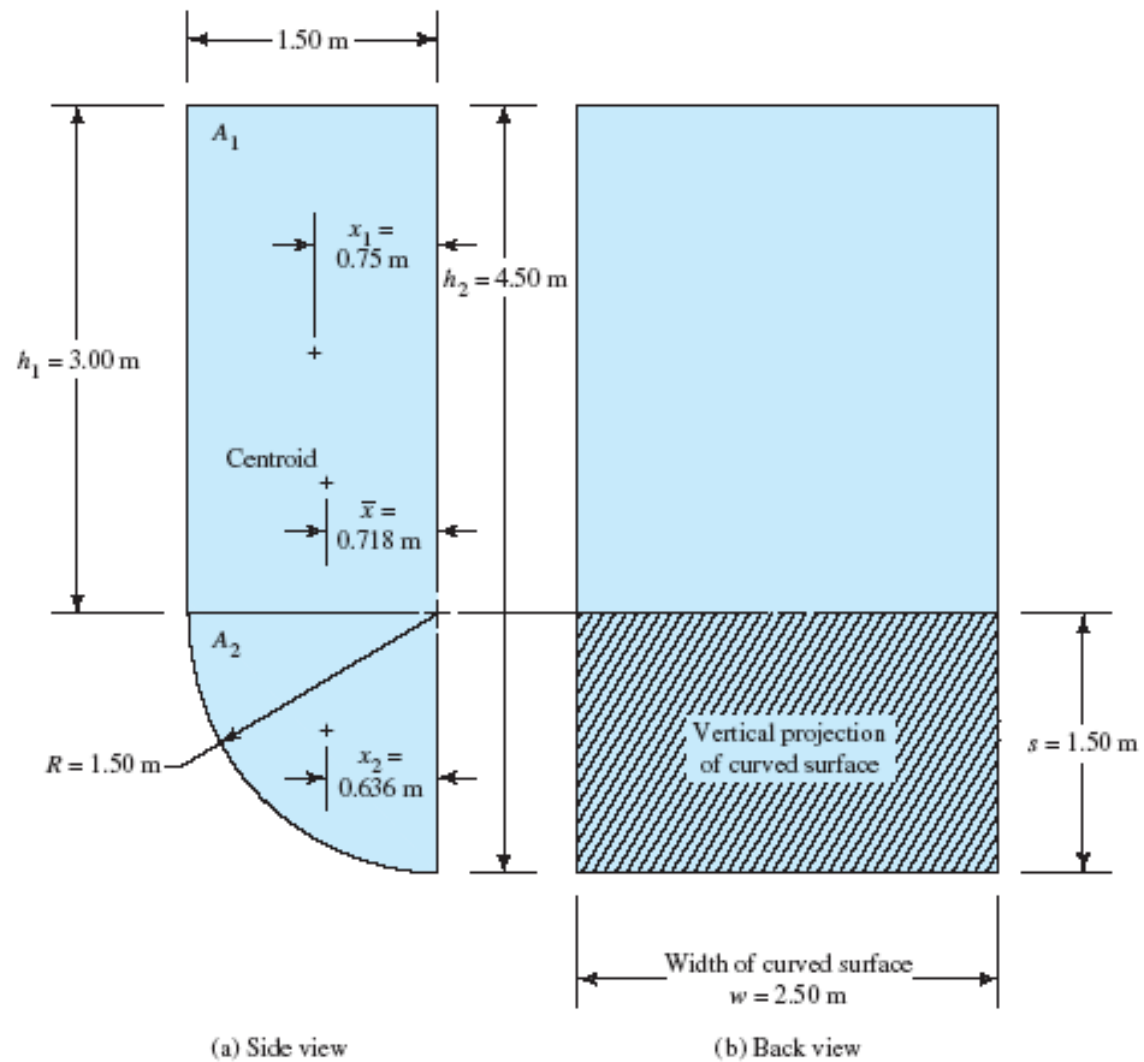
$$\text{Area} = 6.267 \text{ m}^2$$

$$\text{Volume} = \text{area} \cdot w = (6.267 \text{ m}^2)(2.50 \text{ m}) = 15.67 \text{ m}^3$$

$$\text{Weight} = \gamma V = (9.81 \text{ kN/m}^3)(15.67 \text{ m}^3) = 153.7 \text{ kN}$$

4. Forces due to Static Fluid

Example 4.8



4. Forces due to Static Fluid

Example 4.8

3. Then $F_v = 153.7\text{N}$, acting upward through the centroid of the volume. The location of the centroid is found using the composite-area technique. Refer to Fig. 4.15 for the data. Each value should be obvious except the location of the centroid of the quadrant. From Appendix L,

$$x_2 = 0.424R = 0.424(1.50\text{ m}) = 0.636\text{ m}$$

Then, the location of the centroid for the composite area is

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{(4.50)(0.75) + (1.767)(0.636)}{4.50 + 1.767} = 0.718\text{ m}$$

4. Forces due to Static Fluid

Example 4.8

4. The vertical projection of the curved surface is shown in Fig. 4.15. The height s equals 1.50 m.
5. The depth to the centroid of the projected area is

$$h_c = h_1 + s/2 = 3.00 \text{ m} + (1.50 \text{ m})/2 = 3.75 \text{ m}$$

6. The magnitude of the horizontal force is

$$F_H = \gamma s w (h_1 + s/2) = \gamma s w h_c$$

$$F_H = (9.81 \text{ kN/m}^3)(1.50 \text{ m})(2.50 \text{ m})(3.75 \text{ m}) = 138.0 \text{ kN}$$

7. The depth to the line of action of the horizontal component is found from

$$h_p = h_c + s^2/(12h_c)$$

$$h_p = 3.75 \text{ m} + (1.50)^2/[(12)(3.75)] = 3.80 \text{ m}$$

4. Forces due to Static Fluid

Example 4.8

8. The resultant force is computed from

$$F_R = \sqrt{F_V^2 + F_H^2}$$
$$F_R = \sqrt{(153.7 \text{ kN})^2 + (138.0 \text{ kN})^2} = 206.5 \text{ kN}$$

9. The angle of inclination of the resultant force relative to the horizontal is computed from

$$\phi = \tan^{-1}(F_V/F_H)$$
$$\phi = \tan^{-1}(153.7/138.0) = 48.1^\circ$$

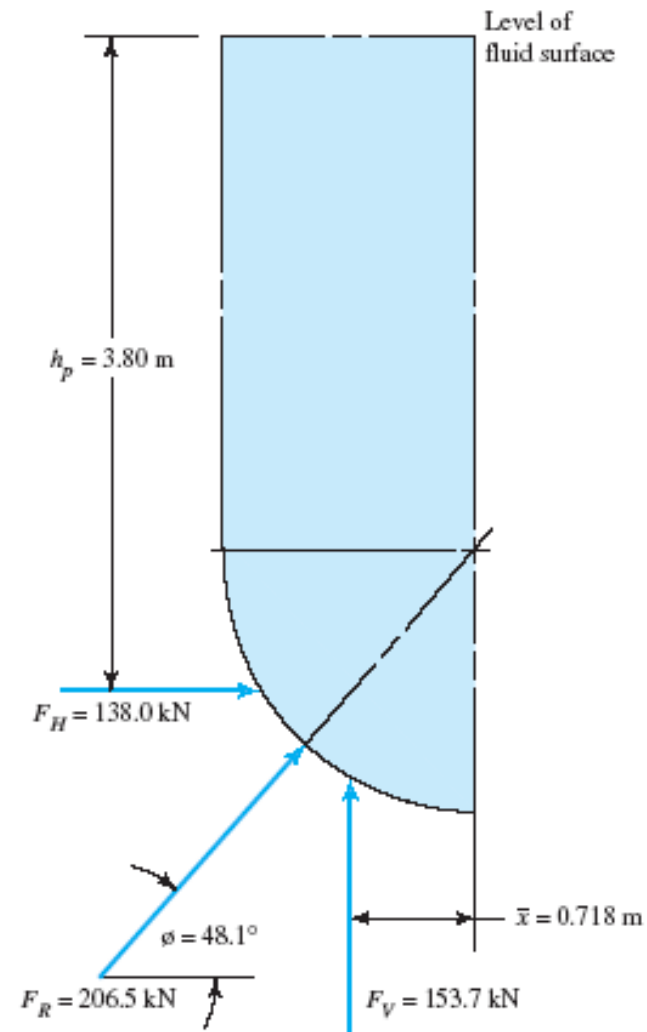
4. Forces due to Static Fluid

Example 4.8

10. The horizontal component, the vertical component, and the resultant force are shown in Fig. 4.16. Note that the line of action F_R of is through the center of curvature of the surface. Also note that the vertical component is acting through the centroid of the volume of liquid above the surface. The horizontal component is acting through the center of pressure of the projected area at a depth h_p from the level of the free surface of the fluid.

4. Forces due to Static Fluid

Example 4.8



4. Forces due to Static Fluid

Example 4.8

10. The horizontal component, the vertical component, and the resultant force are shown in Fig. 4.16. Note that the line of action F_R of is through the center of curvature of the surface. Also note that the vertical component is acting through the centroid of the volume of liquid above the surface. The horizontal component is acting through the center of pressure of the projected area at a depth h_p from the level of the free surface of the fluid.

4. Forces due to Static Fluid

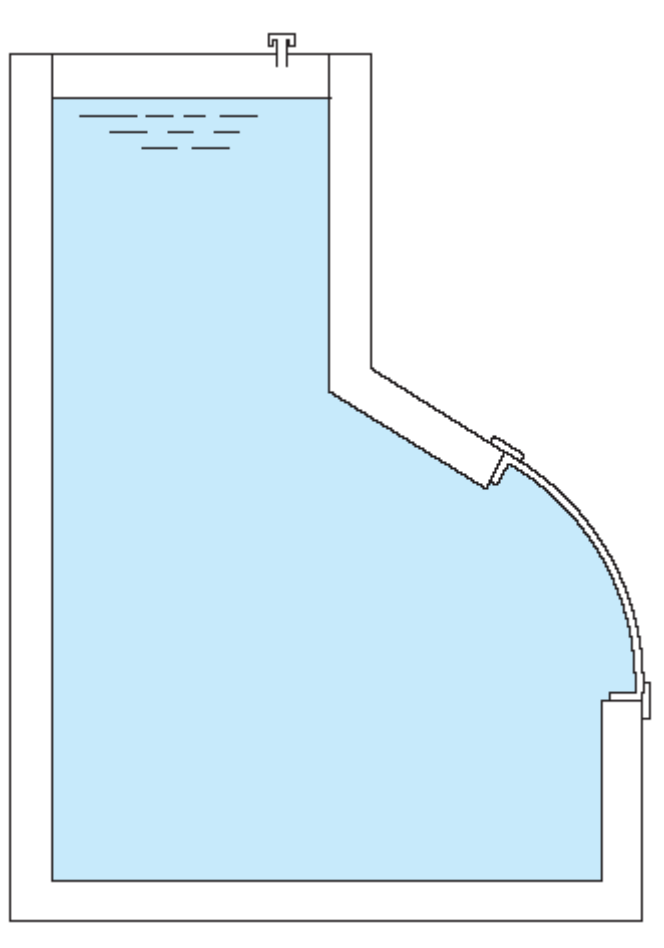
4.9 Effect of Pressure above the Fluid Surface

- If an additional pressure exists above the fluid or if the fluid itself is pressurized, the effect is to add to the actual depth a depth of fluid equivalent to $p/\rho g$.
- This is the same procedure, called *piezometric head*.

4. Forces due to Static Fluid

4.10 Forces on a Curved Surface with Fluid Below It

- Fig 4.17 shows the curved surface restraining a liquid below it.



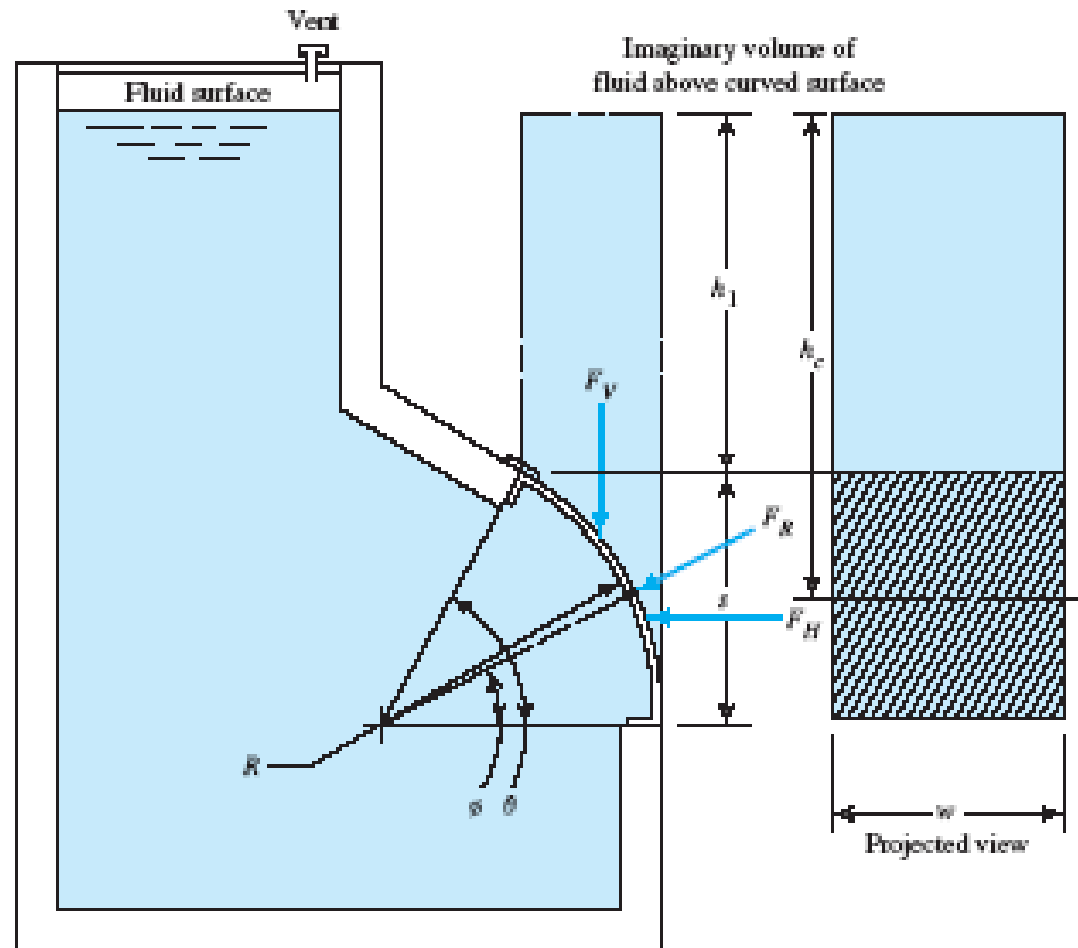
4. Forces due to Static Fluid

4.10 Forces on a Curved Surface with Fluid Below It

- The pressure in the fluid at any point is dependent on the depth of fluid to that point from the level of the free surface.
- This situation is equivalent to having the curved surface supporting a volume of liquid *above* it, except for the direction of the force vectors.
- Fig 4.18 shows the forces exerted by a curved surface on the fluid.
- As before, the horizontal component of the force exerted by the curved surface on the fluid is the force on the projection of the curved surface on a vertical plane.
- The vertical component is equal to the weight of the **imaginary** volume of fluid above the surface.

4. Forces due to Static Fluid

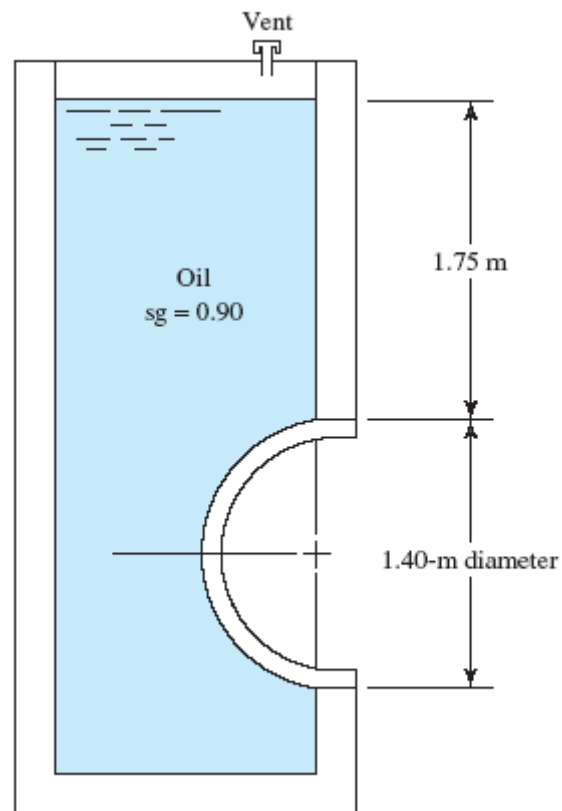
4.10 Forces on a Curved Surface with Fluid Below It



4. Forces due to Static Fluid

4.11 Forces on Curved Surfaces with Fluid above and below

- Fig 4.19 shows the semicylindrical gate.



4. Forces due to Static Fluid

4.11 Forces on Curved Surfaces with Fluid above and below

- The force due to fluid pressure would have a horizontal component acting to the right on the gate.
- This force acts on the projection of the surface on a vertical plane and is computed in the same manner as used in Section 4.8.
- However, there is also a force acting upward on the bottom surface of the gate equal to the total weight of the fluid, both real and imaginary, above that surface.
- The net vertical force is the difference between the two forces, equal to the weight of the semicylindrical volume of fluid displaced by the gate itself.

4. Forces due to Static Fluid

4.11 Forces on Curved Surfaces with Fluid above and below

- Fig 4.19 shows the volumes used to compute the net vertical force on the gate.

