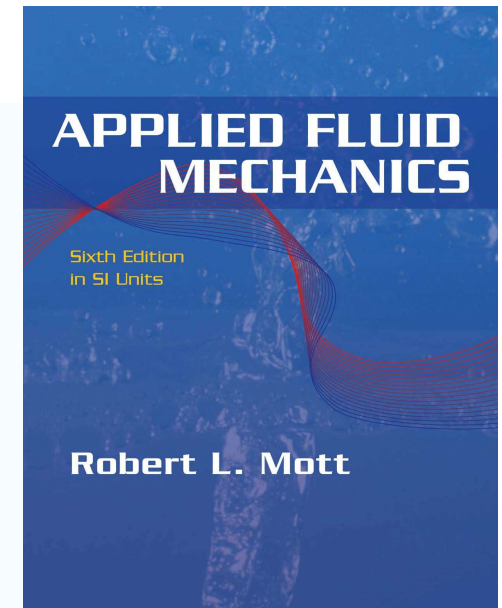


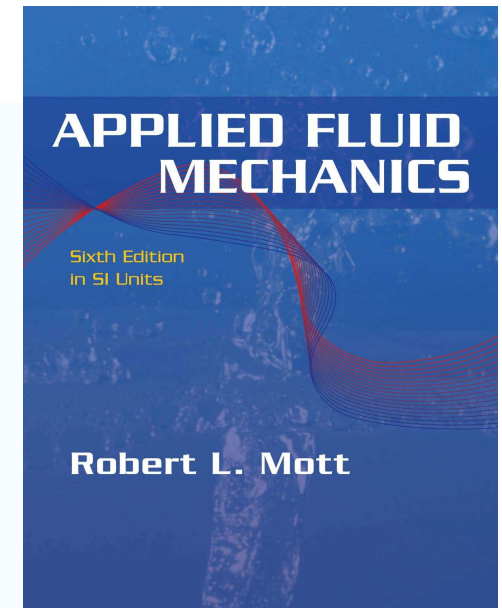
Applied Fluid Mechanics

1. The Nature of Fluid and the Study of Fluid Mechanics
2. Viscosity of Fluid
3. Pressure Measurement
4. Forces Due to Static Fluid
5. Buoyancy and Stability
6. Flow of Fluid and Bernoulli's Equation
7. General Energy Equation
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Applied Fluid Mechanics

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections
10. Minor Losses
11. Series Pipeline Systems
12. Parallel Pipeline Systems
13. Pump Selection and Application
14. Open-Channel Flow
15. Flow Measurement
16. Forces Due to Fluids in Motion

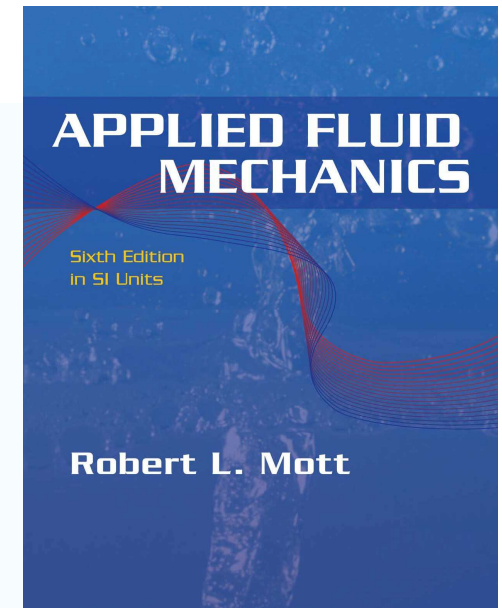


Applied Fluid Mechanics

17. Drag and Lift

18. Fans, Blowers, Compressors
and the Flow of Gases

19. Flow of Air in Ducts



12. Parallel Pipeline System

Chapter Objectives

- Discuss the difference between series pipeline systems and parallel pipeline systems.
- State the general relationships for flow rates and head losses for parallel pipeline systems.
- Compute the amount of flow that occurs in each branch of a two-branch parallel pipeline system and the head loss that occurs across the system when the total flow rate and the description of the system are known.
- Determine the amount of flow that occurs in each branch of a two-branch parallel pipeline system and the total flow if the pressure drop across the system is known.
- Use the Hardy Cross technique to compute the flow rates in all branches of a network having three or more branches.

12. Parallel Pipeline System

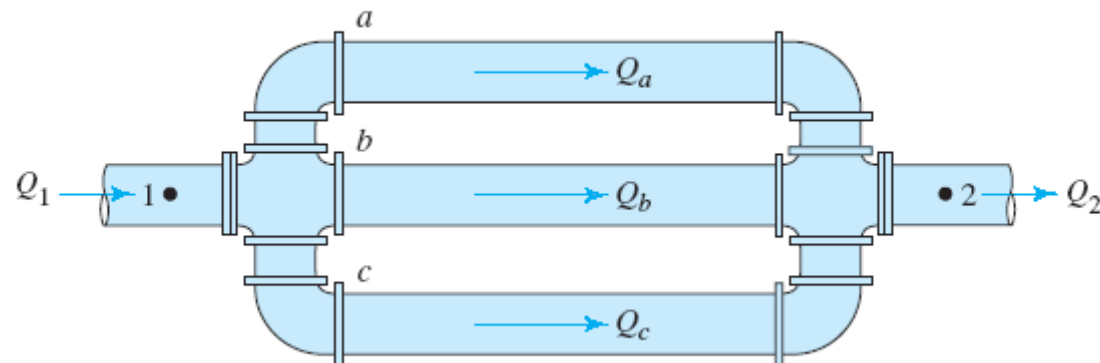
Chapter Outline

1. Introductory Concepts
2. Systems with Two Branches
3. Systems with Three or More Branches – Network

12. Parallel Pipeline System

12.1 Introductory Concepts

- Parallel pipeline systems are those having more than one path for the fluid to take as it flows from a source to a destination point.
- Fig 12.1 shows the example of a parallel pipeline system with three branches.



12. Parallel Pipeline System

12.1 Introductory Concepts

- When we apply the principle of steady flow to a parallel system, we reach the following conclusion:

$$Q_1 = Q_2 = Q_a + Q_b + Q_c \quad (12-1)$$

- To help analyze the pressures, use the energy equation between points 1 and 2:

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

12. Parallel Pipeline System

12.1 Introductory Concepts

- Solving for the pressure drop:

$$p_1 - p_2 = \gamma[(z_2 - z_1) + (v_2^2 - v_1^2)/2g + h_L]$$

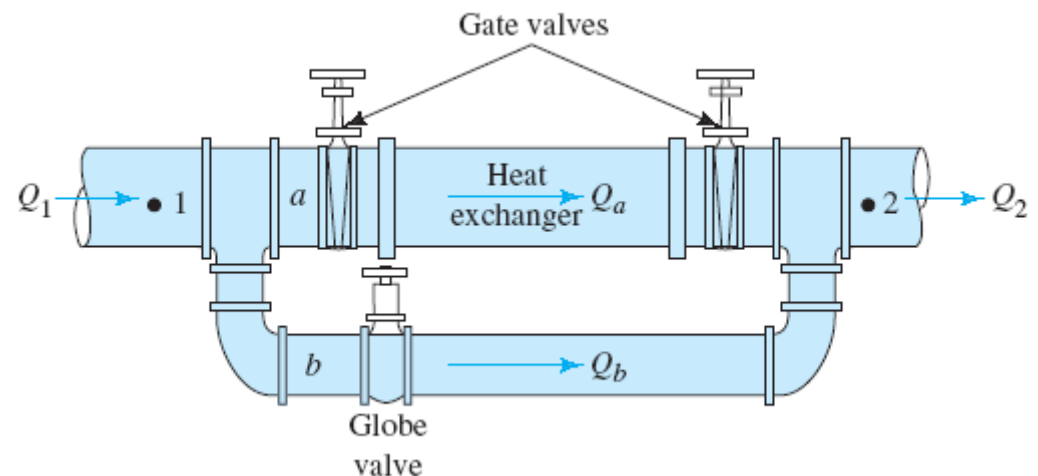
- All elements converging in the junction at the right side of the system have the same total energy per unit weight.
- That is, they all have the same total head. Therefore, each unit weight of fluid must have the same amount of energy. This can be stated mathematically as

$$h_{L\ 1-2} = h_a = h_b = h_c \quad (12-2)$$

12. Parallel Pipeline System

12.2 Systems with Two Branches

- A common parallel piping system includes two branches arranged as shown in Fig. 12.2.
- The analysis of this type of system is relatively simple and straightforward, although some iteration is typically required.
- Because velocities are unknown, friction factors are also unknown.



12. Parallel Pipeline System

12.2 Systems with Two Branches

- We will use the system shown in Fig. 12.2 to illustrate the analysis of flow in two branches.
- The basic relationships that apply here are similar to Eqs. (12–1) and (12–2) except there are only two branches instead of three.
- These relationships are

$$Q_1 = Q_2 = Q_a + Q_b \quad (12-3)$$

$$h_{L_{1-2}} = h_a = h_b \quad (12-4)$$

12. Parallel Pipeline System

12.2 Systems with Two Branches

- Below are the solution method for systems with two branches when the total flow rate and the description of the branches are known:
 1. Equate the total flow rate to the sum of the flow rates in the two branches, as stated in Eq. (12–3). Then express the branch flows as the product of the flow area and the average velocity; that is,

$$Q_a = A_a \bar{v}_a \quad \text{and} \quad Q_b = A_b \bar{v}_b$$

12. Parallel Pipeline System

12.2 Systems with Two Branches

2. Express the head loss in each branch in terms of the velocity of flow in that branch and the friction factor. Include all significant losses due to friction and minor losses.
3. Compute the relative roughness for each branch, estimate the value of the friction factor for each branch, and complete the calculation of head loss in each branch in terms of the unknown velocities.
4. Equate the expression for the head losses in the two branches to each other as stated in Eq. (12–4).
5. Solve for one velocity in terms of the other from the equation in Step 4.

12. Parallel Pipeline System

12.2 Systems with Two Branches

6. Substitute the result from Step 5 into the flow rate equation developed in Step 1 and solve for one of the unknown velocities.
7. Solve for the second unknown velocity from the relationship developed in Step 5.
8. If there is doubt about the accuracy of the value of the friction factor used in Step 2, compute the Reynolds number for each branch and reevaluate the friction factor from the Moody diagram or compute the values for the friction factors from Eq. (8–7) in Chapter 8.

12. Parallel Pipeline System

12.2 Systems with Two Branches

9. If the values for the friction factor have changed significantly, repeat Steps 3–8, using the new values for friction factor.
10. When satisfactory precision has been achieved, use the now-known velocity in each branch to compute the volume flow rate for that branch. Check the sum of the volume flow rates to ensure that it is equal to the total flow in the system.
11. Use the velocity in either branch to compute the head loss across that branch, employing the appropriate relationship from Step 3. This head loss is also equal to the head loss across the entire branched system.

12. Parallel Pipeline System

Example 12.1

In Fig. 12.2, 0.006 m³/s of water at 15°C is flowing in a 2-in Schedule 40 steel pipe at section 1. The heat exchanger in branch *a* has a loss coefficient of $K = 7.5$ based on the velocity head in the pipe. All three valves are wide open. Branch *b* is a bypass line composed of 1½ 4-in Schedule 40 steel pipe. The elbows are standard. The length of pipe between points 1 and 2 in branch *b* is 6 m. Because of the size of the heat exchanger, the length of pipe in branch *a* is very short and friction losses can be neglected. For this arrangement, determine (a) the volume flow rate of water in each branch and (b) the pressure drop between points 1 and 2.

12. Parallel Pipeline System

Example 12.1

If we apply Step 1 of the solution method, Eq. (12–3) relates the two volume flow rates. How many quantities are unknown in this equation?

$$Q_1 = A_a v_a + A_b v_b \quad (12-5)$$

From the given data, $A_a = 2.168 \times 10^{-3} \text{ m}^2$, $A_b = 9.653 \times 10^{-4} \text{ m}^2$, and $Q_1 = 0.006 \text{ m}^3/\text{s}$.

Generate another equation that also relates v_a and v_b using Step 2.

12. Parallel Pipeline System

Example 12.1

Equation (12–4) states that the head losses in the two branches are equal. Because the head losses h_a and h_b are dependent on the velocities v_a and v_b respectively, this equation can be used in conjunction with Eq. (12–5) to solve for the velocities. Now, express the head losses in terms of the velocities for each branch.

$$h_a = 2K_1(v_a^2/2g) + K_2(v_a^2/2g)$$

where

$$K_1 = f_{aT}(L_e/D) = \text{Resistance coefficient for each gate valve}$$

$$K_2 = \text{Resistance coefficient for the heat exchanger} = 7.5 \text{ (given in problem statement)}$$

12. Parallel Pipeline System

Example 12.1

The following data are known:

$$f_{aT} = 0.019 \text{ for a 2-in Schedule 40 pipe (Table 10.5)}$$

$$L_e/D = 8 \text{ for a fully open gate valve (Table 10.4)}$$

Then

$$K_1 = (0.019)(8) = 0.152$$

$$h_a = (2)(0.152)(v_a^2/2g) + 7.5(v_a^2/2g) = 7.80(v_a^2/2g) \quad (12-6)$$

For branch b,

$$h_b = 2K_3(v_b^2/2g) + K_4(v_b^2/2g) + K_5(v_b^2/2g)$$

$$K_3 = f_{bT}(L_e/D) = \text{Resistance coefficient for each elbow}$$

$$K_4 = f_{bT}(L_e/D) = \text{Resistance coefficient for the globe valve}$$

$$K_5 = f_b(L_b/D) = \text{Friction loss in the pipe of branch } b$$

12. Parallel Pipeline System

Example 12.1

The known data are

$$f_{bT} = 0.022 \text{ for a } 1\frac{1}{4}\text{-in Schedule 40 pipe (Table 10.5)}$$

$$L_e/D = 30 \text{ for each elbow (Table 10.4)}$$

$$L_e/D = 340 \text{ for a fully open globe valve (Table 10.4)}$$

Then

$$K_3 = (0.022)(30) = 0.66$$

$$K_4 = (0.022)(340) = 7.48$$

$$K_5 = f_b(6/0.0351) = 170.9f_b$$

12. Parallel Pipeline System

Example 12.1

Then,

$$h_b = (2)(0.66)(v_b^2/2g) + (7.48)(v_b^2/2g) + f_b(170.9)(v_b^2/2g)$$

$$h_b = (8.80 + 170.9f_b)(v_b^2/2g)$$

The relative roughness for branch b will aid in the estimation of the first trial value for f_b

$$D/\epsilon = (0.0351/4.6 \times 10^{-5}) = 763$$

From the Moody diagram in Fig. 8.6, a logical estimate for the friction factor is $f_b = 0.023$. Substituting this into the equation for h_b gives

$$h_b = [8.80 + 170.9(0.023)](v_b^2/2g) = 12.73(v_b^2/2g) \quad (12-7)$$

12. Parallel Pipeline System

Example 12.1

You should have $v_a = 1.28v_b$, obtained as follows:\

$$\begin{array}{l} h_a = h_b \\ 7.80(v_a^2/2g) = 12.73(v_b^2/2g) \end{array} \quad \Bigg|$$

We have

$$Q_1 = A_a v_a + A_b v_b \quad (12-5)$$

$$v_a = 1.28v_b \quad (12-8)$$

$$Q_1 = A_a(1.28v_b) + A_b v_b = v_b(1.28A_a + A_b)$$

12. Parallel Pipeline System

Example 12.1

Solving for v_b we get

$$v_b = \frac{Q_1}{1.28A_a + A_b} = \frac{0.006 \text{ m}^3/\text{s}}{[(1.28)(2.168 \times 10^{-3}) + 9.653 \times 10^{-4}] \text{ m}^2}$$

$$v_b = 1.6 \text{ m/s}$$

$$v_a = (1.28)(1.6 \text{ m/s}) = 2.05 \text{ m/s}$$

Because we made these calculations using an assumed value for f_b we should check the accuracy of the assumption. We can evaluate the Reynolds number for branch b :

$$N_{Rb} = v_b D_b / \nu$$

$$N_{Rb} = (1.6)(0.0351)/(1.124 \times 10^{-6}) = 5 \times 10^4$$

12. Parallel Pipeline System

Example 12.1

Using this value and the relative roughness of 763 from before in the Moody diagram yields a new value, $f_b = 0.025$. Because this is significantly different from the assumed value of 0.023, we can repeat the calculations for Steps 3–8. The results are summarized as follows:

$$h_b = [8.80 + 170.9(0.025)](v_b^2/2g) = 13.07(v_b^2/2g)$$

$$h_a = 7.80(v_a^2/2g) \quad (\text{same as for first trial})$$

Equating the head losses in the two branches gives

$$\begin{array}{l} h_a = h_b \\ 7.80(v_a^2/2g) = 13.07(v_b^2/2g) \end{array} \quad \left| \right.$$

12. Parallel Pipeline System

Example 12.1

Solving for the velocities gives

$$v_a = 1.29v_b$$

Substituting this into the equation for v_b used before gives

$$v_b = \frac{0.006 \text{ m}^3/\text{s}}{[(1.29)(2.168 \times 10^{-3}) + 9.653 \times 10^{-4}] \text{ m}^2} = 1.59 \text{ m/s}$$

$$v_a = 1.29v_b = 1.29(1.59) = 2.05 \text{ m/s}$$

12. Parallel Pipeline System

Example 12.1

Recomputing the Reynolds number for branch b gives

$$N_{Rb} = v_b D_b / \nu$$

$$N_{Rb} = (1.59)(0.0351)/(1.124 \times 10^{-6}) = 5 \times 10^4$$

There is no significant change in the value of f_b .

Therefore the values of the two velocities computed above are correct. We can now complete Steps 10 and 11 of the procedure to find the volume flow rate in each branch and the head loss and the pressure drop across the entire system.

12. Parallel Pipeline System

Example 12.1

You should have

$$Q_a = A_a v_a = (2.168 \times 10^{-3} \text{ m}^2)(2.05 \text{ m/s}) = 0.004 \text{ m}^3/\text{s}$$

$$Q_b = A_b v_b = (9.653 \times 10^{-4} \text{ m}^2)(1.59 \text{ m/s}) = 0.002 \text{ m}^3/\text{s}$$

We can write the energy equation using points 1 and 2 as reference points. Because the velocities and elevations are the same at these points, the energy equation is simply

$$\frac{p_1}{\gamma} - h_L = \frac{p_2}{\gamma}$$

$$p_1 - p_2 = \gamma h_L \quad (12-9)$$

12. Parallel Pipeline System

Example 12.1

Because $h_{L(1-2)} = h_a = h_b$, we can use either Eq. (12–6) or (12–7). Using Eq. (12–6), we get

$$h_a = 7.80(v_a^2/2g) = (7.80)(2.05)^2/19.62 \text{ m} = 1.67 \text{ m}$$

Note that this neglects the minor losses in the two tees. Then we have

$$\begin{aligned} p_1 = p_2 = \gamma h_L &= 9.81 \text{ kN/m}^3 \times 1.67 \text{ m} = 16.38 \text{ kN/m}^2 \\ &= 16.38 \text{ kPa} \end{aligned}$$

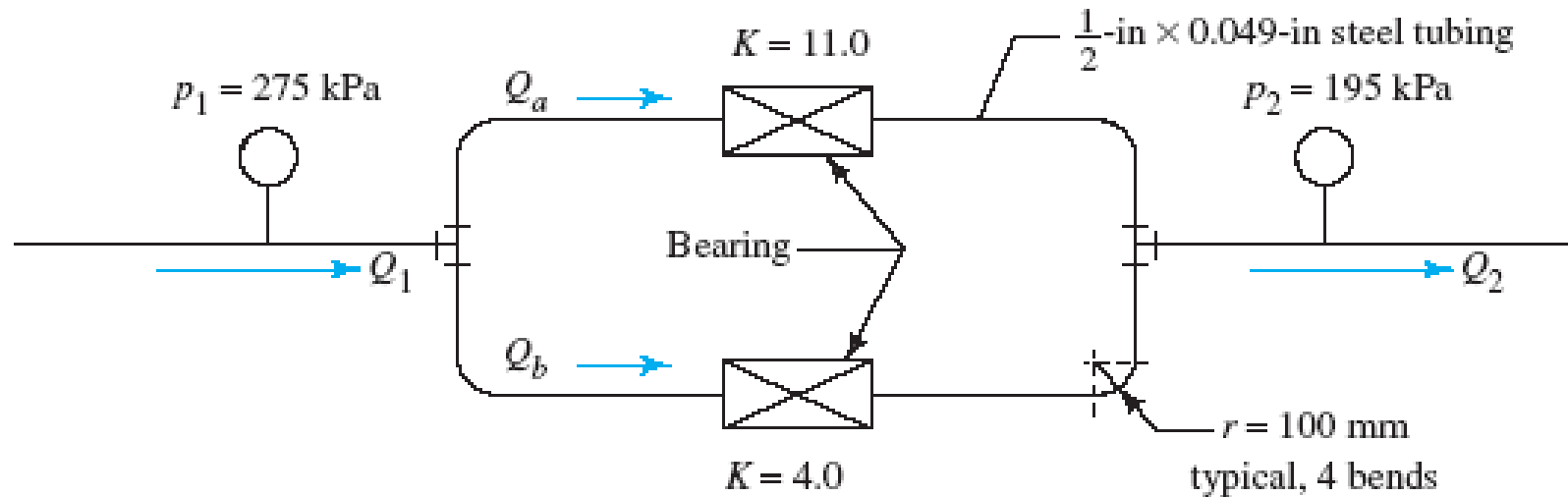
12. Parallel Pipeline System

Example 12.2

The arrangement shown in Fig. 12.3 is used to supply lubricating oil to the bearings of a large machine. The bearings act as restrictions to the flow. The resistance coefficients are 11.0 and 4.0 for the two bearings. The lines in each branch are ~~1 in~~ drawn steel tubing with a wall thickness of 0.049 in. Each of the four bends in the tubing has a mean radius of 100 mm. Include the effect of these bends, but exclude the friction losses because the lines are short. Determine (a) the flow rate of oil in each bearing and (b) the total flow rate in L/min. The oil has a specific gravity of 0.881 and a kinematic viscosity of $2.5 \times 10^{-6} \text{ m}^2/\text{s}$. The system lies in one plane, so all elevations are equal.

12. Parallel Pipeline System

Example 12.2



12. Parallel Pipeline System

Example 12.2

You should have

$$h_L = h_a = h_b \quad (12-10)$$

They are all equal. Determine the magnitude of these head losses by using Step 1.

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

$$\frac{p_1}{\gamma} - h_L = \frac{p_2}{\gamma}$$

$$h_L = (p_1 - p_2)/\gamma \quad (12-11)$$

12. Parallel Pipeline System

Example 12.2

Using the given data, we get

$$h_L = \frac{(275 - 195) \text{ kN}}{\text{m}^2} \times \frac{\text{m}^3}{(0.881)(9.81) \text{ kN}}$$
$$h_L = 9.26 \text{ m}$$

Considering the losses in the bends and in the bearings, you should have

$$h_a = 2K_1(v_a^2/2g) + K_2(v_a^2/2g) \quad (12-12)$$

$$h_b = 2K_1(v_b^2/2g) + K_3(v_b^2/2g) \quad (12-13)$$

12. Parallel Pipeline System

Example 12.2

Where

$K_1 = f_T(L_e/D)$ = Resistance coefficient for each bend

K_2 = Resistance coefficient for the bearing in branch a = 11.0 (given in problem statement)

K_3 = Resistance coefficient for the bearing in branch b = 4.0 (given in problem statement)

f_T = Friction factor in the zone of complete turbulence in the steel tube

(L_e/D) = Equivalent length ratio for each bend (Chapter 10, Fig. 10.27)

We need the relative radius for the bends,

$$r/D = (100 \text{ mm})/(10.21 \text{ mm}) = 9.79$$

From Fig. 10.27 we find

$$L_e/D = 29.5$$

12. Parallel Pipeline System

Example 12.2

The friction factor in the zone of complete turbulence can be determined by using the relative roughness and the Moody diagram, reading at the right end of the relative roughness curve where it approaches a horizontal line:

$$D/\epsilon = 0.01021 \text{ m}/1.5 \times 10^{-6} \text{ m} = 6807$$

12. Parallel Pipeline System

Example 12.2

We can read $f_T = 0.013$ from the Moody diagram. Now we can complete Step 3 by evaluating all of the resistance factors and expressing the energy loss in each branch in terms of the velocity head in the branch:

$$K_1 = f_T(L_e/D) = (0.013)(29.5) = 0.384$$

$$K_2 = 11.0$$

$$K_3 = 4.0$$

$$h_a = (2)(0.384)(v_a^2/2g) + 11.0(v_a^2/2g)$$

$$h_a = 11.77v_a^2/2g \quad (12-14)$$

$$h_b = (2)(0.384)(v_b^2/2g) + 4.0(v_b^2/2g)$$

$$h_b = 4.77v_b^2/2g \quad (12-15)$$

12. Parallel Pipeline System

Example 12.2

Since

$$h_a = 11.77v_a^2/2g$$

$$v_a = \sqrt{\frac{2gh_a}{11.77}} = \sqrt{\frac{(2)(9.81)(9.26)}{11.77}} \text{ m/s} = 3.93 \text{ m/s}$$

$$h_b = 4.77v_b^2/2g$$

$$v_b = \sqrt{\frac{2gh_b}{4.77}} = \sqrt{\frac{(2)(9.81)(9.26)}{4.77}} \text{ m/s} = 6.17 \text{ m/s}$$

For flow rate,

$$Q_a = A_a v_a = 8.189 \times 10^{-5} \text{ m}^2 \times 3.93 \text{ m/s} \times \frac{60\,000 \text{ L/min}}{\text{m}^3/\text{s}}$$

$$Q_a = 19.3 \text{ L/min}$$

12. Parallel Pipeline System

Example 12.2

Similarly,

$$Q_b = A_b v_b = 30.3 \text{ L/min}$$

Then the total flow rate is

$$Q_1 = Q_a + Q_b = (19.3 + 30.3) \text{ L/min} = 49.6 \text{ L/min}$$

12. Parallel Pipeline System

12.3 Systems with Three or More Branches - Network

- When three or more branches occur in a pipe flow system, it is called a *network*.
- The equations available to describe the system are

$$Q_1 = Q_2 = Q_a + Q_b + Q_c \quad (12-16)$$

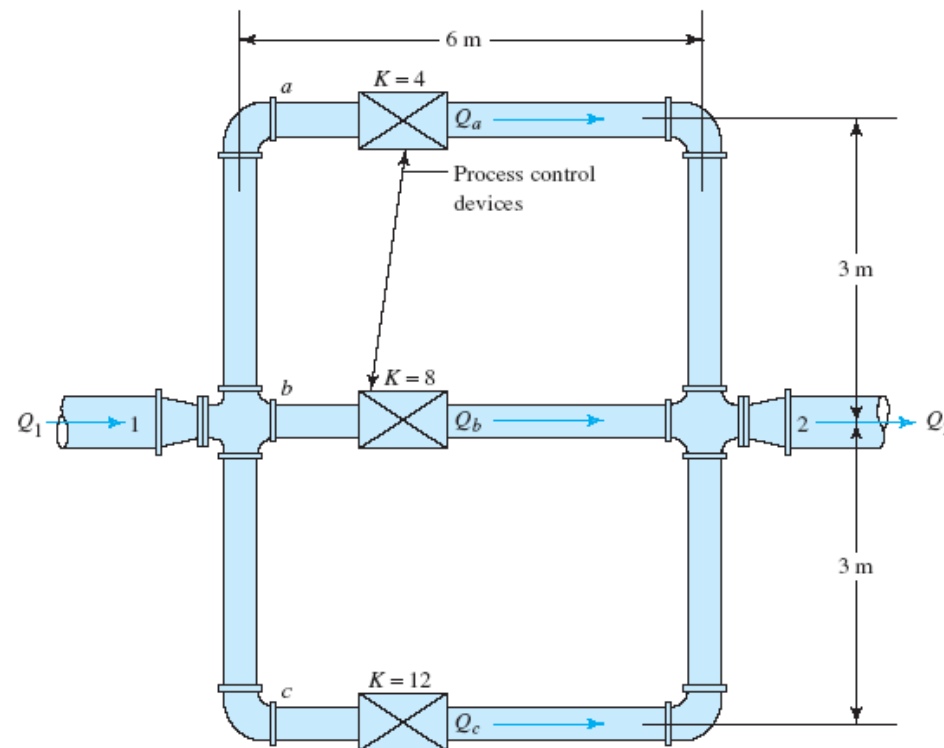
$$h_{L_{1-2}} = h_a = h_b = h_c \quad (12-17)$$

- A third independent equation is required to solve explicitly for the three velocities, and none is available.

12. Parallel Pipeline System

12.3 Systems with Three or More Branches - Network

- A rational approach to complete the analysis of a system such as that shown in Fig. 12.4 employing an iteration procedure.



Note: Inlet and outlet pipes: 2-in Sch. 40
Branch pipes a , b , and c : 1-in Sch. 40
Elbows are standard

12. Parallel Pipeline System

12.3 Systems with Three or More Branches - Network

- The Cross technique requires that the head loss terms for each pipe in the system be expressed in the form

$$h = kQ^n \quad (12-18)$$

- Recall that both friction losses and minor losses are proportional to the velocity head. Then, using the continuity equation, we can express the velocity in terms of the volume flow rate. That is,

$$v = Q/A$$

$$v^2 = Q^2/A^2$$

12. Parallel Pipeline System

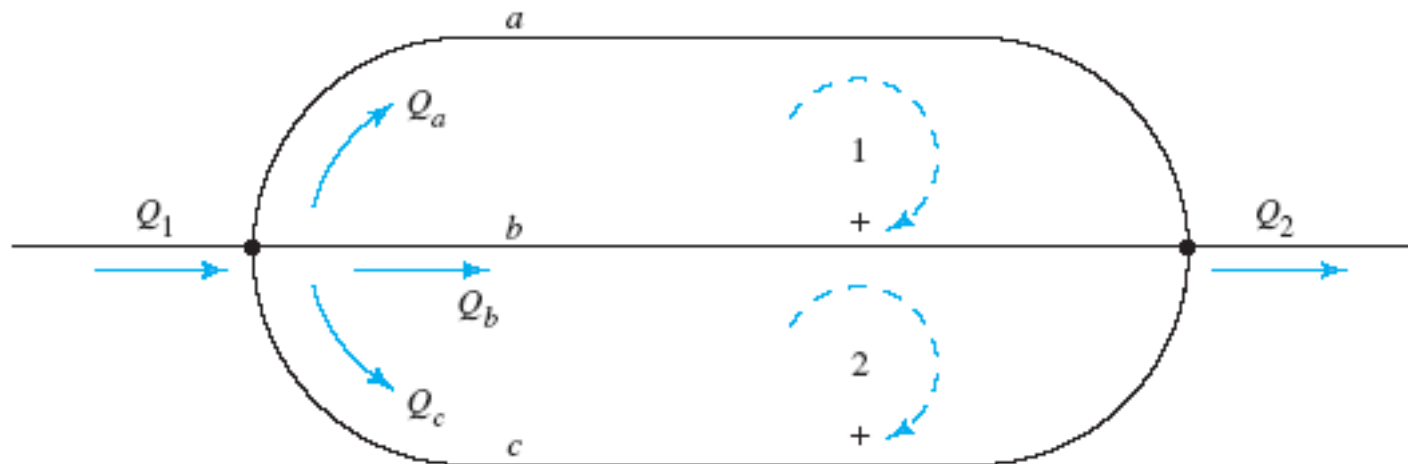
12.3 Systems with Three or More Branches - Network

- The Cross iteration technique requires that initial estimates for the volume flow rate in each branch of the system be made.
- Two factors that help in making these estimates are as follows:
 1. At each junction in the network, the sum of the flow into the junction must equal the flow out.
 2. The fluid tends to follow the path of least resistance through the network. Therefore, a pipe having a lower value of k will carry a higher flow rate than those having higher values.

12. Parallel Pipeline System

12.3 Systems with Three or More Branches - Network

- Figure 12.5 shows a schematic representation of a three-pipe system such as that shown in Fig. 12.4.



12. Parallel Pipeline System

12.3 Systems with Three or More Branches - Network

- ***If the flow in a given pipe of a circuit is clockwise, Q and h are positive. If the flow is counterclockwise, Q and h are negative.***
- The Cross technique for analyzing the flow in pipe networks is presented in step-by-step form below.
 1. Express the energy loss in each pipe in the form

$$h = kQ^2$$

2. Assume a value for the flow rate in each pipe such that the flow into each junction equals the flow out of the junction.
3. Divide the network into a series of closed-loop circuits.

12. Parallel Pipeline System

12.3 Systems with Three or More Branches - Network

4. For each pipe, calculate the head loss using the assumed value of Q .
5. Proceeding around each circuit, algebraically sum all values for h using the following sign convention:
 - i. If the flow is clockwise, h and Q are positive.
 - ii. If the flow is counterclockwise, h and Q are negative.
 - iii. The resulting summation is referred to as h .

12. Parallel Pipeline System

12.3 Systems with Three or More Branches - Network

6. For each pipe, calculate $2kQ$.
7. Sum all values of $2kQ$ for each circuit, assuming all are positive. This summation is referred to as $(2KQ)$.
8. For each circuit, calculate the value of Q from

$$\Delta Q = \frac{\Sigma h}{\Sigma(2kQ)} \quad (12-19)$$

9. For each pipe, calculate a new estimate for Q from

$$Q' = Q - \Delta Q$$

12. Parallel Pipeline System

12.3 Systems with Three or More Branches - Network

10. Repeat Steps 4–8 until Q from Step 8 becomes negligibly small. The Q' value is used for the next cycle of iteration.

12. Parallel Pipeline System

Example 12.3

For the system shown in Fig. 12.4, determine the volume flow rate of water at 15°C through each branch if 600 L/min (0.01 m³/s) is flowing into and out of the system through the 2-in pipes.

The head loss in each pipe should now be expressed in the form of $h = KQ^2$ as Step 1 of the procedure.

Consider branch *a* first and write an expression for the head loss, h_a .

12. Parallel Pipeline System

Example 12.3

The total head loss for the branch is due to the two elbows (each with $L_e/D = 30$), the restriction (with $K = 0.4$ based on the velocity head in the pipe), and friction in the pipe. Then,

$$h_a = \underbrace{2(f_a T)(30)}_{\text{(elbows)}} \left(\frac{v_a^2}{2g}\right) + \underbrace{4.0}_{\text{(restriction)}} \left(\frac{v_a^2}{2g}\right) + \underbrace{f_a(L_a/D_a)}_{\text{(friction)}} \left(\frac{v_a^2}{2g}\right)$$

The friction factor for flow in the pipe is dependent on the Reynolds number and therefore on the volume flow rate. Because that is the objective of the network analysis, we cannot determine that value explicitly at this time.

12. Parallel Pipeline System

Example 12.3

Furthermore, the flow rate will, in general, be different in each segment of the flow system, resulting in different values for the friction factor. We will take that into account in the present analysis by computing the value of the friction factor after assuming the magnitude of the volume flow rate in each pipe, a step that is inherent in the Cross technique. We will use the Swamee–Jain method to compute the friction factor from Eq. (8–7). Then we will recompute the values of the friction factors for each trial as the value of the volume flow rate is refined.

12. Parallel Pipeline System

Example 12.3

We can introduce the volume flow rate Q into the equation by noting that, as shown before,

$$v_a^2 = Q_a^2/A_a^2$$

You should have something like this:

$$h_a = [60(f_a T) + 4.0 + (f_a)(12/0.0266)](v_a^2/2g)$$

$$h_a = [60(f_a T) + 4.0 + 451(f_a)](Q_a^2/2gA_a^2)$$

$$h_a = [60(0.023) + 4.0 + 451(f_a)] \left[\frac{Q_a^2}{2(9.81)(5.574 \times 10^{-4})^2} \right]$$

$$h_a = [5.38 + 451(f_a)](1.64 \times 10^5)Q_a^2 \quad (12-20)$$

12. Parallel Pipeline System

Example 12.3

Because all three branches have the same size and type of pipe, these calculations apply to each branch. If different pipes are used throughout the network, these calculations must be redone for each pipe. For the 1-in steel pipe,

$$D/\epsilon = (0.0266 \text{ m})/(4.6 \times 10^{-5} \text{ m}) = 578$$

We should modify the Reynolds number formula as

$$\begin{aligned} N_{Ra} &= \frac{v_a D_a}{\nu} = \frac{Q_a D_a}{A_a \nu} = \frac{Q_a (0.0266)}{(5.574 \times 10^{-4})(1.15 \times 10^{-6})} \\ N_{Ra} &= (4.15 \times 10^7) Q_a \end{aligned} \quad (12-21)$$

12. Parallel Pipeline System

Example 12.3

Compare your results with these. Note that the pipe size in branches b and c is the same as that in branch a . For branch b :

$$h_b = 8.0(v_b^2/2g) + f_b(L_b/D_b)(v_b^2/2g)$$

(restriction) (friction)

$$h_b = [8.0 + f_b(6/0.0266)](Q_b^2/2gA^2)$$

$$h_b = [8.0 + 225.6(f_b)](1.64 \times 10^5)Q_b^2 \quad (12-22)$$

12. Parallel Pipeline System

Example 12.3

For branch c:

$$h_c = \underbrace{2(f_{cT})(30)(v_c^2/2g)}_{\text{(elbows)}} + \underbrace{12.0(v_c^2/2g)}_{\text{(restriction)}} + \underbrace{f_c(L_c/D_c)(v_c^2/2g)}_{\text{(friction)}}$$

$$h_c = [60(f_{cT}) + 12.0 + f_c(12/0.0266)](v_c^2/2g)$$

$$h_c = [60(0.023) + 12.0 + 451f_c](Q_c^2/2gA^2)$$

$$h_c = [13.38 + 451(f_c)](1.64 \times 10^5)Q_c^2 \quad (12-23)$$

Equations (12–20)–(12–23) will be used in the calculations of head losses as the Cross iteration process continues. When the values for the friction factors are known or assumed, the head loss equations can be reduced to the form of Eq. (12–18). Often it is satisfactory to assume reasonable values for the various friction factors because minor changes have little effect on the flow distribution and the total head loss.

12. Parallel Pipeline System

Example 12.3

We know that

$$Q_a + Q_b + Q_c = Q_1 = 0.01 \text{ m}^3/\text{s}$$

Let's use the initial assumptions

$$Q_a = 0.0033 \text{ m}^3/\text{s} \quad Q_b = 0.0036 \text{ m}^3/\text{s} \quad Q_c = 0.0031 \text{ m}^3/\text{s}$$

You should have, using Eq. (12–21) and $D/\nu = 578$,

$$N_{Ra} = (4.15 \times 10^7)Q_a = (4.15 \times 10^7)(0.0033 \text{ m}^3/\text{s}) = 1.37 \times 10^5$$

$$N_{Rb} = (4.15 \times 10^7)Q_b = (4.15 \times 10^7)(0.0036 \text{ m}^3/\text{s}) = 1.49 \times 10^5$$

$$N_{Rc} = (4.15 \times 10^7)Q_c = (4.15 \times 10^7)(0.0031 \text{ m}^3/\text{s}) = 1.29 \times 10^5$$

12. Parallel Pipeline System

Example 12.3

We now use Eq. (9–5) to compute the friction factor for each pipe:

$$f_a = \frac{0.25}{\left[\log_{10} \left(\frac{1}{3.7(D/\epsilon)} + \frac{5.74}{N_{Ra}^{0.9}} \right) \right]^2}$$
$$f_a = \frac{0.25}{\left[\log_{10} \left(\frac{1}{3.7(578)} + \frac{5.74}{(1.37 \times 10^5)^{0.9}} \right) \right]^2} = 0.0241$$

Insert the friction factors and the assumed values for Q into Eqs. (12–20), (12–22) and (12–23):

$$h_a = [5.38 + 451(f_a)](1.64 \times 10^5)Q_a^2 = k_a Q_a^2$$
$$h_a = [5.38 + 451(0.0241)](1.64 \times 10^5)Q_a^2 = 2.67 \times 10^6 Q_a^2$$

12. Parallel Pipeline System

Example 12.3

Then $k_a = 2.67 \times 10^6$. Completing the calculation gives

$$h_a = 2.67 \times 10^6(0.0033)^2 = 29.05$$

Similarly, for branch b :

$$h_b = [8.0 + 225.6(f_b)](1.64 \times 10^5)Q_b^2 = k_b Q_b^2$$

$$h_b = [8.0 + 225.6(0.0240)](1.64 \times 10^5)Q_b^2 = 2.20 \times 10^6 Q_b^2$$

$$h_b = 2.20 \times 10^6(0.0036)^2 = 28.53$$

For branch c :

$$h_c = [13.38 + 451(f_c)](1.64 \times 10^5)Q_c^2 = k_c Q_c^2$$

$$h_c = [13.38 + 451(0.0242)](1.64 \times 10^5)Q_c^2 = 3.99 \times 10^6 Q_c^2$$

$$h_c = 3.99 \times 10^6(0.0031)^2 = 38.31$$

12. Parallel Pipeline System

Example 12.3

For circuit 1,

$$\sum h_1 = h_a - h_b = 29.05 - 28.53 = 0.52.$$

For circuit 2,

$$\sum h_2 = h_b - h_c = 28.53 - 38.31 = -9.78$$

Here are the correct values for the three pipes:

$$2k_a Q_a = (2)(2.67 \times 10^6)(0.0033) = 17\,609$$

$$2k_b Q_b = (2)(2.20 \times 10^6)(0.0036) = 15\,850$$

$$2k_c Q_c = (2)(3.99 \times 10^6)(0.0031) = 24\,717$$

12. Parallel Pipeline System

Example 12.3

For circuit 1,

$$\Sigma(2kQ)_1 = 17\,609 + 15\,850 = 33\,459$$

$$\Delta Q_1 = \frac{\Sigma h_1}{\Sigma(2kQ)_1} = \frac{0.52}{33\,459} = 1.56 \times 10^{-5}$$

For circuit 2,

$$\Sigma(2kQ)_2 = 15\,850 + 24\,717 = 40\,567$$

$$\Delta Q_2 = \frac{\Sigma h_2}{\Sigma(2kQ)_2} = \frac{-9.78}{40\,567} = -2.41 \times 10^{-4}$$

12. Parallel Pipeline System

Example 12.3

The values for Q are estimates of the error in the originally assumed values for Q . We recommended that the process be repeated until the magnitude of Q is less than 1 percent of the assumed value of Q . Special circumstances may warrant using a different criterion for judging Q . The calculation is as follows:

$$\begin{aligned} Q'_a &= Q_a - \Delta Q_1 = 0.0033 - 1.56 \times 10^{-5} \\ &= 0.00328 \text{ m}^3/\text{s} \end{aligned}$$

$$\begin{aligned} Q'_c &= Q_c - \Delta Q_2 = -0.0031 - (-2.41 \times 10^{-4}) \\ &= -0.00286 \text{ m}^3/\text{s} \end{aligned}$$

12. Parallel Pipeline System

Example 12.3

Notice that Q_c is negative because it flows in a counterclockwise direction in circuit 2. We can interpret the calculation for Q'_c as indicating that the magnitude of Q_c must be decreased in absolute value.

Both Q_1 and Q_2 must be applied to Q_b . For circuit 1,

$$Q'_b = Q_b - \Delta Q_1 = -0.0036 - 1.56 \times 10^{-5}$$

This would result in an increase in the absolute value of Q_b . For circuit 2,

$$Q'_b = Q_b - \Delta Q_2 = +0.0036 - (-2.41 \times 10^{-4})$$

12. Parallel Pipeline System

Example 12.3

Thus,

$$\begin{aligned}Q'_b &= 0.0036 + 1.56 \times 10^{-5} + 2.41 \times 10^{-4} \\ &= 0.00386 \text{ m}^3/\text{s}\end{aligned}$$

Remember that the sum of the absolute values of the flow rates in the three pipes must equal 0.01 m³/s, the total Q.

Table 12.1 were found using a spreadsheet on a computer.

12. Parallel Pipeline System

Example 12.3

TABLE 12.1

Trial	Circuit	Pipe	Q	N_k	f	k	$h = kQ^2$	$2kQ$	ΔQ	% Change
1	1	a	3.300×10^{-3}	1.37×10^5	0.0241	2.67×10^6	29.054	17609	1.568×10^{-5}	0.48
		b	-3.600×10^{-3}			2.20×10^6	-28.530	15850		-0.44
	Sum of h and $2kQ =$					0.524	33459			
	2	b	3.600×10^{-3}	1.49×10^5	0.0240	2.20×10^6	28.530	15850		-6.70
c		-3.100×10^{-3}	1.29×10^5	0.0242	3.99×10^6	-38.312	24717	7.78		
Sum of h and $2kQ =$					-9.782	40567	-2.411×10^{-4}			
2	1	a	3.284×10^{-3}	1.36×10^5	0.0241	2.67×10^6	28.784	17528	-1.135×10^{-4}	-3.46
		b	-3.857×10^{-3}			2.20×10^6	-32.700	16957		2.94
	Sum of h and $2kQ =$					-3.916	34485			
	2	b	3.857×10^{-3}	1.60×10^5	0.0239	2.20×10^6	32.700	16957		0.03
c		-2.859×10^{-3}	1.19×10^5	0.0243	4.00×10^6	-32.654	22844	-0.04		
Sum of h and $2kQ =$					0.046	39801	1.151×10^{-6}			
3	1	a	3.398×10^{-3}	1.41×10^5	0.0241	2.67×10^6	30.770	18112	-9.176×10^{-7}	-0.03
		b	-3.742×10^{-3}			2.20×10^6	-30.802	16462		0.02
	Sum of h and $2kQ =$					-0.032	34574			
	2	b	3.742×10^{-3}	1.55×10^5	0.0240	2.20×10^6	30.802	16462		-1.28
c		-2.860×10^{-3}	1.19×10^5	0.0243	4.00×10^6	-32.680	22853	1.67		
Sum of h and $2kQ =$					-1.878	39315	-4.776×10^{-5}			
4	1	a	3.399×10^{-3}	1.41×10^5	0.0241	2.67×10^6	30.787	18116	-2.252×10^{-5}	-0.66
		b	-3.789×10^{-3}			2.20×10^6	-31.570	16664		0.59
	Sum of h and $2kQ =$					-0.783	34780			
	2	b	3.789×10^{-3}	1.57×10^5	0.0240	2.20×10^6	31.570	16664		-0.03
c		-2.812×10^{-3}	1.17×10^5	0.0244	4.00×10^6	-31.612	22482	0.04		
Sum of h and $2kQ =$					-0.042	39146	-1.073×10^{-6}			