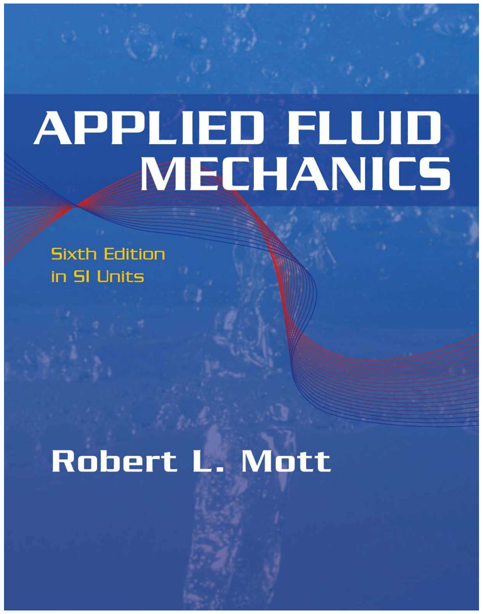
Applied Fluid Mechanics



1. The Nature of Fluid and the Study of Fluid Mechanics
2. Viscosity of Fluid
3. Pressure Measurement
4. Forces Due to Static Fluid
5. Buoyancy and Stability
6. Flow of Fluid and Bernoulli's Equation
7. General Energy Equation
8. Reynolds Number, Laminar Flow, Turbulent Flow and Energy Losses Due to Friction

Applied Fluid Mechanics

9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections
10. Minor Losses
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12. Parallel Pipeline Systems
13. Pump Selection and Application
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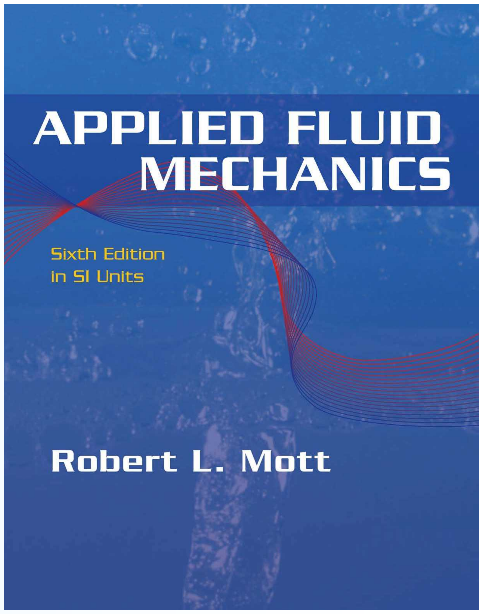
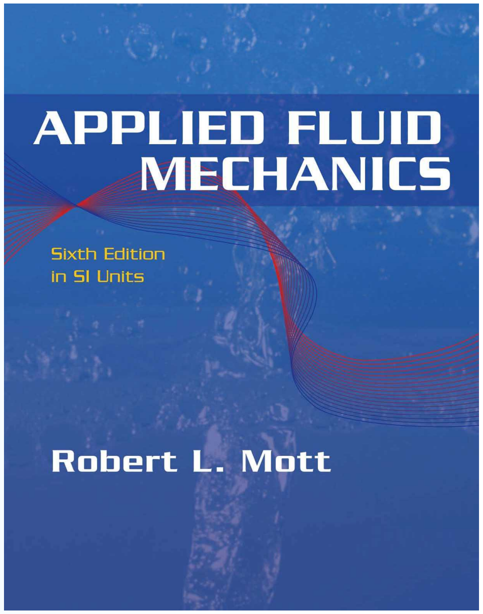




17. Drag and Lift
18. Fans, Blowers, Compressors and the Flow of Gases
19. Flow of Air in Ducts

14. Open Channel Flow

Chapter Objectives

- Compute the hydraulic radius for open channels.
- Describe *uniform flow* and *varied flow*.
- Use Manning’s equation to analyze uniform flow.
- Define the slope of an open channel and compute its value.
- Compute the normal discharge for an open channel.
- Compute the normal depth of flow for an open channel.
- Design an open channel to transmit a given discharge with uniform flow.

14. Open Channel Flow

Chapter Objectives

- Define the *Froude number*.
- Describe *critical flow*, *subcritical flow*, and *supercritical flow*.
- Define the specific energy of the flow in open channels.
- Define the terms *critical depth*, *alternate depth*, and *sequent depth*.
- Describe the term *hydraulic jump*.
- Describe *weirs* and *flumes* as they are used for measuring flow in open channels, and perform the necessary computations.

14. Open Channel Flow

Chapter Outline

1. Introductory Concepts
2. Classification of Open-Channel Flow
3. Hydraulic Radius and Reynolds Number in Open-Channel Flow
4. Kinds of Open-Channel Flow
5. Uniform Steady Flow in Open Channels
6. The Geometry of Typical Open Channels
7. The Most Efficient Shapes for Open Channels
8. Critical Flow and Specific Energy
9. Hydraulic Jump
10. Open-Channel Flow Measurement

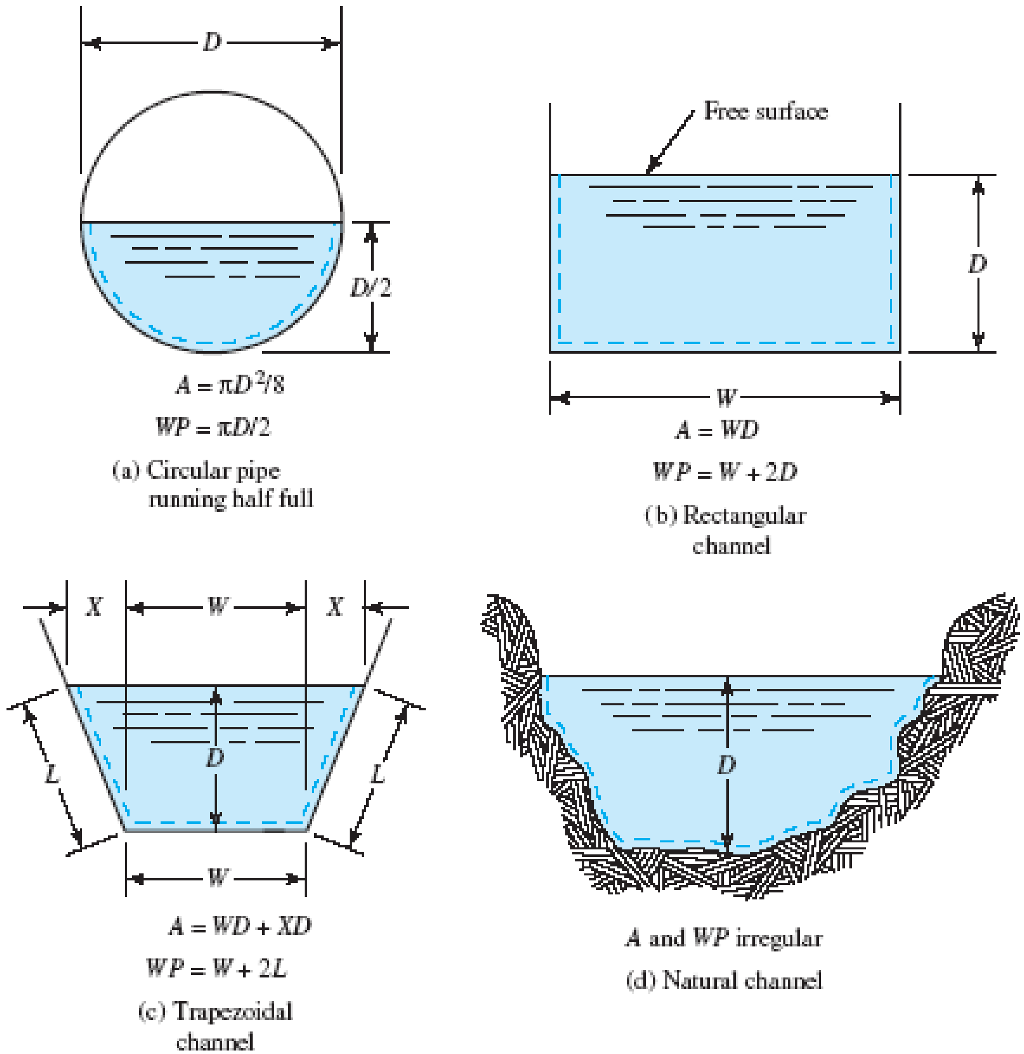
14. Open Channel Flow

14.1 Introductory Concepts

- Many examples of open channels occur in nature and in systems designed to supply water to communities or to carry storm drainage and sewage safely away.
- Fig 14.1 shows the examples of cross sections of open channels.

14. Open Channel Flow

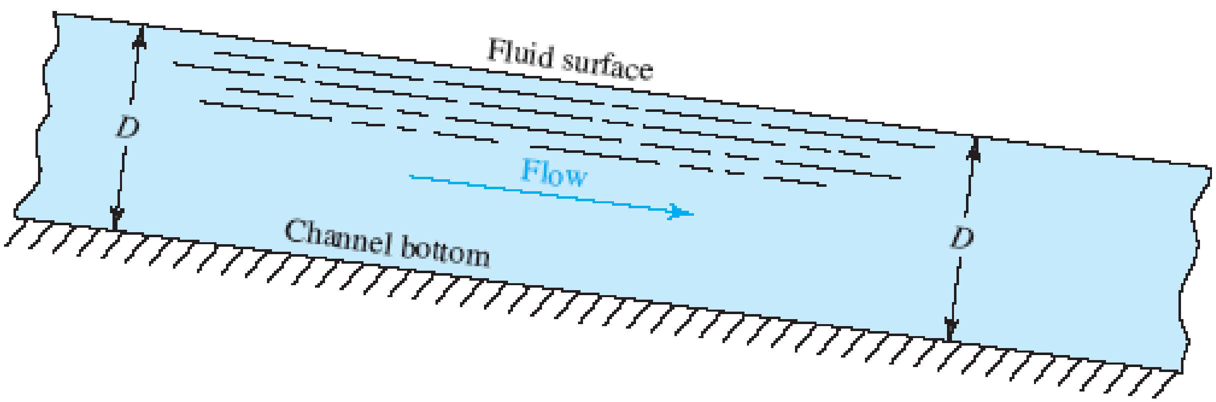
14.1 Introductory Concepts



14. Open Channel Flow

14.2 Classification of Open-Channel Flow

- *Uniform steady flow* occurs when the volume flow rate (typically called *discharge* in open-channel flow analysis) remains constant in the section of interest and the depth of the fluid in the channel does not vary.
- Figure 14.2 shows uniform flow in a side view.

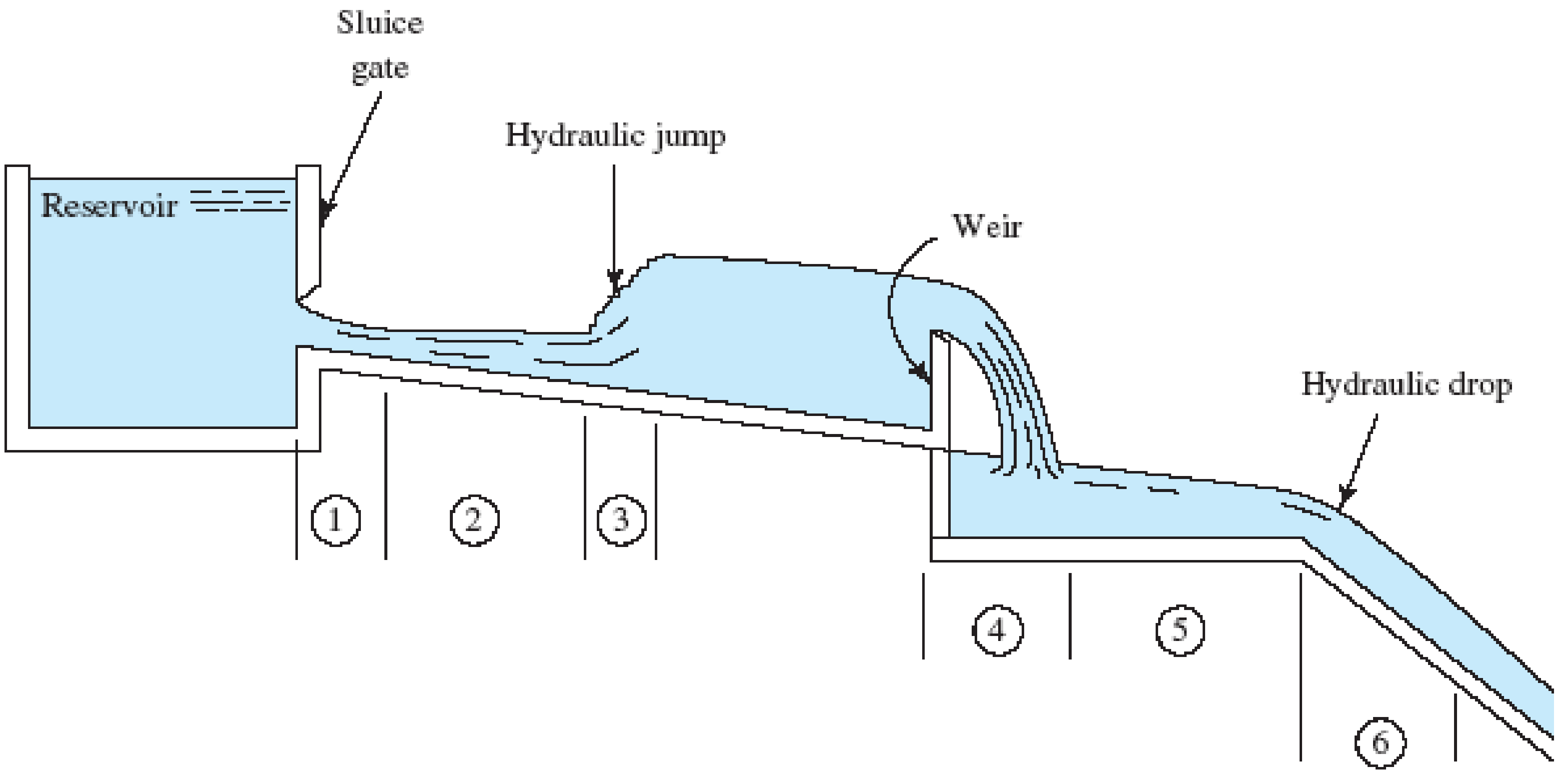


14.2 Classification of Open-Channel Flow

- *Varied steady flow* occurs when the discharge remains constant but the depth of the fluid varies along the section of interest.
- *Unsteady varied flow* occurs when the discharge varies with time, resulting in changes in the depth of the fluid along the section of interest whether the channel is prismatic or not.
- Varied flow can be further classified into *rapidly varying flow* or *gradually varying flow*.
- Figure 14.3 illustrates a series of conditions in which varied flow occurs.

14. Open Channel Flow

14.2 Classification of Open-Channel Flow



14.3 Hydraulic Radius and Reynolds Number in Open-Channel Flow

- The characteristic dimension of open channels is the *hydraulic radius*, defined as the ratio of the net cross-sectional area of a flow stream to the wetted perimeter of the section.
- That is,

$$R = \frac{A}{WP} = \frac{\text{Area}}{\text{Wetted perimeter}} \qquad (14\text{–}1)$$

- The unit for *R* is the meter in the SI unit system and feet in the English system.

Example 14.1

Determine the hydraulic radius of the trapezoidal section shown in Fig. 14.1(c) if W=1.22 m, X=0.305 m, and D=0.61 m.

The net flow area is

$$A = WD + 2(XD/2) = WD + XD$$

$$A = (1.22)(0.61) + (0.305)(0.61) = 0.93 \text{ m}^2$$

Example 14.1

To find the wetted perimeter, we must determine the value of *L*:

$$WP = W + 2L$$

$$L = \sqrt{X^2 + D^2} = \sqrt{(0.305)^2 + (0.61)^2} = 0.682 \text{ m}$$

$$WP = 1.22 + 0.61(0.682) = 1.64 \text{ m}$$

Thus

$$R = A/WP = 0.93 \text{ m}^2/1.64 \text{ m} = 0.57 \text{ m}$$

14.2 Classification of Open-Channel Flow

- Recall that the Reynolds number for closed circular cross sections running full is

$$N_R = \frac{vD}{\nu} \qquad (14\text{–}2)$$

where v = Average velocity of flow, D = pipe diameter, and *v* = Kinematic viscosity of the fluid.

- The Reynolds number for open-channel flow is then

$$N_R = \frac{vR}{\nu} \qquad (14\text{–}3)$$

14.3 Kinds of Open-Channel Flow

- The Reynolds number and the terms *laminar* and *turbulent* are not sufficient to characterize all kinds of open-channel flow.
- In addition to the viscosity-versus-inertial effects, the ratio of inertial forces to gravity forces is also important, given by the *Froude number* defined as

$$N_F = \frac{v}{\sqrt{g y_h}} \qquad (14\text{–}4)$$

where y_h called the hydraulic depth, is given by

$$y_h = A/T \qquad (14\text{–}5)$$

and *T* is the width of the free surface of the fluid at the top of the channel.

14.3 Kinds of Open-Channel Flow

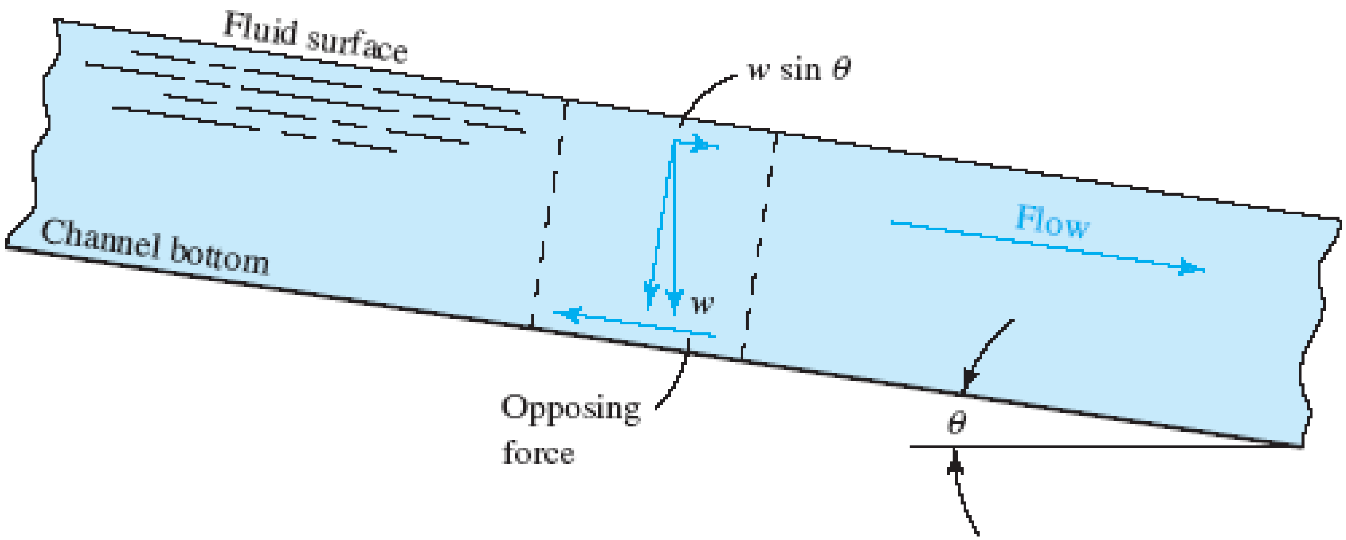
- The following kinds of flow are possible:

1. Subcritical-laminar: $N_R < 500$ and $N_F < 1.0$
2. Subcritical-turbulent: $N_R > 2000$ and $N_F < 1.0$
3. Supercritical-turbulent: $N_R > 2000$ and $N_F > 1.0$
4. Supercritical-laminar: $N_R < 500$ and $N_F > 1.0$

14. Open Channel Flow

14.4 Uniform Steady Flow in Open Channels

- In uniform flow, the driving force for the flow is provided by the component of the weight of the fluid that acts along the channel, as shown in Fig. 14.4.



14. Open Channel Flow

14.4 Uniform Steady Flow in Open Channels

- By equating the expressions for the driving force and the opposing force, we can derive an expression for the average velocity of uniform flow.

$$v = \frac{1.00}{n} R^{2/3} S^{1/2} \qquad (14\text{–}6)$$

- The average velocity of flow will be in m/s when the hydraulic radius *R* is in m.
- The channel slope, *S*, is dimensionless.
- The final term *n* is a resistance factor sometimes called *Manning's n*. The value of *n* depends on the condition of the channel surface and is therefore somewhat analogous to the pipe wall roughness used previously.

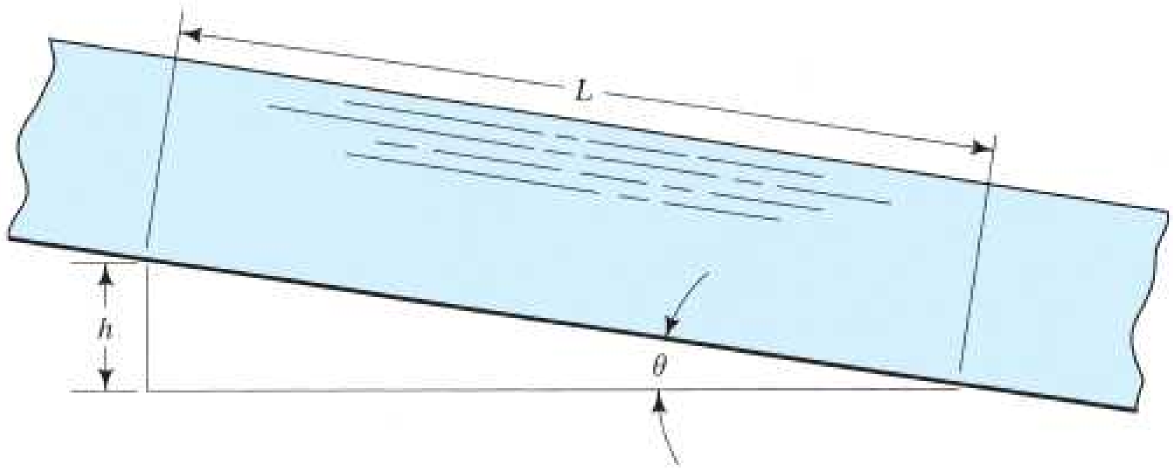
14.4 Uniform Steady Flow in Open Channels

- Typical design values of *n* are listed in Table 14.1 for materials commonly used for artificial channels and natural streams.

Channel Description	*n*
Glass, copper, plastic, or other smooth surfaces	0.010
Smooth, unpainted steel, planed wood	0.012
Painted steel or coated cast iron	0.013
Smooth asphalt, common clay drainage tile, trowel-finished concrete, glazed brick	0.013
Uncoated cast iron, black wrought iron pipe, vitrified clay sewer tile	0.014
Brick in cement mortar, float-finished concrete, concrete pipe	0.015
Formed, unfinished concrete, spiral steel pipe	0.017
Smooth earth	0.018
Clean excavated earth	0.022
Corrugated metal storm drain	0.024
Natural channel with stones and weeds	0.030
Natural channel with light brush	0.050
Natural channel with tall grasses and reeds	0.060
Natural channel with heavy brush	0.100

14.4 Uniform Steady Flow in Open Channels

- For small slopes, which are typical in open-channel flow, it is more practical to use *h*>*L*, where *L* is the length of the channel as shown in Fig. 14.5.



14.4 Uniform Steady Flow in Open Channels

- We can calculate the volume flow rate in the channel from the continuity equation, which is the same as that used for pipe flow:

$$Q = Av \qquad (14\text{–}7)$$

- In open-channel flow analysis, *Q* is typically called the *discharge*. Substituting Eq. (14–6) into (14–7) gives an equation that directly relates the discharge to the physical parameters of the channel:

$$Q = \left(\frac{1.00}{n}\right)AR^{2/3}S^{1/2} \qquad (14\text{–}8)$$

14.4 Uniform Steady Flow in Open Channels

- Another useful form of this equation is

$$AR^{2/3} = \frac{nQ}{S^{1/2}} \qquad (14\text{–}9)$$

- The term on the left side of Eq. (14–9) is solely dependent on the geometry of the section.
- Therefore, for a given discharge, slope, and surface type, we can determine the geometrical features of a channel.

Example 14.2

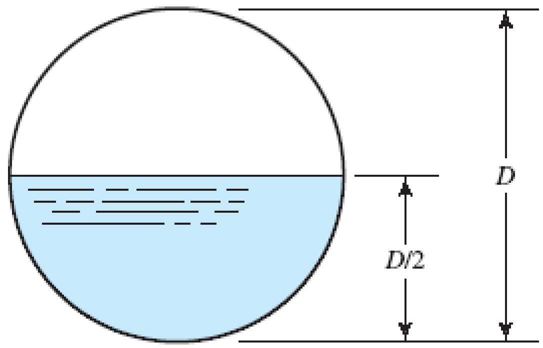
Determine the normal discharge for a 200-mm-inside-diameter common clay drainage tile running half full if it is laid on a slope that drops 1 m over a run of 1000 m.

Equation (14–8) will be used:

$$Q = \left(\frac{1.00}{n}\right) A R^{2/3} S^{1/2}$$

The slope S = 1/1000 = 0.001. From Table 14.1 we find n = 0.013. Figure 14.6 shows the cross section of the tile half full.

Example 14.2



Write

$$A = \frac{1}{2}\left(\frac{\pi D^2}{4}\right) = \frac{\pi D^2}{8} = \frac{\pi(200)^2}{8}\,\text{mm}^2 = 5000\pi\ \text{mm}^2$$

$$A = 15\,708\ \text{mm}^2 = 0.0157\ \text{m}^2$$

$$WP = \pi D/2 = 100\,\pi\ \text{mm}$$

Then we have

$$R = A/WP = 5000\pi\ \text{mm}^2/100\pi\ \text{mm} = 50\ \text{mm} = 0.05\ \text{m}$$

Example 14.2

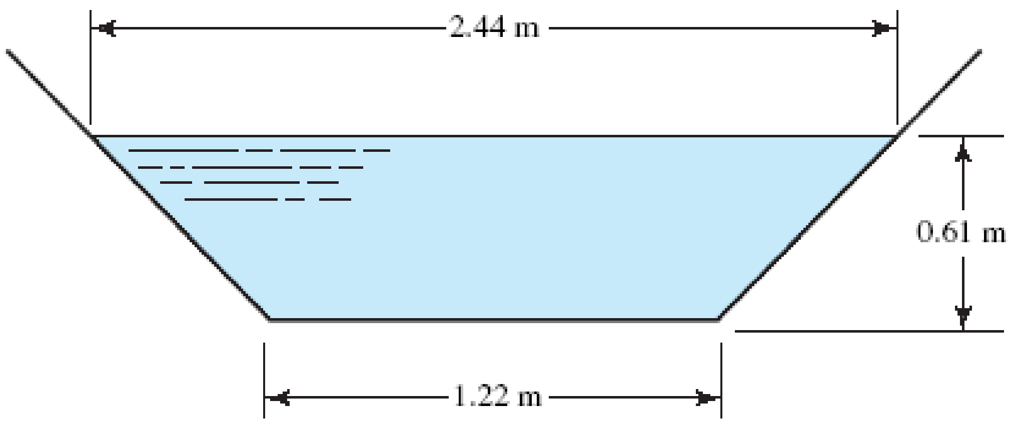
Then in Eq. (14–8),

$$Q = \frac{(0.0157)(0.05)^{2/3}(0.001)^{1/2}}{0.013}$$

$$Q = 5.18 \times 10^{-3}\ \mathrm{m^3/s}$$

Example 14.3

Calculate the minimum slope on which the channel shown in Fig. 14.7 must be laid if it is to carry 1.416 m3/s of water with a depth of 0.61 m. The sides and bottom of the channel are made of formed, unfinished concrete.



Example 14.3

Equation (14–11) can be solved for the slope *S*:

$$Q = \left(\frac{1.00}{n}\right) A R^{2/3} S^{1/2}$$

$$S = \left(\frac{Qn}{AR^{2/3}}\right)^2 \qquad (14\text{–}13)$$

From Table 14.1 we find n = 0.017. The values of *A* and *R* can be calculated from the geometry of the section:

$$A = (1.22)(0.61) + \frac{2(0.61)(0.61)}{2} = 1.116 \text{ m}$$

$$WP = (1.22) + 2\sqrt{0.61^2 + 0.61^2} = 2.945 \text{ m}$$

$$R = A / WP = \frac{1.116}{2.945} = 0.379 \text{ m}$$

Example 14.3

Then from Eq. (14–13) we have

$$S = \left[\frac{(1.416)(0.017)}{(1.116)(0.379)^{2/3}} \right]^2 = 0.0017$$

Therefore, the channel must drop at least 1.7 m per 1000 m of length.

Example 14.4

Design a rectangular channel to be made of formed, unfinished concrete to carry 5.75 m^3/s of water when laid on a 1.2-percent slope. The normal depth should be one-half the width of the channel bottom.

Because the geometry of the channel is to be determined, Eq. (14–9) is most convenient:

$$AR^{2/3} = \frac{nQ}{S^{1/2}} = \frac{(0.017)(5.75)}{(0.012)^{1/2}} = 0.892$$

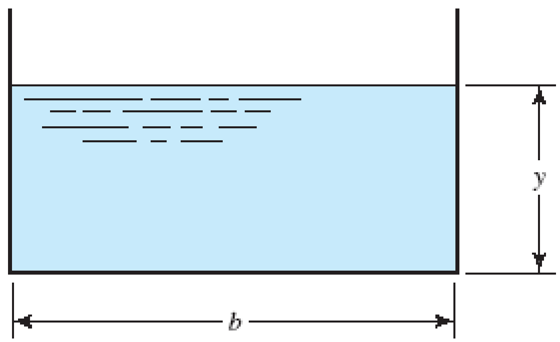
Example 14.4

Figure 14.8 shows the cross section. Because only y= b/2, *b* must be determined. Both *A* and *R* can be expressed in terms of *b*:

$$A = by = \frac{b^2}{2}$$

$$WP = b + 2y = 2b$$

$$R = A/WP = \frac{b^2}{(2)(2b)} = \frac{b}{4}$$



$$AR^{2/3} = 0.892$$

$$\frac{b^2}{2}\left(\frac{b}{4}\right)^{2/3} = 0.892$$

$$\frac{b^{8/3}}{5.04} = 0.892$$

$$b = (4.50)^{3/8} = 1.76\,\text{m}$$

The width of the channel must be 1.76 m.

Example 14.5

In the final design of the channel described in Example Problem 14.4, the width was made 2 m. The maximum expected discharge for the channel is 12 m^3/s. Determine the normal depth for this discharge.

Equation (14–9) will be used again:

$$AR^{2/3} = \frac{nQ}{S^{1/2}} = \frac{(0.017)(12)}{(0.012)^{1/2}} = 1.86$$

Both *A* and *R* must be expressed in terms of the dimension *y* in Fig. 14.8, with b = 2.0 m

$$A = 2y$$
$$WP = 2 + 2y$$
$$R = A/WP = 2y/(2 + 2y)$$

Example 14.5

Then we have

$$1.86 = AR^{2/3} = 2y\left(\frac{2y}{2 + 2y}\right)^{2/3}$$

Algebraic solution for *y* is not simply done. A trial-and-error approach can be used. The results are as follows:

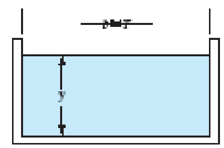
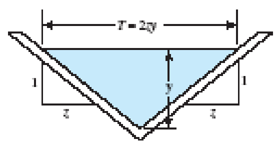
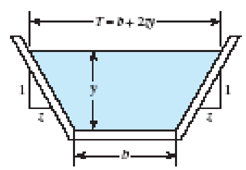
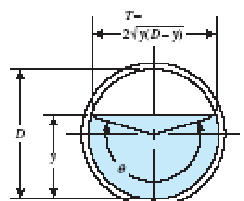
y (m)	A (m^2)	WP (m)	R (m)	$R^{2/3}$	$AR^{2/3}$	Required Change in y
2.0	4.0	6.0	0.667	0.763	3.05	Make y lower
1.5	3.0	5.0	0.600	0.711	2.13	Make y lower
1.35	2.7	4.7	0.574	0.691	1.86	y is OK

Therefore, the channel depth would be 1.35 m when the discharge is 12 m^3/s.

14.6 The Geometry of Typical Open Channel

- Table 14.2 gives the formulas for computing the geometric features pertinent to open-channel flow calculations.

TABLE 14.2. Geometry of open channel sections

Section	Area A	Wetted Perimeter WP	Hydraulic Radius R
Rectangle 	by	$b + 2y$	$\frac{by}{b+2y}$
Triangle 	zy^2	$2y\sqrt{1+z^2}$	$\frac{zy}{2\sqrt{1+z^2}}$
Trapezoid 	$(b+zy)y$	$b+2y\sqrt{1+z^2}$	$\frac{(b+zy)y}{b+2y\sqrt{1+z^2}}$
Circle  θ is in radians	$\frac{(\theta - \sin\theta)D^2}{8}$	$\theta D/2$	$\left[\frac{(\theta - \sin\theta)}{\theta}\right]\frac{D}{4}$

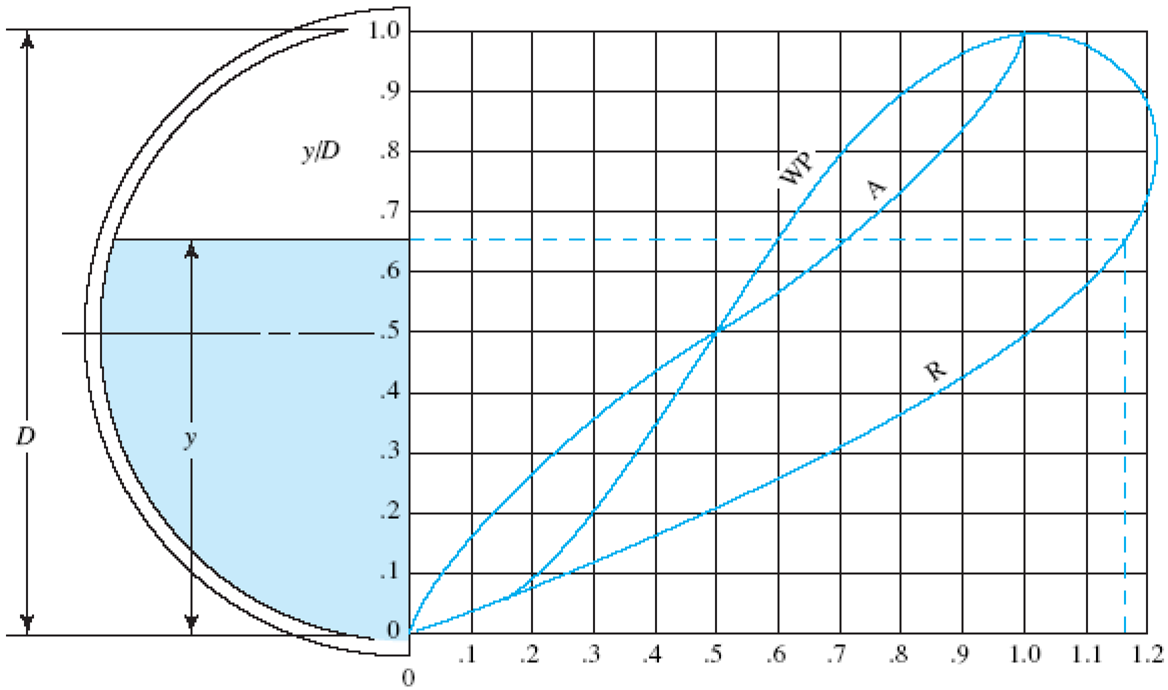
Note: θ must be in radians.
For $y < D/2$, $\theta = \pi - 2\sin^{-1}[1 - (2y/D)]$
For $y > D/2$, $\theta = \pi + 2\sin^{-1}[(2y/D) - 1]$

14.6 The Geometry of Typical Open Channel

- The trapezoid is popular for several reasons.
- It is an efficient shape because it gives a large flow area relative to the wetted perimeter.
- The slope of the sides can be defined by the angle with respect to the horizontal or by means of the *pitch*, the ratio of the horizontal distance to the vertical distance.
- The computation of the data for circular sections at various depths can be facilitated by the graph in Fig. 14.9.

14. Open Channel Flow

14.6 The Geometry of Typical Open Channel



Half-section only shown

Curve A: Ratio of A/A_f; $A_f = \pi D^2/4$
Curve WP: Ratio of WP/WP_f; $WP_f = \pi D$
Curve R: Ratio of R/R_f; $R_f = D/4$

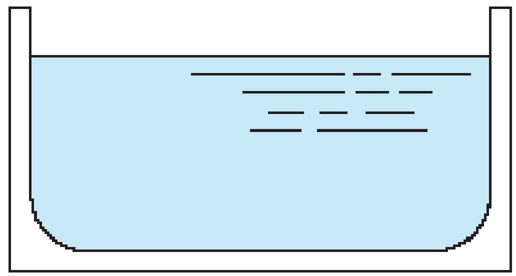
Example: $D = 0.6$ m; $y = 0.4$ m; $y/D = 0.67$

$A_f = 0.283\ \text{m}^2$; $A/A_f = .7$; $A = 0.7(0.283) = 0.198\ \text{m}^2$
$WP_f = 1.885$ m; $WP/WP_f = 0.6$; $WP = 0.6(1.885) = 1.13$ m
$R_f = 0.15$ m; $R/R_f = 1.16$; $R = 1.16(0.15) = 0.174$ m

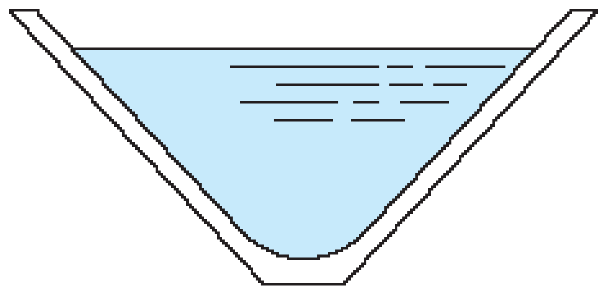
14.6 The Geometry of Typical Open Channel

- Figure 14.10 shows three other shapes used for open channels.
- Natural streams frequently can be approximated as shallow parabolas.
- The triangle with a rounded bottom is more practical to make in the earth than the sharp-V triangle.
- The round cornered rectangle performs somewhat better than the square-cornered rectangle and is easier to maintain.

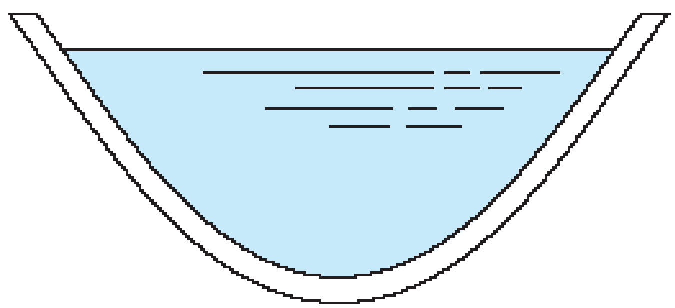
14.6 The Geometry of Typical Open Channel



(a) Round-cornered rectangle



(b) Round-bottom triangle



(c) Parabola

14. Open Channel Flow

14.7 The Most Efficient Shapes for Open Channel

- The term *conveyance* is used to indicate the carrying capacity of open channels.
- Its value can be deduced from Manning's equation. In SI metric units, we have Eq. (14–8),

$$Q = \left(\frac{1.00}{n}\right) A R^{2/3} S^{1/2}$$

- We can then define the conveyance *K* to be

$$K = \left(\frac{1.00}{n}\right) A R^{2/3} \qquad (14\text{–}14)$$

14.7 The Most Efficient Shapes for Open Channel

- In U.S. Customary units,

$$K = \left(\frac{1.49}{n}\right) A R^{2/3} \qquad (14\text{–}15)$$

- Manning's equation is then

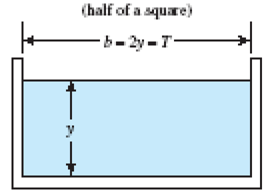
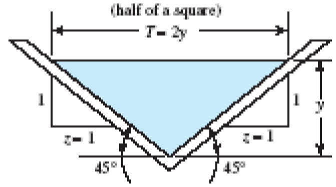
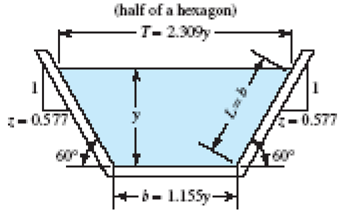
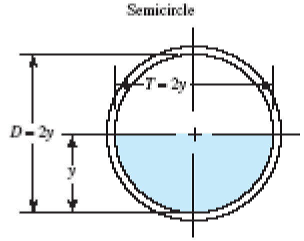
$$Q = K S^{1/2} \qquad (14\text{–}16)$$

- Table 14.3 shows the most efficient designs of other shapes.

14. Open Channel Flow

14.7 The Most Efficient Shapes for Open Channel

TABLE 14.3. Most efficient sections for open channels

Section	Area A	Wetted Perimeter WP	Hydraulic Radius R
Rectangle (half of a square) 	$2.0y^2$	$4y$	$y/2$
Triangle (half of a square) 	y^2	$2.83y$	$0.354y$
Trapezoid (half of a hexagon) 	$1.73y^2$	$3.46y$	$y/2$
Semicircle 	$\frac{1}{2}\pi y^2$	πy	$y/2$

14.8 Critical Flow and Specific Energy

- The total energy is measured relative to the channel bottom and is composed of potential energy due to the depth of the fluid plus kinetic energy due to its velocity.
- Letting *E* denote the total energy, we get

$$E = y + v^2/2g \qquad (14\text{–}17)$$

where *y* is the depth and is the average velocity of flow.

- For a given discharge *Q*, the velocity is *Q*/*A*. Then

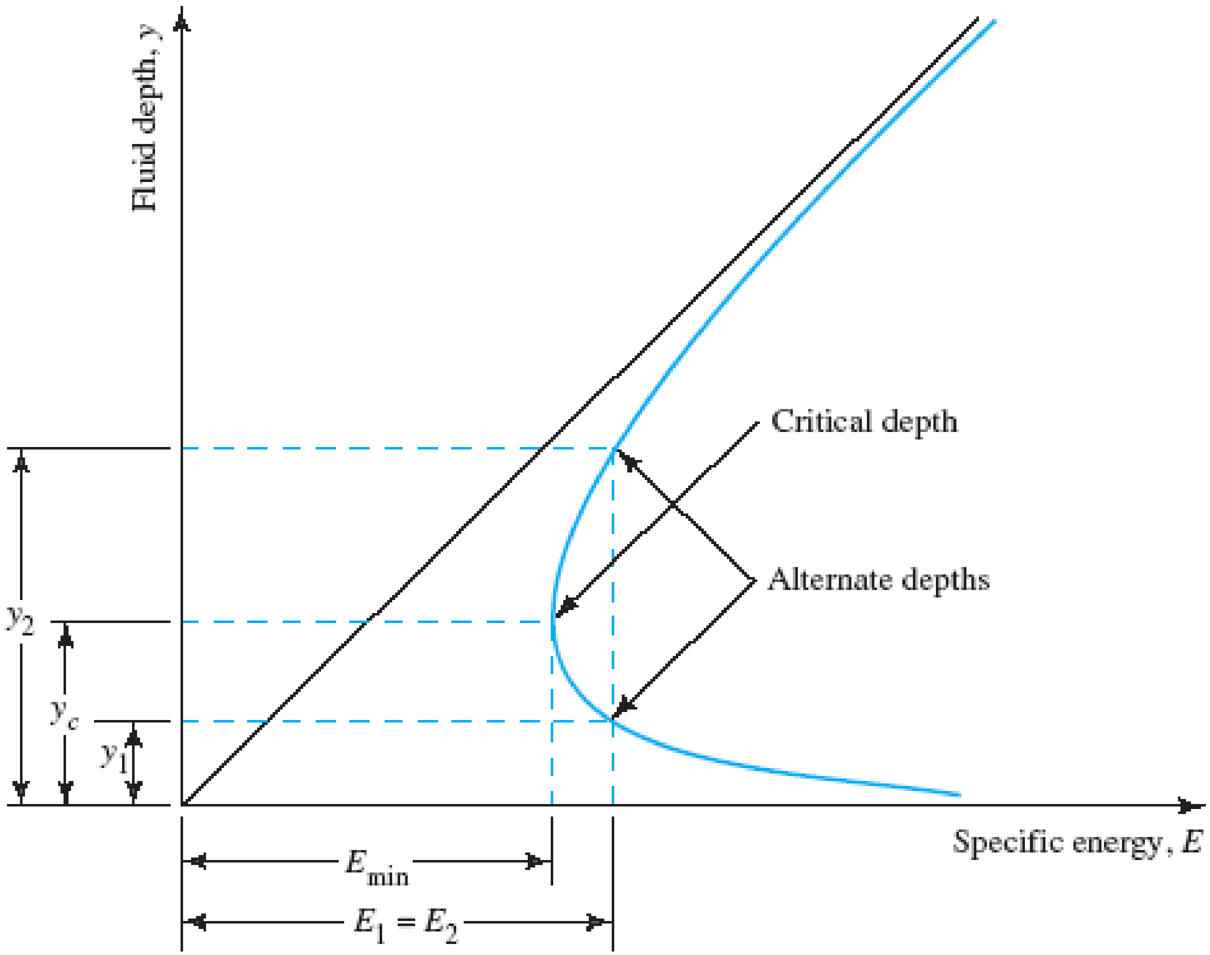
$$E = y + Q^2/2gA^2 \qquad (14\text{–}18)$$

14.8 Critical Flow and Specific Energy

- The depth corresponding to the minimum specific energy is therefore called the *critical depth*, y_c.
- In Fig. 14.12, both y_1 below the critical depth y_c , and y_2 above y_c have the same energy.
- The two depths y_1 and y_2 are called the *alternate depths* for the specific energy E.

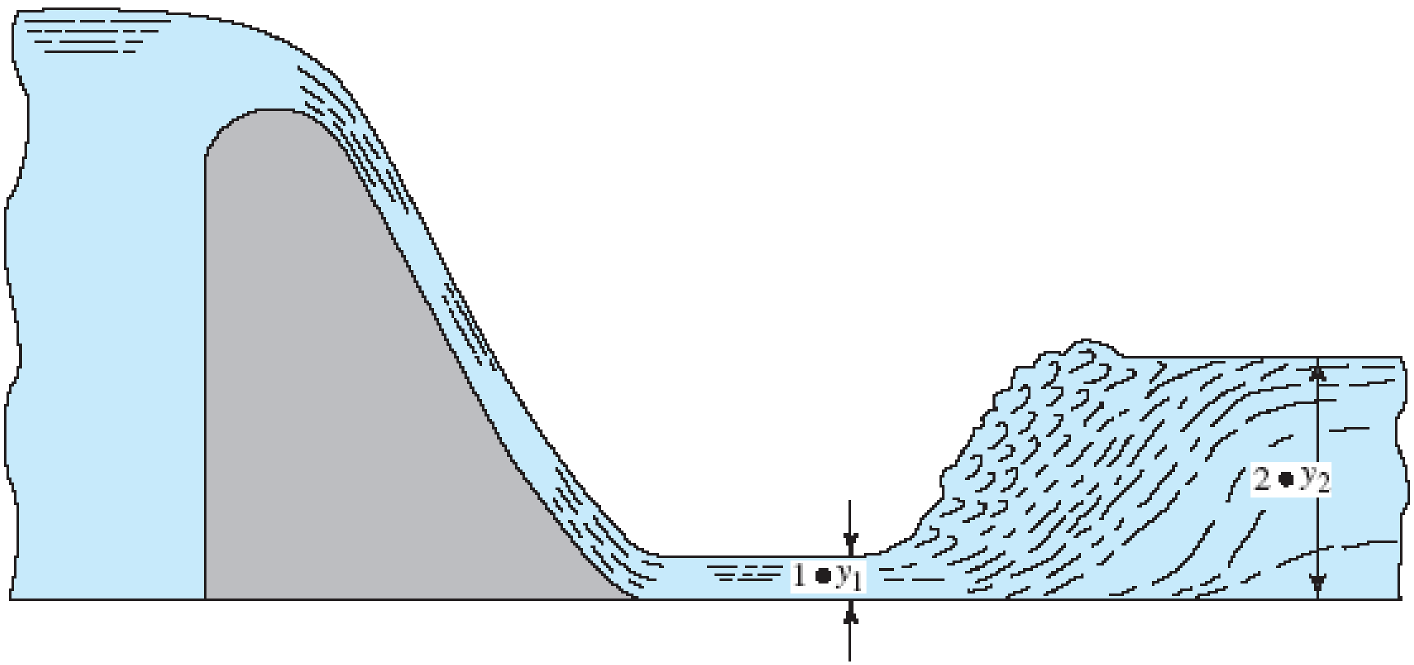
14. Open Channel Flow

14.8 Critical Flow and Specific Energy



14.9 Hydraulic Jump

- To understand the significance of the phenomenon known as *hydraulic jump*, consider one of its most practical uses, illustrated in Fig. 14.13.



14.9 Hydraulic Jump

- For a hydraulic jump to occur, the flow before the jump must be the supercritical range.
- The depth at section 2 after the jump can be calculated from the equation

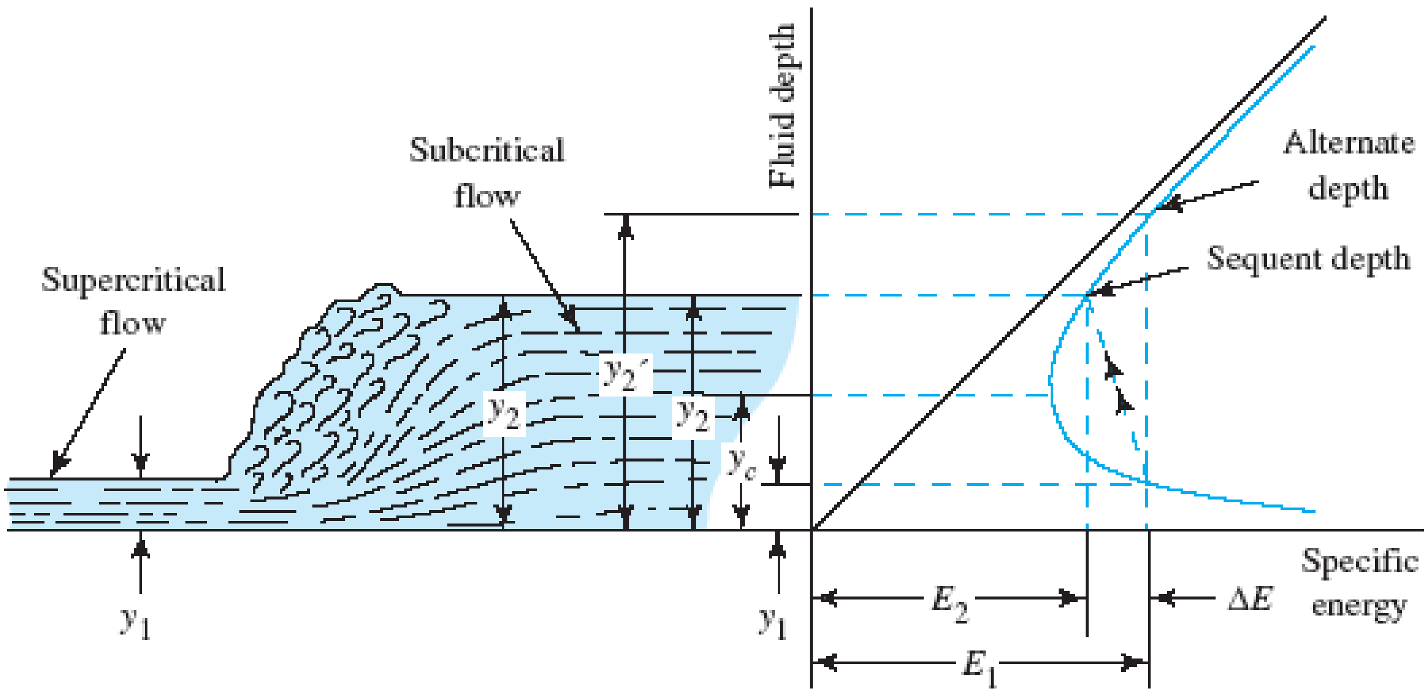
$$y_2 = (y_1/2)(\sqrt{1 + 8N_{F_1}^2} - 1) \quad (14\text{–}19)$$

- The energy loss in the jump is dependent on the two depths

$$E_1 - E_2 = \Delta E = (y_2 - y_1)^3/4y_1y_2 \quad (14\text{–}20)$$

14.9 Hydraulic Jump

- Figure 14.14 illustrates what happens in a hydraulic jump by using a specific energy curve.

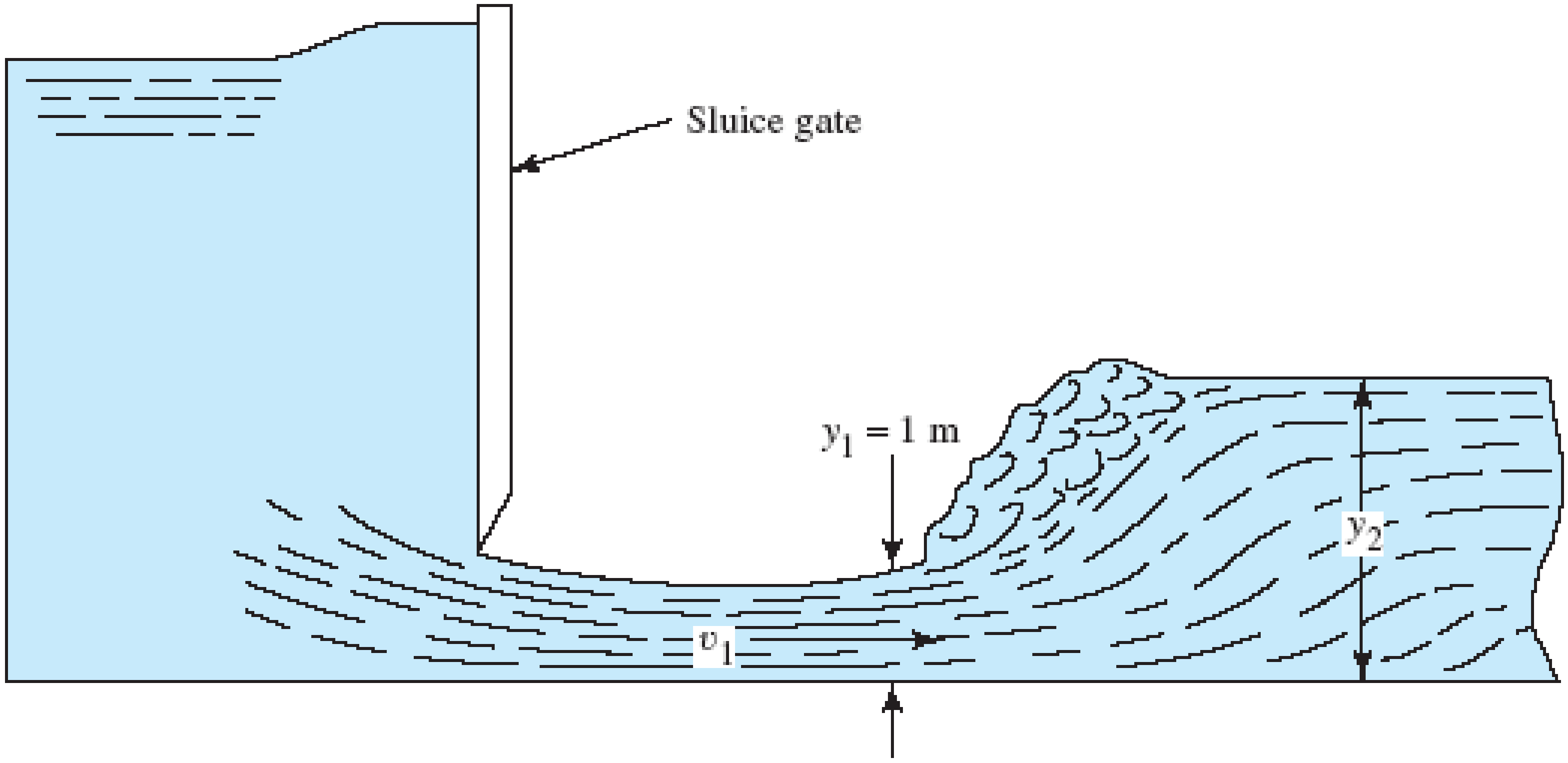


Example 14.6

As shown in Fig. 14.15, water is being discharged from a reservoir under a sluice gate at the rate of 18 m^3/s into a horizontal rectangular channel, 3 m wide, made of unfinished formed concrete. At a point where the depth is 1 m, a hydraulic jump is observed to occur. Determine the following:

a. The velocity before the jump
b. The depth after the jump
c. The velocity after the jump
d. The energy dissipated in the jump

Example 14.6



14. Open Channel Flow

Example 14.6

a. The velocity before the jump is

$$v_1 = Q/A_1$$
$$A_1 = (3)(1) = 3\ \text{m}^2$$
$$v_1 = (18\ \text{m}^3/\text{s})/3\ \text{m}^2 = 6.0\ \text{m/s}$$

b. Equation (13–19) can be used to determine the depth after the jump y_2,

$$y_2 = (y_1/2)(\sqrt{1 + 8N_{F_1}^2} - 1)$$
$$N_{F_1} = v_1/\sqrt{gy_h}$$

Example 14.6

The hydraulic depth is equal to *A>T*, where *T* is the width of the free surface. Then for a rectangular channel, $y_h = y$. Then we have

$$N_{F_1} = 6.0/\sqrt{(9.81)(1)} = 1.92$$

The flow is in the supercritical range. We have

$$y_2 = (1/2)(\sqrt{1 + (8)(1.92)^2} - 1) = 2.26\ \text{m}$$

c. Because of continuity,

$$v_2 = Q/A_2 = (18\ \text{m}^3/\text{s})/(3)(2.26)\ \text{m}^2 = 2.65\ \text{m/s}$$

Example 14.6

d. From Eq. (14–20), we get

$$\Delta E = (y_2 - y_1)^3/4y_1y_2$$
$$= \frac{(2.26 - 1.0)^3}{(4)(1.0)(2.26)}\,\mathrm{m} = 0.221\,\mathrm{m}$$

This means that 0.221 Nm of energy is dissipated from each newton of water as it flows through the jump.

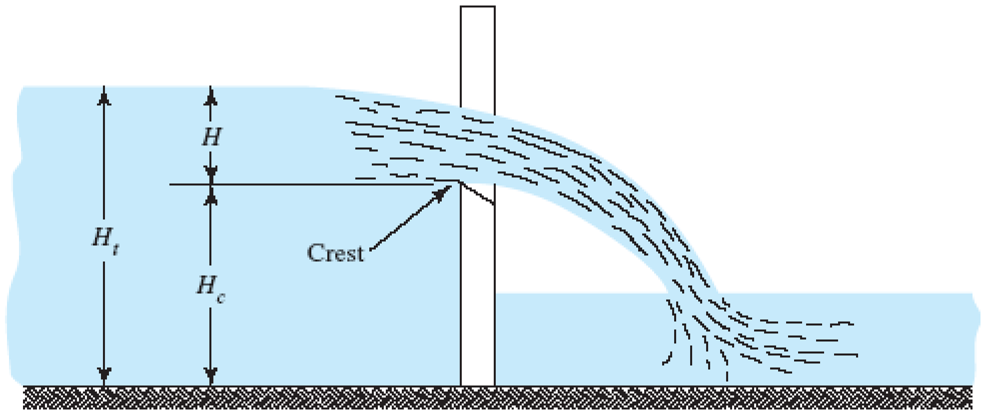
14.10 Open-Channel Flow Measurement

- An open channel is one that has its top surface open to the prevailing atmosphere.
- Two widely used devices for open-channel flow measurement are *weirs* and *flumes*.
- Each causes the area of the stream to change, which in turn changes the level of the fluid surface.
- The resulting level of the surface relative to some feature of the device is related to the quantity of flow.

14. Open Channel Flow

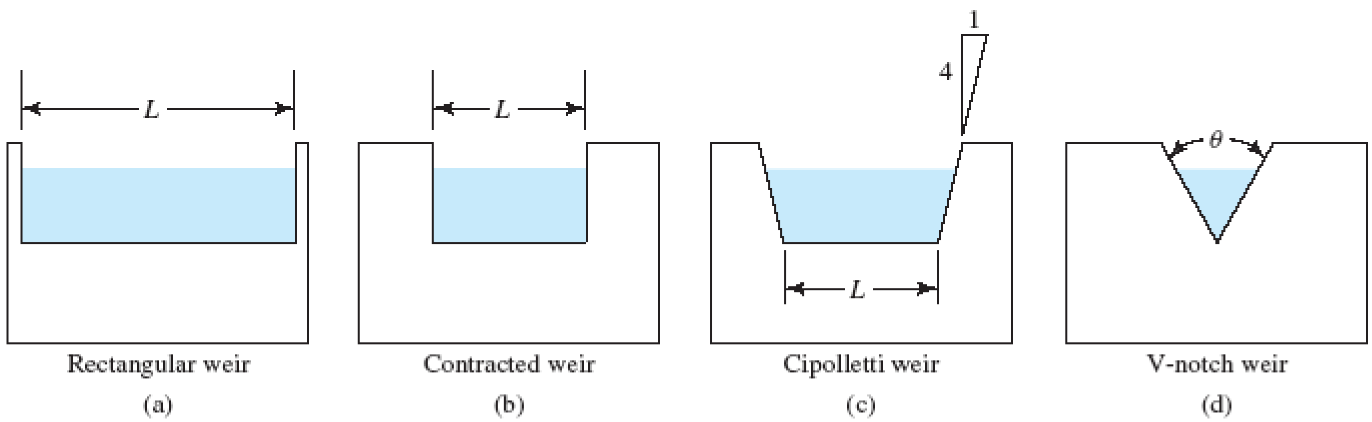
14.10.1 Weirs

- A *weir* is a specially shaped barrier installed in an open channel over which the fluid flows as a free jet into a stream beyond the barrier.
- Figure 14.16 shows a side view of the typical design of a weir.



14.10.1 Weirs

- Figure 14.17 shows four common shapes for weirs for which rating equations have been developed to enable the calculation of discharge *Q* as a function of the dimensions of the weir and the head of fluid above the crest *H*.



14.10.1 Weirs

- Measurement of the head can be by a fixed gage, called a *staff gage*, mounted at the side of the stream for which the zero reading is at the level of the crest of the weir.
- A *rectangular weir*, also called a *suppressed weir*, has a crest length *L* that extends the full width of the channel into which it is installed.

14.10.1 Weirs

- The standard design requires

1. The crest height above the bottom of the channel *Hc* $\geq$ $3H_{max}$.
2. The minimum head above the crest $H_{min} > 0.06$ m
3. The maximum head above the crest $H_{max} < L/3$

- The rating equation is

$$Q = 1.84LH^{3/2} \tag{14–21}$$

where L and H are in m and Q is in m^3/s.

14.10.1 Weirs

- A *contracted weir* is a rectangular weir having sides extended inward from the sides of the channel by a distance of at least $2H_{max}$.
- The fluid stream must then contract as it flows around the sides of the weir, decreasing slightly the effective length of the weir.
- The standard design requires

1. The crest height above the bottom of the channel $Hc >= 2H_{max}$.
2. The minimum head above the crest H_{min}>0.06 m
3. The maximum head above the crest H_{max}<L/3

14.10.1 Weirs

- The rating equation is

$$Q = 1.84(L - 0.2H)H^{3/2} \qquad (14\text{–}22)$$

where L and H are in m and Q is in m^3/s.

- A *Cipolletti weir* is also contracted from the sides of the stream by a distance at least $2H_{max}$ and has sides that are sloped outward as shown in Fig. 14.17(c).
- The same requirements listed for the contracted rectangular weir apply. The rating equation is

$$Q = 1.86LH^{3/2} \qquad (14\text{–}23)$$

14.10.1 Weirs

- The *triangular weir* is used primarily for low flow rates because the V-notch produces a larger head *H* than can be obtained with a rectangular notch.
- The theoretical equation for a triangular weir is

$$Q = \tfrac{8}{15} C \sqrt{2g} \tan(\theta/2) H^{5/2} \qquad (14\text{–}24)$$

- An additional reduction of this equation gives

$$Q = 4.28 C \tan(\theta/2) H^{5/2} \qquad (14\text{–}25)$$

14.10.1 Weirs

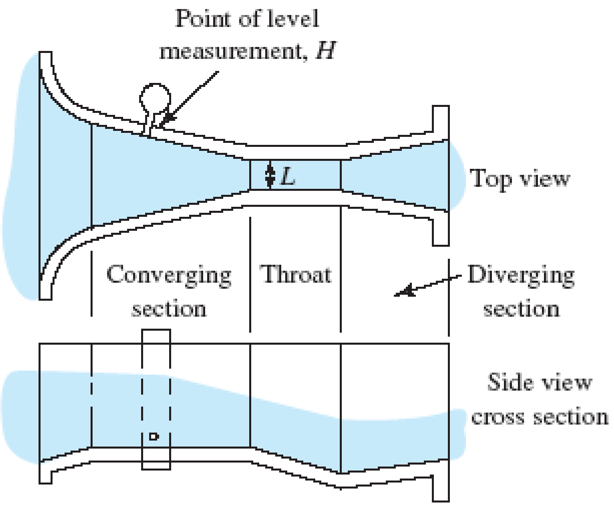
- The value of *C* is somewhat dependent on the head *H*, but a nominal value is 0.58.
- Using this and the common values of 60° and 90° for θ we get

$$Q = 0.79H^{5/2} \qquad \text{(60° notch)} \qquad (14\text{–}26)$$

$$Q = 1.37H^{5/2} \qquad \text{(90° notch)} \qquad (14\text{–}27)$$

14.10.2 Flumes

- *Critical flow flumes* are contractions in the stream that cause the flow to achieve its critical depth within the structure.
- There is a definite relationship between depth and discharge when critical flow exists.
- *Parshall flume*, the geometry of which is shown in Fig. 14.18.



14. Open Channel Flow

14.10.2 Flumes

- The discharge equations for the Parshall flume were developed empirically for flumes designed and constructed for several sizes of flumes as shown in Table 14.4.

TABLE 14.4. Discharge equations for Parshall flumes

Throat Width L		Flow Range Q, m^3/s Min.	Max.	ft^3/s Min.	Max.	Equation (H and L in m, Q in m^3/s)	Equation (H and L in ft, Q in ft^3/s)
7.62 cm	3 in	0.0008	0.03	0.0538	1.9	$Q = 0.158H^{1.547}$	$Q = 0.992H^{1.547}$
15.24 cm	6 in	0.0014	0.05	0.1105	3.9	$Q = 0.382H^{1.58}$	$Q = 2.06H^{1.548}$
22.86 cm	9 in	0.0025	0.09	0.2521	8.9	$Q = 0.536H^{1.53}$	$Q = 3.07H^{1.53}$
0.305 m	1 ft	0.0031	0.11	0.456	16.1	$Q = 2.34LH^{1.55}$	$Q = 4.00LH^n$, n=1.55
0.61 m	2 ft	0.0119	0.42	0.937	33.1	$Q = 2.34LH^{1.55}$	n=1.55
1.22 m	4 ft	0.0368	1.3	1.923	67.9	$Q = 2.43LH^{1.58}$	n=1.58
1.83 m	6 ft	0.0736	2.6	2.931	103.5	$Q = 2.46LH^{1.59}$	n=1.59
2.44 m	8 ft	0.1	3.5	3.951	139.5	$Q = 2.52LH^{1.61}$	n=1.61
3.05 m	10 ft	0.17	6	5.664	200	$Q = (2.2927L + 0.4738)H^{1.6}$	$Q = (3.6875L + 2.5)H^{1.6}$
6.1 m	20 ft	0.283	10	28.321	1000		
9.14 m	30 ft	0.425	15	42.481	1500		
12.2 m	40 ft	0.566	20	56.641	2000		
15.24 m	50 ft	0.708	25	84.962	3000		

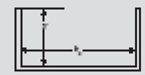
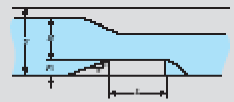
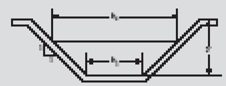
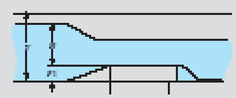
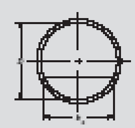
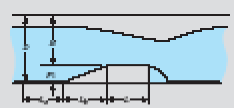
14.10.2 Flumes

- *Long-throated flumes*, rather than Parshall flumes, are recommended for new construction because they are simpler and less expensive to build and can more easily be adapted to channels with a variety of shapes.
- Table 14.5 shows the general shape consisting of a straight ramp up from the bottom of the channel, a flat throat section, and a sudden drop.

14.10.2 Flumes

TABLE 14.5. Long-throated flume data

Design A			Design B			Design C		
Rectangular channels: $Q = b_c K_1 (H + K_2)^n$ in ft³/s $\quad Q = m b_c K_1 (H + K_2)^n$ in m³/s								
b_c	0.500 ft	0.152 m	b_c	1.000 ft	0.305 m	b_c	1.500 ft	0.497 m
L	0.750 ft	0.229 m	L	1.000 ft	0.305 m	L	2.250 ft	0.689 m
p_1	0.125 ft	0.038 m	p_1	0.250 ft	0.076 m	p_1	0.500 ft	0.152 m
K_1	3.996		K_1	3.696		K_1	3.375	
K_2	0.000		K_2	0.004		K_2	0.011	
n	1.612		n	1.617		n	1.625	
m	0.664		m	0.539		m	0.507	
H_{min}	0.057 ft	0.017 m	H_{min}	0.082 ft	0.025 m	H_{min}	0.148 ft	0.045 m
H_{max}	0.462 ft	0.141 m	H_{max}	0.701 ft	0.214 m	H_{max}	1.500 ft	0.457 m
Q_{min}	0.020 ft³/s	0.0006 m³/s	Q_{min}	0.070 ft³/s	0.002 m³/s	Q_{min}	0.255 ft³/s	0.007 m³/s
Q_{max}	0.575 ft³/s	0.017 m³/s	Q_{max}	2.100 ft³/s	0.06 m³/s	Q_{max}	9.900 ft³/s	0.228 m³/s
Trapezoidal channels: $Q = K_1 (H + K_2)^n$ in ft³/s $\quad Q = m K_1 (H + K_2)^n$ in m³/s								
b_1	1.000 ft	0.305 m	b_1	1.000 ft	0.305 m	b_1	2.000 ft	0.61 m
b_c	2.000 ft	0.61 m	b_c	4.000 ft	1.219 m	b_c	5.000 ft	1.524 m
L	0.750 ft	0.229 m	L	1.000 ft	0.305 m	L	1.000 ft	0.305 m
p_1	0.500 ft	0.152 m	p_1	1.500 ft	0.457 m	p_1	1.500 ft	0.457 m
K_1	9.290		K_1	14.510		K_1	16.180	
K_2	0.030		K_2	0.053		K_2	0.035	
n	1.878		n	1.855		n	1.784	
m	0.199		m	0.192		m	0.189	
H_{min}	0.400 ft	0.122 m	H_{min}	0.579 ft	0.176 m	H_{min}	0.580 ft	0.177 m
H_{max}	0.893 ft	0.272 m	H_{max}	0.808 ft	0.246 m	H_{max}	1.456 ft	0.444 m
Q_{min}	1.900 ft³/s	0.054 m³/s	Q_{min}	6.200 ft³/s	0.176 m³/s	Q_{min}	6.800 ft³/s	0.193 m³/s
Q_{max}	8.000 ft³/s	0.195 m³/s	Q_{max}	11.000 ft³/s	0.287 m³/s	Q_{max}	33.000 ft³/s	0.823 m³/s
Circular channels: $Q = D^{2.5} K_1 (H/D + K_2)^n$ in ft³/s $\quad Q = 0.552 D^{2.5} K_1 (H/D + K_2)^n$ in m³/s								
D	1.000 ft	0.305 m	D	2.000 ft	0.61 m	D	3.000 ft	0.914 m
b_c	0.866 ft	0.264 m	b_c	1.834 ft	0.56 m	b_c	2.940 ft	0.896 m
L_a	0.600 ft	0.183 m	L_a	1.100 ft	0.335 m	L_a	1.350 ft	0.411 m
L_b	0.750 ft	0.229 m	L_b	1.800 ft	0.549 m	L_b	3.600 ft	1.097 m
L	1.125 ft	0.343 m	L	2.100 ft	0.64 m	L	2.700 ft	0.823 m
p_1	0.250 ft	0.076 m	p_1	0.600 ft	0.183 m	p_1	1.200 ft	0.366 m
K_1	3.970		K_1	3.780		K_1	3.507	
K_2	0.004		K_2	0.000		K_2	0.000	
n	1.689		n	1.625		n	1.573	
H_{min}	0.069 ft	0.021 m	H_{min}	0.140 ft	0.043 m	H_{min}	0.180 ft	0.055 m
H_{max}	0.599 ft	0.183 m	H_{max}	1.102 ft	0.336 m	H_{max}	1.343 ft	0.409 m
Q_{min}	0.048 ft³/s	0.0013 m³/s	Q_{min}	0.283 ft³/s	0.008 m³/s	Q_{min}	0.655 ft³/s	0.019 m³/s

Example 14.7

Select a design from Table 14.5 for a long-throated flume for measuring a flow rate within the range of 0.07 to 0.17 m3/s of water. Then compute the discharge *Q* for several values of head *H*.

Either design C for a rectangular channel, design A for a trapezoidal channel, or design B for a circular channel is appropriate for the given desired flow range. The trapezoidal channel will be illustrated here. The rating equation and the values for its variables are found in Table 14.5. We have

$$Q = (0.199)K_1(H + K_2)^n$$

$$Q = (0.199)9.29(H + 0.03)^{1.878}$$

Example 14.7

Evaluating this equation for H=0.152 m to 0.244 m gives the following results:

Head H (m)	Flow Q (m^3/s)
0.152	0.075
0.183	0.101
0.213	0.130
0.244	0.163

Example 14.7

The flow equation can also be solved for the value of *H* that will give a desired *Q*,

$$H = \left(\frac{Q}{(0.199)K_1}\right)^{1/n} = K_2$$

Now we can determine what values of head correspond to the ends of the desired range of flow:

$$\text{For } Q = 0.07 \text{ m}^3/\text{s},\ H = 0.145 \text{ m}$$

$$\text{For } Q = 0.17 \text{ m}^3/\text{s},\ H = 0.251 \text{ m}$$