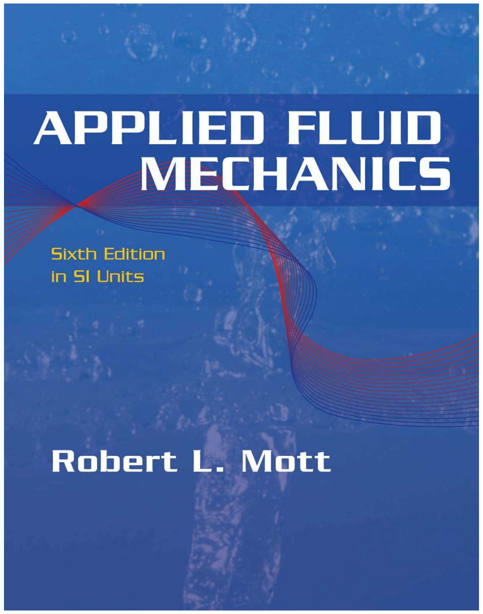
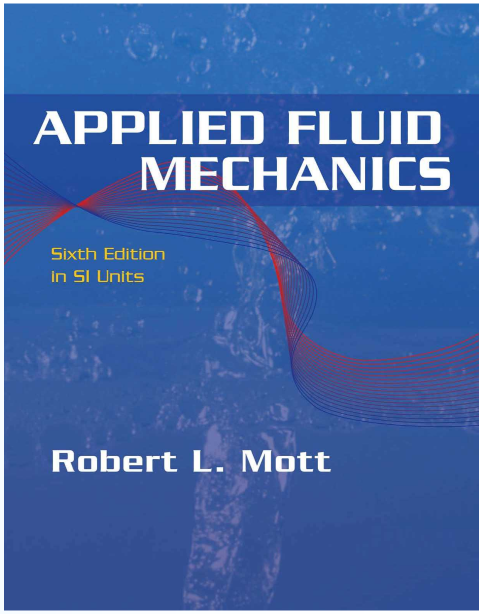
Applied Fluid Mechanics



1. The Nature of Fluid and the Study of Fluid Mechanics
2. Viscosity of Fluid
3. Pressure Measurement
4. Forces Due to Static Fluid
5. Buoyancy and Stability
6. Flow of Fluid and Bernoulli’s Equation
7. General Energy Equation
8. Reynolds Number, Laminar Flow, Turbulent Flow and Energy Losses Due to Friction

Applied Fluid Mechanics



9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections
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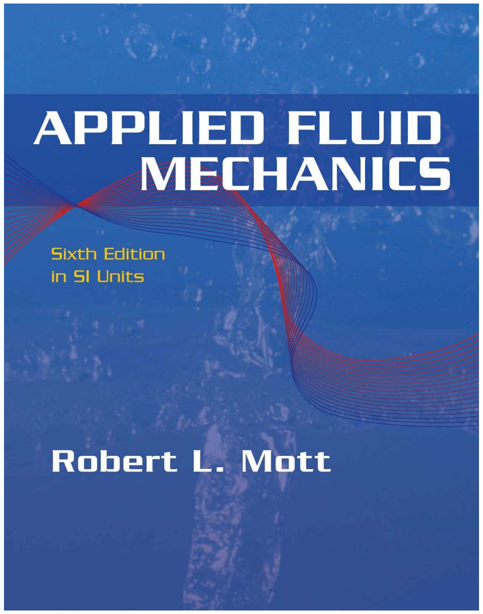
Applied Fluid Mechanics



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7. General Energy Equation

Chapter Objectives

- Identify the conditions under which energy losses occur in fluid flow systems.
- Identify the means by which energy can be added to a fluid flow system.
- Identify the means by which energy can be removed from a fluid flow system.
- Expand Bernoulli's equation to form the general energy equation by considering energy losses, energy additions, and energy removals.
- Apply the general energy equation to a variety of practical problems.
- Compute the power added to a fluid by pumps.

7. General Energy Equation

Chapter Objectives

- Define the *efficiency of pumps*.
- Compute the power required to drive pumps.
- Compute the power delivered by a fluid to a fluid motor.
- Define the *efficiency of fluid motors*.
- Compute the power output from a fluid motor.

7. General Energy Equation

Chapter Outline

1. Introductory Concepts
2. Energy Losses and Applications
3. Nomenclature of Energy Losses and Additions
4. General Energy Equation
5. Power Required by Pumps
6. Power Delivered to Fluid Motors

7. General Energy Equation

7.1 Introductory Concepts

- We use the following form of the continuity equation involving volume flow rate most often when liquids are flowing in the system:

$$Q_1 = Q_2$$

- Because Q = vA, we can write this as

$$A_1 v_1 = A_2 v_2$$

- These relationships allow us to determine the velocity of flow at any point in a system if we know the volume flow rate and the areas of the pipes at the sections of interest.

7. General Energy Equation

7.1 Introductory Concepts

- You should also be familiar with the terms that express the energy possessed by a fluid per unit weight of the fluid flowing in the system:

p/γ is the pressure head.

z is the elevation head.

$v^2/2g$ is the velocity head.

The sum of these three terms is called the total head.

7. General Energy Equation

7.1 Introductory Concepts

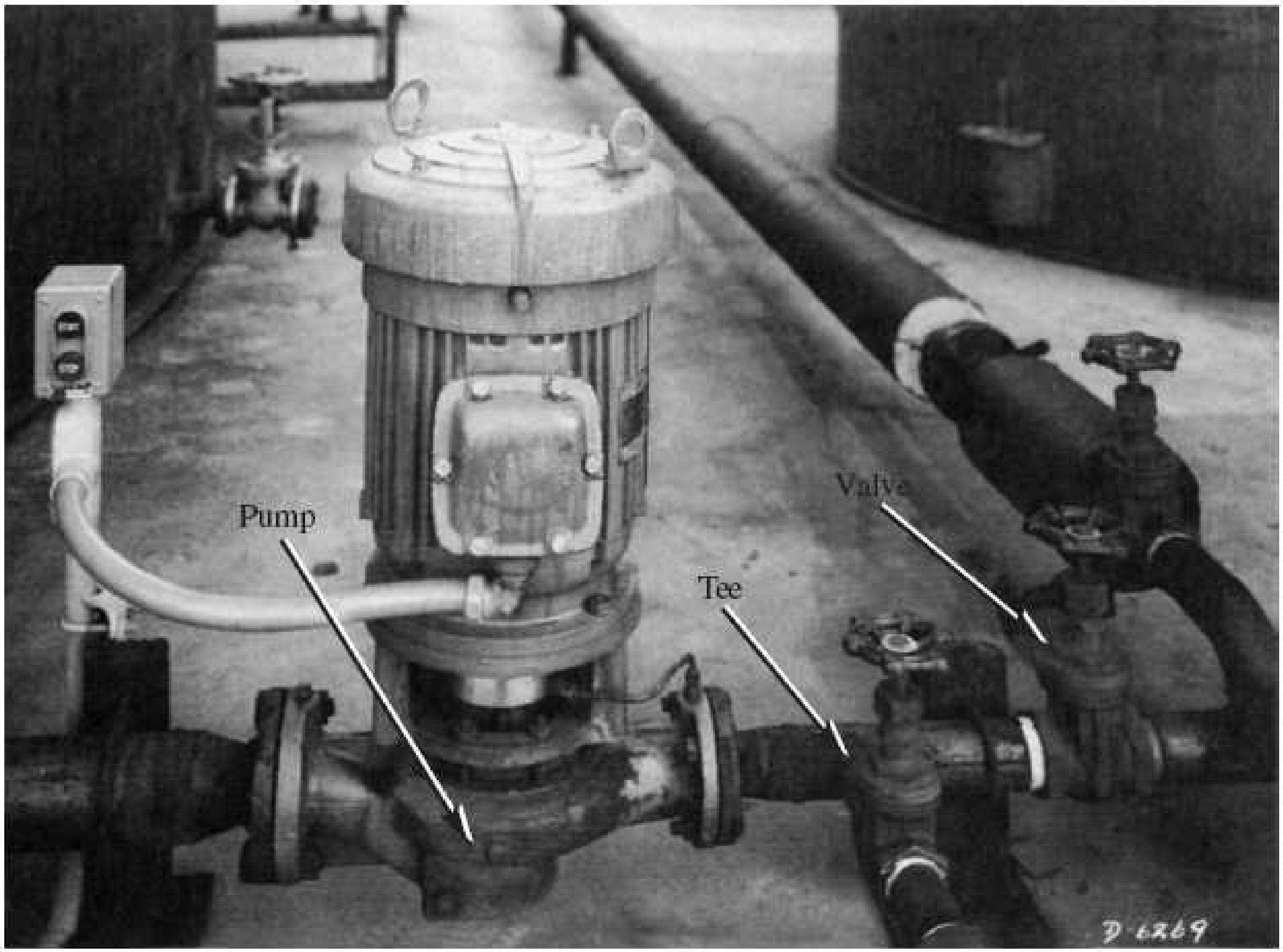
- All of this comes together in Bernoulli's equation,

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

- Fig 7.1 shows the typical pipeline installation, showing a pump, valves, tees, and other fittings.
- The fluid enters from the lower left, where the suction line draws fluid from a storage tank.
- The inline pump adds energy to the fluid and causes it to flow into the discharge line and then through the rest of the piping system.

7. General Energy Equation

7.1 Introductory Concepts



7. General Energy Equation

7.2 Energy Losses and Additions

- We discuss pumps, fluid motors, friction losses as fluid flows in pipes and tubes, energy losses from changes in the size of the flow path, and energy losses from valves and fittings.
- In later chapters you will learn more details about how to compute the amount of energy losses in pipes and specific types of valves and fittings.
- You will learn the method of using performance curves for pumps to apply them properly.

7. General Energy Equation

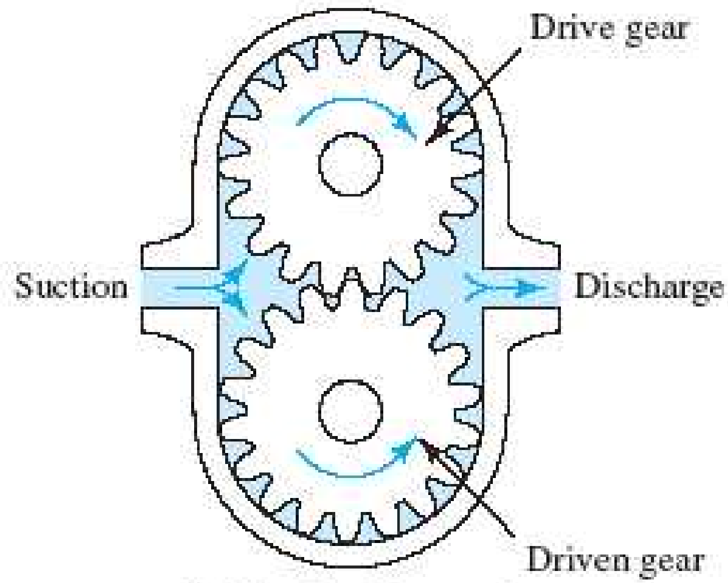
7.2.1 Pumps

- A pump is a common example of a mechanical device that adds energy to a fluid.
- An electric motor or some other prime power device drives a rotating shaft in the pump.
- The pump then takes this kinetic energy and delivers it to the fluid, resulting in fluid flow and increased fluid pressure.
- Fig 7.2 shows the gear pump.
- Fig 7.3 shows the piston pump.

7. General Energy Equation

7.2.1 Pumps

(a) Cutaway

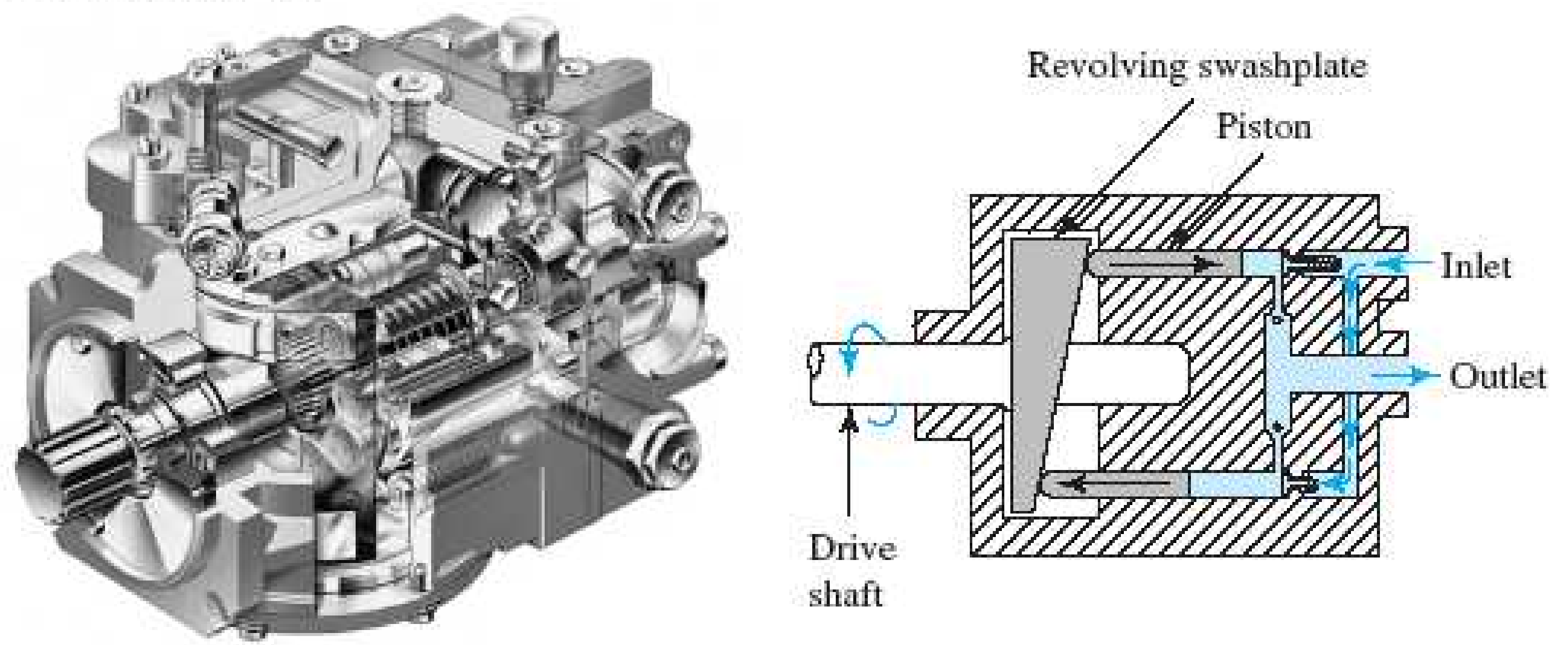


(b) Sketch of flow path

7. General Energy Equation

7.2.1 Pumps

Series 90 Variable Pump



(a) Cutaway

(b) Sketch of flow path

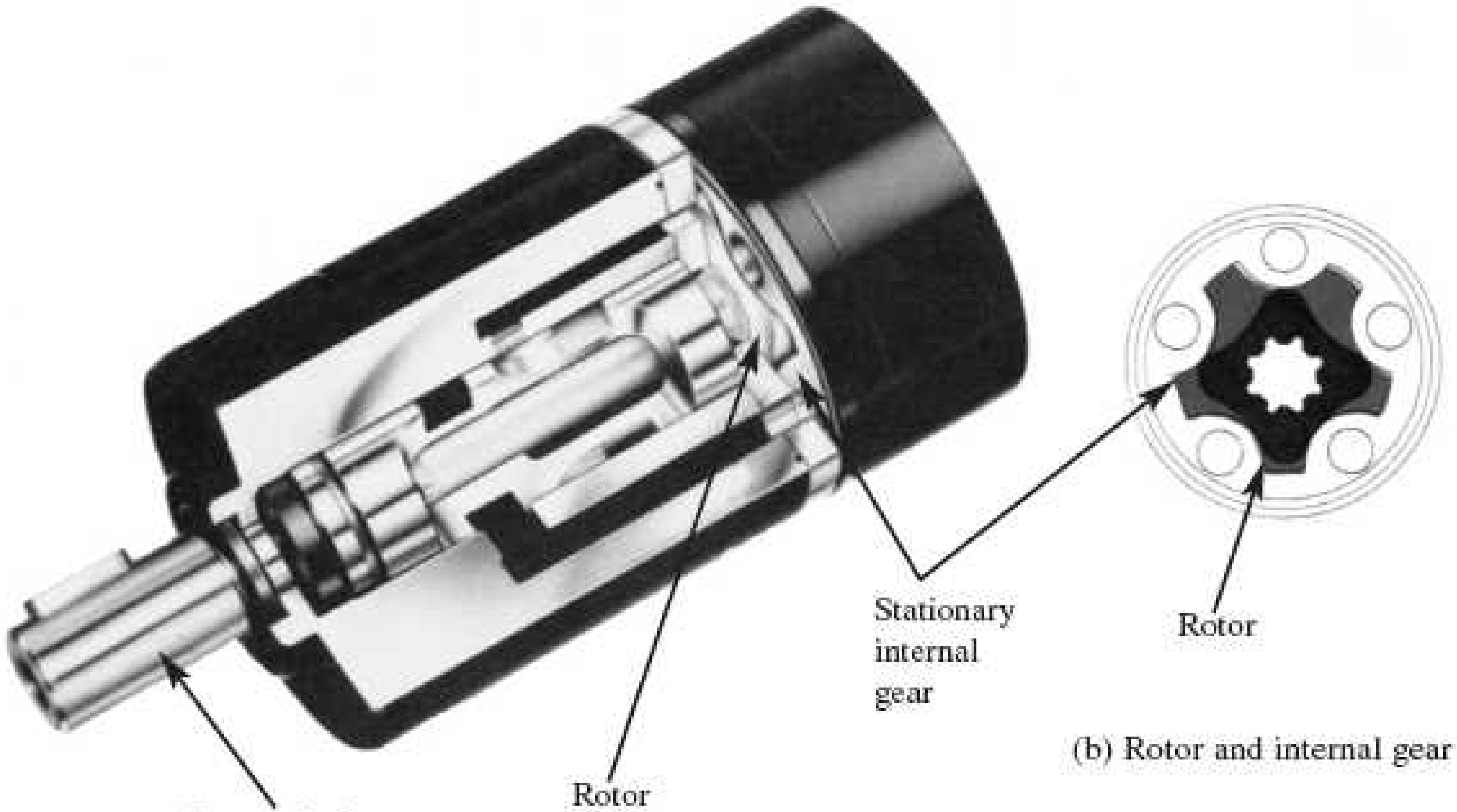
7. General Energy Equation

7.2.2 Fluid Motors

- Fluid motors, turbines, rotary actuators, and linear actuators are examples of devices that take energy from a fluid and deliver it in the form of work, causing the rotation of a shaft or the linear movement of a piston.
- The hydraulic motor shown in Fig. 7.4 is often used as a drive for the wheels of construction equipment and trucks and for rotating components of material transfer systems, conveyors, agricultural equipment, special machines, and automation equipment.

7. General Energy Equation

7.2.2 Fluid Motors

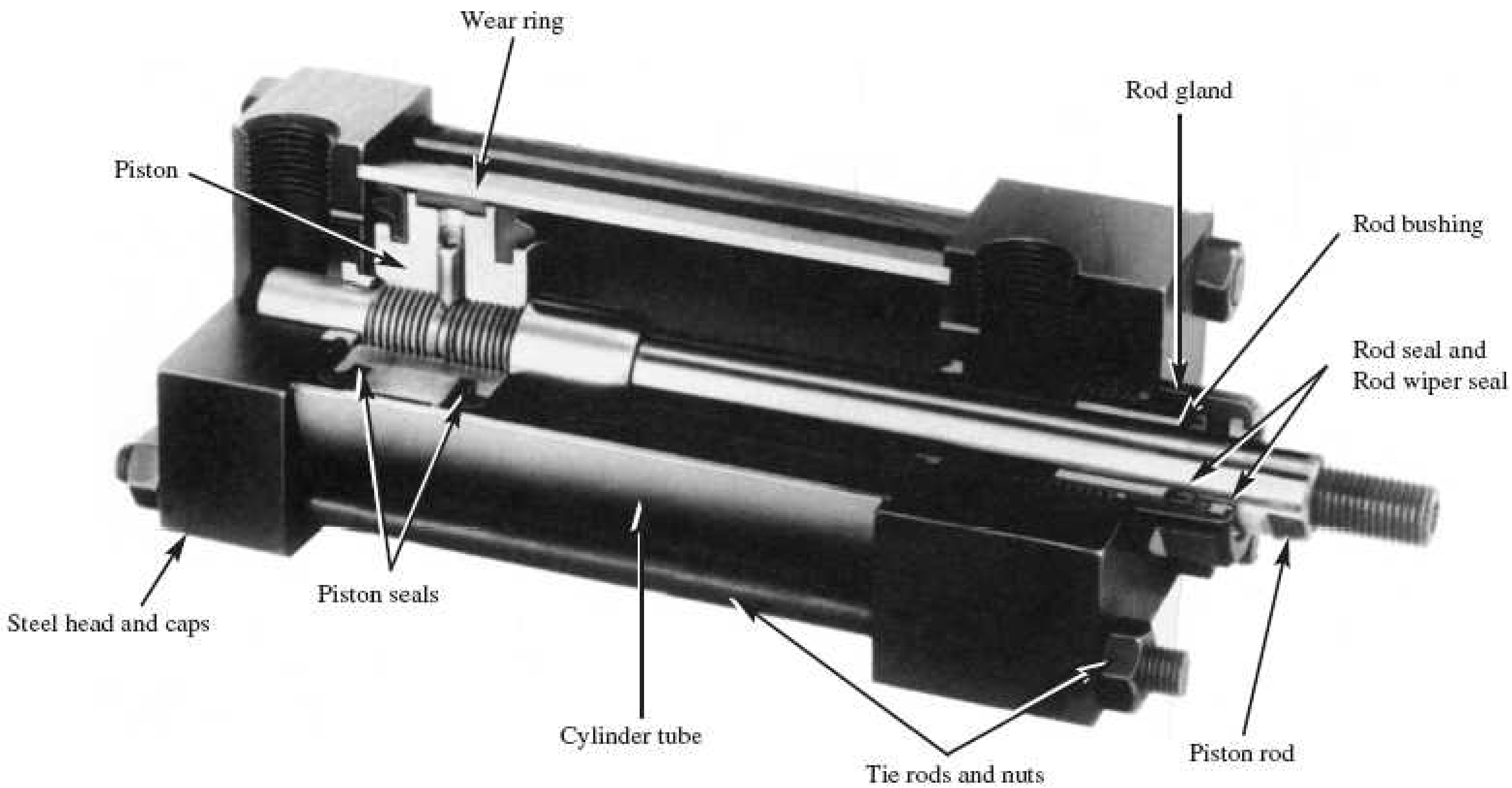


(a) Cutaway

(b) Rotor and internal gear

7.2.2 Fluid Motors

- Figure 7.5 is a photograph of a cutaway model of a fluid power cylinder or linear actuator.



7. General Energy Equation

7.2.3 Fluid Friction

- A fluid in motion offers frictional resistance to flow.
- Part of the energy in the system is converted into *thermal energy* (heat), which is dissipated through the walls of the pipe in which the fluid is flowing.
- The magnitude of the energy loss is dependent on the properties of the fluid, the flow velocity, the pipe size, the smoothness of the pipe wall, and the length of the pipe

7. General Energy Equation

7.2.4 Valves and Fittings

- Elements that control the direction or flow rate of a fluid in a system typically set up local turbulence in the fluid, causing energy to be dissipated as heat.
- Whenever there is a restriction, a change in flow velocity, or a change in the direction of flow, these energy losses occur. In a large system the magnitude of losses due to valves and fittings is usually small compared with frictional losses in the pipes.
- Therefore, such losses are referred to as *minor losses*.

7. General Energy Equation

7.3 Nomenclature of Energy Losses and Additions

- We will account for energy losses and additions in a system in terms of energy per unit weight of fluid flowing in the system. This is also known as “head”.
- Specifically, we will use the following terms

h_A = *Energy added* to the fluid with a mechanical device such as a pump; this is often referred to as the *total head* on the pump

h_R = *Energy removed* from the fluid by a mechanical device such as a fluid motor

h_L = *Energy losses* from the system due to friction in pipes or minor losses due to valves and fittings

7. General Energy Equation

7.3 Nomenclature of Energy Losses and Additions

- The magnitude of energy losses produced by fluid friction, valves, and fittings is directly proportional to the velocity head of the fluid.
- This can be expressed mathematically as

$$h_L = K(v^2/2g)$$

- The term *K* is the *resistance coefficient*.

7. General Energy Equation

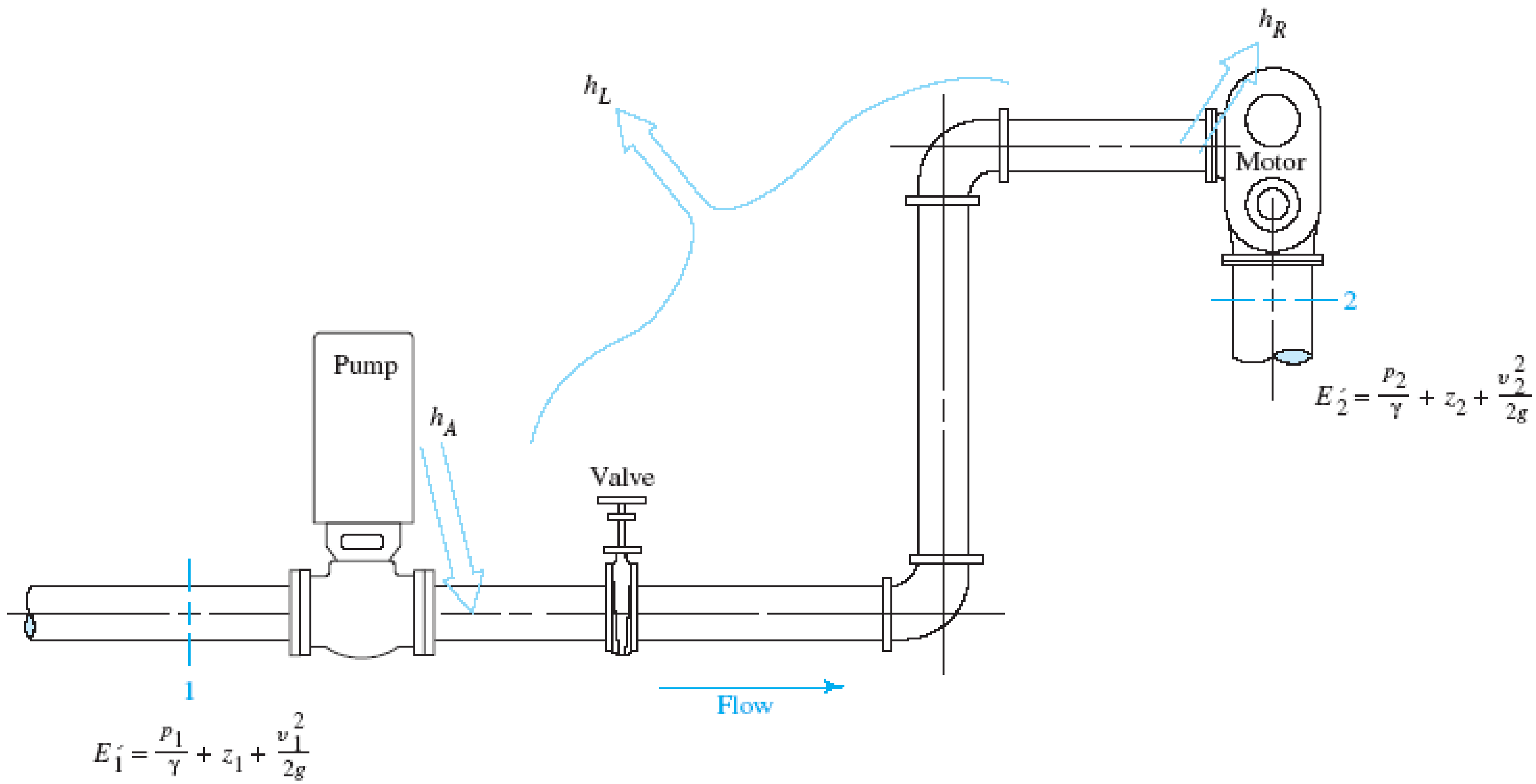
7.4 General Energy Equation

- The general energy equation as used in this text is an expansion of Bernoulli's equation, which makes it possible to solve problems in which energy losses and additions occur.
- Fig 7.6 shows the fluid flow system illustrating the general energy equation.
- For such a system the expression of the principle of conservation of energy is

$$E_1' + h_A - h_R - h_L = E_2' \qquad (7\text{–}1)$$

7. General Energy Equation

7.4 General Energy Equation



7. General Energy Equation

7.4 General Energy Equation

- The energy possessed by the fluid per unit weight is

$$E' = \frac{p}{\gamma} + z + \frac{v^2}{2g} \qquad (7\text{–}2)$$

- Equation (7–1) then becomes

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} + h_A - h_R - h_L = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g} \qquad (7\text{–}3)$$

7. General Energy Equation

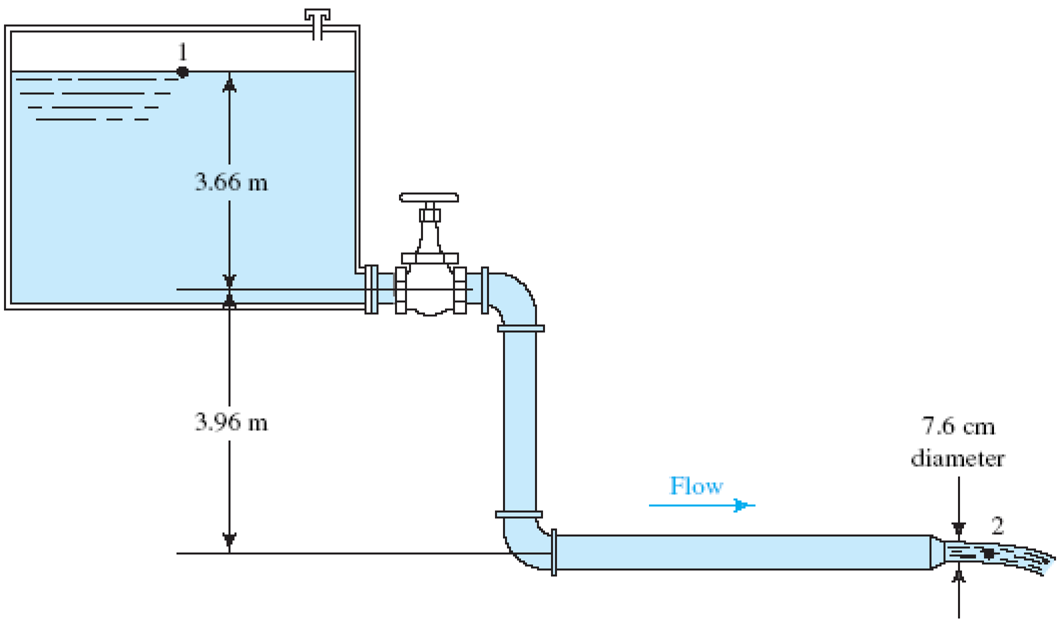
7.4 General Energy Equation

- It is essential that the general energy equation be written *in the direction of flow*, that is, *from* the reference point on the left side of the equation *to* that on the right side.

7. General Energy Equation

Example 7.1

Water flows from a large reservoir at the rate of 0.034 m^3/s through a pipe system as shown in Fig. 7.7. Calculate the total amount of energy lost from the system because of the valve, the elbows, the pipe entrance, and fluid friction.



7. General Energy Equation

Example 7.1

Using an approach similar to that used with Bernoulli's equation, select two sections of interest and write the general energy equation before looking at the next panel.

The sections at which we know the most information about pressure, velocity, and elevation are the surface of the reservoir and the free stream of fluid at the exit of the pipe. Call these section 1 and section 2, respectively. Then, the complete general energy equation [Eq. (7–3)] is

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} + h_A - h_R - h_L = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

7. General Energy Equation

Example 7.1

The value of some of these terms is zero. Determine which are zero and simplify the energy equation accordingly.

The value of the following terms is zero:

$p_1 = 0$	Surface of reservoir exposed to the atmosphere
$p_2 = 0$	Free stream of fluid exposed to the atmosphere
$v_1 = 0$	(Approximately) Surface area of reservoir is large
$h_A = h_R = 0$	No mechanical device in the system

7. General Energy Equation

Example 7.1

Then, the energy equation becomes

$$\cancelto{0}{\frac{p_1}{\gamma}} + z_2 + \cancelto{0}{\frac{v_1^2}{2g}} + \cancelto{0}{h_A} - \cancelto{0}{h_R} - h_L = \cancelto{0}{\frac{p_2}{\gamma}} + z_2 + \frac{v_2^2}{2g}$$

$$z_1 - h_L = z_2 + v_2^2/2g$$

Because we are looking for the total energy lost from the system, solve this equation for h_L, you should have

$$h_L = (z_1 - z_2) - v_2^2/2g$$

Now evaluate the terms on the right side of the equation to determine h_L in the units of m.

7. General Energy Equation

Example 7.1

The answer is h_L=4.8 m. Here is how it is found. First,

$$z_1 - z_2 = 7.62 \text{ m}$$
$$v_2 = Q/A_2$$

Because Q was given as 0.034 m3/s and the area of a 7.6-cm-diameter jet is 0.0045 m2, we have

$$v_2 = \frac{Q}{A} = \frac{0.034m^3}{s} \times \frac{1}{0.0045m^2} = 7.56m/s$$
$$\frac{v_2^2}{2g} = \frac{(7.56)^2 m^2}{s^2} \times \frac{s^2}{(2)(9.81)m} = 2.913m$$

7. General Energy Equation

Example 7.1

Then the total amount of energy lost from the system is

$$h_L = (z_2 - z_1) - v_2^2/2g = 7.62m - 2.913m$$

$$h_L = 4.707m$$

7. General Energy Equation

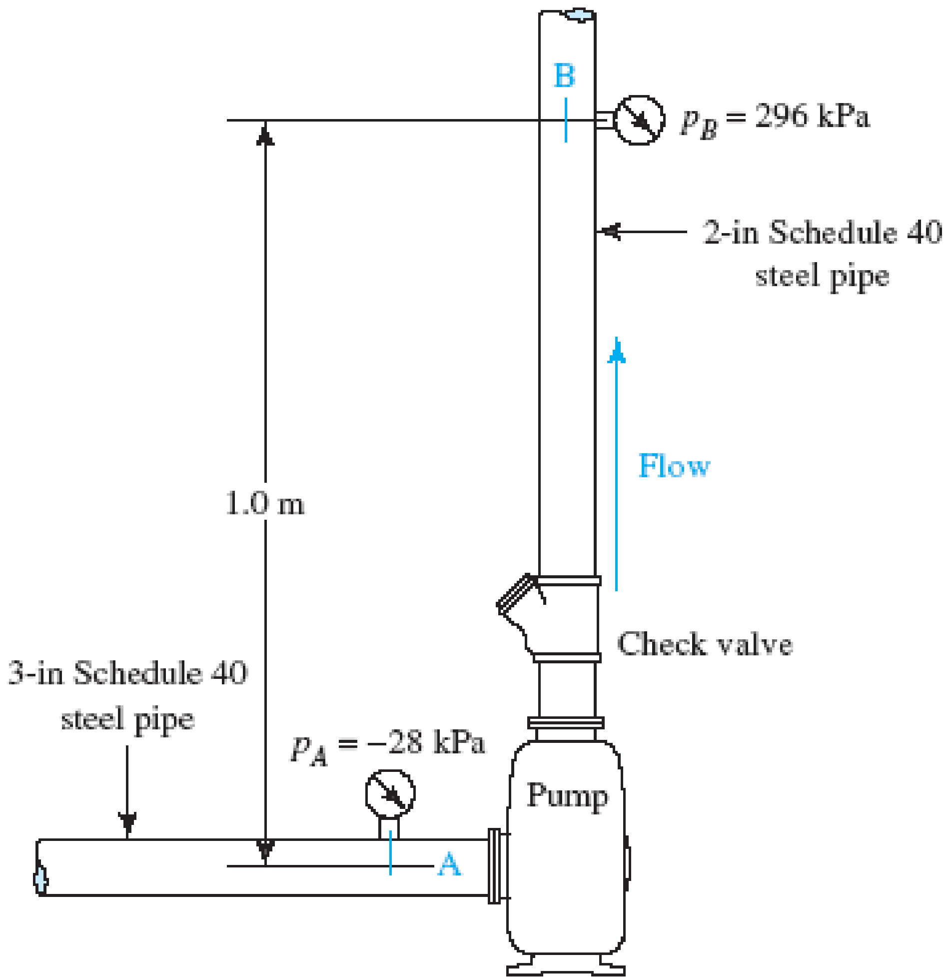
Example 7.1

The volume flow rate through the pump shown in Fig. 7.8 is $0.014 m^3/s$. The fluid being pumped is oil with a specific gravity of 0.86. Calculate the energy delivered by the pump to the oil per unit weight of oil flowing in the system. Energy losses in the system are caused by the check valve and friction losses as the fluid flows through the piping. The magnitude of such losses has been determined to be 1.86Nm/N.

Using the sections where the pressure gages are located as the sections of interest, write the energy equation for the system, including only the necessary terms.

7. General Energy Equation

Example 7.1



7. General Energy Equation

Example 7.1

You should have

$$\frac{p_A}{\gamma} + z_A + \frac{v_A^2}{2g} + h_A - h_L = \frac{p_B}{\gamma} + z_B + \frac{v_B^2}{2g}$$

Notice that the term h_R has been left out of the general energy equation. The objective of the problem is to calculate the energy added to the oil by the pump. Solve for h_A before looking at the next panel.

$$h_A = \frac{p_B - p_A}{\gamma} + (z_B - z_A) + \frac{v_B^2 - v_A^2}{2g} + h_L \qquad (7\text{–}4)$$

7. General Energy Equation

Example 7.1

Notice that similar terms have been grouped. This will be convenient when performing the calculations. Equation (7–4) should be studied well. It indicates that the total head on the pump is a measure of all of the tasks the pump is required to do in a system. It must increase the pressure from that at point A at the inlet to the pump to the pressure at point B. It must raise the fluid by the amount of the elevation difference between points A and B. It must supply the energy to increase the velocity of the fluid from that in the larger pipe at the pump inlet (called the suction pipe) to the velocity in the smaller pipe at the pump outlet (called the discharge pipe). In addition, it must overcome any energy losses that occur in the system such as those due to the check valve and friction in the discharge pipe.

7. General Energy Equation

Example 7.1

We recommend that you evaluate each of the terms in Eq. (7–4) separately and then combine them at the end. The first term is the difference between the pressure head at point A and that at point B. What is the value of γ?

Remember that the specific weight of the fluid being pumped must be used. In this case, the specific weight of the oil is

$$\gamma = (sg)(\gamma_w) = (0.86)(9.81\ \mathrm{kN/m^3}) = 8.44\ \mathrm{kN/m^3}$$

7. General Energy Equation

Example 7.1

Because p_B = 296kPa and p_A = -28kPa, we have

$$\frac{p_B - p_A}{\gamma} = \frac{[296 - (-28)]\ \text{kN}}{\text{m}^2} \times \frac{\text{m}^3}{8.44\ \text{kN}} = 38.4\ \text{m}$$

Notice that point B is at a higher elevation than point A and, therefore the result of $z_B - z_A$ is that is a positive number.

We can use the definition of volume flow rate and the continuity equation to determine each velocity:

$$Q = Av = A_A v_A = A_B v_B$$

Example 7.1

Then, solving for the velocities and using the flow areas for the suction and discharge pipes from Appendix F gives

$$v_A = Q/A_A = (0.014\,\text{m}^3/\text{s})/(4.768 \times 10^{-3}\,\text{m}^2) = 2.94\,\text{m/s}$$
$$v_B = Q/A_B = (0.014\,\text{m}^3/\text{s})/(2.168 \times 10^{-3}\,\text{m}^2) = 6.46\,\text{m/s}$$

Finally,

$$\frac{v_B^2 - v_A^2}{2g} = \frac{[(6.46)^2 - (2.94)^2]\,\text{m}^2/\text{s}^2}{2(9.81\,\text{m/s}^2)} = 1.69\,\text{m}$$

The only remaining term in Eq. (7–4) is the energy loss h_L which is given to be 1.86 Nm/N or 1.86m. We can now combine all of these terms and complete the calculation of h_A.

7. General Energy Equation

Example 7.1

The energy added to the system is

$$h_A = 38.4\text{ m} + 1.0\text{ m} + 1.69\text{ m} + 1.86\text{ m} = 42.9\text{ m, or } 42.9\text{ N}\cdot\text{m/N}$$

That is, the pump delivers 42.9Nm of energy to each newton of oil flowing through it. This completes the programmed instruction.

7. General Energy Equation

7.5 Power Required by Pumps

- Power is defined as the rate of doing work. In fluid mechanics we can modify this statement and consider that power is the rate at which energy is being transferred.
- Power is calculated by multiplying the energy transferred per newton of fluid by the weight flow rate. This is

$$P_A = h_A W$$

- Because W = γQ, we can also write

$$P_A = h_A \gamma Q \qquad (7\text{–}5)$$

7.5 Power Required by Pumps

where P_A denotes power added to the fluid, γ is the specific weight of the fluid flowing through the pump, and Q is the volume flow rate of the fluid.

7. General Energy Equation

7.5.1 Power in the US Customary System

- Because it is common practice to refer to power in horsepower (hp), the conversion factor required is

$$1 \text{ hp} = 550 \text{ lb-ft/s}$$

- To convert these units to the SI system we use the factors

$$1 \text{ lb-ft/s} = 1.356 \text{ W}$$
$$1 \text{ hp} = 745.7 \text{ W}$$

7. General Energy Equation

7.5.2 Mechanical Efficiency of Pumps

- The term *efficiency* is used to denote the ratio of the power delivered by the pump to the fluid to the power supplied to the pump.
- Because of energy losses due to mechanical friction in pump components, fluid friction in the pump, and excessive fluid turbulence in the pump, not all of the input power is delivered to the fluid.
- Then, using the symbol e_M for mechanical efficiency, we have

$$e_M = \frac{\text{Power delivered to fluid}}{\text{Power put into pump}} = \frac{P_A}{P_I} \qquad (7\text{–}6)$$

7. General Energy Equation

7.5.2 Mechanical Efficiency of Pumps

- The value of e_M will always be less than 1.0.
- The value of the mechanical efficiency of pumps depends not only on the design of the pump, but also on the conditions under which it is operating, particularly the total head and the flow rate.
- Efficiency values for positive-displacement fluid power pumps are reported differently from those for centrifugal pumps.
- Three values are often used: *overall efficiency* e_o, *volumetric efficiency* e_v and *torsional efficiency* e_T

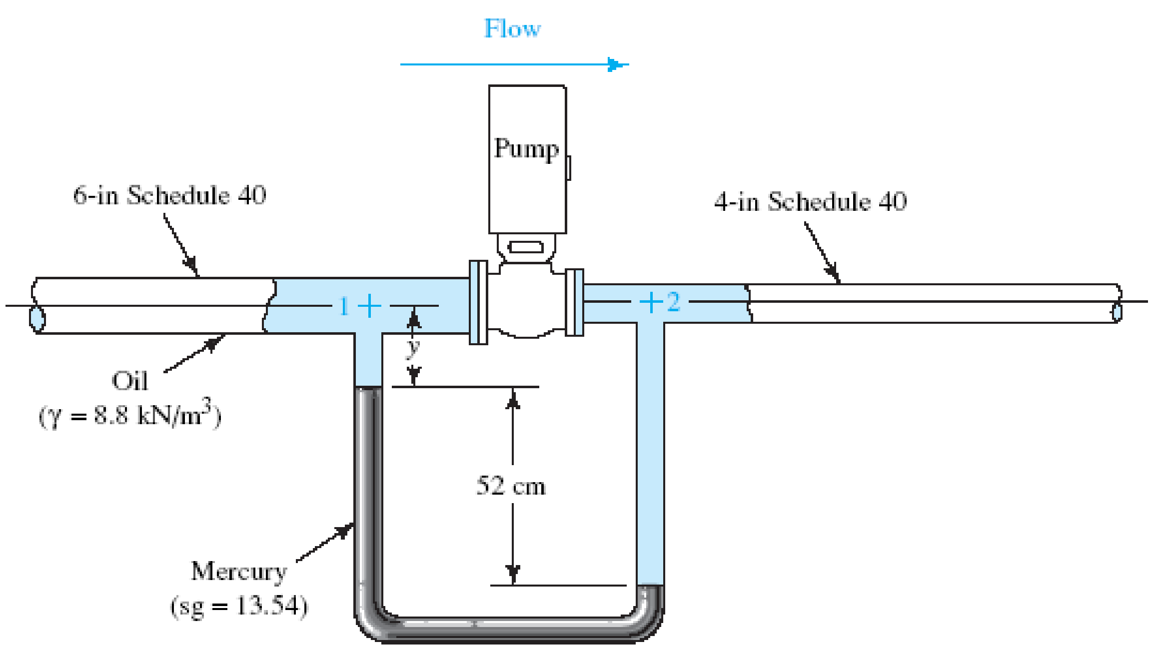
7. General Energy Equation

7.5.2 Mechanical Efficiency of Pumps

- In general the *overall efficiency* is analogous to the mechanical efficiency discussed for other types of pumps in this section.
- *Volumetric efficiency* is a measure of the actual delivery from the pump compared with the ideal delivery found from the displacement per revolution times the rotational speed of the pump.
- *Torsional efficiency* is a measure of the ratio of the ideal torque required to drive the pump against the pressure it is developing to the actual torque.

Example 7.3

For the pump test arrangement shown in Fig. 7.9, determine the mechanical efficiency of the pump if the power input is measured to be 2.87 kW when pumping 125 m^3/h or oil (γ=8.8 kN/m3).



7. General Energy Equation

Example 7.3

To begin, write the energy equation for this system. Using the points identified as 1 and 2 in Fig. 7.9, we have

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} + h_A = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

We use the following equation

$$h_A = \frac{p_2 - p_1}{\gamma} + (z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g} \qquad (7\text{–}7)$$

7. General Energy Equation

Example 7.3

It is convenient to solve for each term individually and then combine the results. The manometer enables us to (p2 – p1)/γ calculate because it measures the pressure difference. Using the procedure outlined in Chapter 3, write the manometer equation between points 1 and 2.

$$p_1 + \gamma_o y + \gamma_m(0.52 \text{ m}) - \gamma_o(0.52 \text{ m}) - \gamma_o y = p_2$$

where *y* is the unknown distance from point 1 to the top of the mercury column in the left leg of the manometer. The terms involving *y* cancel out. Also, in this equation γ_0 is the specific weight of the oil and γ_m is the specific weight of the mercury gage fluid.

7. General Energy Equation

Example 7.3

First, write

$$Q = \left(125 m^3 / hr\right)\left(\frac{10}{3600}\right) = 0.035 m^3 / s$$

Using A_1=1.864 x 10^{-2} m^2 and A_2=8.213 x 10^{-3} m^2 from Appendix F, we get

$$v_1 = \frac{Q}{A_1} = \frac{0.035 m^3}{s} \times \frac{1}{1.864 \times 10^{-2} m^2} = 1.878 m/s$$

$$v_2 = \frac{Q}{A_2} = \frac{0.035 m^3}{s} \times \frac{1}{8.213 \times 10^{-2} m^2} = 4.262 m/s$$

$$\frac{v_2^2 - v_1^2}{2g} = 0.746 m$$

7. General Energy Equation

Example 7.3

Solving h_A for we get

$$h_A = 7.3\text{ m} + 0 + 0.746\text{ m} = 8.046\text{ m}$$

Solving for pressure

$$P_A = h_a \gamma Q = 8.046m\left(\frac{8.8kN}{m^3}\right)\left(\frac{0.035m^3}{s}\right)$$

$$P_A = 2.478kNm/s = 2.478kW$$

From Eq. (7–6) we get

$$e_M = P_A/P_I = 2.478/2.87 = 0.86$$

Expressed as a percentage, the pump is 86 percent efficient at the stated conditions.

7. General Energy Equation

7.6 Power Delivered to Fluid Motor

- The energy delivered by the fluid to a mechanical device such as a fluid motor or a turbine is denoted in the general energy equation by the term h_R.
- This is a measure of the energy delivered by each unit weight of fluid as it passes through the device.
- We find the power delivered by multiplying h_R by the weight flow rate *W*:

$$P_R = h_R W = h_R \gamma Q \qquad (7\text{–}8)$$

where P_R is the power delivered by the fluid to the fluid motor.

7. General Energy Equation

7.6.1 Mechanical Efficiency of Fluid Motors

- Not all the power delivered to the motor is ultimately converted to power output from the device.
- Mechanical efficiency is then defined as

$$e_M = \frac{\text{Power output from motor}}{\text{Power delivered by fluid}} = \frac{P_O}{P_R} \qquad (7\text{–}9)$$

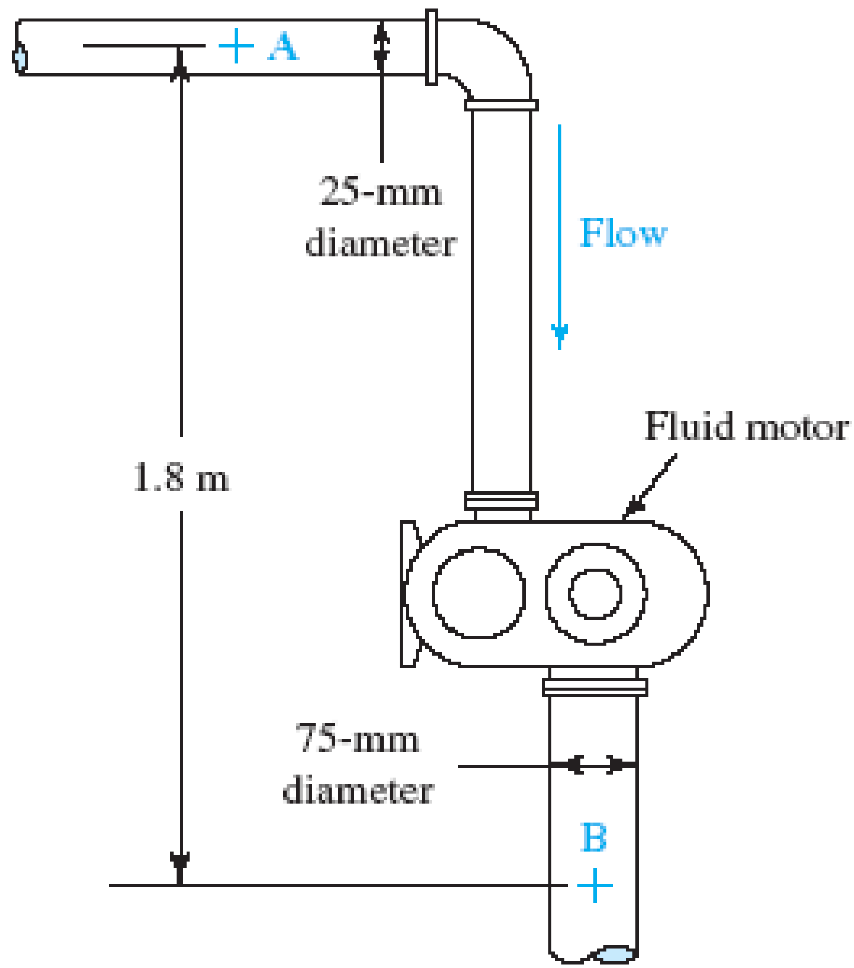
- Here again, the value is always less than 1.0.

Example 7.4

Water at 10°C is flowing at a rate of 115 L/min through the fluid motor shown in Fig. 7.10. The pressure at A is 700 kPa and the pressure at B is 125 kPa. It is estimated that due to friction in the piping there is an energy loss of 4 Nm/N of water flowing. (a) Calculate the power delivered to the fluid motor by the water. (b) If the mechanical efficiency of the fluid motor is 85 percent, calculate the power output.

7. General Energy Equation

Example 7.4



7. General Energy Equation

Example 7.4

Choosing points A and B as our reference points, we get

$$\frac{p_A}{\gamma} + z_A + \frac{v_A^2}{2g} - h_R - h_L = \frac{p_B}{\gamma} + z_B + \frac{v_B^2}{2g}$$

The correct results are as follow

1. $\frac{p_A - p_B}{\gamma} = \frac{(700 - 125)(10^3)\text{N}}{\text{m}^2} \times \frac{\text{m}^3}{9.81 \times 10^3 \text{ N}} = 58.6 \text{ m}$

2. $z_A - z_B = 1.8 \text{ m}$

3. Solving for $(v_A^2 - v_B^2)/2g$, we obtain

$$Q = 115 \text{ L/min} \times \frac{1.0 \text{ m}^3/\text{s}}{60\,000 \text{ L/min}} = 1.92 \times 10^{-3} \text{ m}^3/\text{s}$$

$$v_A = \frac{Q}{A_A} = \frac{1.92 \times 10^{-3} \text{ m}^3}{\text{s}} \times \frac{1}{4.909 \times 10^{-4} \text{ m}^2} = 3.91 \text{ m/s}$$

$$v_B = \frac{Q}{A_B} = \frac{1.92 \times 10^{-3} \text{ m}^3}{\text{s}} \times \frac{1}{4.418 \times 10^{-3} \text{ m}^2} = 0.43 \text{ m/s}$$

$$\frac{v_A^2 - v_B^2}{2g} = \frac{(3.91)^2 - (0.43)^2}{(2)(9.81)} \frac{\text{m}^2}{\text{s}^2} \frac{\text{s}^2}{\text{m}} = 0.77 \text{ m}$$

4. $h_L = 4.0 \text{ m (given)}$

7. General Energy Equation

Example 7.4

The energy delivered by the water to the turbine is

$$h_R = (58.6 + 1.8 + 0.77 - 4.0)\,\text{m} = 57.2\,\text{m}$$

Substituting the known values into Eq. (7–8), we get

$$P_R = h_R \gamma Q$$

$$P_R = 57.2\,\text{m} \times \frac{9.81 \times 10^3\,\text{N}}{\text{m}^3} \times \frac{1.92 \times 10^{-3}\,\text{m}^3}{\text{s}} = 1080\,\text{N}\cdot\text{m/s}$$

$$P_R = 1.08\,\text{kW}$$

Because the efficiency of the motor is 85 percent, we get 0.92 kW of power out.

$$P_O = e_M P_R$$

$$= (0.85)(1.08\,\text{kW})$$

$$P_O = 0.92\,\text{kW}$$