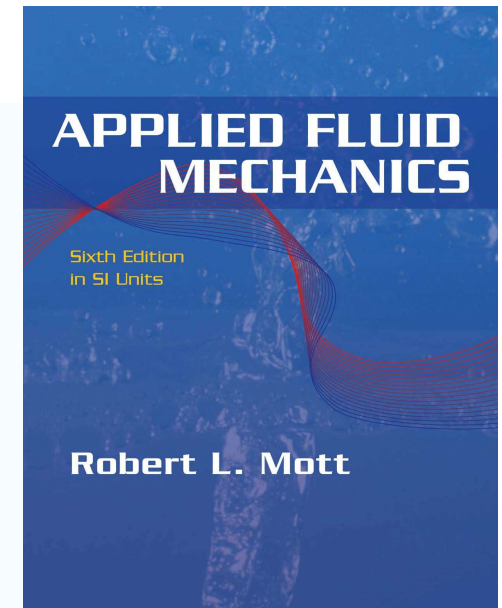


Applied Fluid Mechanics

1. The Nature of Fluid and the Study of Fluid Mechanics
2. Viscosity of Fluid
3. Pressure Measurement
4. Forces Due to Static Fluid
5. Buoyancy and Stability
6. Flow of Fluid and Bernoulli's Equation
7. General Energy Equation
8. Reynolds Number, Laminar Flow, Turbulent Flow and Energy Losses Due to Friction



Applied Fluid Mechanics

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

10. Minor Losses

11. Series Pipeline Systems

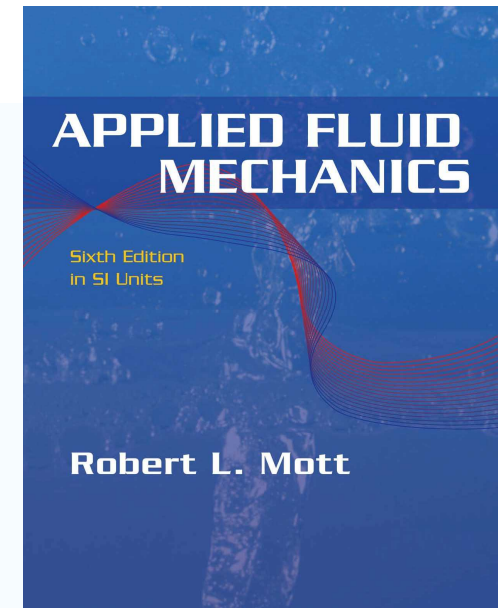
12. Parallel Pipeline Systems

13. Pump Selection and Application

14. Open-Channel Flow

15. Flow Measurement

16. Forces Due to Fluids in Motion

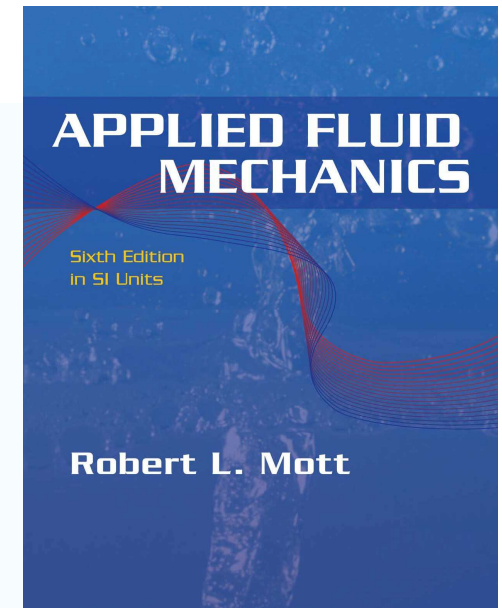


Applied Fluid Mechanics

17. Drag and Lift

18. Fans, Blowers, Compressors
and the Flow of Gases

19. Flow of Air in Ducts



9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Chapter Objectives

- Describe the velocity profile for laminar and turbulent flow in circular pipes, tubes, or hose.
- Describe the laminar boundary layer as it occurs in turbulent flow.
- Compute the local velocity of flow at any given radial position in a circular cross section.
- Compute the average velocity of flow in noncircular cross sections.
- Compute the Reynolds number for flow in noncircular cross sections using the *hydraulic radius* to characterize the size of the cross section.
- Determine the energy loss for the flow of a fluid in a noncircular cross section, considering special forms for the **relative roughness** and Darcy's equation.

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

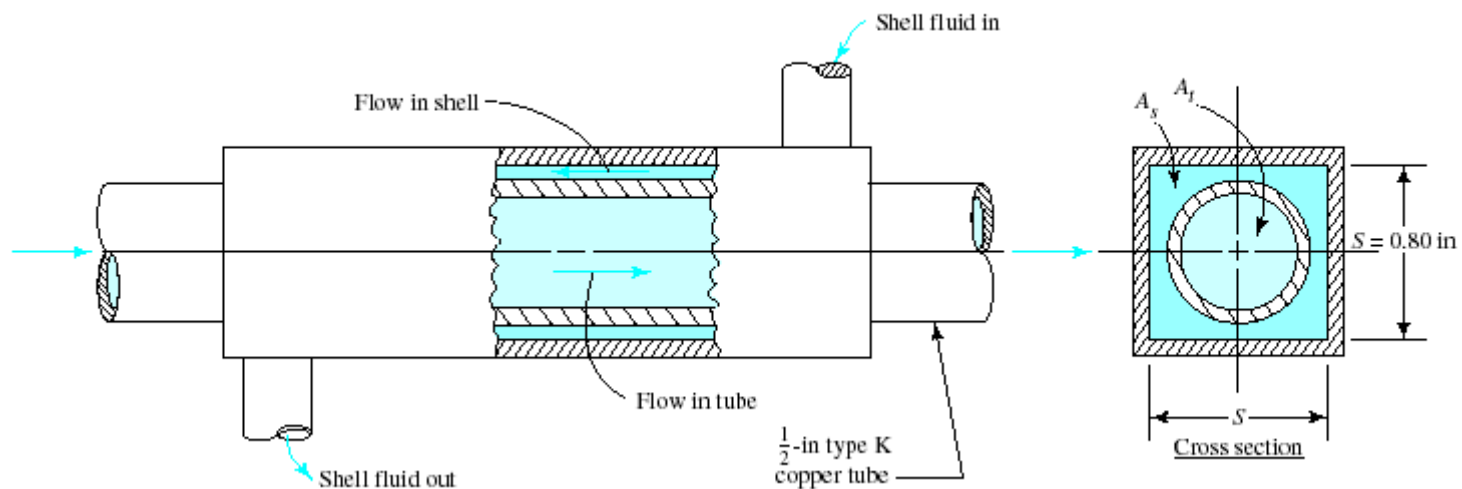
Chapter Outline

1. Introductory Concepts
2. Velocity Profiles
3. Velocity Profile For Laminar Flow
4. Velocity Profile For Turbulent Flow
5. Flow in Noncircular Sections
6. Computational Fluid Dynamics

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.1 Introductory Concepts

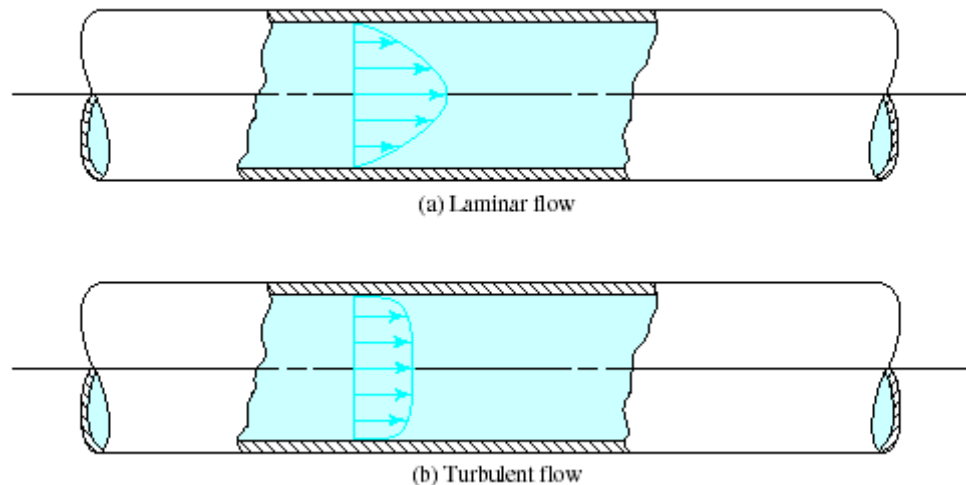
- There are many practical flow situations where the cross section is not circular.
- An example is the shell-and-tube heat exchanger shown in Fig. 9.1, in which, for example, hot water from an industrial process may be flowing to the right inside the inner tube.



9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.2 Velocity Profiles

- Figure 9.2 shows the general shape of the velocity profiles for laminar and turbulent flow.
- The velocity at the pipe wall is zero.
- The local velocity increases rapidly over a short distance from the wall.



9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.3 Velocity Profiles for Laminar Flow

- Because of the regularity of the velocity profile in laminar flow, we can define an equation for the local velocity at any point within the flow path.
- If we call the local velocity U at a radius r , the maximum radius r_o and the average velocity v , then

$$U = 2v[1 - (r/r_o)^2] \quad (9-1)$$

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.1

In Example Problem 8.1 we found that the Reynolds number is 708 when glycerine at 25°C flows with an average flow velocity of 3.6 m/s in a pipe having a 150-mm inside diameter. Thus the flow is laminar. Compute points on the velocity profile from the pipe wall to the centerline of the pipe in increments of 15 mm. Plot the data for the local velocity U versus the radius r . Also show the average velocity on the plot.

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.1

We can use Eq. (9–1) to compute U . First we compute the maximum radius r_o :

$$r_o = D/2 = 150/2 = 75 \text{ mm}$$

At $r = 75 \text{ mm} = r_o$ at the pipe wall, $r/r_o = 1$, and $U = 0$ from Eq. (9–1). This is consistent with the observation that the velocity of a fluid at a solid boundary is equal to the velocity of that boundary.

At $r = 60 \text{ mm}$,

$$U = 2(3.6 \text{ m/s})[1 - (60/75)^2] = 2.59 \text{ m/s}$$

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.1

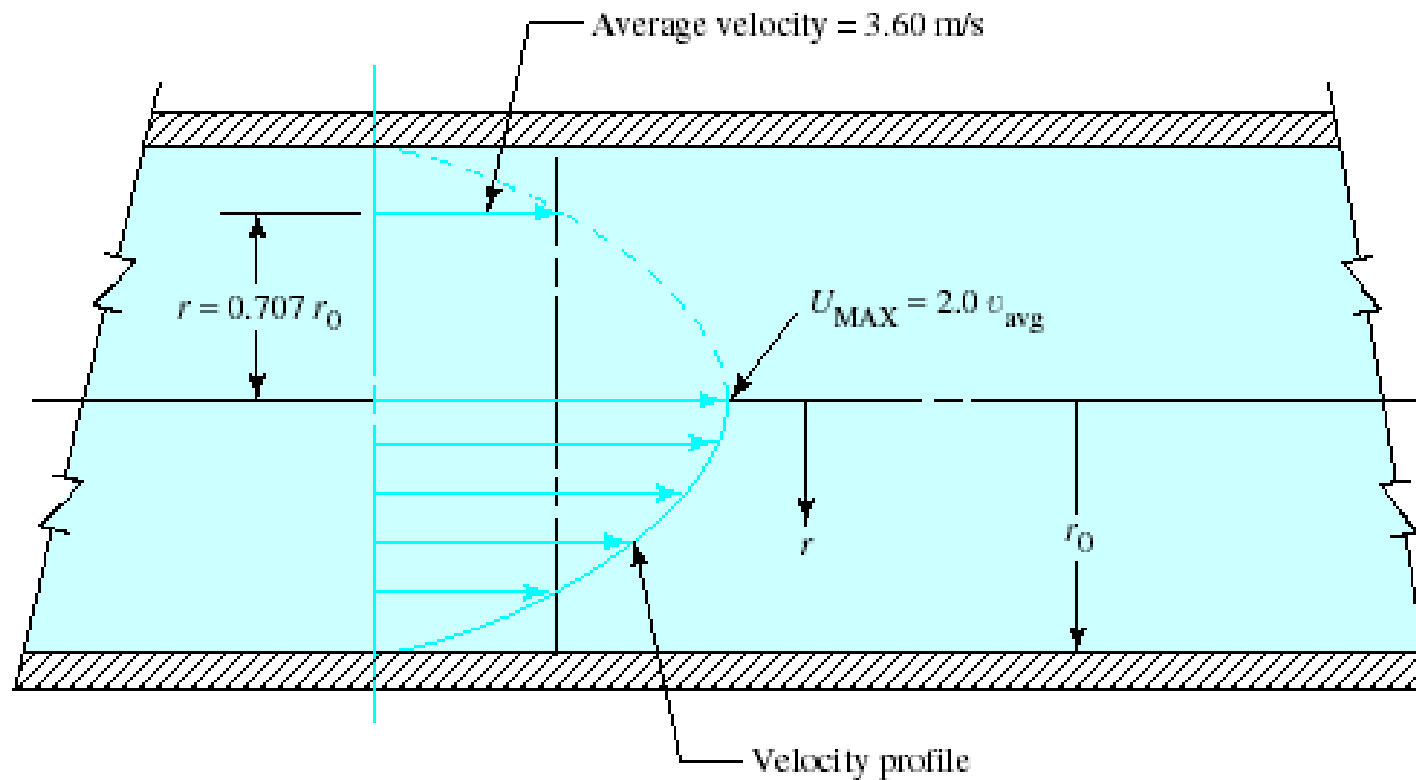
Using a similar technique, we can compute the following values:

r (mm)	r/r_o	U (m/s)	
75	1.0	0	(at the pipe wall)
60	0.8	2.59	
45	0.6	4.61	
30	0.4	6.05	
15	0.2	6.91	
0	0.0	7.20	(middle of the pipe)

Notice that the local velocity at the middle of the pipe is 2.0 times the average velocity. Figure 9.3 shows the plot of U versus r .

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.1



9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.2

Compute the radius at which the local velocity U would equal the average velocity v for laminar flow.

$$1 = 2[1 - (r/r_o)^2]$$

$$r = \sqrt{0.5}r_o = 0.707r_o \quad (9-2)$$

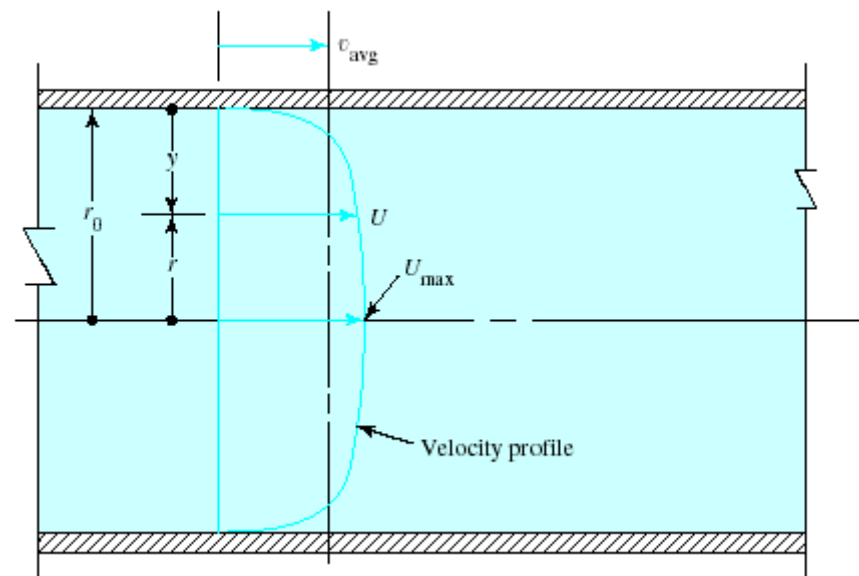
For the data from Example Problem 9.1, the local velocity is equal to the average velocity 3.6 m/s at

$$r = 0.707(75 \text{ mm}) = 53.0 \text{ mm} \quad |$$

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.4 Velocity Profiles for Turbulent Flow

- The velocity profile for turbulent flow is far different from the parabolic distribution for laminar flow.
- As shown in Fig. 9.4, the fluid velocity near the wall of the pipe changes rapidly from zero at the wall to a nearly uniform velocity distribution throughout the bulk of the cross section.



9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.4 Velocity Profiles for Turbulent Flow

- The governing equation is

$$U = v[1 + 1.43\sqrt{f} + 2.15\sqrt{f} \log_{10}(1 - r/r_o)] \quad (9-2)$$

- An alternate form of this equation can be developed by defining the distance from the wall of the pipe as $y = r_o - r$.
- Then, the argument of the logarithm term becomes

$$1 - \frac{r}{r_o} = \frac{r_o - r}{r_o} = \frac{y}{r_o}$$

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.4 Velocity Profiles for Turbulent Flow

- Equation (9–2) is then

$$U = v[1 + 1.43\sqrt{f} + 2.15\sqrt{f} \log_{10}(y/r_o)] \quad (9-3)$$

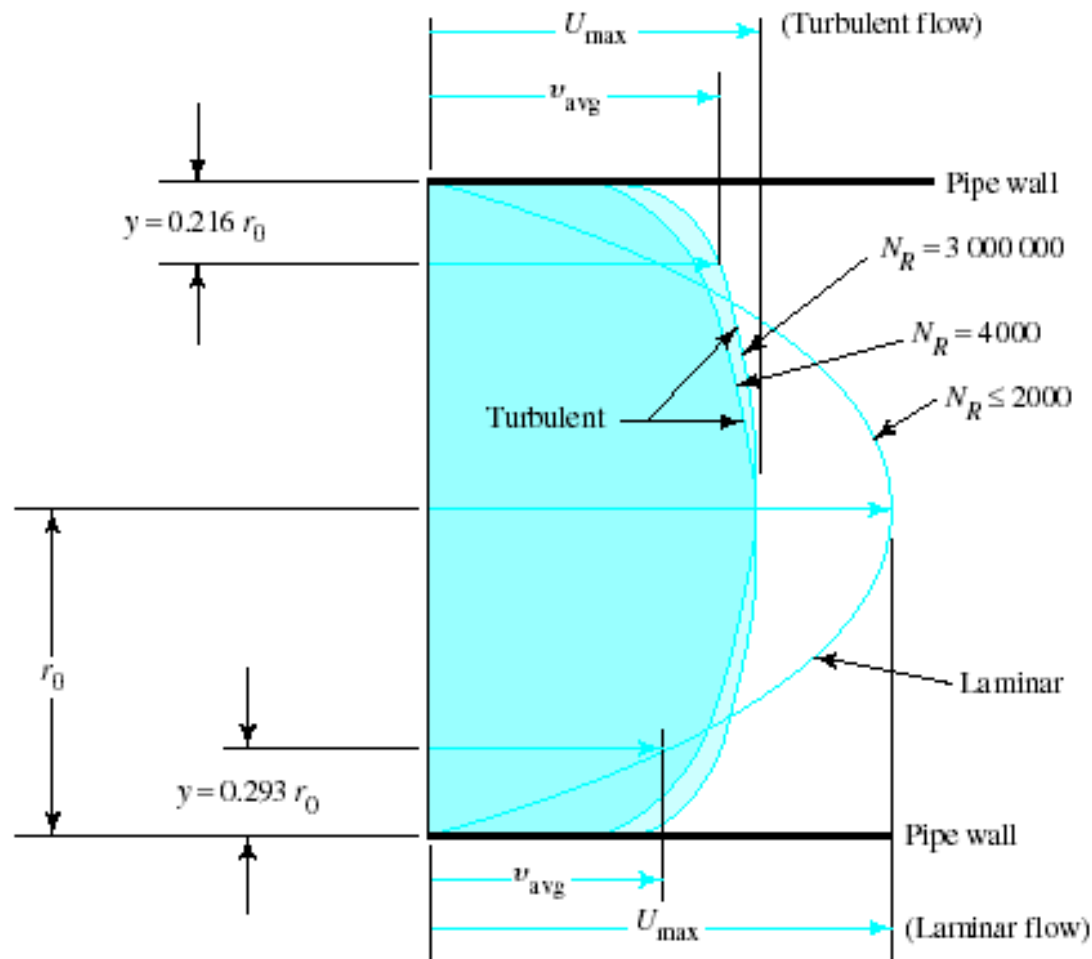
- The maximum velocity occurs at the center of the pipe and its value can be computed from ($r = 0$ or $y = r_o$),

$$U_{\max} = v(1 + 1.43\sqrt{f}) \quad (9-4)$$

- Figure 9.5 compares the velocity profiles for laminar flow and for turbulent flow at a variety of Reynolds numbers.

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.4 Velocity Profiles for Turbulent Flow



9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.3

For the data from Example Problem 8.8, compute the expected maximum velocity of flow, and compute several points on the velocity profile. Plot the velocity versus the distance from the pipe wall.

From Example Problem 8.8, we find the following data:

$$\begin{aligned} D &= 50 \text{ mm} = \text{pipe inside diameter} \\ v &= 0.932 \text{ m/s} = \text{average velocity} \\ f &= 0.018 = \text{friction factor} \\ N_R &= 9.54 \times 10^4 \quad (\text{turbulent}) \end{aligned}$$

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.3

Now, from Eq. (9–4), we see that the maximum velocity of flow is

$$U_{\max} = v(1 + 1.43 \sqrt{f}) = (0.932 \text{ m/s})(1 + 1.43 \sqrt{0.018})$$

$$U_{\max} = 1.111 \text{ m/s} \quad \text{at the center of the pipe}$$

Equation (9–3) can be used to determine the points on the velocity profile. We know that the velocity equals zero at the pipe wall ($y = 0$). Also, the rate of change of velocity with position is greater near the wall than near the center of the pipe.

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.3

Therefore, increments of 0.5 mm will be used from $y = 0.5$ to $y = 2.5$ mm. Then, increments of 2.5 mm will be used up to $y = 10$ mm. Finally, increments of 5.0 mm will provide sufficient definition of the profile near the center of the pipe. At $y = 1.0$ m and $r_o = 25$ mm,

$$U = v[1 + 1.43\sqrt{f} + 2.15\sqrt{f} \log_{10}(y/r_o)]$$

$$U = (0.932 \text{ m/s})[1 + 1.43\sqrt{0.018} + 2.15\sqrt{0.018} \log_{10}(1/25)]$$

$$U = 0.735 \text{ m/s}$$

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.3

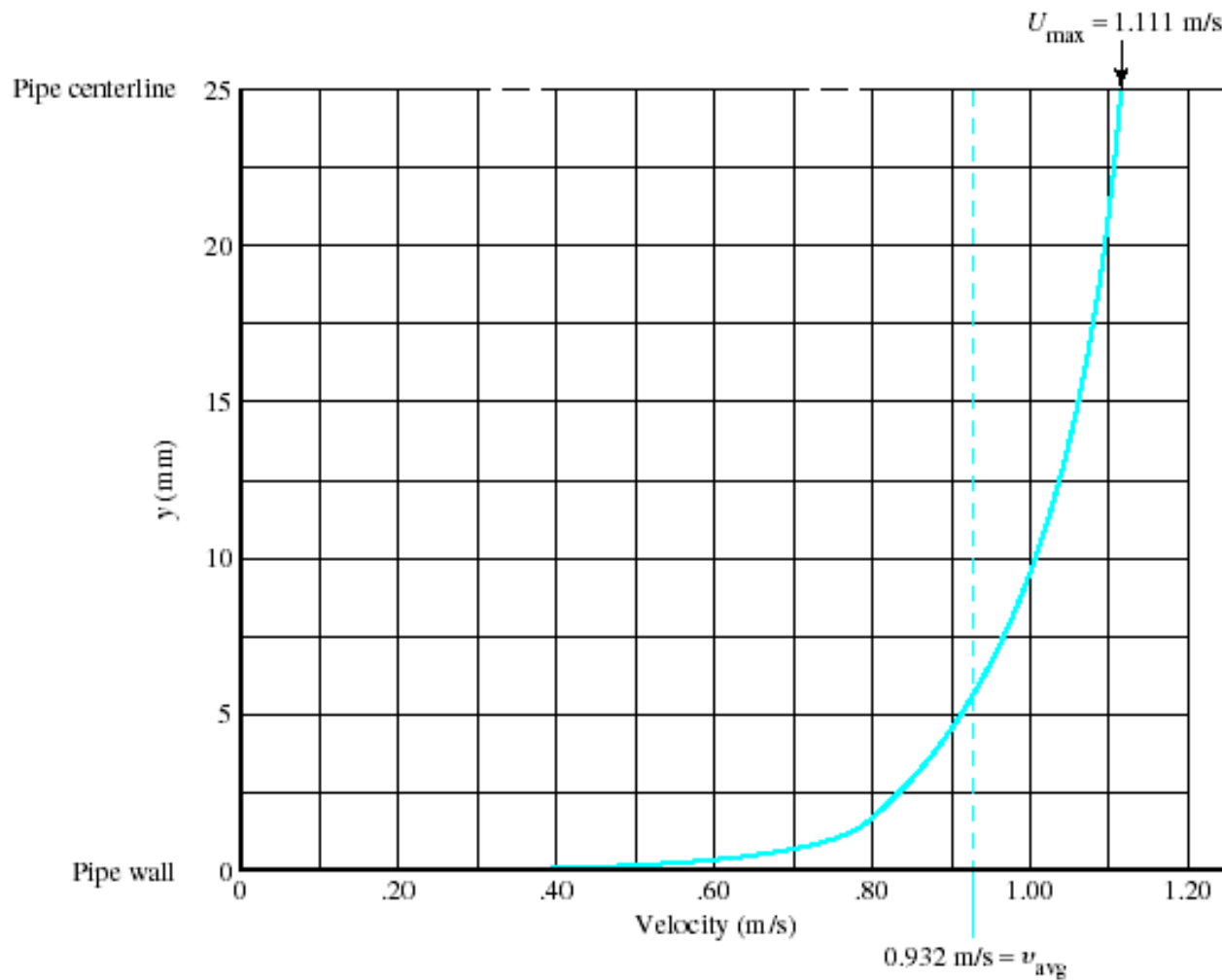
Using similar calculations, we can compute the following values:

y (mm)	y/r_o	U (m/s)	
0.5	0.02	0.654	
1.0	0.04	0.735	
1.5	0.06	0.782	
2.0	0.08	0.816	
2.5	0.10	0.842	
5.0	0.20	0.923	
7.5	0.30	0.970	
10.0	0.40	1.004	
15.0	0.60	1.051	
20.0	0.80	1.085	
25.0	1.00	1.111	(U_{\max} at center of pipe)

Figure 9.6 is the plot of y versus velocity in the form in which the velocity profile is normally shown. Because the plot is symmetrical, only one-half of the profile is shown.

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.3



9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

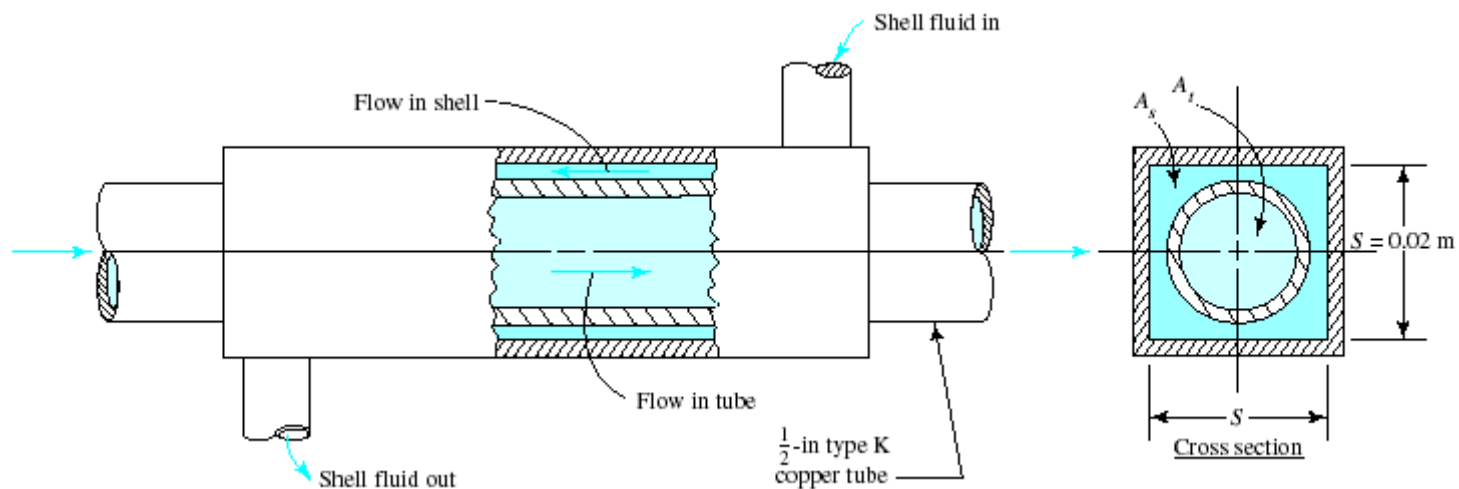
9.5 Flow in Noncircular Sections

- We discuss average velocity, hydraulic radius used as the characteristic size of the section, Reynolds number, and energy loss due to friction.
- All sections considered here are full of liquid.
- Noncircular sections for open-channel flow or partially filled sections.

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.5.1 Average Velocity

- Figure 9.7 shows a heat exchanger used to transfer heat from the fluid flowing inside the inner tube to that flowing in the space between the outside of the tube and the inside of the square shell that surrounds the tube.



9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.5.1 Average Velocity

- The definition of volume flow rate and the continuity equation is

$$Q = Av$$

$$v = Q/A$$

$$A_1v_1 = A_2v_2$$

- Care must be exercised to compute the net cross-sectional area for flow from the specific geometry of the noncircular section.

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.4

Figure 9.7 shows a heat exchanger used to transfer heat from the fluid flowing inside the inner tube to that flowing in the space between the outside of the tube and the inside of the square shell that surrounds the tube. Such a device is often called a *shell-and-tube heat exchanger*. Compute the volume flow rate that would produce a velocity of 2.44 m/s both inside the tube and in the shell.

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.4

We use the formula $Q = Av$, for volume flow rate, for each part.

(a) Inside the 0.5in Type K copper tube: From Appendix H, we can read

$$OD = 0.016 \text{ m}$$

$$ID = 0.013 \text{ m}$$

$$\text{Wall thickness} = 0.00124 \text{ m}$$

$$A_f = 1.407 \times 10^{-4} \text{ m}^2$$

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.4

Then, the volume flow rate inside the tube is

$$Q_t = A_t v = (1.407 \times 10^{-4} \text{ m}^2)(2.44 \text{ m/s}) = 3.433 \times 10^{-4} \text{ m}^3/\text{s}$$

(b) In the shell: The net flow area is the difference between the area *inside* the square shell and the *outside* of the tube. Then,

$$A_s = S^2 - \pi OD^2/4$$

$$A_s = (0.02 \text{ m})^2 - \pi(0.016 \text{ m})^2/4 = 1.99 \times 10^{-4} \text{ m}^2$$

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.4

The required volume flow rate is then

$$Q_s = A_s v = (1.99 \times 10^{-4} \text{ m}^2)(2.44 \text{ m/s}) = 4.856 \times 10^{-4} \text{ m}^3/\text{s}$$

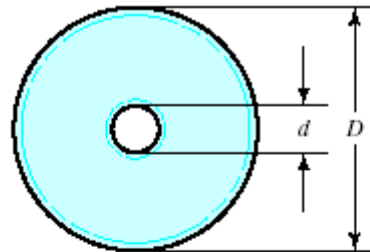
The ratio of the flow in the shell to the flow in the tube is

$$\text{Ratio} = Q_s/Q_t = \frac{4.856 \times 10^{-4}}{3.433 \times 10^{-4}} = 1.415$$

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.5.2 Hydraulic Radius for Noncircular Cross Section

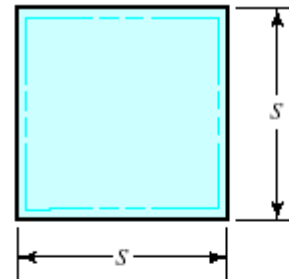
- Examples of typical closed, noncircular cross sections are shown in Fig. 9.8.



$$A = \frac{\pi}{4}(D^2 - d^2)$$

$$WP = \pi(D + d)$$

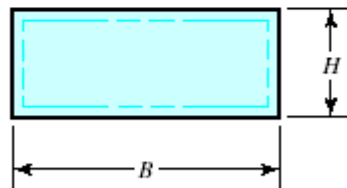
(a)



$$A = S^2$$

$$WP = 4S$$

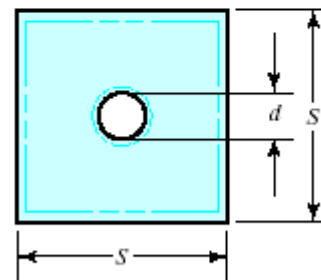
(b)



$$A = BH$$

$$WP = 2B + 2H$$

(c)



$$A = S^2 - \pi d^2/4$$

$$WP = 4S + \pi d$$

(d)

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.5.2 Hydraulic Radius for Noncircular Cross Section

- The characteristic dimension of noncircular cross sections is called the *hydraulic radius* R , defined as the ratio of the net cross-sectional area of a flow stream to the wetted perimeter of the section.
- That is,

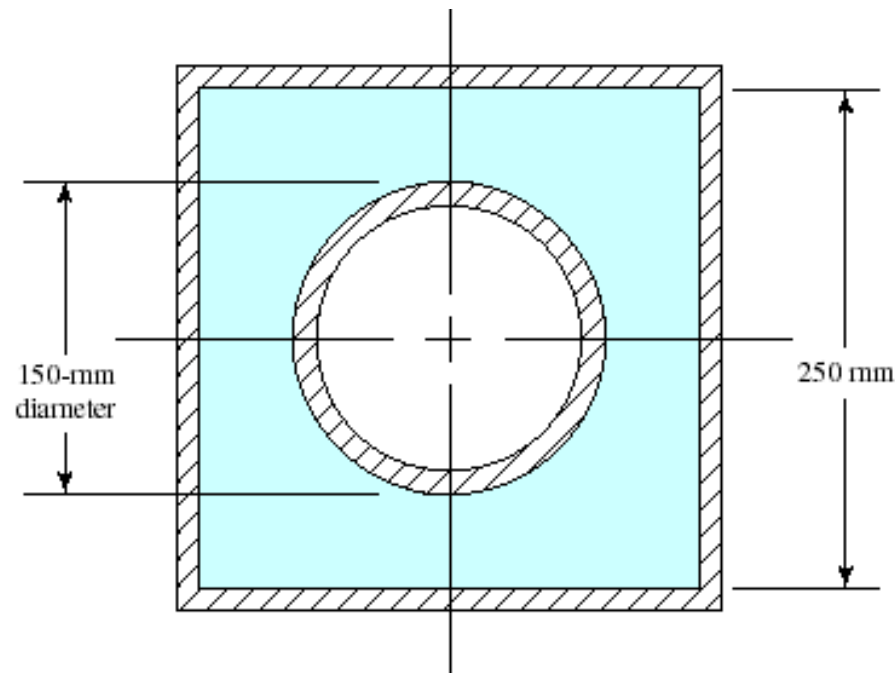
$$R = \frac{A}{WP} = \frac{\text{Area}}{\text{Wetted perimeter}} \quad (9-5)$$

- ***The wetted perimeter is defined as the sum of the length of the boundaries of the section actually in contact with (that is, wetted by) the fluid.***

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.5

Determine the hydraulic radius of the section shown in Fig. 9.9 if the inside dimension of each side of the square is 250 mm and the outside diameter of the tube is 150 mm.



9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.5

The net flow area is the difference between the area of the square and the area of the circle:

$$A = S^2 - \pi d^2/4 = (250)^2 - \pi(150)^2/4 = 44\,829 \text{ mm}^2$$

The wetted perimeter is the sum of the four sides of the square and the circumference of the circle:

$$WP = 4S + \pi d = 4(250) + \pi(150) = 1471 \text{ mm}$$

Then, the hydraulic radius R is

$$R = \frac{A}{WP} = \frac{44\,829 \text{ mm}^2}{1471 \text{ mm}} = 30.5 \text{ mm} = 0.0305 \text{ m}$$

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.5.3 Reynolds Number for Closed Noncircular Cross Sections

- When the fluid completely fills the available cross-sectional area and is under pressure, the average velocity of flow is determined by using the volume flow rate and the net flow area in the familiar equation,

$$v = Q/A$$

- The Reynolds number for flow in noncircular sections is computed in a very similar manner to that used for circular pipes and tubes.

$$N_R = \frac{v(4R)\rho}{\mu} = \frac{v(4R)}{\nu}$$

(9-6)

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.5.3 Reynolds Number for Closed Noncircular Cross Sections

- The validity of this substitution can be demonstrated by calculating the hydraulic radius for a circular pipe:

$$R = \frac{A}{WP} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4}$$

$$D = 4R$$

- Therefore, $4R$ is equivalent to D for the circular pipe.
- Thus, by analogy, the use of $4R$ as the characteristic dimension for noncircular cross sections is appropriate.
- This approach will give reasonable results as long as the cross section has an *aspect ratio* not much different from that of the circular cross section.

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.6

Compute the Reynolds number for the flow of ethylene glycol at 25°C through the section shown in Fig. 9.8(d). The volume flow rate is 0.16 m³/s. Use the dimensions given in Example Problem 9.5.

The result for the hydraulic radius for the section from Example Problem 9.5 can be used: $R = 0.0305$ m. Now the Reynolds number can be computed from Eq. (9–6). We can use $\mu = 1.62 \times 10^{-2}$ Pa•s and (from Appendix B). The area must be converted to m². We have $\rho = 1100$ kg/m³

$$A = (44\,829 \text{ mm}^2)(1 \text{ m}^2/10^6 \text{ mm}^2) = 0.0448 \text{ m}^2$$

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.6

The average velocity of flow is

$$v = \frac{Q}{A} = \frac{0.16 \text{ m}^3/\text{s}}{0.0448 \text{ m}^2} = 3.57 \text{ m/s}$$

The Reynolds number can now be calculated:

$$N_R = \frac{v(4R)\rho}{\mu} = \frac{(3.57)(4)(0.0305)(1100)}{1.62 \times 10^{-2}}$$

$$N_R = 2.96 \times 10^4$$

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.5.4 Friction Loss in Noncircular Cross Sections

- Darcy's equation for friction loss can be used for noncircular cross sections if the geometry is represented by the hydraulic radius instead of the pipe diameter, as is used for circular sections.
- After computing the hydraulic radius, we can compute the Reynolds number from Eq. (9–6).
- In Darcy's equation, replacing D with $4R$ gives

$$h_L = f \frac{L}{4R} \frac{v^2}{2g} \quad (9-7)$$

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.7

Determine the pressure drop for a 50-m length of a duct with the cross section shown in Fig. 9.9. Ethylene glycol at 25°C is flowing at the rate of 0.16 m³/s. The inside dimension of the square is 250 mm and the outside diameter of the tube is 150 mm. Use $\epsilon = 3 \times 10^{-5}$ m, somewhat smoother than commercial steel pipe.

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.7

The area, velocity, hydraulic radius, and Reynolds number were computed in Example Problems 9.5 and 9.6. The results are

$$A = 0.0448 \text{ m}^2$$

$$v = 3.57 \text{ m/s}$$

$$R = 0.0305 \text{ m}$$

$$N_R = 2.96 \times 10^4$$

The flow is turbulent, and Darcy's equation can be used to calculate the energy loss between two points 50 m apart. To determine the friction factor, we must first find the relative roughness:

$$4R/\epsilon = (4)(0.0305)/(3 \times 10^{-5}) = 4067$$

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.7

From the Moody diagram, $f = 0.0245$. Then, we have

$$h_L = f \times \frac{L}{4R} \times \frac{v^2}{2g} = 0.0245 \times \frac{50}{(4)(0.0305)} \times \frac{(3.57)^2}{(2)(9.81)} \text{ m}$$
$$h_L = 6.52 \text{ m}$$

If the duct is horizontal,

$$h_L = \Delta p / \gamma$$
$$\Delta p = \gamma h_L$$

where Δp is the pressure drop caused by the energy loss.

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

Example 9.7

Use $\gamma = 10.79 \text{ kN/m}^3$ from Appendix B. Then, we have

$$\Delta p = \frac{10.79 \text{ kN}}{\text{m}^3} \times 6.52 \text{ m} = 70.4 \text{ kPa}$$

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.6 Computational Fluid Dynamics

- The partial differential equations governing fluid flow and heat transfer include the continuity equation (conservation of mass), the Navier–Stokes equations (conservation of momentum, or Newton’s second law) and the energy equation (conservation of energy, or the first law of thermodynamics).
- These equations are complex, intimately coupled, and nonlinear, making a general analytic solution impossible except for a limited number of special problems where the equations can be reduced to yield analytic solutions.

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

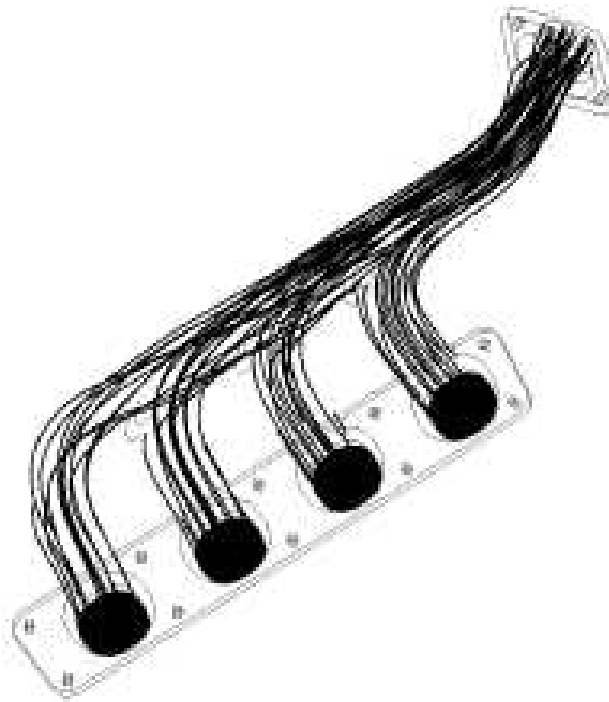
9.6 Computational Fluid Dynamics

- There are numerous methods available for doing so, called collectively *computational fluid dynamics* (*CFD*), which use the finite-element method to reduce the complex governing equations to a set of algebraic equations at discrete points or nodes on each small element within the fluid.
- In addition to characterizing the flow phenomena, CFD can analyze the heat transfer occurring in the fluid.

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.6 Computational Fluid Dynamics

- Fig 9.10 shows the flow through an exhaust manifold.



9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.6 Computational Fluid Dynamics

- The steps required to use CFD include the following:
 1. Establish the boundary conditions that define known values of pressure, velocity, temperature, and heat transfer coefficients in the fluid.
 2. Assign a mesh size to each element, with a nominal size being 0.10 mm.
 3. Most commercially available CFD software will then automatically create the finite-element model.
 4. Specify material types for solid components (such as steel, aluminum, and plastic) and fluids (such as air, water, and oil). The software typically includes the necessary properties of such materials.

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

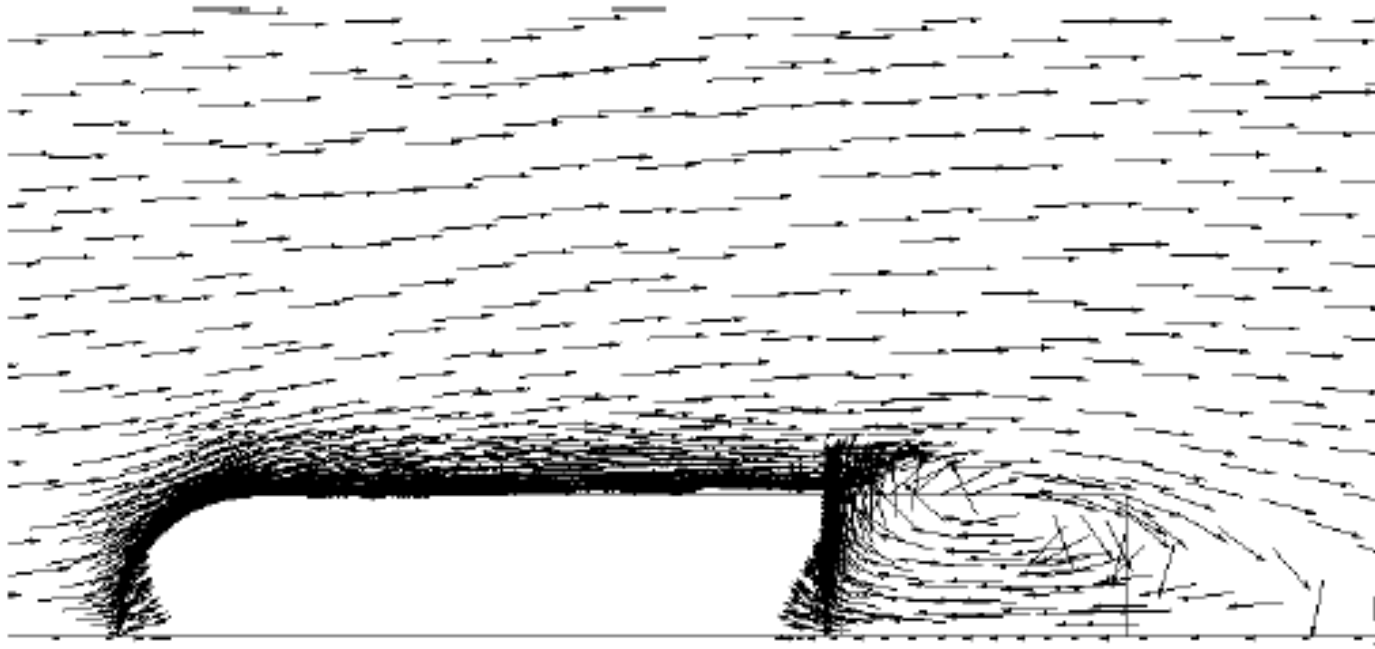
9.6 Computational Fluid Dynamics

5. Initiate the computational process. Because there is a huge number of calculations to be made, this process may take a significant amount of time, depending on the complexity of the model.
6. When the analysis is completed, the user can select the type of display pertinent to the factors being investigated. It may be fluid trajectories, velocity profiles, isothermal temperature plots, pressure distributions, or others.

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.6 Computational Fluid Dynamics

- Figure 9.11 shows the flow of air around a bullet.



9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections

9.6 Computational Fluid Dynamics

- The use of CFD software provides a dramatic reduction in the time needed to develop a new product, the virtual prototyping of components, and a reduction of the number of test models required to prove out a design before committing it to production.