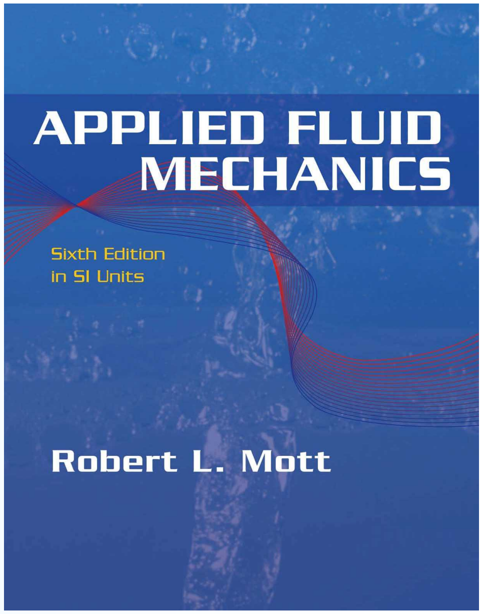
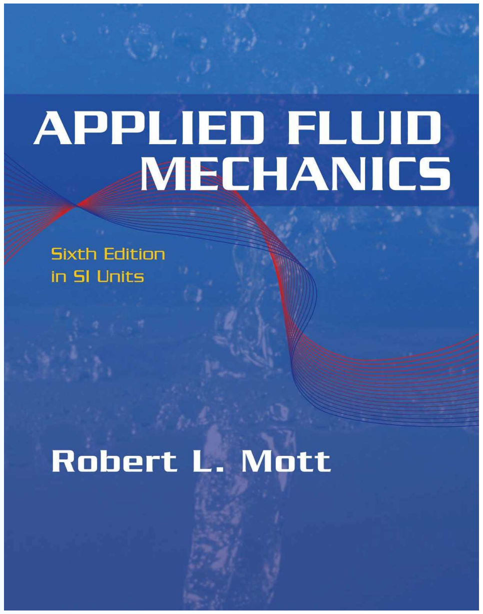
Applied Fluid Mechanics



1. The Nature of Fluid and the Study of Fluid Mechanics
2. Viscosity of Fluid
3. Pressure Measurement
4. Forces Due to Static Fluid
5. Buoyancy and Stability
6. Flow of Fluid and Bernoulli’s Equation
7. General Energy Equation
8. Reynolds Number, Laminar Flow, Turbulent Flow and Energy Losses Due to Friction

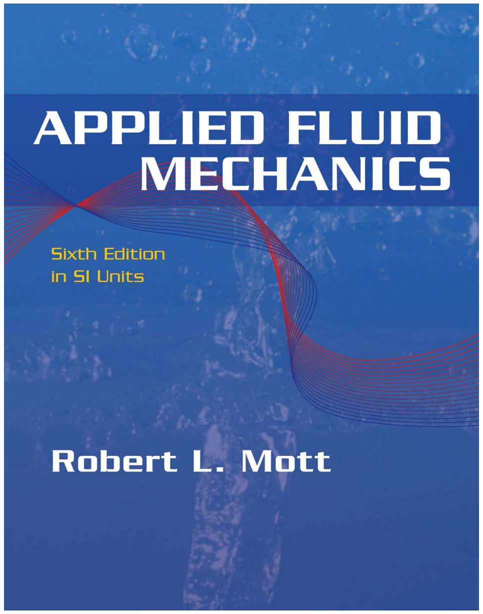
Applied Fluid Mechanics



9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections
10. Minor Losses
11. Series Pipeline Systems
12. Parallel Pipeline Systems
13. Pump Selection and Application
14. Open-Channel Flow
15. Flow Measurement
16. Forces Due to Fluids in Motion

Applied Fluid Mechanics

17. Drag and Lift
18. Fans, Blowers, Compressors and the Flow of Gases
19. Flow of Air in Ducts



16. Forces Due to Fluids in Motion

Chapter Objectives

- Use Newton's second law of motion, $F = ma$, to develop the *force equation*, which is used to compute the force exerted by a fluid as its direction of motion or its velocity is changed.
- Relate the force equation to *impulse–momentum*.
- Use the force equation to compute the force exerted on a stationary object that causes the change in direction of a fluid flow stream.
- Use the force equation to compute the force exerted on bends in pipelines.
- Use the force equation to compute the force on moving objects, such as the vanesof a pump impeller.

16. Forces Due to Fluids in Motion

Chapter Outline

1. Introductory Concepts
2. Force Equation
3. Impulse-Momentum Equation
4. Problem-Solving Method Using the Force Equations
5. Forces on Stationary Objects
6. Forces on Bends in Pipelines
7. Forces on Moving Objects

16. Forces Due to Fluids in Motion

16.1 Introductory Concepts

- Whenever a fluid stream is deflected from its initial direction or if its velocity is changed, a force is required to accomplish the change.
- Sometimes the force is desired, sometimes it is destructive.

16.2 Force Equation

- Whenever the magnitude or direction of the velocity of a body is changed, a force is required to accomplish the change.
- Newton's second law of motion is often used to express this concept in mathematical form; the most common form is

$$F = ma \qquad (16\text{–}1)$$

- Force equals mass times acceleration. Acceleration is the time rate of change of velocity.

16.2 Force Equation

- In fluid flow problems, a continuous flow is caused to undergo the acceleration, and a different form of Newton’s equation is desirable.
- Because acceleration is the time rate of change of velocity, Eq. (16–1) can be written as

$$F = ma = m\frac{\Delta v}{\Delta t} \qquad (16\text{–}2)$$

- The term $m/\Delta t$ can be interpreted as the mass flow rate, that is, the amount of mass flowing in a given amount of time.

16.2 Force Equation

- *M* is related to the volume flow rate *Q* by the relationship

$$M = \rho Q \qquad \text{(16–3)}$$

- Then Eq. (16–2) becomes

$$F = (m/\Delta t)\Delta v = M\,\Delta v = \rho Q\,\Delta v \qquad \text{(16–4)}$$

- This is the general form of the force equation for use in fluid flow problems because it involves the velocity and volume flow rate, items generally known in a fluid flow system.

16.3 Impulse-Momentum Equation

- The force equation, Eq. (16–4), is related to another principle of fluid dynamics, the *impulse–momentum equation*.
- Impulse is defined as a force acting on a body for a period of time, and it is indicated by

$$\text{Impulse} = F(\Delta t)$$

- When conditions vary, the instantaneous form of the equation is used:

$$\text{Impulse} = F(dt)$$

16. Forces Due to Fluids in Motion

16.3 Impulse-Momentum Equation

- *Momentum* is defined as the product of the mass of a body and its velocity.
- The *change* in momentum is

$$\text{Change in momentum} = m(\Delta v)$$

- In an instantaneous sense,

$$\text{Change in momentum} = m(dv)$$

- Now Eq. (16–2) can be rearranged to the form

$$F(\Delta t) = m(\Delta v)$$

16. Forces Due to Fluids in Motion

16.3 Impulse-Momentum Equation

- Here we have shown the impulse–momentum equation for steady flow conditions.
- In an instantaneous sense,

$$F(dt) = m(dv)$$

16. Forces Due to Fluids in Motion

16.4 Problem-Solving Method Using the Force Equations

- In general, if three perpendicular directions are called *x*, *y*, and *z*, a separate equation can be written for each direction:

$$F_x = \rho Q\,\Delta v_x = \rho Q(v_{2_x} - v_{1_x}) \qquad (16\text{–}5)$$

$$F_y = \rho Q\,\Delta v_y = \rho Q(v_{2_y} - v_{1_y}) \qquad (16\text{–}6)$$

$$F_z = \rho Q\,\Delta v_z = \rho Q(v_{2_z} - v_{1_z}) \qquad (16\text{–}7)$$

- In a particular direction, say *x*, the term F_x refers to the net external force that acts on the fluid in that direction.

16.4 Problem-Solving Method Using the Force Equations

- Below is the procedure for using the force equations:

1. Identify a portion of the fluid stream to be considered a free body. This will be the part where the fluid is changing direction or where the geometry of the flow stream is changing.
2. Establish reference axes for directions of forces. Usually one axis is chosen to be parallel to one part of the flow stream. In the example problems to follow, the positive *x* and *y* directions are chosen to be in the same direction as the reaction forces.

16.4 Problem-Solving Method Using the Force Equations

3. Identify and show on the free-body diagram all external forces acting on the fluid. All solid surfaces that affect the direction of the flow stream exert forces. Also, the fluid pressure acting on the cross-sectional area of the stream exerts a force in a direction parallel to the stream at the boundary of the free body.
4. Show the direction of the velocity of flow as it enters the free body and as it leaves the free body.

16.4 Problem-Solving Method Using the Force Equations

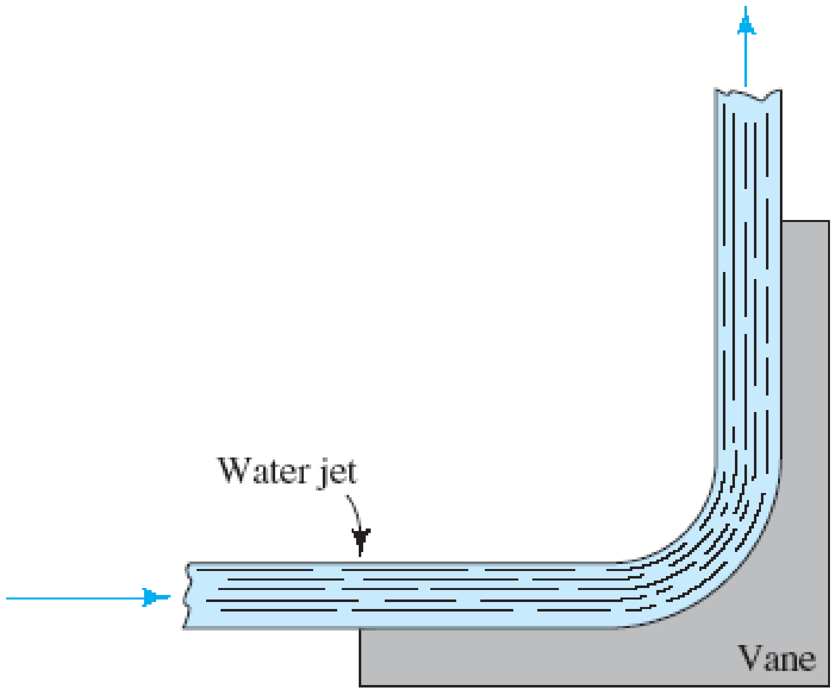
5. Using the data thus shown for the free body, write the force equations in the pertinent directions. Use Eq. (16–5), (16–6), or (16–7).
6. Substitute data and solve for the desired quantity.

16.5 Forces on Stationary Objects

- When free streams of fluid are deflected by stationary objects, external forces must be exerted to maintain the object in equilibrium.

Example 16.1

A 25-mm-diameter jet of water having a velocity of 6 m/s is deflected 90° by a curved vane, as shown in Fig. 16.1. The jet flows freely in the atmosphere in a horizontal plane. Calculate the *x* and *y* forces exerted on the water by the vane.

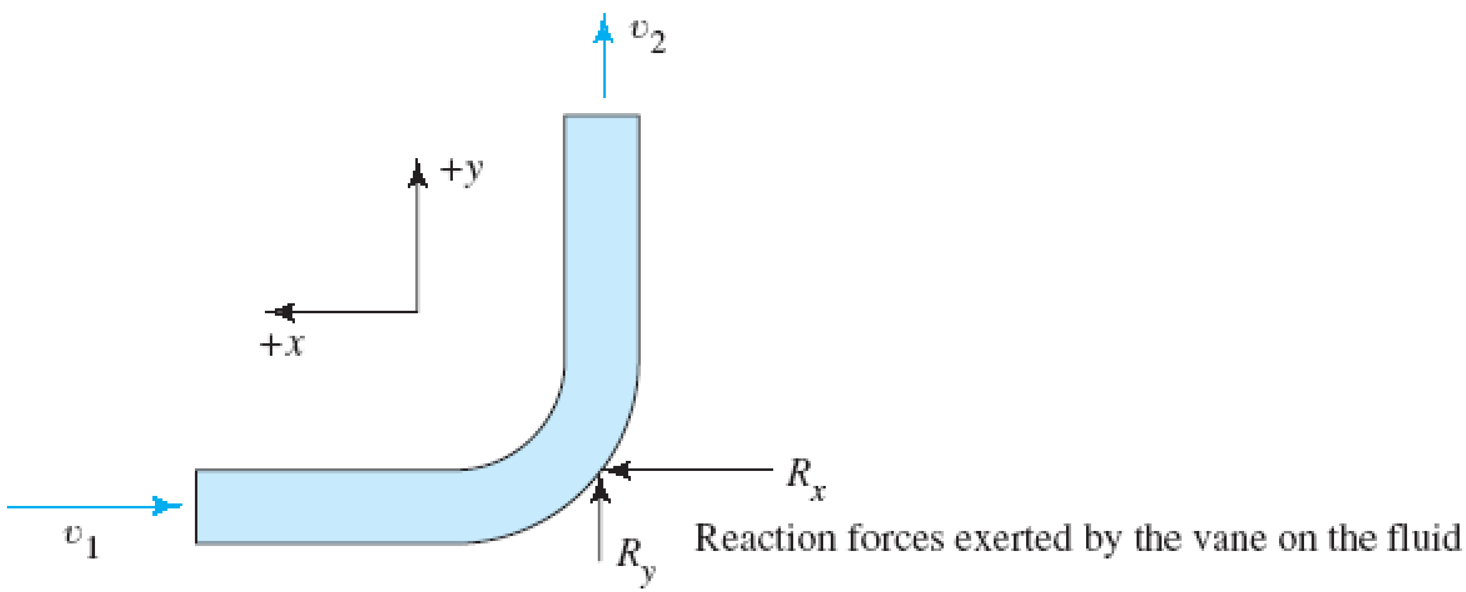


Example 16.1

Using the force diagram of Fig. 16.2, we can write the force equation for the *x* direction

$$F_x = \rho Q(v_{2_x} - v_{1_x})$$

$$R_x = \rho Q[0 - (-v_1)] = \rho Q v_1$$



Example 16.1

We know that

$$Q = Av = (0.0005\ \text{m}^2)(6\ \text{m/s}) = 0.003\ \text{m}^3/\text{s}$$

Then, assuming ρ =1000 kg/m^3, we write

$$R_x = \rho Q v_1 = \frac{1000kg}{m^3} \times \frac{0.003m^3}{s} \times \frac{6m}{s} = 18.0N$$

For the *y* direction, assuming $v_2 = v_1$, the force is

$$F_y = \rho Q(v_{2_y} - v_{1_y})$$
$$R_y = \rho Q(v_2 - 0) = (1000)(0.003)(6) = 18.0\ \text{N}$$

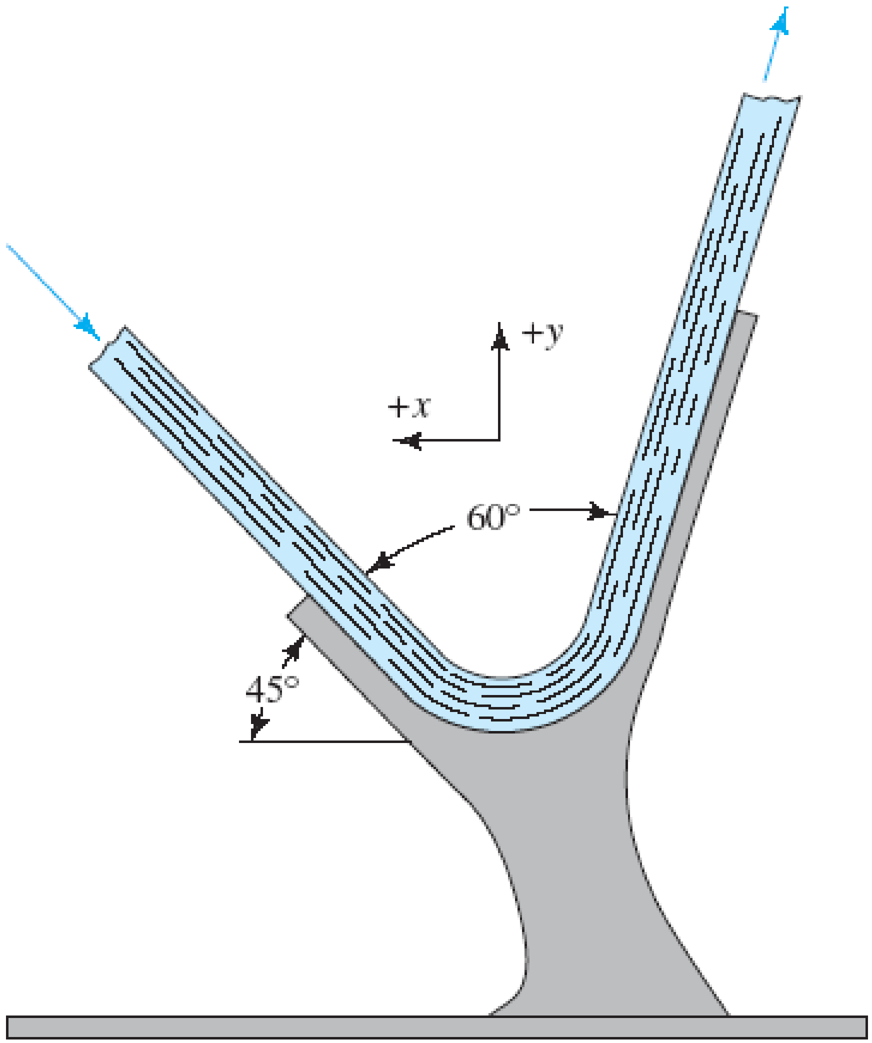
Example 16.2

In a decorative fountain, 0.05 m^3/s of water having a velocity of 8 m/s is being deflected by the angled chute shown in Fig. 16.3. Determine the reactions on the chute in the x and y directions shown. Also calculate the total resultant force and the direction in which it acts. Neglect elevation changes.

Figure 16.4 shows the x and y components of the velocity vectors and the assumed directions for R_x and R_y. The force equation in the x direction is

$$F_x = \rho Q(v_{2_x} - v_{1_x})$$

Example 16.2



Example 16.2

We know that

$$v_{2_x} = -v_2 \sin 15° \qquad \text{(toward the right)}$$
$$v_{1_x} = -v_1 \cos 45° \qquad \text{(toward the right)}$$

Neglecting friction in the chute, we can assume that $v_1 = v_2$. The only external force is R_x. Then we have

$$\begin{aligned} R_x &= \rho Q[-v_2 \sin 15° - (-v_1 \cos 45°)] \\ &= \rho Q v(-\sin 15° + \cos 45°) = 0.448\, \rho Q v \end{aligned}$$

16. Forces Due to Fluids in Motion

Example 16.2

Using ρ = 1000 kg/m³ for water, we get

$$R_x = \frac{(0.448)(1000\,\text{kg})}{\text{m}^3} \times \frac{0.05\,\text{m}^3}{\text{s}} \times \frac{8\,\text{m}}{\text{s}} = \frac{179\,\text{kg}\cdot\text{m}}{\text{s}^2} = 179\,\text{N}$$

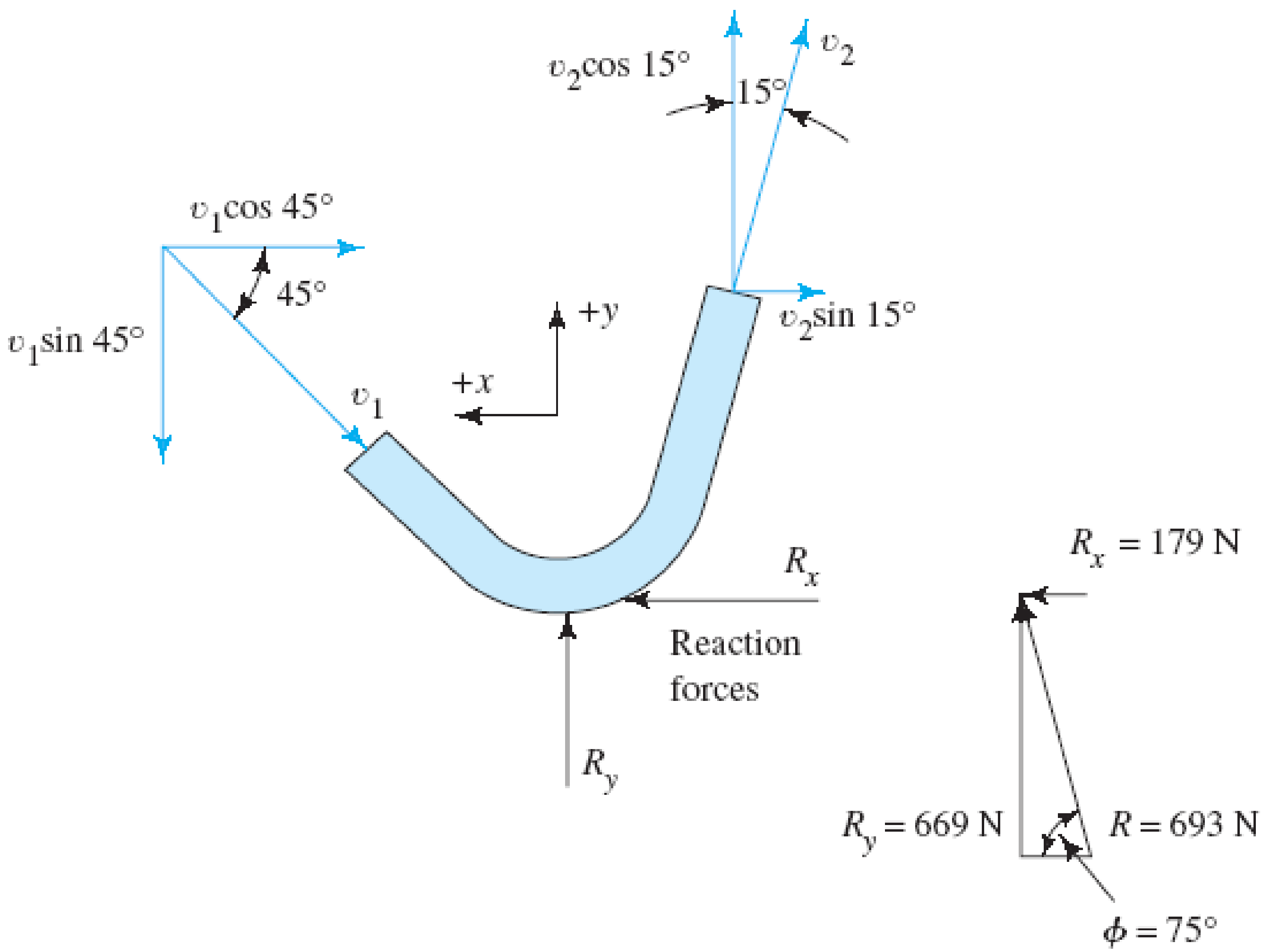
In the *y* direction, the force equation is

$$F_y = \rho Q(v_{2_y} - v_{1_y})$$

We know that

$$\begin{aligned} v_{2_y} &= v_2 \cos 15^\circ \qquad &\text{(upward)} \\ v_{1_y} &= -v_1 \sin 45^\circ \qquad &\text{(downward)} \end{aligned}$$

Example 16.2



$v_2\cos 15°$
$15°$
v_2
$v_1\cos 45°$
$45°$
$v_1\sin 45°$
$+y$
$v_2\sin 15°$
$+x$
v_1
R_x
Reaction
forces
R_y
$R_x = 179$ N
$R_y = 669$ N
$R = 693$ N
$\phi = 75°$

16. Forces Due to Fluids in Motion

Example 16.2

Then we have

$$
\begin{aligned}
R_y &= \rho Q[v_2 \cos 15^\circ - (-v_1 \sin 45^\circ)] \\
&= \rho Q v(\cos 15^\circ + \sin 45^\circ) \\
&= (1000)(0.05)(8)(0.966 + 0.707)\ \mathrm{N} \\
R_y &= 699\ \mathrm{N}
\end{aligned}
$$

The resultant force *R* is

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{179^2 + 669^2} = 693\ \mathrm{N}$$

Example 16.2

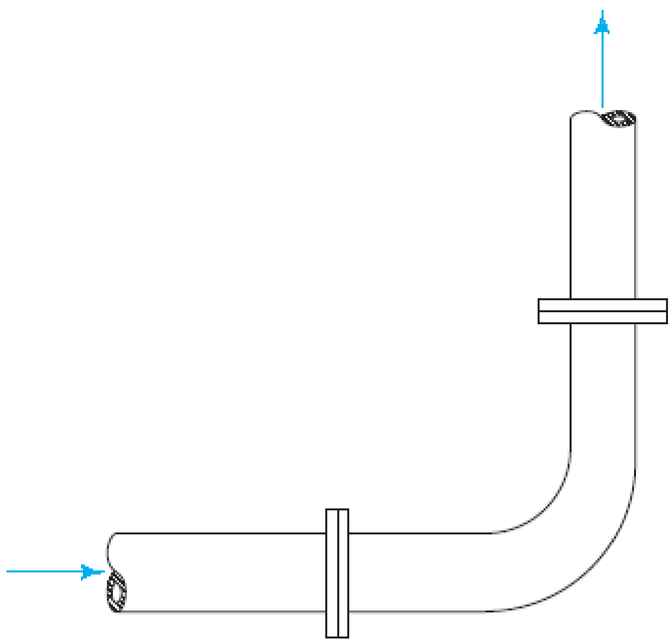
For the direction of *R*, we get

$$\tan \phi = R_y/R_x = 669/179 = 3.74$$
$$\phi = 75.0°$$

Therefore, the resultant force that the chute must exert on the water is 693 N acting 75° from the horizontal, as shown in Fig. 16.4.

16.6 Forces on Bends in Pipeline

- Figure 16.5 shows a typical 90° elbow in a pipe carrying a steady volume flow rate *Q*.
- To ensure proper installation, it is important to know how much force is required to hold it in equilibrium.



Example 16.3

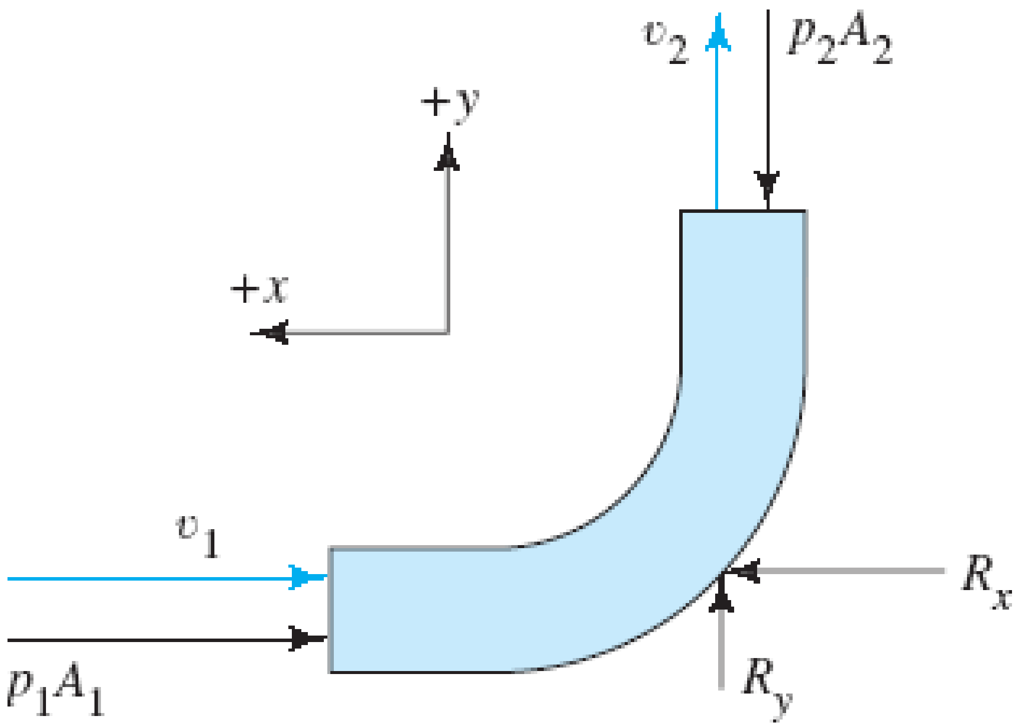
Calculate the force that must be exerted on the pipe shown in Fig. 16.5 to hold it in equilibrium. The elbow is in a horizontal plane and is connected to two 4-in Schedule 40 pipes carrying 3000 L/min of water at 15°C. The inlet pressure is 550 kPa.

Example 16.3

The problem may be visualized by considering the fluid within the elbow to be a free body, as shown in Fig. 16.6. Forces are shown in black vectors, and the direction of the velocity of flow is shown by blue vectors. A convention must be set for the directions of all vectors. Here we assume that the positive *x* direction is to the left and the positive *y* direction is up. The forces R_x and R_y are the external reactions required to maintain equilibrium. The forces p_1A_1 and p_2A_2 are the forces due to the fluid pressure. The two directions will be analyzed separately.

16. Forces Due to Fluids in Motion

Example 16.3



Example 16.3

We find the net external force in the *x* direction by using the equation

$$F_x = \rho Q(v_{2_x} - v_{1_x})$$

We know that

$$F_x = R_x - p_1 A_1$$
$$v_{2_x} = 0$$
$$v_{1_x} = -v_1$$

Then we have

$$R_x - p_1 A_1 = \rho Q[0 - (-v_1)]$$
$$R_x = \rho Q v_1 + p_1 A_1 \qquad (16\text{–}8)$$

16. Forces Due to Fluids in Motion

Example 16.3

From the given data,

$$Q = 3000\ \text{L/min} \times \frac{1\ \text{m}^3/\text{s}}{60\,000\ \text{L/min}} = 0.05\ \text{m}^3/\text{s}$$

$$v_1 = \frac{Q}{A_1} = \frac{0.05\ \text{m}^3/\text{s}}{8.213 \times 10^{-3}\ \text{m}^2} = 6.09\ \text{m/s}$$

$$\rho Q v_1 = \frac{1000\ \text{kg}}{\text{m}^3} \times \frac{0.05\ \text{m}^3}{\text{s}} \times \frac{6.09\ \text{m}}{\text{s}} = 305\ \text{kg}\cdot\text{m/s}^2 = 305\ \text{N}$$

$$p_1 A_1 = \frac{550 \times 10^3\ \text{N}}{\text{m}^2} \times (8.213 \times 10^{-3}\ \text{m}^2) = 4517\ \text{N}$$

Example 16.3

Substituting these values into Eq. (16–8) gives

$$R_x = (305 + 4517)\,\text{N} = 4822\,\text{N}$$

In the *y* direction, the equation for the net external force is

$$F_y = \rho Q(v_{2_y} - v_{1_y})$$

We know that

$$F_y = R_y - p_2 A_2$$

$$v_{2_y} = +v_2$$

$$v_{1_y} = 0$$

Example 16.3

Then we have

$$R_y - p_2A_2 = \rho Q v_2$$
$$R_y = \rho Q v_2 + p_2A_2$$

If energy losses in the elbow are neglected, $v_2 = v_1$ and $p_2 = p_1$ because the sizes of the inlet and outlet are equal. Then,

$$\rho Q v_2 = 305\ \text{N}$$
$$p_2A_2 = 4517\ \text{N}$$
$$R_y = (305 + 4517)\ \text{N} = 4822\ \text{N}$$

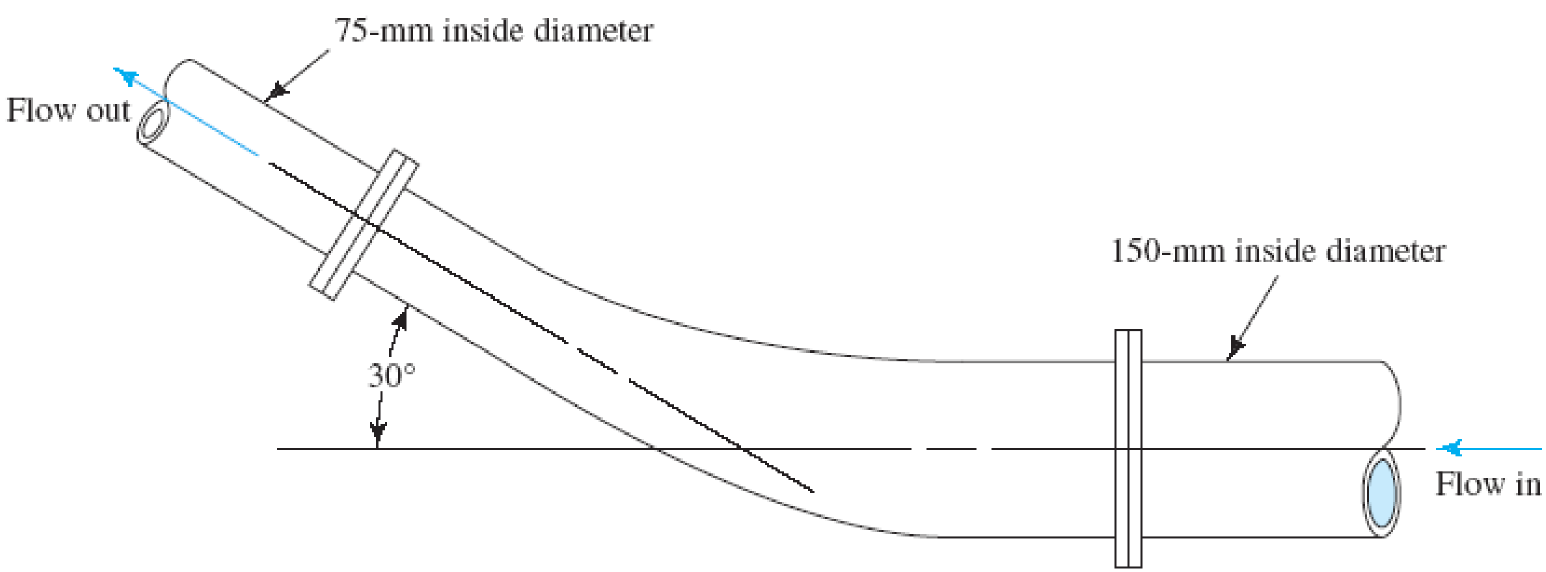
16. Forces Due to Fluids in Motion

Example 16.3

The forces R_x and R_y are the reactions caused at the elbow as the fluid turns 90°. They may be supplied by anchors on the elbow or taken up through the flanges into the main pipes.

Example 16.4

Linseed oil with a specific gravity of 0.93 enters the reducing bend shown in Fig. 16.7 with a velocity of 3 m/s and a pressure of 275 kPa. The bend is in a horizontal plane. Calculate the *x* and *y* forces required to hold the bend in place. Neglect energy losses in the bend.



Example 16.4

The fluid in the bend is shown as a free body in Fig. 16.8. We must first develop the force equations for the *x* and *y* directions shown. The force equation for the *x* direction is

$$F_x = \rho Q(v_{2_x} - v_{1_x})$$

$$R_x - p_1A_1 + p_2A_2\cos 30° = \rho Q[-v_2\cos 30° - (-v_1)] \qquad (16\text{–}9)$$

$$R_x = p_1A_1 - p_2A_2\cos 30° - \rho Q v_2\cos 30° + \rho Q v_1$$

Algebraic signs must be carefully included according to the sign convention shown in Fig. 16.8. Notice that all forces and velocity terms are the components *in the x direction*.

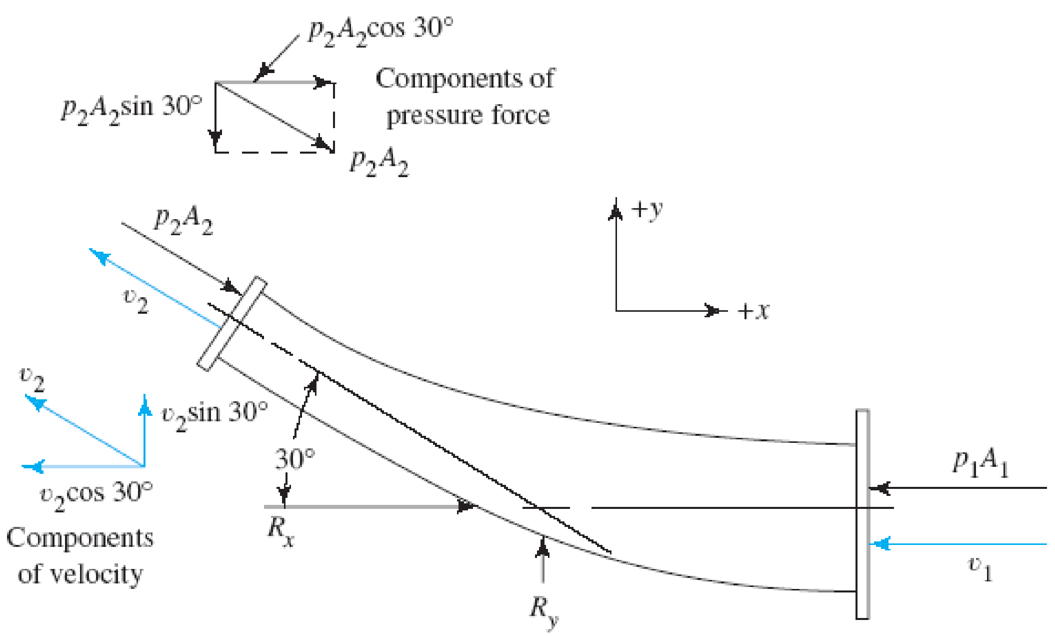
Example 16.4

In the *y* direction, the force equation is

$$F_y = \rho Q(v_{2_y} - v_{1_y})$$

$$R_y - p_2A_2\sin 30° = \rho Q(v_2\sin 30° - 0)$$

$$R_y = p_2A_2\sin 30° + \rho Qv_2\sin 30° \quad (16\text{–}10)$$



Example 16.4

The numerical values of several items must now be calculated. We have

$$A_1 = 1.767 \times 10^{-2}\,\mathrm{m}^2$$

$$A_2 = 4.418 \times 10^{-3}\,\mathrm{m}^2$$

$$\rho = (\mathrm{sg})(\rho_w) = (0.93)(1000\,\mathrm{kg/m^3}) = 930\,\mathrm{kg/m^3}$$
$$\gamma = (\mathrm{sg})(\gamma_w) = (0.93)(9.81\,\mathrm{kN/m^3}) = 9.12\,\mathrm{kN/m^3}$$
$$Q = A_1 v_1 = (1.767 \times 10^{-2}\,\mathrm{m}^2)(3\,\mathrm{m/s}) = 0.053\,\mathrm{m^3/s}$$

Example 16.4

Because of continuity,

$$A_1 v_1 = A_2 v_2.$$

$$v_2 = v_1(A_1/A_2) = (3\,\text{m/s})(1.767 \times 10^{-2}/4.418 \times 10^{-3}) = 12\,\text{m/s}$$

Bernoulli's equation can be used to find p_2:

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

But $z_2 = z_1$.Then we have

$$\begin{aligned} p_2 &= p_1 + \gamma(v_1^2 - v_2^2)/2g \\ &= 275\,\text{kPa} + \left[\frac{(9.12)(3^2 - 12^2)}{(2)(9.81)} \times \frac{\text{kN}}{\text{m}^3} \times \frac{\text{m}^2}{\text{s}^2} \times \frac{\text{s}^2}{\text{m}}\right] \\ &= 275\,\text{kPa} - 62.8\,\text{kPa} \\ p_2 &= 212.2\,\text{kPa} \end{aligned}$$

16. Forces Due to Fluids in Motion

Example 16.4

The quantities needed for Eqs. (16–9) and (16–10) are

$$p_1A_1 = (275\ \mathrm{kN/m^2})(1.767 \times 10^{-2}\ \mathrm{m^2}) = 4859\ \mathrm{N}$$
$$p_2A_2 = (212.2\ \mathrm{kN/m^2})(4.418 \times 10^{-3}\ \mathrm{m^2}) = 938\ \mathrm{N}$$
$$\rho Q v_1 = (930\ \mathrm{kg/m^3})(0.053\ \mathrm{m^3/s})(3\ \mathrm{m/s}) = 148\ \mathrm{N}$$
$$\rho Q v_2 = (930\ \mathrm{kg/m^3})(0.053\ \mathrm{m^3/s})(12\ \mathrm{m/s}) = 591\ \mathrm{N}$$

From Eq. (16–9), we get

$$R_x = (4859 - 938\cos 30^\circ - 591\cos 30^\circ + 148)\ \mathrm{N} = 3683\ \mathrm{N}$$

From Eq. (16–10), we get

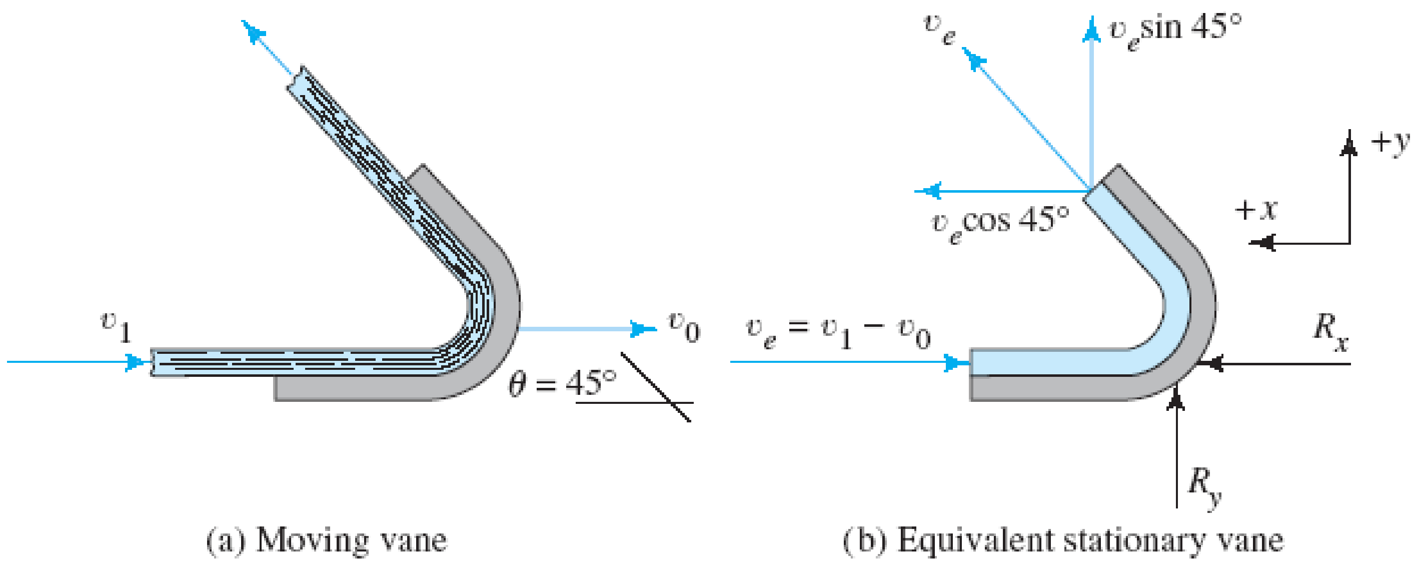
$$R_y = (938\sin 30^\circ + 591\sin 30^\circ)\ \mathrm{N} = 765\ \mathrm{N}$$

16.7 Forces on Moving Objects

- The vanes of turbines and other rotating machinery are familiar examples of moving objects that are acted on by high-velocity fluids.
- A jet of fluid with a velocity greater than that of the blades of the turbine exerts a force on the blades, causing them to accelerate or to generate useful mechanical energy.
- When dealing with forces on moving bodies, the *relative motion* of the fluid with respect to the body must be considered.

Example 16.5

Figure 16.9(a) shows a jet of water with a velocity v_1 striking a vane that is moving with a velocity v_0. Determine the forces exerted by the vane on the water if v_1 = 20 m/s and v_0 = 8 m/s. The jet is 50 mm in diameter.



Example 16.5

The system with a moving vane can be converted into an equivalent stationary system as shown in Fig. 16.9(b) by defining an effective velocity v_e and an effective volume flow rate Q_e. We then have

$$v_e = v_1 - v_0 \qquad (16\text{–}11)$$

$$Q_e = A_1 v_e \qquad (16\text{–}12)$$

where A_1 is the area of the jet as it enters the vane. It is only the difference between the jet velocity and the vane velocity that is effective in creating a force on the vane.

Example 16.5

The force equations can be written in terms of v_e and Q_e. In the *x* direction,

$$R_x = \rho Q_e v_e \cos\theta - (-\rho Q_e v_e)$$
$$= \rho Q_e v_e (1 + \cos\theta) \qquad \textbf{(16–13)}$$

In the *y* direction,

$$R_y = \rho Q_e v_e \sin\theta - 0 \qquad \textbf{(16–14)}$$

We know that

$$v_e = v_1 - v_0 = (20 - 8)\,\text{m/s} = 12\,\text{m/s}$$
$$Q_e = A_1 v_e = (1.964 \times 10^{-3}\,\text{m}^2)(12\,\text{m/s}) = 0.0236\,\text{m}^3/\text{s}$$

Example 16.5

Then the reactions are calculated from Eqs. (16–13) and (16–14):

$$R_x = (1000)(0.0236)(12)(1 + \cos 45^\circ) = 483\ \text{N}$$

$$R_y = (1000)(0.0236)(12)(\sin 45^\circ) = 200\ \text{N}$$