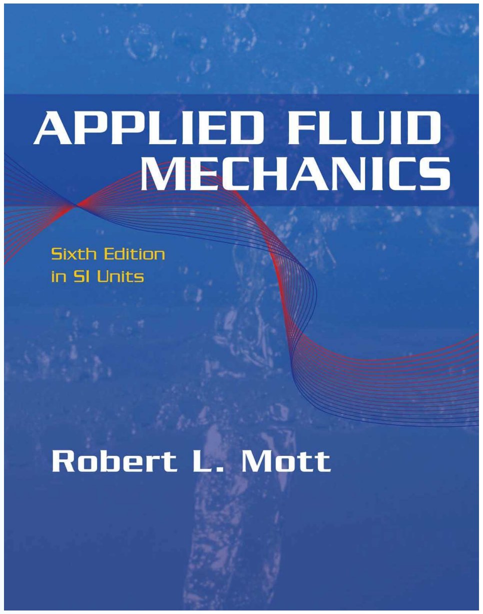
Applied Fluid Mechanics



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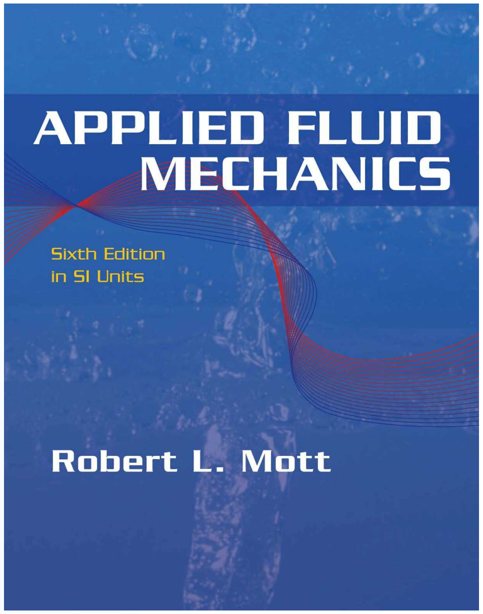
Applied Fluid Mechanics



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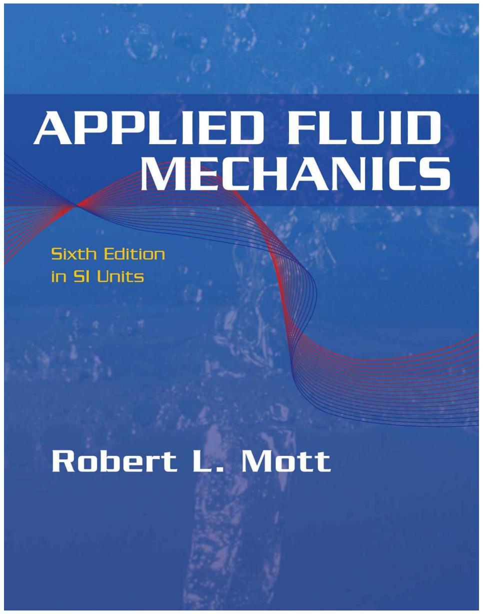


5. Buoyancy and Stability

Chapter Objectives

- Write the equation for the buoyant force.
- Analyze the case of bodies floating on a fluid.
- Use the principle of static equilibrium to solve for the forces involved in buoyancy problems.
- Define the conditions that must be met for a body to be stable when completely submerged in a fluid.
- Define the conditions that must be met for a body to be stable when floating on a fluid.
- Define the term *metacenter* and compute its location.

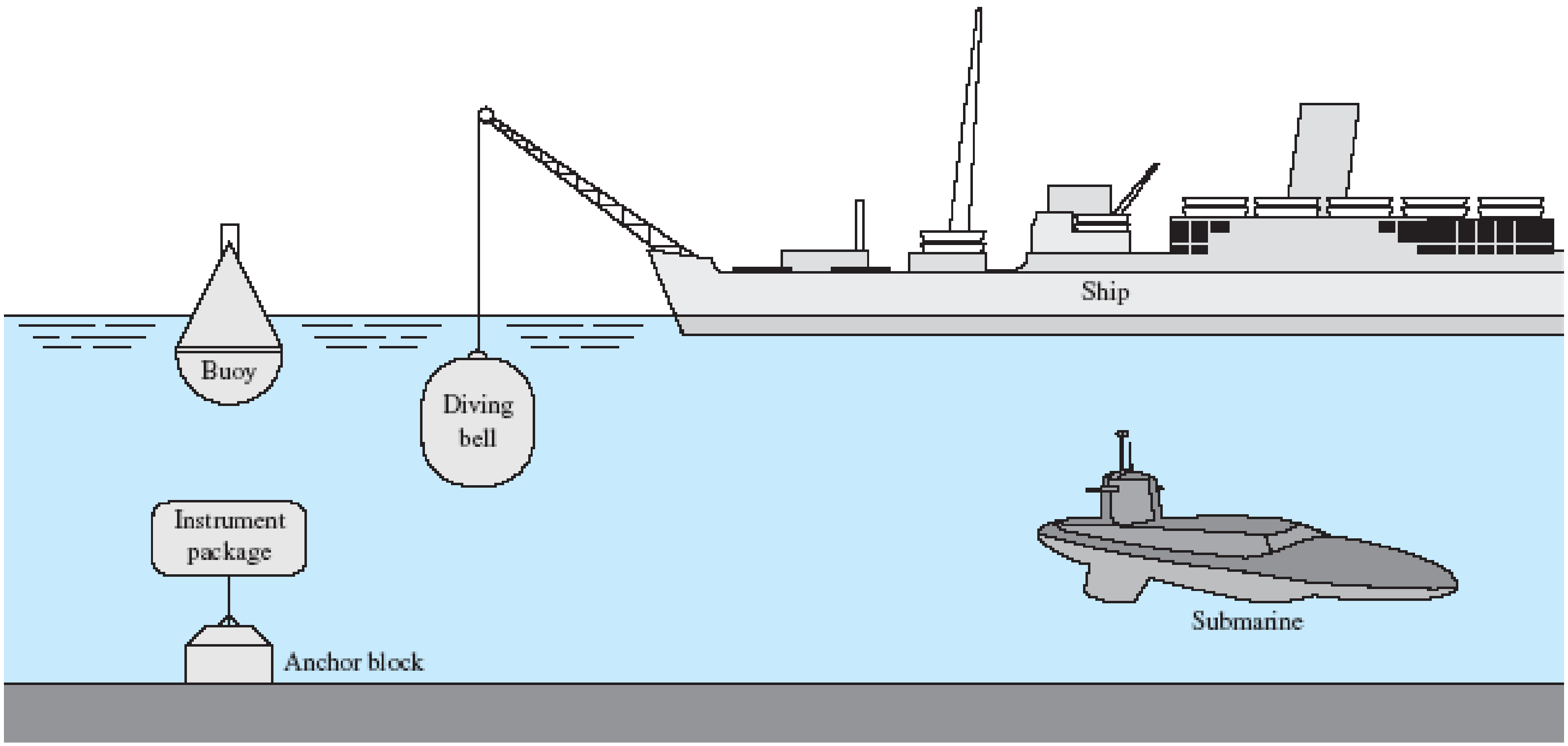
Chapter Outline

1. Introductory Concepts
2. Buoyancy
3. Buoyancy Materials
4. Stability of Completely Submerged Bodies
5. Stability of Floating Bodies
6. Degree of Stability

5. Buoyancy and Stability

5.1 Introductory Concepts

- Fig 5.1 shows the examples of types of buoyancy problems.



5. Buoyancy and Stability

5.1 Introductory Concepts

- The buoy and the ship are obviously designed to float.
- The diving bell would tend to sink unless supported by the cable from the crane on the ship.
- Consider any kind of boat, raft, or other floating object that is expected to maintain a particular orientation when placed in a fluid.

5. Buoyancy and Stability

5.2 Buoyancy

- ***A body in a fluid, whether floating or submerged, is buoyed up by a force equal to the weight of the fluid displaced.***
- The buoyant force acts vertically upward through the centroid of the displaced volume and can be defined mathematically by Archimedes' principle as follows:

$$F_b = \gamma_f V_d \qquad (5\text{–}1)$$

F_b = Buoyant force

γ_f = Specific weight of the fluid

V_d = Displaced volume of the fluid

5.2 Buoyancy

- Below are the procedure for solving buoyancy problems:

1. Determine the objective of the problem solution. Are you to find a force, a weight, a volume, or a specific weight?
2. Draw a free-body diagram of the object in the fluid. Show all forces that act on the free body in the vertical direction, including the weight of the body, the buoyant force, and all external forces. If the direction of some force is not known, assume the most probable direction and show it on the free body.

5.2 Buoyancy

3. Write the equation of static equilibrium in the vertical direction, $\Sigma F_v = 0$, assuming the positive direction to be upward.
4. Solve for the desired force, weight, volume, or specific weight, remembering the following concepts:

a. The buoyant force is calculated from $F_b = \gamma_f V_d$.

b. The weight of a solid object is the product of its total volume and its specific weight; that is $w = \gamma V$.

c. An object with an average specific weight less than that of the fluid will tend to float because $w < F_b$ with the object submerged.

5. Buoyancy and Stability

5.2 Buoyancy

- An object with an average specific weight greater than that of the fluid will tend to sink because $w > F_b$ with the object submerged.
- *Neutral buoyancy* occurs when a body stays in a given position wherever it is submerged in a fluid. An object whose average specific weight is equal to that of the fluid is neutrally buoyant.

Example 5.1

A cube 0.50 m on a side is made of bronze having a specific weight of 86.9kN/m^3. Determine the magnitude and direction of the force required to hold the cube in equilibrium completely submerged (a) in water and (b) in mercury. The specific gravity of mercury is 13.54.

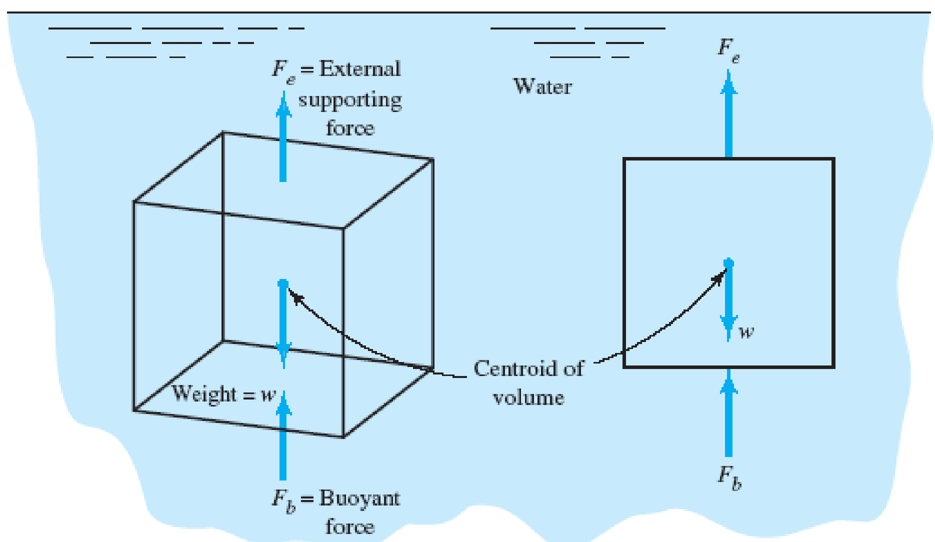
Example 5.1

Consider part (a) first. Imagine the cube of bronze submerged in water. Now do Step 1 of the procedure.

On the assumption that the bronze cube will not stay in equilibrium by itself, some external force is required. The objective is to find the magnitude of this force and the direction in which it would act—that is, up or down. Now do Step 2 of the procedure before looking at the next panel.

Example 5.1

The free body is simply the cube itself. There are three forces acting on the cube in the vertical direction, as shown in Fig. 5.2: the weight of the cube *w*, acting downward through its center of gravity; the buoyant force F_b acting upward through the centroid of the displaced volume; and the externally applied supporting force F_e .



(a) Forces acting on the cube
(b) Two-dimensional free-body diagram

Example 5.1

Part (a) of Fig. 5.2 shows the cube as a three-dimensional object with the three forces acting along a vertical line through the centroid of the volume. This is the preferred visualization of the free-body diagram. However, for most problems it is suitable to use a simplified two-dimensional sketch as shown in part (b). How do we know to draw the force F_e in the upward direction?

Example 5.1

We really do not know for certain. However, experience should indicate that without an external force the solid bronze cube would tend to sink in water. Therefore, an upward force seems to be required to hold the cube in equilibrium. If our choice is wrong, the final result will indicate that to us.

Now, assuming that the forces are as shown in Fig. 5.2, go on to Step 3.

Example 5.1

The equation should look as follows (assume that positive forces act upward):

$$\Sigma F_v = 0$$

$$F_b + F_e - w = 0 \qquad (5\text{–}2)$$

As a part of Step 4, solve this equation algebraically for the desired term. You should now have

$$F_e = w - F_b \qquad (5\text{–}3)$$

because the objective is to find the external force.

How do we calculate the weight of the cube *w*?

Example 5.1

Item b under Step 4 of the procedure indicates that $w = \gamma_B V$, where γ_B is the specific weight of the bronze cube and V is its total volume. For the cube, because each side is 0.50 m, we have

$$V = (0.50\ \text{m})^3 = 0.125\ \text{m}^3$$

$$w = \gamma_B V = (86.9\ \text{kN/m}^3)(0.125\ \text{m}^3) = 10.86\ \text{kN}$$

There is another unknown on the right side of Eq. (5–3). How do we calculate F_b?

Example 5.1

Check Step 4a of the procedure if you have forgotten. Write

$$F_b = \gamma_f V_d$$

In this case γ_f is the specific weight of the water (9.81 kN/m^3), and the displaced volume V_d is equal to the total volume of the cube, which we already know to be 0.123m^3. Then, we have

$$F_b = \gamma_f V_d = (9.81\ \mathrm{kN/m^3})(0.125\ \mathrm{m^3}) = 1.23\ \mathrm{kN}$$

Now we can complete our solution for F_e.

$$F_e = w - F_b = 10.86\ \mathrm{kN} - 1.23\ \mathrm{kN} = 9.63\ \mathrm{kN}$$

Example 5.1 – Results Part a

Notice that the result is positive. This means that our assumed direction for F_e was correct. Then the solution to the problem is that an upward force of 9.63 kN is required to hold the block of bronze in equilibrium under water.

What about part (b) of the problem, where the cube is submerged in mercury? Our objective is the same as before—to determine the magnitude and direction of the force required to hold the cube in equilibrium.

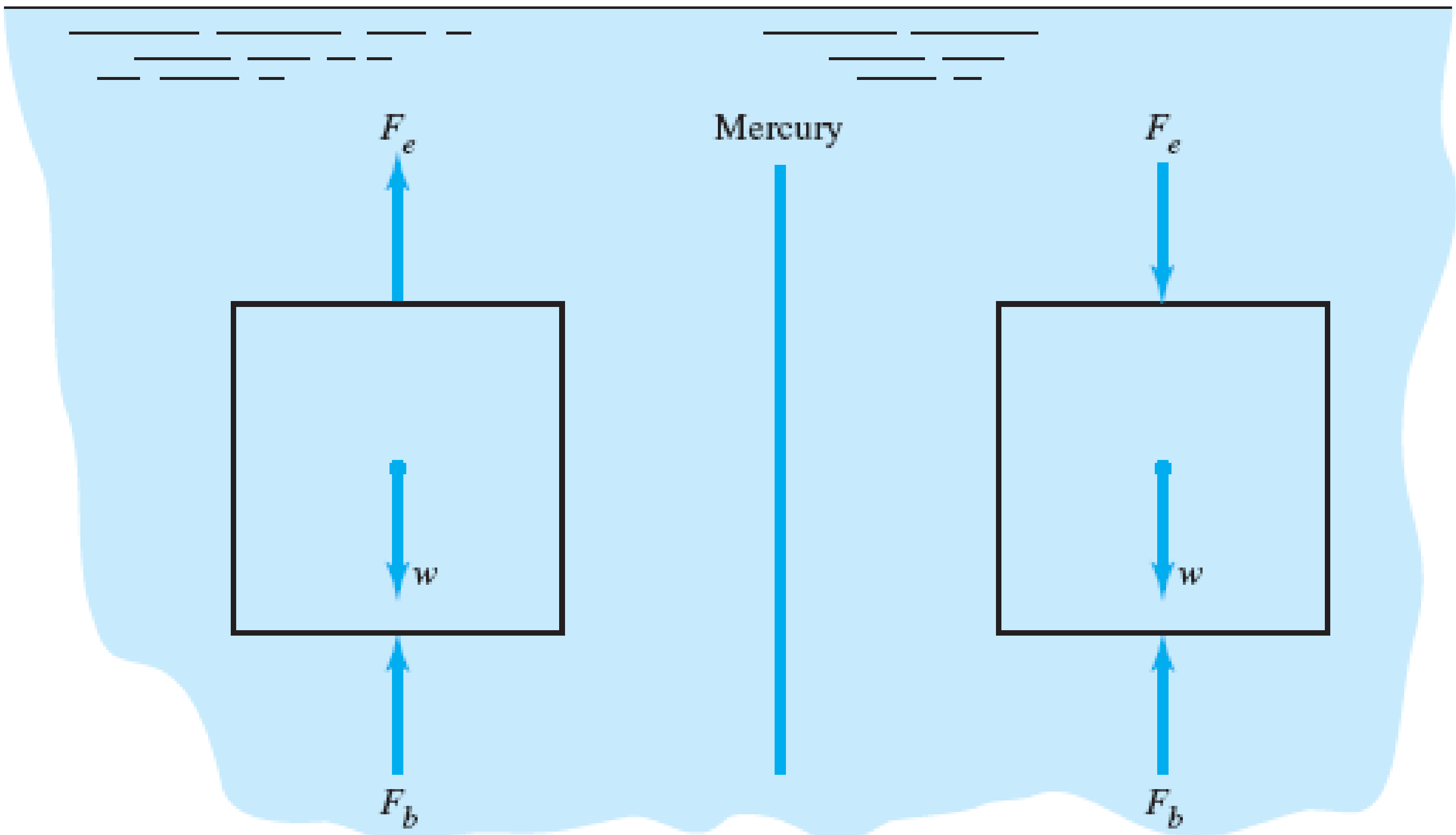
Now do Step 2 of the procedure.

Example 5.1 – Results Part a

Either of two free-body diagrams is correct as shown in Fig. 5.3, depending on the assumed direction for the external force F_e. The solution for the two diagrams will be carried out simultaneously so you can check your work regardless of which diagram looks like yours and to demonstrate that either approach will yield the correct answer.

Now do Step 3 of the procedure.

Example 5.1 – Results Part a



(a) Assuming cube would sink (b) Assuming cube would float

Example 5.1 – Results Part a

The following are the correct equations of equilibrium. Notice the differences and relate them to the figures:

$$F_b + F_e - w = 0 \quad \Big| \quad F_b - F_e - w = 0$$

$$F_e = w - F_b \quad \Big| \quad F_e = F_b - w$$

Because the magnitudes of w and F_b are the same for each equation, they can now be calculated.

As in part (a) of the problem, the weight of the cube is

$$w = \gamma_B V = (86.9\ \mathrm{kN/m^3})(0.125\ \mathrm{m^3}) = 10.86\ \mathrm{kN}$$

Example 5.1 – Results Part a

For the buoyant force you should have

$$F_b = \gamma_m V = (\mathrm{sg})_m(9.81\ \mathrm{kN/m^3})(V)$$

$$F_b = (13.54)(9.81\ \mathrm{kN/m^3})(0.125\ \mathrm{m^3}) = 16.60\ \mathrm{kN}$$

The correct answers are

$$\begin{aligned} F_e &= w - F_b \\ &= 10.86\ \mathrm{kN} - 16.60\ \mathrm{kN} \\ &= -5.74\ \mathrm{kN} \end{aligned}$$

$$\begin{aligned} F_e &= F_b - w \\ &= 16.60\ \mathrm{kN} - 10.86\ \mathrm{kN} \\ &= +5.74\ \mathrm{kN} \end{aligned}$$

Notice that both solutions yield the same numerical value, but they have opposite signs. The negative sign for the solution on the left means that the assumed direction for F_e in Fig. 5.3(a) was wrong. Therefore, both approaches give the same result.

Example 5.1 – Results Part b

The required external force is a downward force of 5.74 kN. How could you have reasoned from the start that a downward force would be required?

Items c and d of Step 4 of the procedure suggest that the specific weight of the cube and the fluid be compared. In this case we have the following results:

$$\text{For the bronze cube } \gamma_B = 86.9\ \text{kN/m}^3$$

$$\text{For the fluid (mercury) } \gamma_m = (13.54)(9.81\ \text{kN/m}^3)$$

$$= 132.8\ \text{kN/m}^3$$

Example 5.1 – Comment

Because the specific weight of the cube is less than that of the mercury, it would tend to float without an external force. Therefore, a downward force, as pictured in Fig. 5.3(b), would be required to hold it in equilibrium under the surface of the mercury.

This example problem is concluded.

5. Buoyancy and Stability

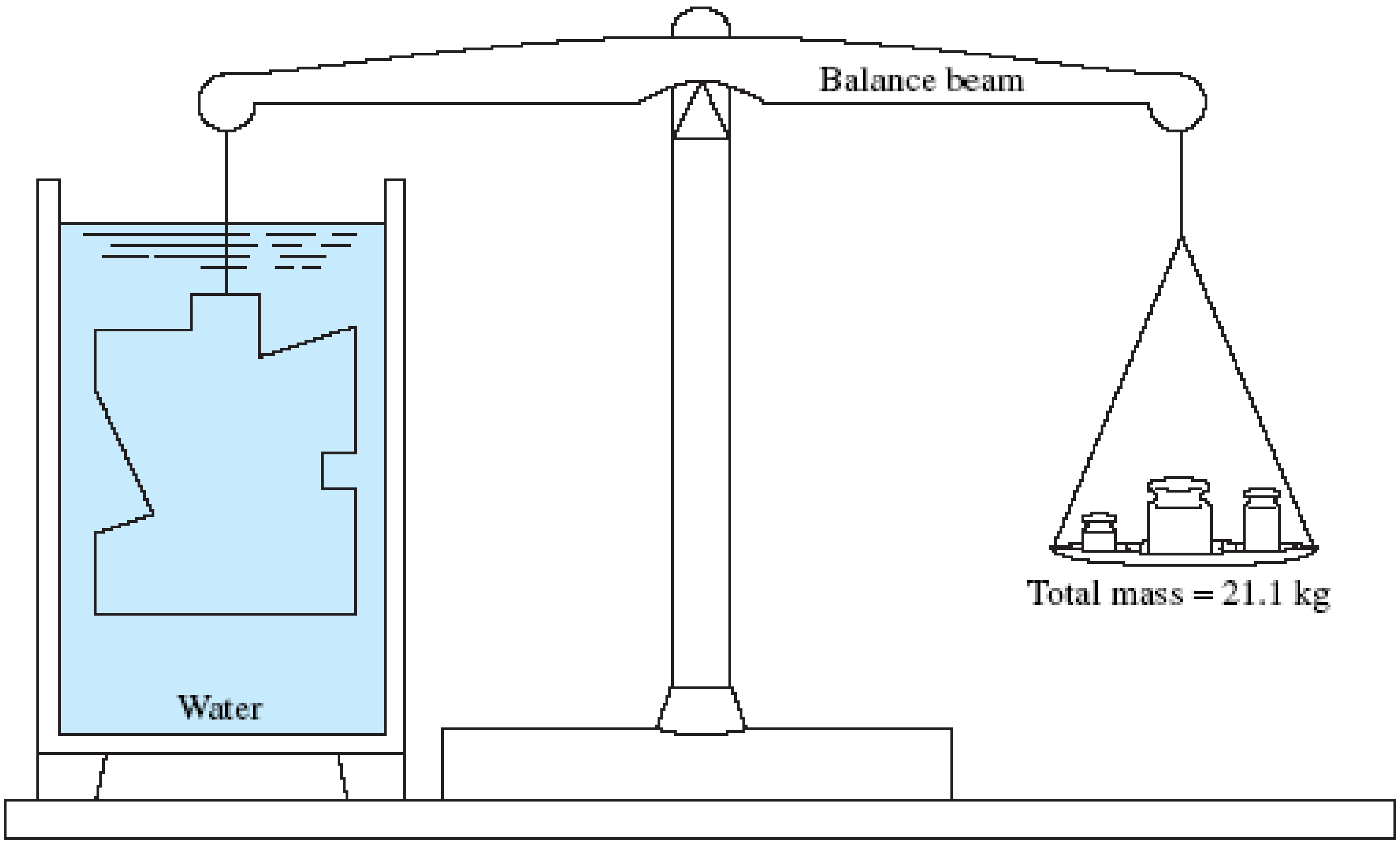
Example 5.2

A certain solid metal object has such an irregular shape that it is difficult to calculate its volume by geometry. Use the principle of buoyancy to calculate its volume and specific weight.

First, the mass of the object is determined in the normal manner to be 27.2 kg. Then, using a setup similar to that in Fig. 5.4, we find its apparent mass while submerged in water to be 21.1 kg. Using these data and the procedure for analyzing buoyancy problems, we can find the volume of the object. ($W=mg$)

5. Buoyancy and Stability

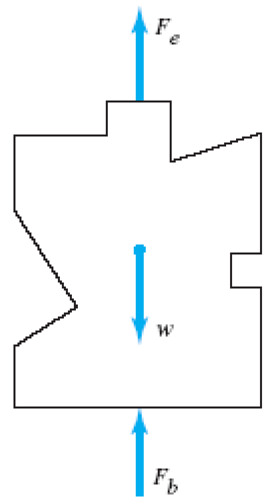
Example 5.2



Example 5.2

Now apply Step 2 of the procedure and draw the free-body diagram of the object while it is suspended in the water.

The free-body diagram of the object while it is suspended in the water should look like Fig. 5.5. In this figure, what are the two forces F_e and w?



Example 5.2

We know that w=266.8 N, the weight of the object in air, and F_e=207 N, the supporting force exerted by the balance shown in Fig. 5.4. Now do Step 3 of the procedure.

Using $\Sigma F_v = 0$, we get

$$F_b + F_e - w = 0$$

Our objective is to find the total volume V of the object. How can we get V into this equation?

5. Buoyancy and Stability

Example 5.2

We use this equation from Step 4a,

$$F_b = \gamma_f V$$

where is the specific weight of the water, 9.81 kN/m^3. Substitute this into the preceding equation and solve for *V*.

$$F_b + F_e - w = 0$$

$$\gamma_f V + F_e - w = 0$$

$$\gamma_f V = w - F_e$$

$$V = \frac{w - F_e}{\gamma_f}$$

Example 5.2

The result is V=0.0061 m^3. This is how it is done:

$$V = \frac{w - F_e}{\gamma_f} = \frac{(266.8 - 207)\text{ N}}{9810\text{ N/m}^3} = 0.0061\text{ m}^3$$

$$\gamma = \frac{w}{V} = \frac{(27.2\text{ kg})(9.81\text{ m/s}^2)}{0.0061\text{ m}^3} = 43.74\text{ kN/m}^3$$

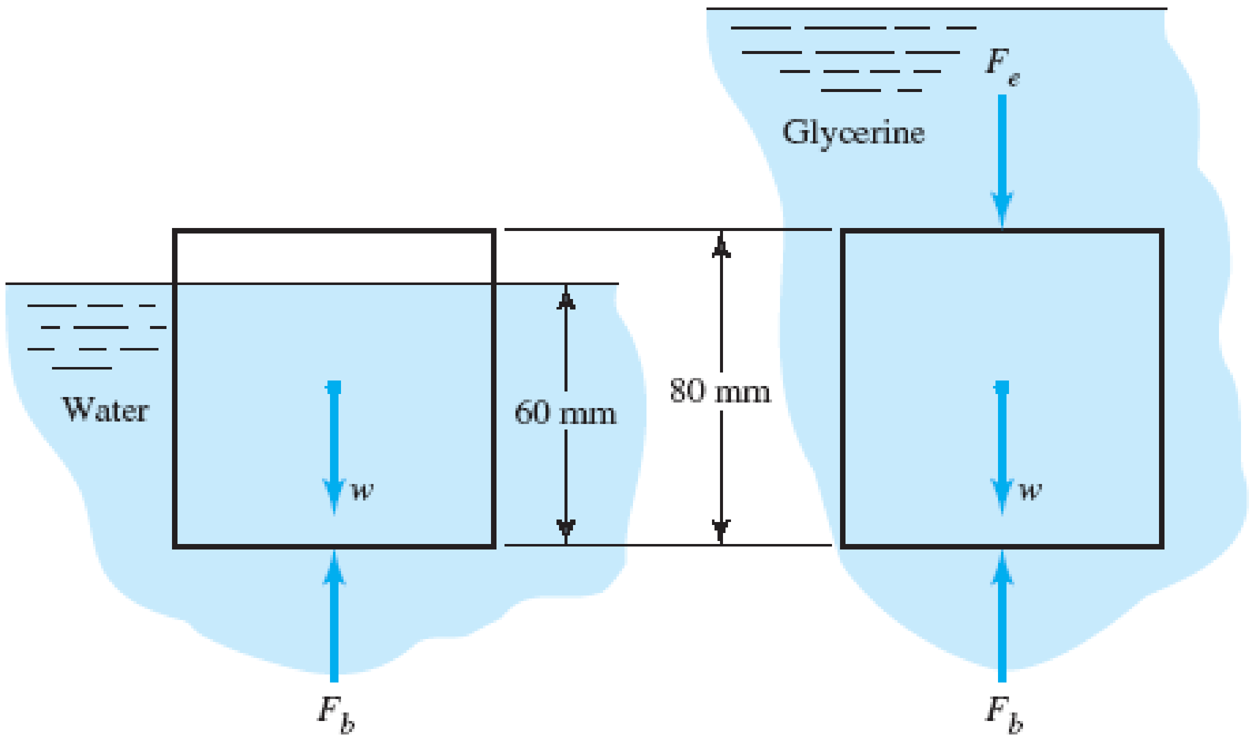
This is approximately the specific weight of a titanium alloy.

Example 5.3

A cube 80 mm on a side is made of a rigid foam material and floats in water with 60 mm of the cube below the surface. Calculate the magnitude and direction of the force required to hold it completely submerged in glycerine, which has a specific gravity of 1.26.

Complete the solution before looking at the next panel.

Example 5.3



(a) Floating (b) Submerged

Example 5.3

From Fig. 5.6(a), we have

$$\sum F_v = 0$$

$$F_b - w = 0$$

$$w = F_b = \gamma_f V_d$$

$$V_d = (80\text{ mm})(80\text{ mm})(60\text{ mm}) = 384 \times 10^3\text{ mm}^3$$

(submerged volume of cube)

$$w = \left(\frac{9.81 \times 10^3\text{ N}}{\text{m}^3}\right)(384 \times 10^3\text{ mm}^3)\left(\frac{1\text{ m}^3}{(10^3\text{ mm})^3}\right)$$

$$= 3.77\text{ N}$$

Example 5.3

From Fig. 5.6(b), we have

$$\sum F_v = 0$$

$$F_b - F_e - w = 0$$

$$F_e = F_b - w = \gamma_f V_d - 3.77\ \text{N}$$

$$V_d = (80\ \text{mm})^3 = 512 \times 10^3\ \text{mm}^3$$

(total volume of cube)

$$\gamma_f = (1.26)(9.81\ \text{kN/m}^3) = 12.36\ \text{kN/m}^3$$

$$F_e = \gamma_f V_d - 3.77\ \text{N}$$

$$= \left(\frac{12.36 \times 10^3\ \text{N}}{\text{m}^3}\right)(512 \times 10^3\ \text{mm}^3)\left(\frac{1\ \text{m}^3}{(10^3\text{mm})^3}\right) - 3.77\ \text{N}$$

$$F_e = 6.33\ \text{N} - 3.77\ \text{N} = 2.56\ \text{N}$$

A downward force of 2.56 N is required to hold the cube submerged in glycerine.

Example 5.4

A brass cube 152.4 mm on a side weighs 298.2 N. We want to hold this cube in equilibrium under water by attaching a light foam buoy to it. If the foam weighs 707.3 N/m^3, what is the minimum required volume of the buoy?

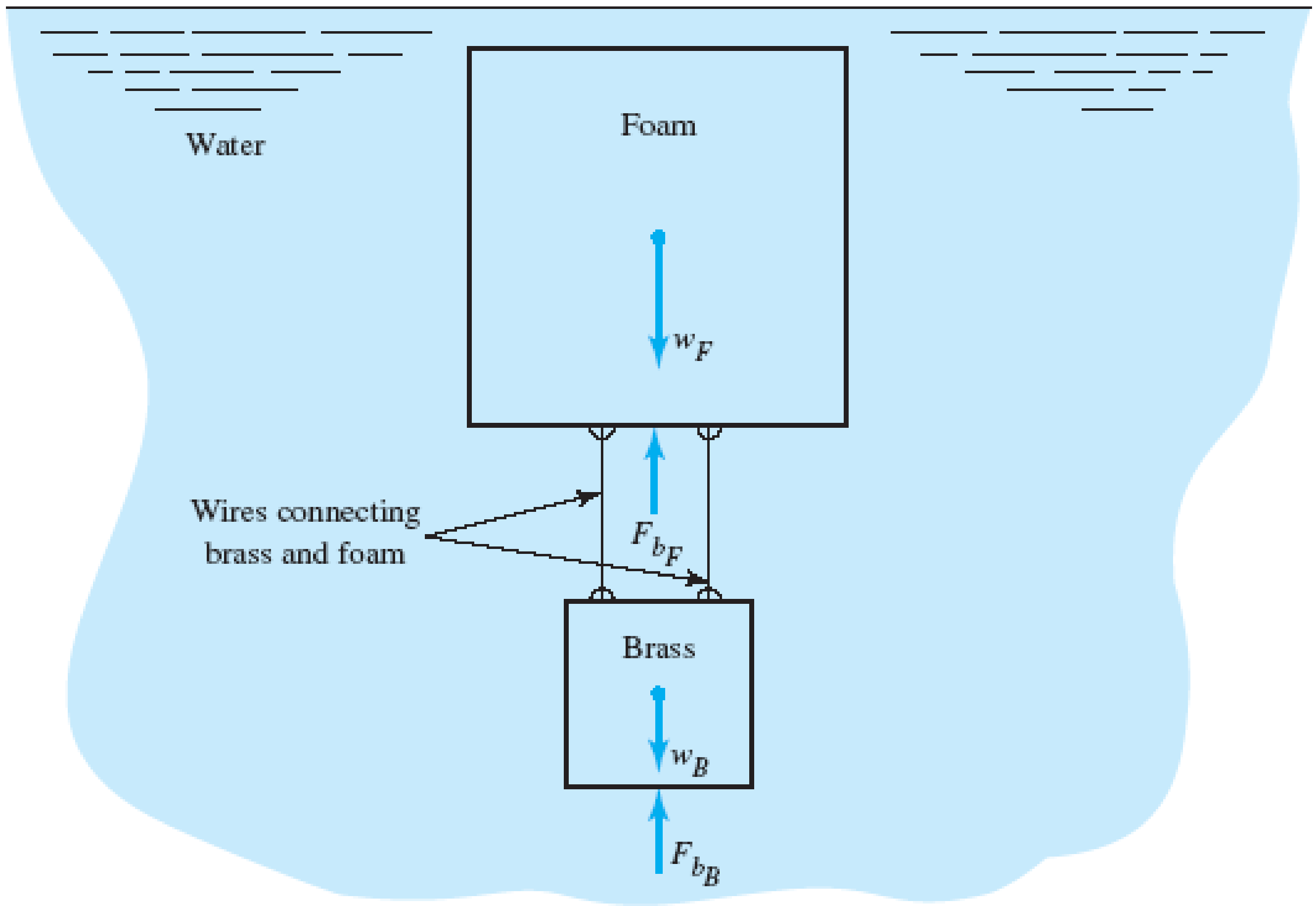
Complete the solution before looking at the next panel.

Example 5.4

Calculate the minimum volume of foam to hold the brass cube in equilibrium. Notice that the foam and brass in Fig. 5.7 are considered as parts of a single system and that there is a buoyant force on each. The subscript F refers to the foam and the subscript B refers to the brass. No external force is required.

5. Buoyancy and Stability

Example 5.4



Example 5.4

The equilibrium equation is

$$\sum F_v = 0$$

$$0 = F_{b_B} + F_{b_F} - w_B - w_F \qquad (5\text{–}4)$$

$$w_B = 298.2 \text{ N} \quad \text{(given)}$$

$$F_{b_B} = \gamma_f V_{d_B} = 9.81 \text{ kN/m}^3 \times (0.152 \text{ m})^3 = 34.5 \text{ N}$$

$$w_F = \gamma_F V_F$$

$$F_{b_F} = \gamma_f V_F$$

Example 5.4

Substitute these quantities into Eq. (5–4):

$$F_{b_B} + F_{b_F} - w_B - w_F = 0$$

$$34.5\ \text{N} + \gamma_f V_F - 298.2\ \text{N} - \gamma_F V_F = 0$$

Solve for V_F, using $\gamma_f = 9.810\ \text{N/m}^3$ and $\gamma_F = 707.3\ \text{N/m}^3$:

$$\gamma_f V_F - \gamma_F V_F = 298.2\ \text{N} - 34.5\ \text{N} = 263.7\ \text{N}$$

$$V_F(\gamma_f - \gamma_F) = 263.7\ \text{N}$$

$$V_F = \frac{263.7\ \text{N}}{\gamma_f - \gamma_F} = \frac{263.7\ \text{N}}{(9810 - 707.3)\ \text{N/m}^3}$$

$$V_F = 0.029\ \text{m}^3$$

Example 5.4

This means that if 0.029 m^3 of foam were attached to the brass cube, the combination would be in equilibrium in water without any external force. It would be neutrally buoyant.

5.3 Buoyancy Materials

- The design of floating bodies often requires the use of lightweight materials that offer a high degree of buoyancy.
- The buoyancy material should typically have the following properties:

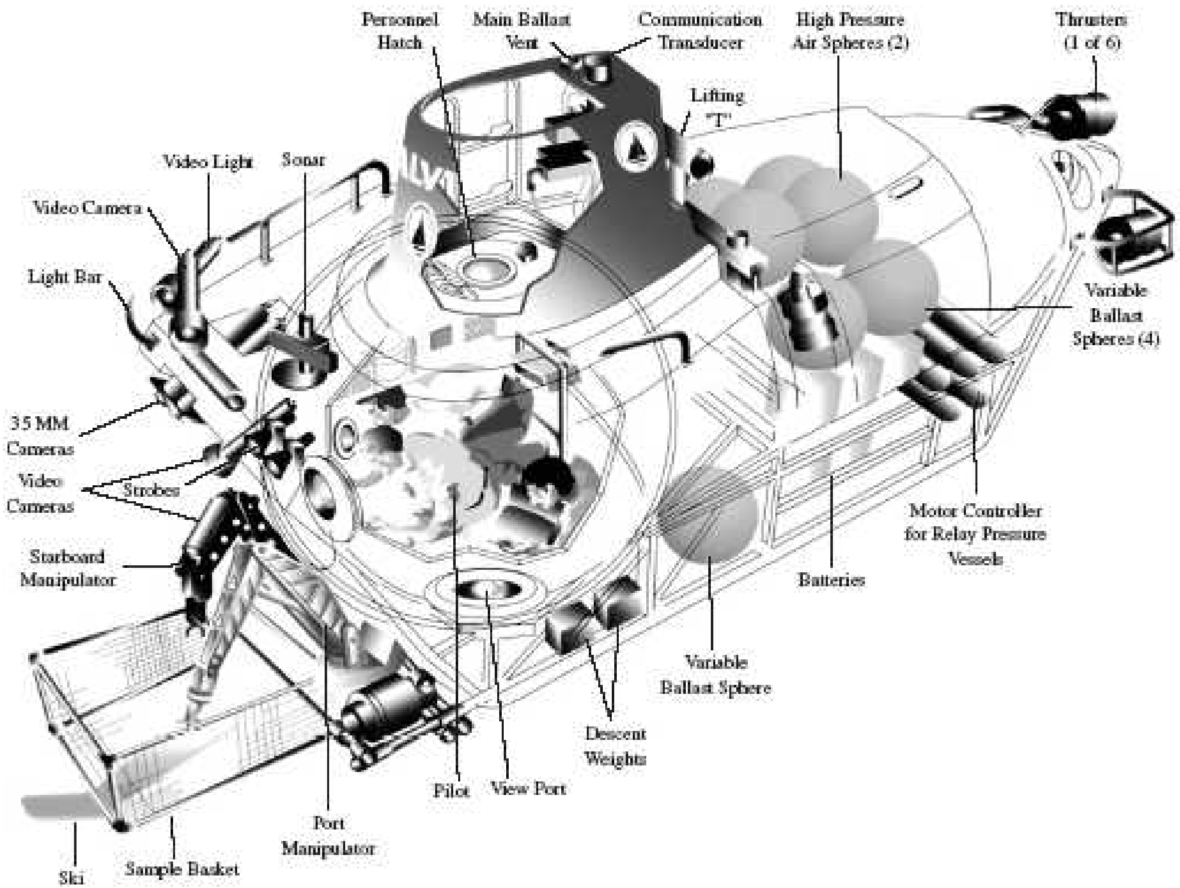
1. Low specific weight and density
2. Little or no tendency to absorb the fluid
3. Compatibility with the fluid in which it will operate
4. Ability to be formed to appropriate shapes
5. Ability to withstand fluid pressures to which it will be subjected
6. Abrasion resistance and damage tolerance

5.4 Stability of Completely Submerged Bodies

- A body in a fluid is considered stable if it will return to its original position after being rotated a small amount about a horizontal axis.
- ***The condition for stability of bodies completely submerged in a fluid is that the center of gravity of the body must be below the center of buoyancy.***
- The center of buoyancy of a body is at the centroid of the displaced volume of fluid, and it is through this point that the buoyant force acts in a vertical direction.
- The weight of the body acts vertically downward through the center of gravity.

5.4 Stability of Completely Submerged Bodies

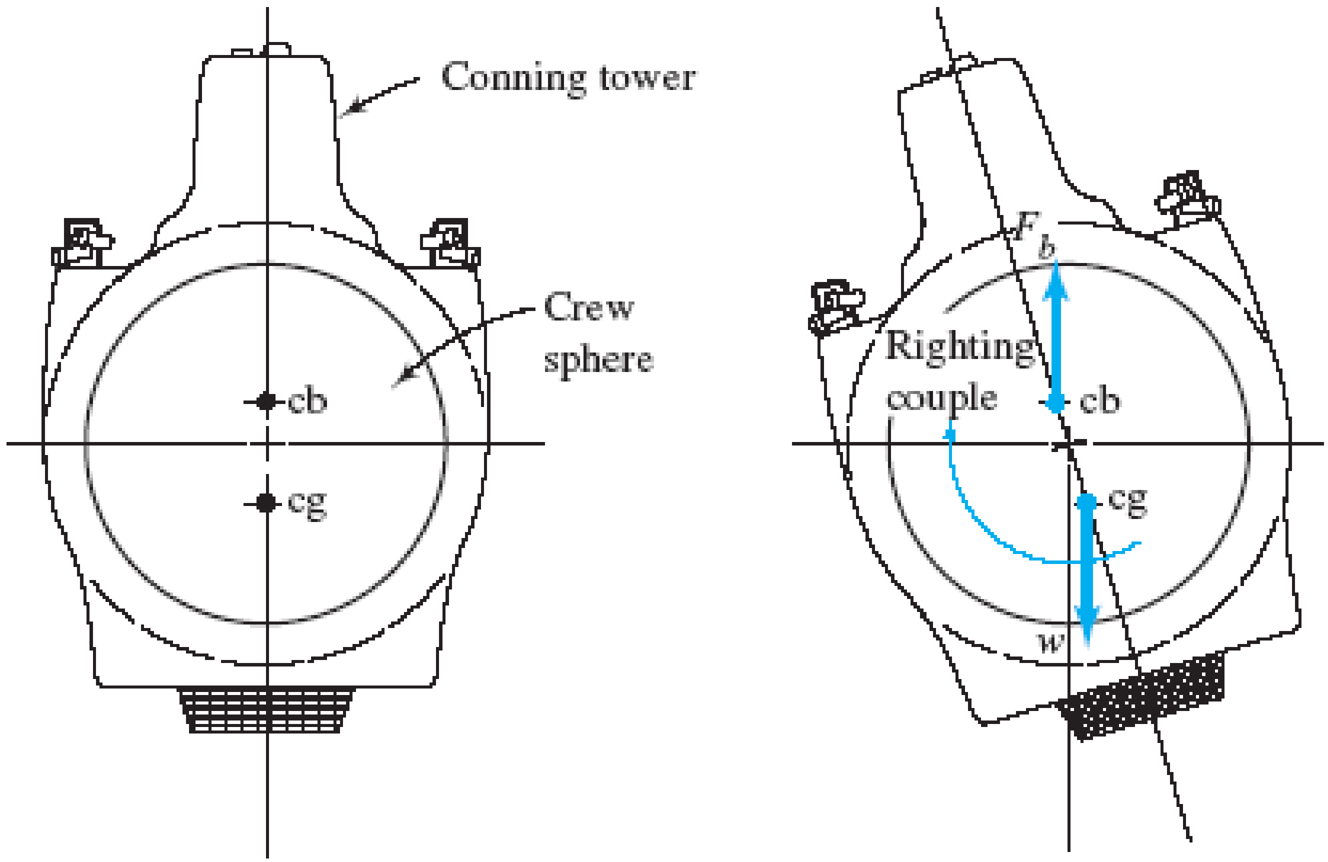
- Fig 5.8 shows the deep submergence vehicle *Alvin*, cutaway drawing showing major components.



5.4 Stability of Completely Submerged Bodies

- Much of the upper structure is filled with light syntactic foam to provide buoyancy. This causes the center of gravity (cg) to be lower than the center of buoyancy (cb), achieving stability.
- Fig 5.9 shows the stability of a submerged submarine.
- Because their lines of action are now offset, these forces create a *righting couple* that brings the vehicle back to its original orientation, demonstrating stability.

5.4 Stability of Completely Submerged Bodies



(a)
Normal orientation

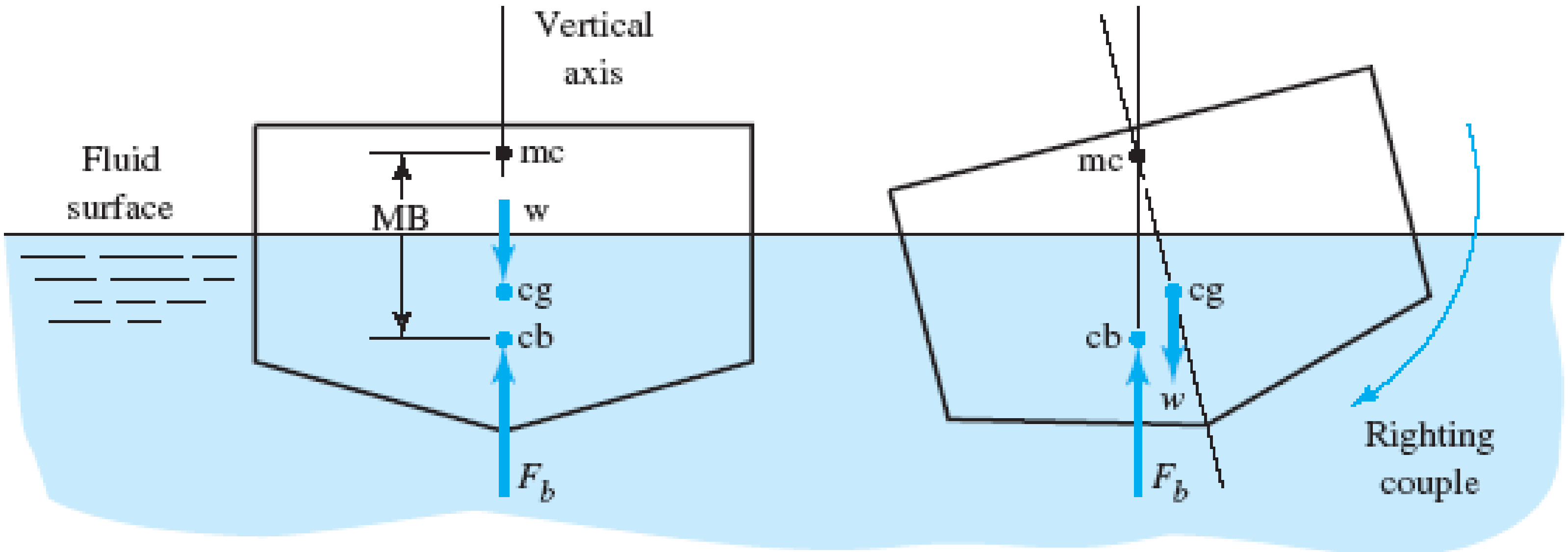
(b)
Tilted position

5.4 Stability of Completely Submerged Bodies

- If the cg is above the cb, the couple created when the body is tilted would produce an *overturning couple* that would cause it to capsize.
- Solid objects have the cg and cb coincident and they exhibit *neutral stability* when completely submerged, meaning that they tend to stay in whatever position they are placed.

5.5 Stability of Floating Bodies

- Fig 5.10 shows the method of finding the metacenter.



(a) Original position (b) Tilted position

5.5 Stability of Floating Bodies

- In part (a) of the figure, the floating body is at its equilibrium orientation and the center of gravity (cg) is above the center of buoyancy (cb).
- A vertical line through these points will be called the *vertical axis* of the body.
- Figure 5.10(b) shows that if the body is rotated slightly, the center of buoyancy shifts to a new position because the geometry of the displaced volume has changed.
- The buoyant force and the weight now produce a righting couple that tends to return the body to its original orientation.
- Thus, the body is stable.

5. Buoyancy and Stability

5.5 Stability of Floating Bodies

- ***A floating body is stable if its center of gravity is below the metacenter.***
- The distance to the *m*etacenter from the center of *b*uoyancy is called MB and is calculated from

$$\mathrm{MB} = I/V_d \qquad (5\text{–}5)$$

- In this equation, V_d is the displaced volume of fluid and *I* is the *least* moment of inertia of a horizontal section of the body taken at the surface of the fluid.
- *If the distance MB places the metacenter above the center of gravity, the body is stable.*

5.5 Stability of Floating Bodies

- Below is the procedure for evaluating the stability of floating bodies:

1. Determine the position of the floating body, using the principles of buoyancy.
2. Locate the center of buoyancy, cb; compute the distance from some reference axis to cb, called y_{cb}. Usually, the bottom of the object is taken as the reference axis.
3. Locate the center of gravity, cg; compute y_{cg} measured from the same reference axis.

5.5 Stability of Floating Bodies

- Determine the shape of the area at the fluid surface and compute the *smallest* moment of inertia *I* for that shape.
- Compute the displaced volume V_d.
- Compute MB = I/V_d.
- Compute $y_{mc} = y_{cb}$ + MB.
- If $y_{mc} > y_{mg}$, the body is stable.
- If $y_{mc} < y_{mg}$, the body is unstable.

5.5 Stability of Floating Bodies

- The conditions for stability of bodies in a fluid can be summarized as follows.

1. *Completely submerged bodies are stable if the center of gravity is below the center of buoyancy.*
2. *Floating bodies are stable if the center of gravity is below the metacenter.*

Example 5.5

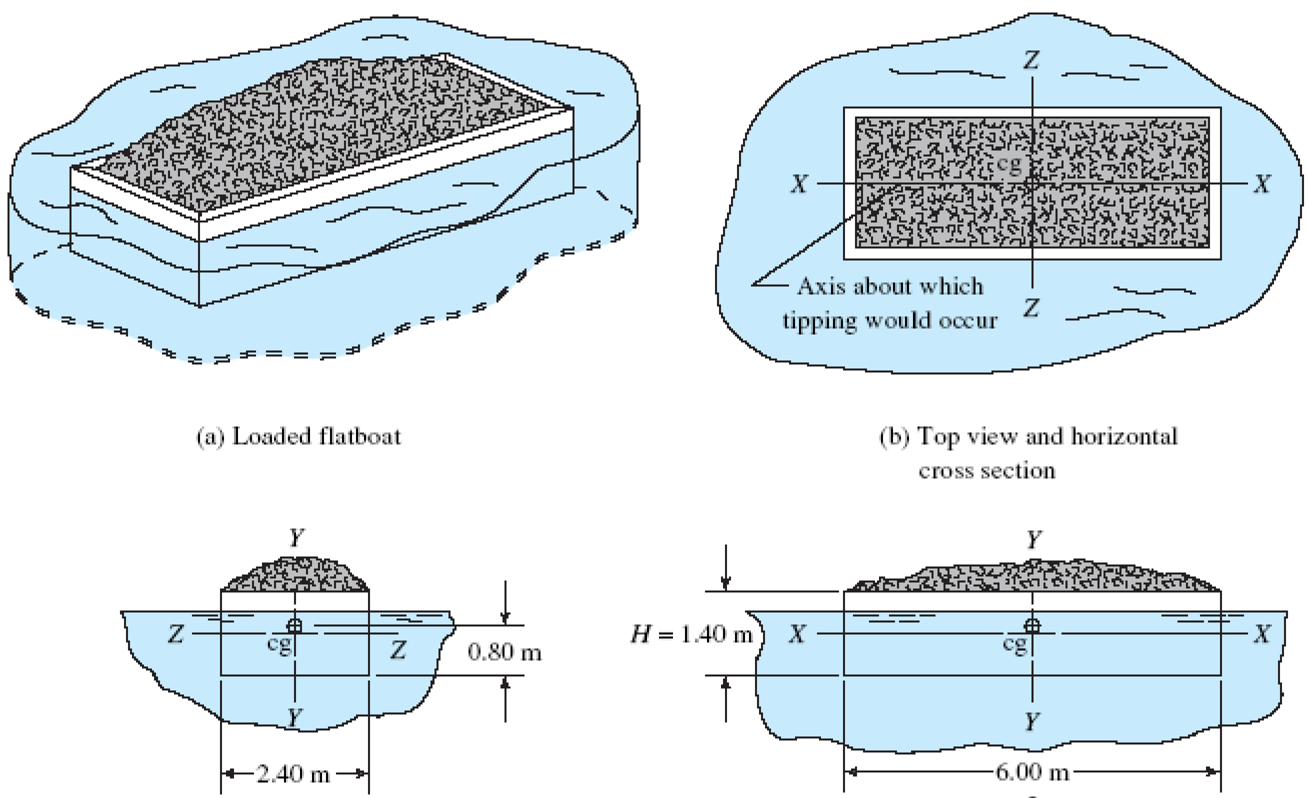
Figure 5.11(a) shows a flatboat hull that, when fully loaded, weighs 150 kN. Parts (b)–(d) show the top, front, and side views of the boat, respectively. Note the location of the center of gravity, cg. Determine whether the boat is stable in fresh water.

First, find out whether the boat will float.

This is done by finding how far the boat will sink into the water, using the principles of buoyancy stated in Section 5.2. Complete that calculation before going to the next panel.

Example 5.5

The depth of submergence or draft of the boat is 1.06 m, as shown in Fig. 5.12, found by the following method:



(a) Loaded flatboat

(b) Top view and horizontal cross section

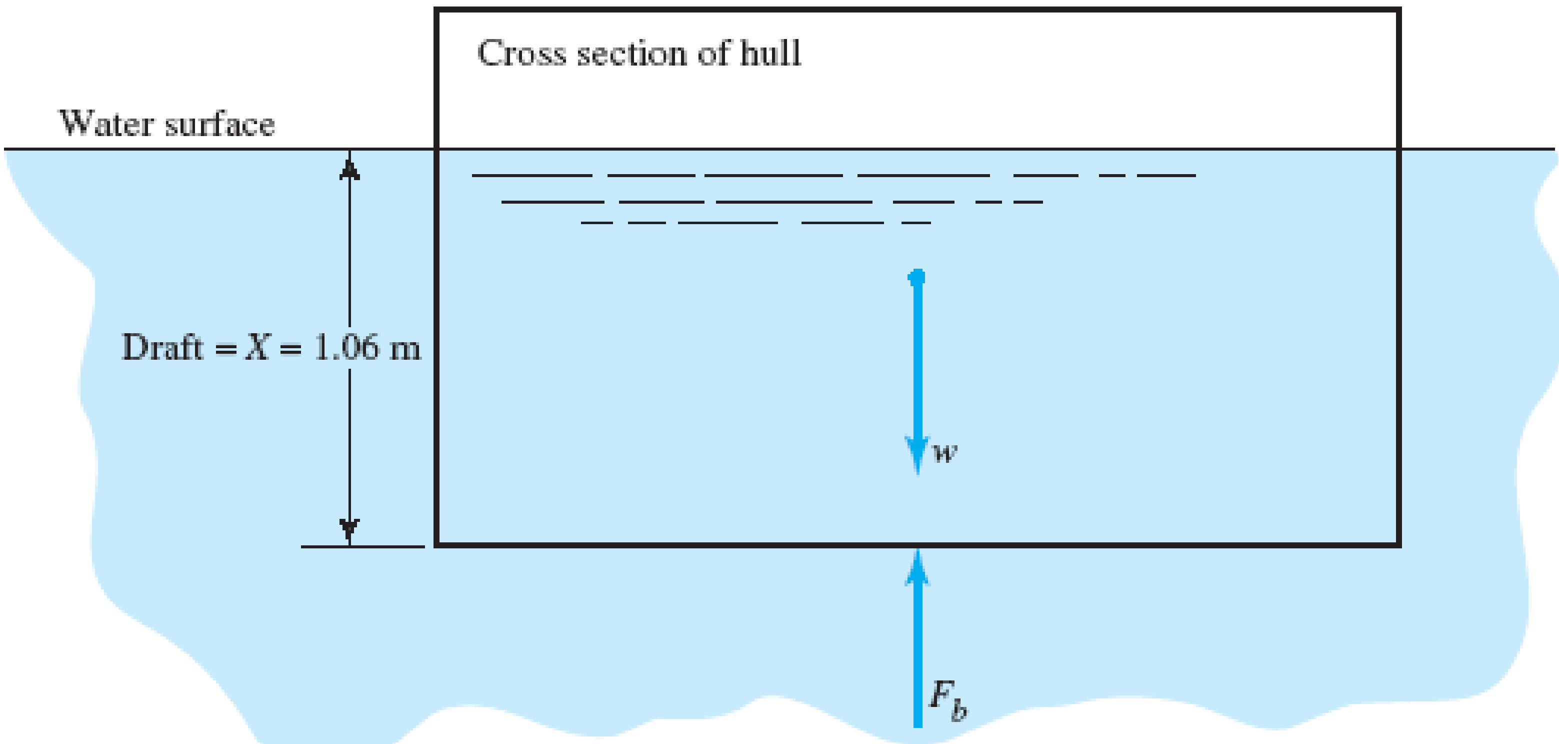
(c) Front view and vertical cross section

(d) Side view

5. Buoyancy and Stability

Example 5.5

The depth of submergence or draft of the boat is 1.06 m, as shown in Fig. 5.12, found by the following method:



Example 5.5

Equation of equilibrium:

$$\sum F_v = 0 = F_b - w$$
$$w = F_b$$

Submerged volume:

$$V_d = B \times L \times X$$

Buoyant force:

$$F_b = \gamma_f V_d = \gamma_f \times B \times L \times X$$

Example 5.5

Then we have

$$w = F_b = \gamma_f \times B \times L \times X$$

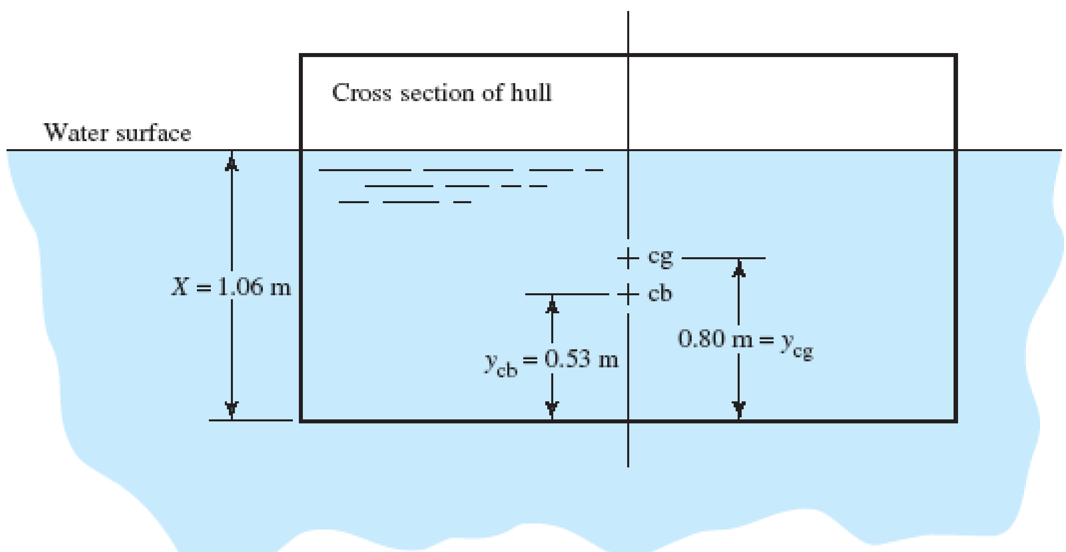
$$X = \frac{w}{B \times L \times \gamma_f} = \frac{150\ \text{kN}}{(2.4\ \text{m})(6.0\ \text{m})} \times \frac{\text{m}^3}{(9.81\ \text{kN})} = 1.06\ \text{m}$$

It floats with 1.06 m submerged. Where is the center of buoyancy?

Example 5.5

It is at the center of the displaced volume of water. In this case, as shown in Fig. 5.13, it is on the vertical axis of the boat at a distance of 0.53 m from the bottom. That is half of the draft, *X.*

Then $y_{cb} = 0.53$ m.



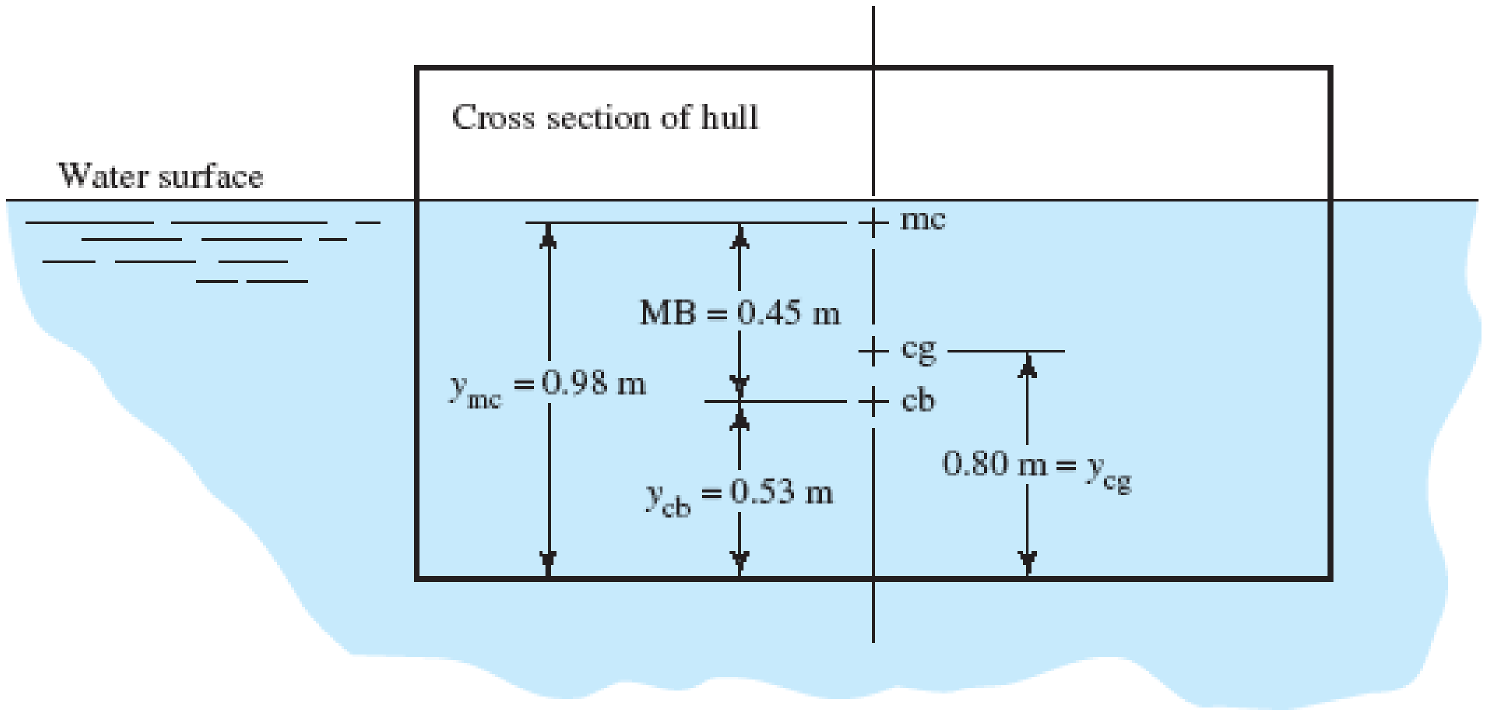
Example 5.5

Because the center of gravity is above the center of buoyancy, we must locate the metacenter to determine whether the boat is stable.

Using Eq. (5–5), calculate the distance MB and show it on the sketch.

Example 5.5

The result is as shown in Fig. 5.14.



Example 5.5

Here is how it is done:

$$MB = I/V_d$$
$$V_d = L \times B \times X = (6.0\text{ m})(2.4\text{ m})(1.06\text{ m}) = 15.26\text{ m}^3$$

The moment of inertia *I* is determined about the axis *X*–*X* in Fig. 5.11(b) because this would yield the smallest value for *I:*

$$I = \frac{LB^3}{12} = \frac{(6.0\text{ m})(2.4\text{ m})^3}{12} = 6.91\text{ m}^4$$

$$MB = I/V_d = 6.91\text{ m}^4/15.26\text{ m}^3 = 0.45\text{ m}$$

$$y_{mc} = y_{cb} + MB = 0.53\text{ m} + 0.45\text{ m} = 0.98\text{ m}$$

Example 5.5

Is the boat stable?

Yes, it is. Because the metacenter is above the center of gravity, as shown in Fig. 5.14, the boat is stable. That is, $y_{mc} > y_{cg}$.

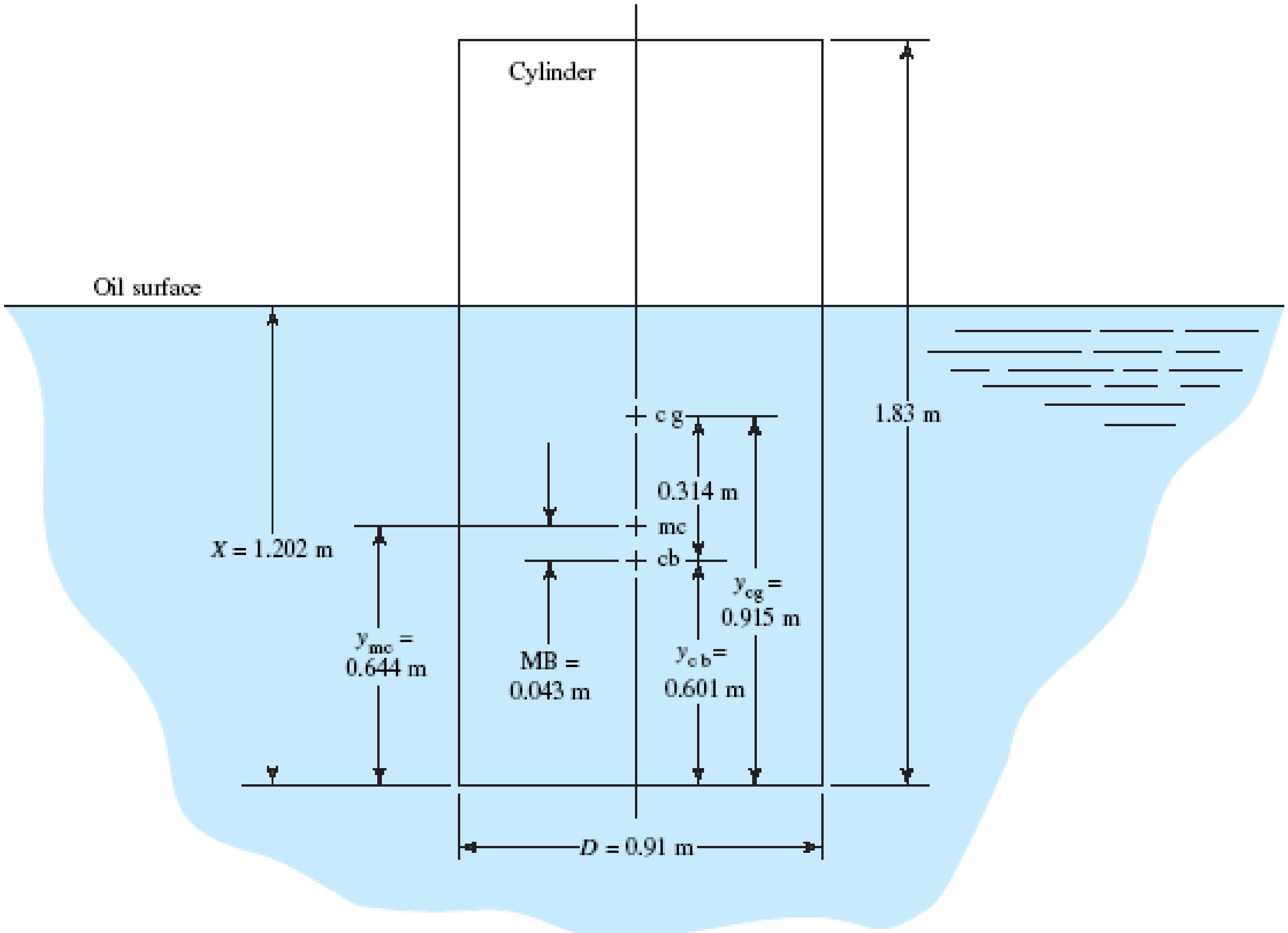
Example 5.6

A solid cylinder is 0.91 m in diameter, is 1.83 m high, and weighs 6.90 kN. If the cylinder is placed in oil (sg = 0.1) with its axis vertical, would it be stable?

The complete solution is shown in the next panel. Do this problem and then look at the solution.

Example 5.6

Position of cylinder in oil (Fig. 5.15):



Example 5.6

$$V_d = \text{submerged volume} = AX = \frac{\pi D^2}{4}(X)$$

Equilibrium equation:

$$\sum F_v = 0$$

$$w = F_b = \gamma_o V_d = \gamma_o \frac{\pi D^2}{4}(X)$$

$$X = \frac{4w}{\pi D^2 \gamma_o} = \frac{(4)6.90 \text{ kN}}{(\pi)(0.91 \text{ m})^2 (0.90)(9810 \text{ N/m}^3)} = 1.202 \text{ m}$$

Example 5.6

The center of buoyancy cb is at a distance *X/2* from the bottom of the cylinder:

$$y_{cb} = X/2 = 1.202 \text{ m}/2 = 0.601 \text{ m}$$

The center of gravity cg is at *H*/2=0.915 m from the bottom of the cylinder, assuming the material of the cylinder is of uniform specific weight. The position of the metacenter mc using Eq. (5–5), is

$$\text{MB} = I/V_d$$

Example 5.6

$$I = \frac{\pi D^4}{64} = \frac{\pi(0.91\text{ m})^4}{64} = 0.034\text{ m}^4$$

$$V_d = AX = \frac{\pi D^2}{4}(X) = \frac{\pi(0.91\text{ m})^2}{4}(1.202\text{ m}) = 0.782\text{ m}^3$$

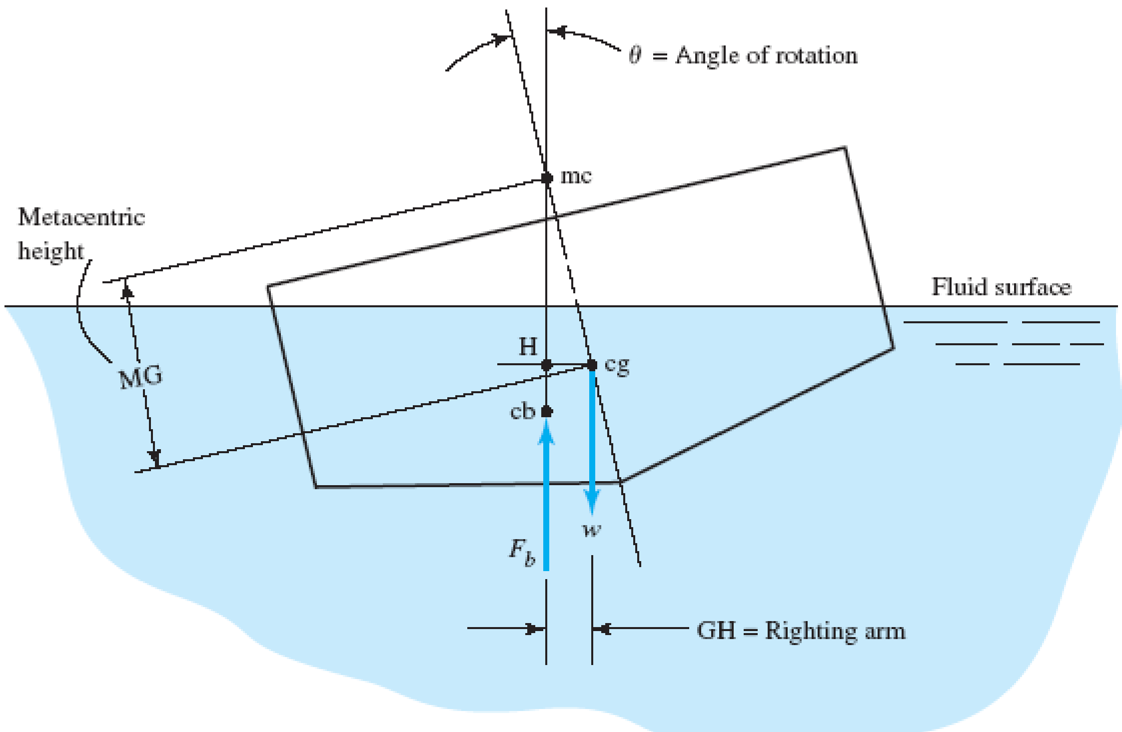
$$\text{MB} = I/V_d = 0.034\text{ m}^4/0.782\text{ m}^3 = 0.043\text{ m}$$

$$y_{mc} = y_{cb} + \text{MB} = 0.601\text{ m} + 0.043\text{ m} = 0.644\text{ m}$$

Because the metacenter is below the center of gravity ($y_{mc} < y_{cg}$)the cylinder is not stable in the position shown. It would tend to fall to one side until it reached a stable orientation, probably with the axis horizontal or nearly so.

5.6 Degree of Stability

- Fig 5.16 shows the degree of stability as indicated by the metacentric height and the righting arm.



5.6 Degree of Stability

- We can compute MG from

$$\mathrm{MG} = y_{\mathrm{mc}} - y_{\mathrm{cg}} \qquad (5\text{–}6)$$

- The metacentric height should not be too large, however, because the ship may then have the uncomfortable rocking motions that cause seasickness.

Example 5.7

Compute the metacentric height for the flatboat hull described in Example Problem 5.5.

From the results of Example Problem 5.5,

$$y_{mc} = 0.98 \text{ m from the bottom of the hull}$$
$$y_{cg} = 0.80 \text{ m}$$

$$MG = y_{mc} - y_{cg} = 0.98 \text{ m} - 0.80 \text{ m} = 0.18 \text{ m}$$

5.6.1 Static Stability

- Another measure of the stability of a floating object is the amount of offset between the line of action of the weight of the object acting through the center of gravity and that of the buoyant force acting through the center of buoyancy.
- Fig 5.17 shows the static stability curve for a floating body.
- It shows a characteristic plot of the righting arm versus the angle of rotation for a ship.
- As long as the value of GH remains positive, the ship is stable. Conversely, when GH becomes negative, the ship is unstable and it will overturn.

5. Buoyancy and Stability

5.6.1 Static Stability