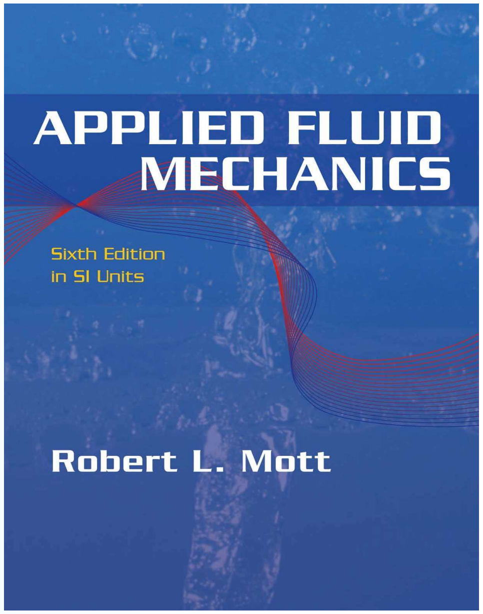
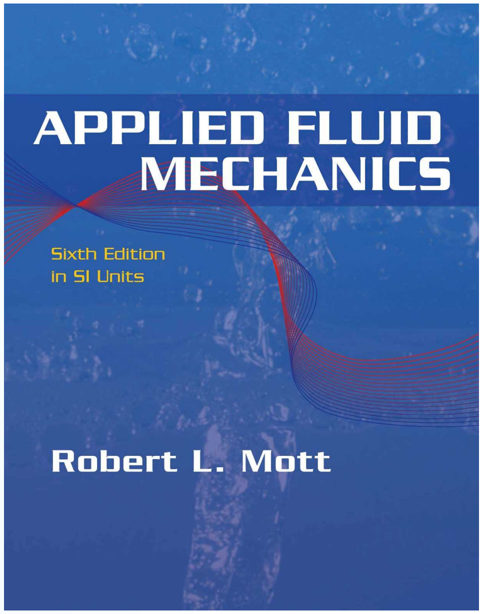
Applied Fluid Mechanics



1. The Nature of Fluid and the Study of Fluid Mechanics
2. Viscosity of Fluid
3. Pressure Measurement
4. Forces Due to Static Fluid
5. Buoyancy and Stability
6. Flow of Fluid and Bernoulli's Equation
7. General Energy Equation
8. Reynolds Number, Laminar Flow, Turbulent Flow and Energy Losses Due to Friction

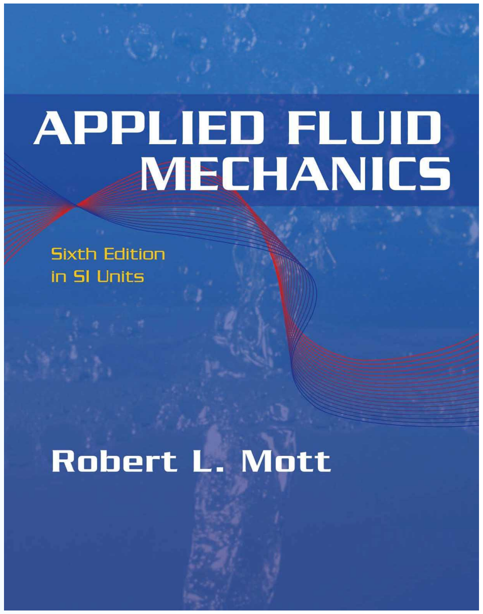
Applied Fluid Mechanics



9. Velocity Profiles for Circular Sections and Flow in Noncircular Sections
10. Minor Losses
11. Series Pipeline Systems
12. Parallel Pipeline Systems
13. Pump Selection and Application
14. Open-Channel Flow
15. Flow Measurement
16. Forces Due to Fluids in Motion

Applied Fluid Mechanics

17. Drag and Lift
18. Fans, Blowers, Compressors and the Flow of Gases
19. Flow of Air in Ducts



18. Fan, Blowers, Compressors and the Flow of Gases

Chapter Objectives

- Describe the general characteristics of fans, blowers, and compressors.
- Describe propeller fans, duct fans, and centrifugal fans.
- Describe blowers and compressors of the centrifugal, axial, vane–axial, reciprocating, lobed, vane, and screw types.
- Specify suitable sizes for pipes carrying steam, air, and other gases at higher pressures.
- Compute the flow rate of air and other gases through nozzles.

18. Fan, Blowers, Compressors and the Flow of Gases

Chapter Outline

1. Introductory Concepts
2. Gas Flow Rates and Pressures
3. Classification of Fans, Blowers and Compressors
4. Flow of Compressed Air and Other Gases in Pipes
5. Flow of Air and Other Gases Through Nozzles

18. Fan, Blowers, Compressors and the Flow of Gases

18.1 Introductory Concepts

- Fans, blowers, and compressors are used to increase pressure and to cause the flow of air and other gases in a gas flow system.
- Special techniques for the design of flow systems carrying gases, such as air, have been developed by professionals based on years of experience.
- The detailed analysis of the phenomena involved requires knowledge of thermodynamics.

18.2 Gas Flow Rates and Pressures

- For systems carrying relatively low flow rates, the unit of L/s is sometimes used.
- Convenient conversions are listed below:

$$1.0\ \text{ft}^3/\text{s} = 60\ \text{ft}^3/\text{min} = 60\ \text{cfm}$$

$$1.0\ \text{m}^3/\text{s} = 2120\ \text{ft}^3/\text{min} = 2120\ \text{cfm}$$

$$1.0\ \text{ft/s} = 60\ \text{ft/min}$$

$$1.0\ \text{m/s} = 3.28\ \text{ft/s}$$

$$1.0\ \text{m/s} = 197\ \text{ft/min}$$

18.2 Gas Flow Rates and Pressures

- Some useful conversion factors are listed below:

$$
\begin{aligned}
1.0\,\text{bar} &= 100\,\text{kPa} \\
1.0\,\text{psi} &= 6895\,\text{Pa} \\
1.0\,\text{inH}_2\text{O} &= 248.8\,\text{Pa} \\
1.0\,\text{mm H}_2\text{O} &= 9.81\,\text{Pa} \\
1.0\,\text{mm Hg} &= 132.8\,\text{Pa}
\end{aligned}
$$

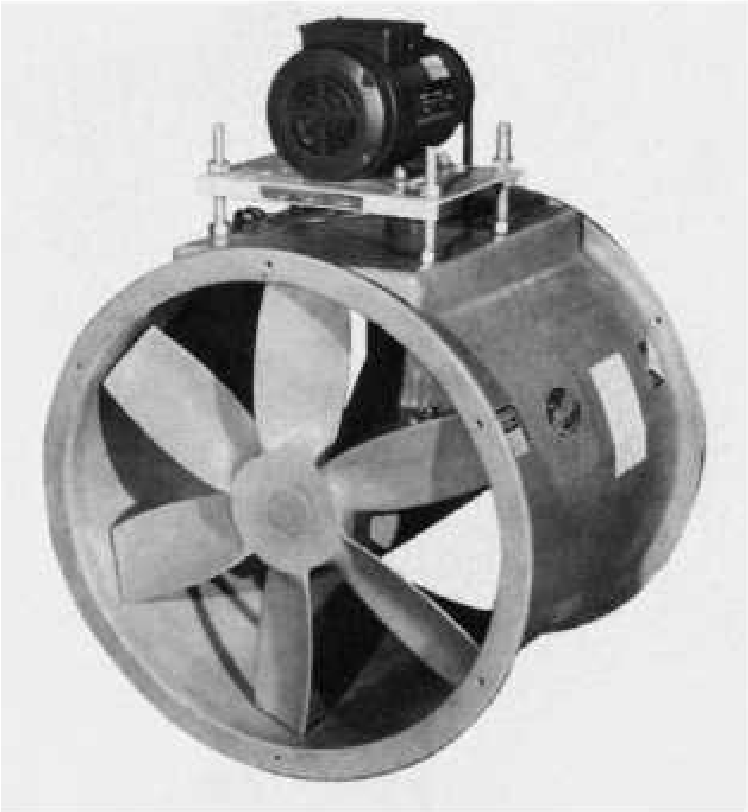
18.3 Classification of Fans, Blowers and Compressors

- Fans, blowers, and compressors are all used to increase the pressure of and move air or other gases.
- The primary differences among them are their physical construction and the pressures that they are designed to develop.
- Fans are used to circulate air within a space, to bring air into or exhaust it from a space, or to move air through ducts in ventilation, heating, or air conditioning systems.

18.3 Classification of Fans, Blowers and Compressors

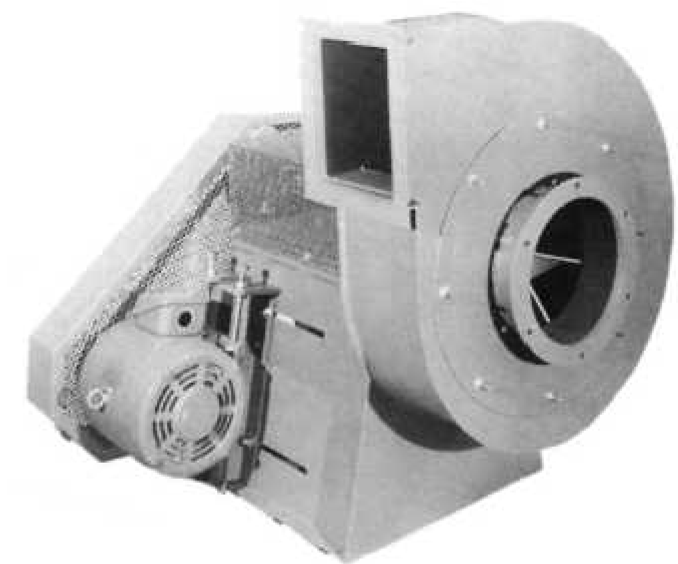
- *Propeller fans* operate at virtually zero static pressure and are composed of two to six blades with the appearance of aircraft propellers.
- Thus, they draw air in from one side and discharge it from the other side in an approximately axial direction.
- *Duct fans* have a construction similar to that of propeller fans, except that the fan is mounted inside a cylindrical duct, as shown in Fig. 18.1.

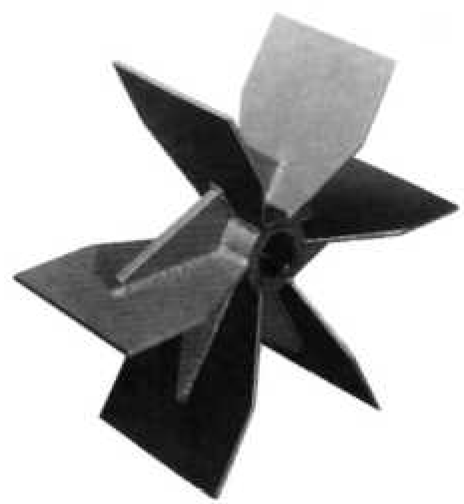
18.3 Classification of Fans, Blowers and Compressors



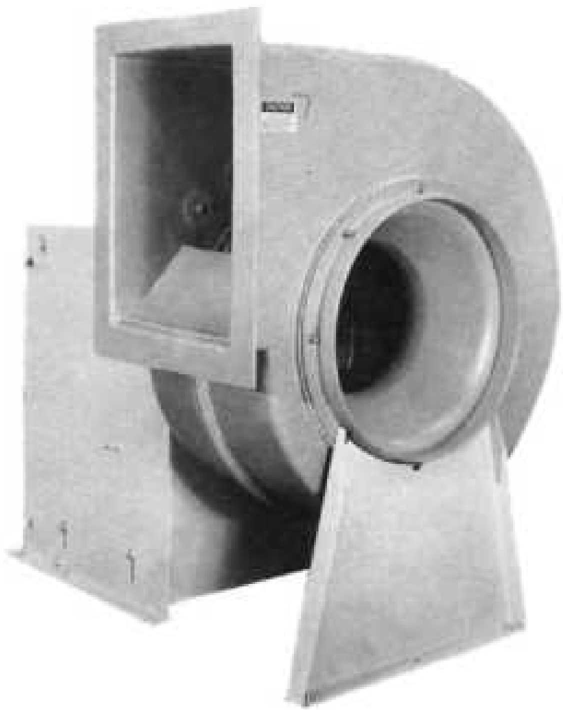
18.3 Classification of Fans, Blowers and Compressors

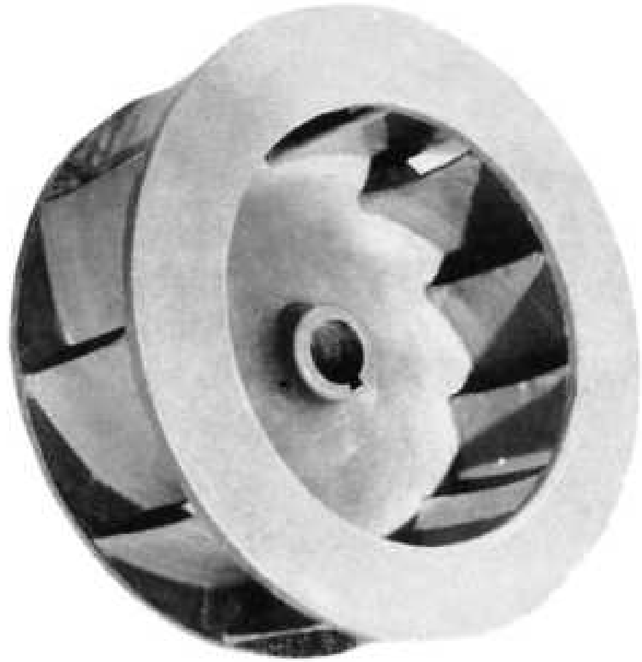
- Two examples of *centrifugal fans* or *centrifugal blowers,* along with their rotors, are shown in Figs. 18.2 and 18.3.





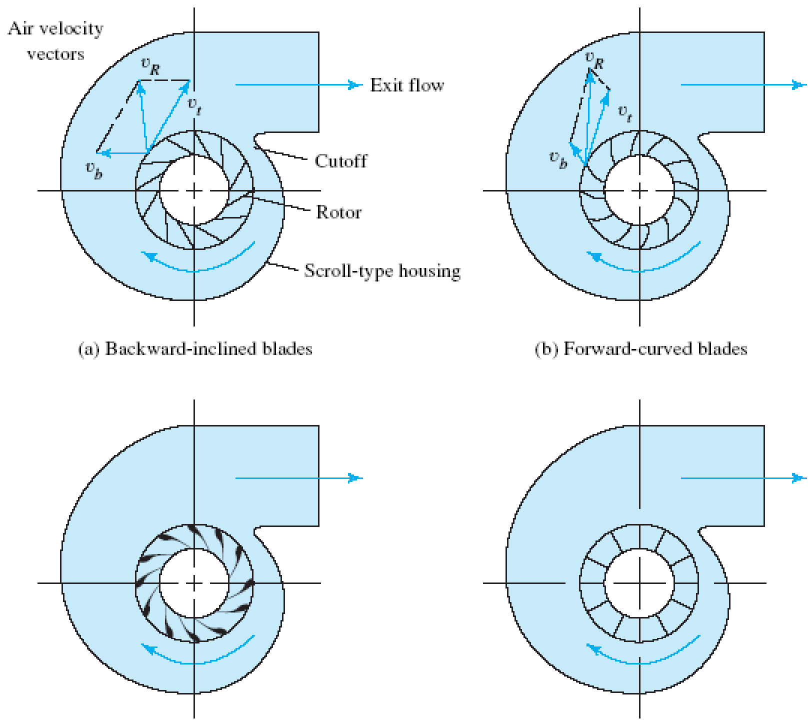
18.3 Classification of Fans, Blowers and Compressors





18.3 Classification of Fans, Blowers and Compressors

- The construction of the rotor is typically one of four basic designs, as shown in Fig. 18.4.
- The *backward-inclined blade* is often made with simple flat plates.



(a) Backward-inclined blades
(b) Forward-curved blades
(c) Airfoil-shaped backward-inclined blades
(d) Straight radial blades

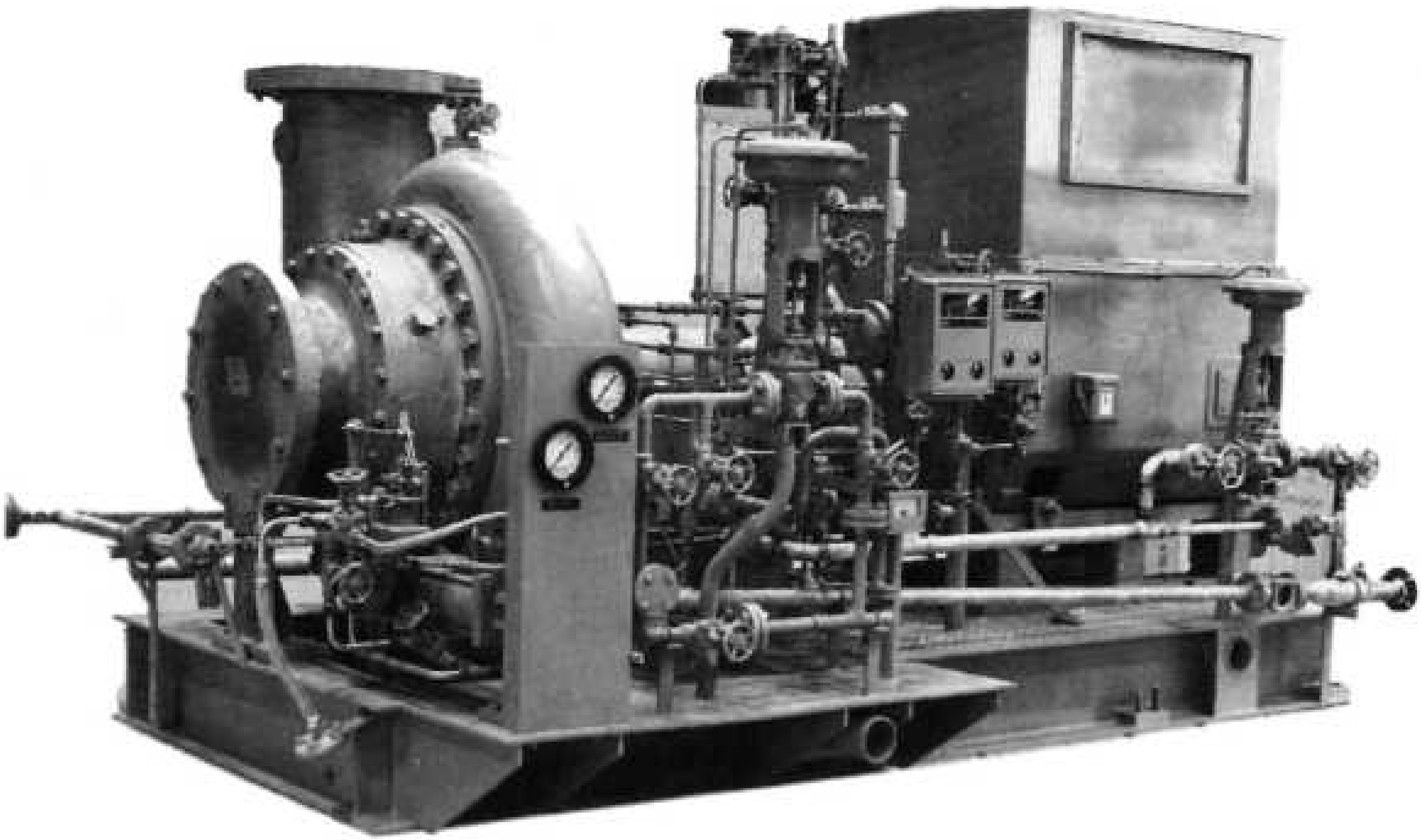
18.3 Classification of Fans, Blowers and Compressors

- *Airfoil-shaped*, *backward inclined fan blades* operate more quietly and efficiently than flat, backward-inclined blades.
- All these types of fans are used for ventilation systems and some industrial process uses.
- *Radial-blade* fans have many applications in industry for supplying large volumes of air at moderate pressures for boilers, cooling towers, material dryers, and bulk material conveying.

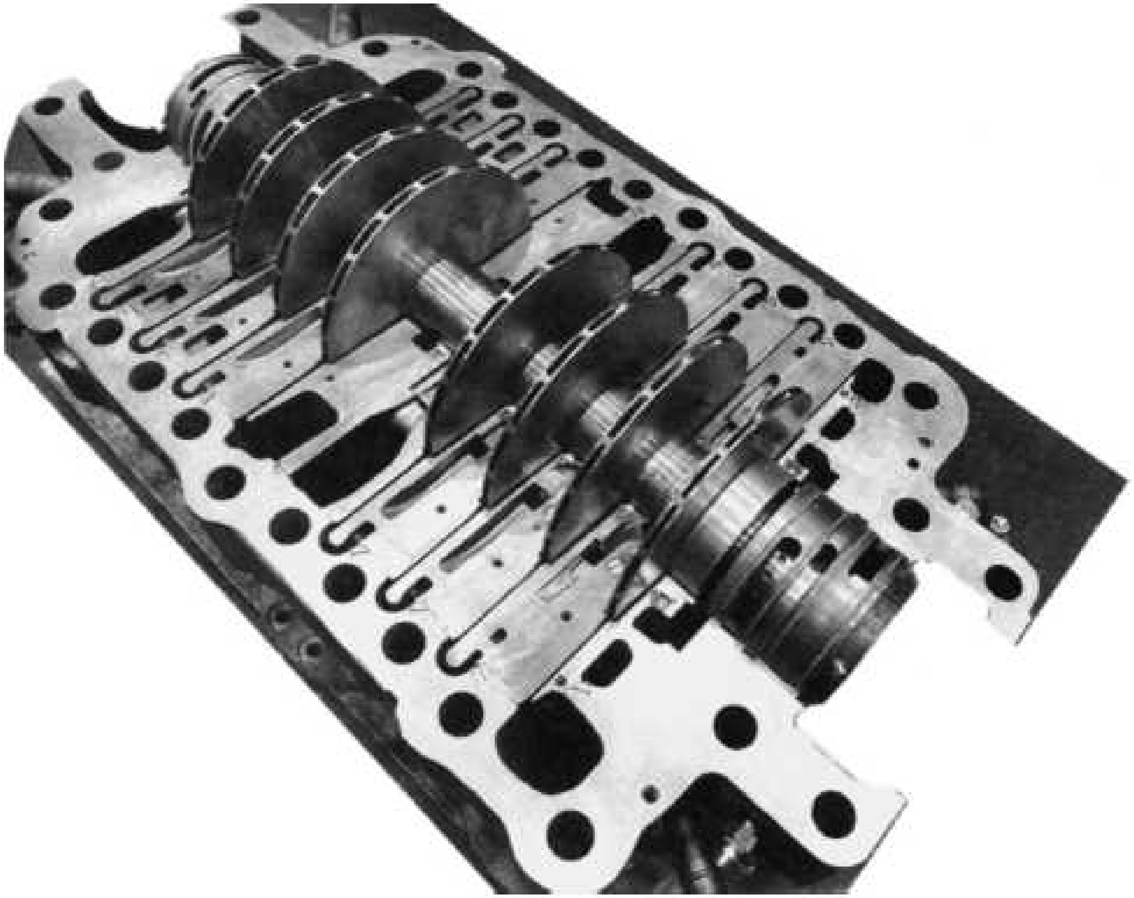
18.3 Classification of Fans, Blowers and Compressors

- *Centrifugal compressors* employ impellers similar to those in centrifugal pumps.
- Figure 18.5 shows a large, single-stage, centrifugal compressor.
- When a single-rotor compressor cannot develop a sufficiently high pressure, a multistage compressor, as shown in Fig. 18.6, is used.
- A multistage *axial compressor* is shown in Fig. 18.7.

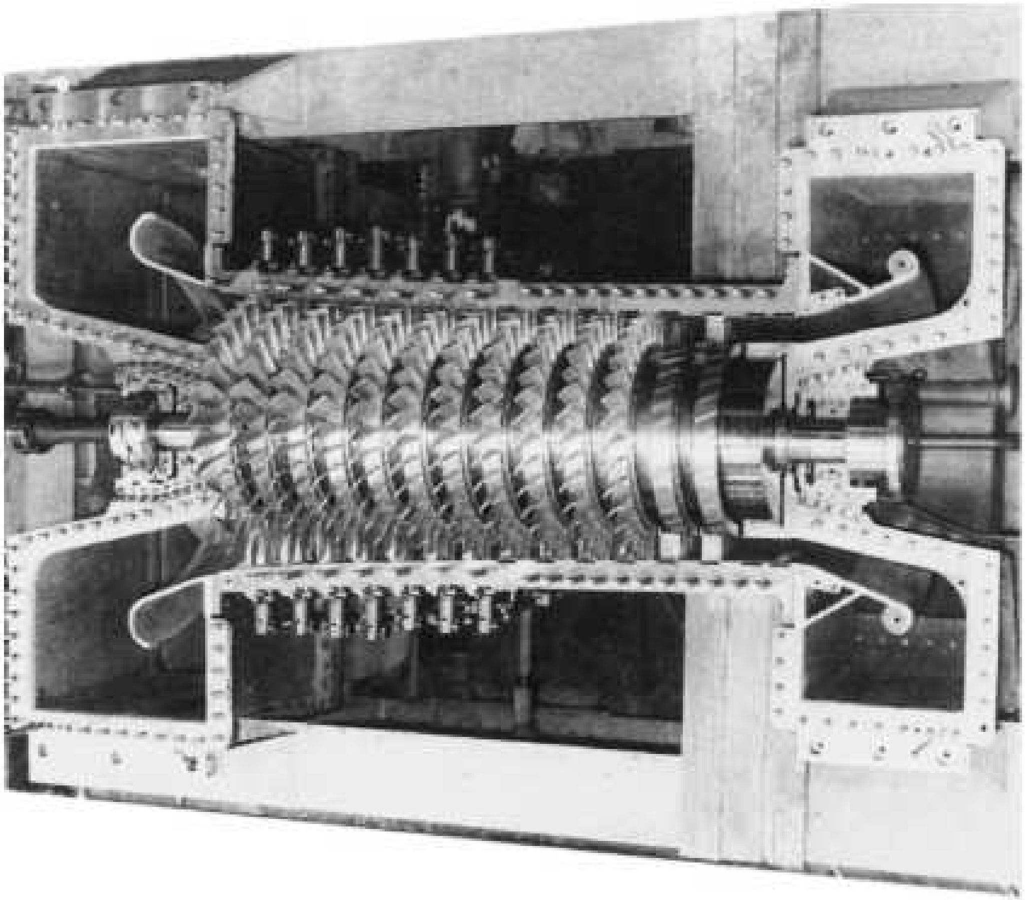
18.3 Classification of Fans, Blowers and Compressors



18.3 Classification of Fans, Blowers and Compressors



18.3 Classification of Fans, Blowers and Compressors



18.3 Classification of Fans, Blowers and Compressors

- *Vane–axial blowers* are similar to duct fans, described earlier, except they typically have blades that are airfoil shaped and include vanes within the cylindrical housing to redirect the flow axially within the following duct.
- *Positive-displacement blowers and compressors* come in a variety of designs:

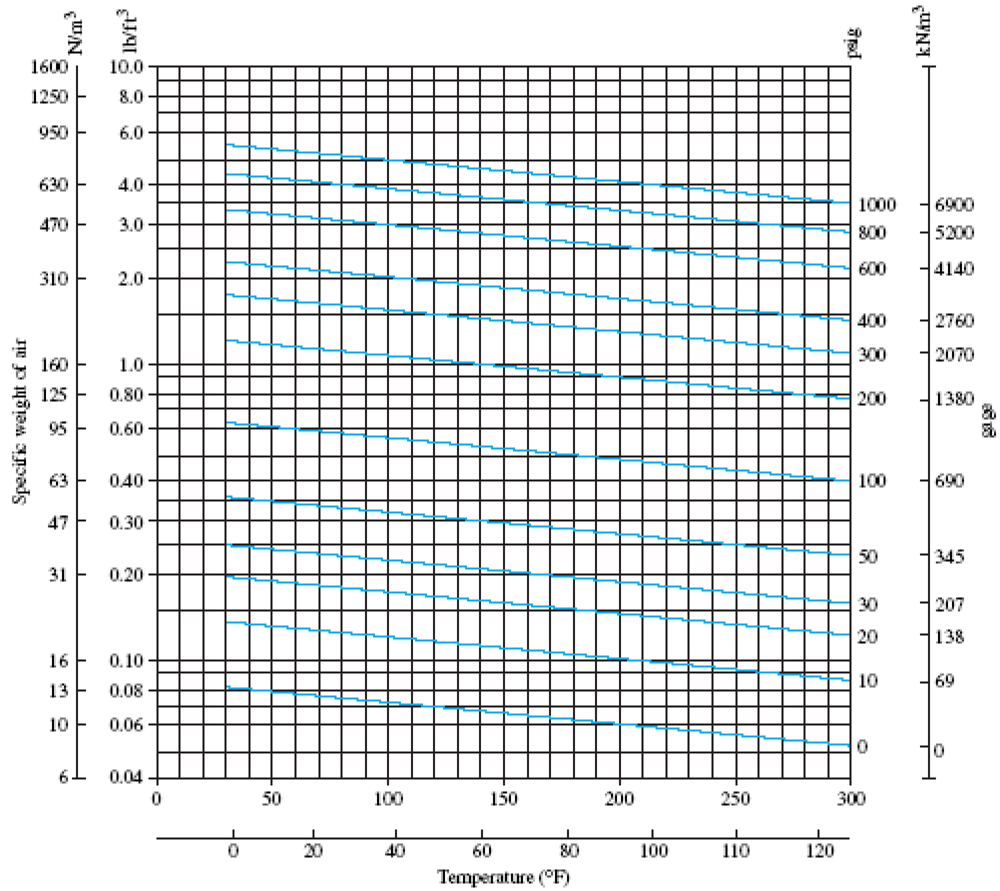
1. Reciprocating—single-acting or double-acting
2. Rotary—lobe, vane, or screw

18.4 Flow of Compressed Air and Other Gases in Pipes

- Many industries use compressed air in fluid power systems to power production equipment, material handling devices, and automation machinery.
- Performance and productivity of the equipment are degraded if the pressure drops below the design pressure.
- When large changes in pressure or temperature of the compressed air occur along the length of a flow system, the corresponding changes in the specific weight of the air should be taken into account.

18.4.1 Specific Weight of Air

- Figure 18.8 shows the variation of the specific weight for air as a function of changes in pressure and temperature.



18.4.1 Specific Weight of Air

- The specific weight for any conditions of pressure and temperature can be computed from the *ideal gas law* from thermodynamics, which states

$$\frac{p}{\gamma T} = \text{constant} = R \tag{18–1}$$

where

p = Absolute pressure of the gas

γ = Specific weight of the gas

T = Absolute temperature of the gas, that is, the temperature above absolute zero

R = Gas constant for the gas being considered

18.4.1 Specific Weight of Air

- Equation (18–1) also can be solved for the specific weight:

$$\gamma = \frac{p}{RT} \qquad (18\text{–}2)$$

- In SI units,

$$T = (t\,°\mathrm{C} + 273)\ \mathrm{K}$$

where K (kelvin) is the standard SI unit for absolute temperature.

Example 18.1

Compute the specific weight of air at 100 psig and 80°F.

Using Eq. (18–2), we find

$$p = p_{\text{atm}} + p_{\text{gage}} = 14.7\ \text{psia} + 100\ \text{psig} = 114.7\ \text{psia}$$

$$T = t + 460 = 80°\text{F} + 460 = 540°\text{R}$$

Then,

$$\gamma = \frac{p}{RT} = \frac{114.7\ \text{lb}}{\text{in}^2} \cdot \frac{\text{lb} \cdot °\text{R}}{53.3\ \text{ft} \cdot \text{lb}} \cdot \frac{1}{540°\text{R}} \cdot \frac{144\ \text{in}^2}{\text{ft}^2}$$

$$\gamma = 0.574\ \text{lb/ft}^3$$

Example 18.1

Note that the quantities 53.3 and 144 will always be used for air in this type of calculation. Then, a convenient, unit-specific equation for specific weight of air can be derived as follows:

$$\gamma = 2.70p/T \qquad (18\text{–}3)$$

Example 18.2

Compute the specific weight of air at 690 kPa gage and 27°C.

Using Eq. (18–2), we get

$$p = p_{atm} + p_{gage} = 101.3 \text{ kPa} + 690 \text{ kPa} = 791.3 \text{ kPa}$$

$$T = t + 273 = 27°\text{C} + 273 = 300 \text{ K}$$

Then,

$$\gamma = \frac{p}{RT} = \frac{791.3 \times 10^3 \text{ N}}{\text{m}^2} \cdot \frac{\text{N} \cdot \text{K}}{29.2 \text{ N} \cdot \text{m}} \cdot \frac{1}{300 \text{ K}} = 90.3 \text{ N/m}^3$$

18.4.2 Flow Rates for Compressed Air Lines

- Ratings for equipment using compressed air and for compressors delivering the air are given in terms of *free air*, sometimes called *free air delivery* (f.a.d.).
- This gives the quantity of air delivered per unit time assuming that the air is at standard atmospheric pressure 101.3 kPa (14.7 psia) absolute and at the standard temperature of 15°C (60°F) (absolute temperatures of 520°R or 285 K).

18.4.2 Flow Rates for Compressed Air Lines

- To determine the flow rate at other conditions, the following equation can be used:

$$Q_a = Q_s \cdot \frac{p_{\text{atm}-s}}{p_{\text{atm}} + p_a} \cdot \frac{T_a}{T_s} \qquad (18\text{–}4)$$

where

Q_a = Volume flow rate at actual conditions

Q_s = Volume flow rate at standard conditions

$p_{\text{atm}-s}$ = Standard absolute atmospheric pressure

p_{atm} = Actual absolute atmospheric pressure

p_a = Actual gage pressure

T_a = Actual absolute pressure

T_s = Standard absolute temperature = 285 K or 520°R

18.4.2 Flow Rates for Compressed Air Lines

- Using these values and those of the standard atmosphere, we can write Eq. (18–4) as follows.
- In U.S. Customary System units:

$$Q_a = Q_s \cdot \frac{14.7 \text{ psia}}{p_{\text{atm}} + p_a} \cdot \frac{(t + 460)°\text{R}}{520°\text{R}} \qquad \textbf{(18–4a)}$$

- In SI units:

$$Q_a = Q_s \cdot \frac{101.3\text{kPa}}{p_{\text{atm}} + p_a} \cdot \frac{(t + 273)\text{K}}{285 \text{ K}} \qquad \textbf{(18–4b)}$$

Example 18.3

An air compressor has a rating of 0.24 m^3/s free air. Compute the flow rate in a pipeline in which the pressure is 689 kPa gage and the temperature is 27°C.

Using Eq. (18–4a) and assuming that the local atmospheric pressure is 101.3 kPa abs, we get

$$Q_a = 0.24 \text{ m}^3/\text{s} \cdot \frac{101.3 \text{ kPa}}{101.3 + 689} \cdot \frac{(27 + 273)}{285} = 0.032 \text{ m}^3/\text{s}$$

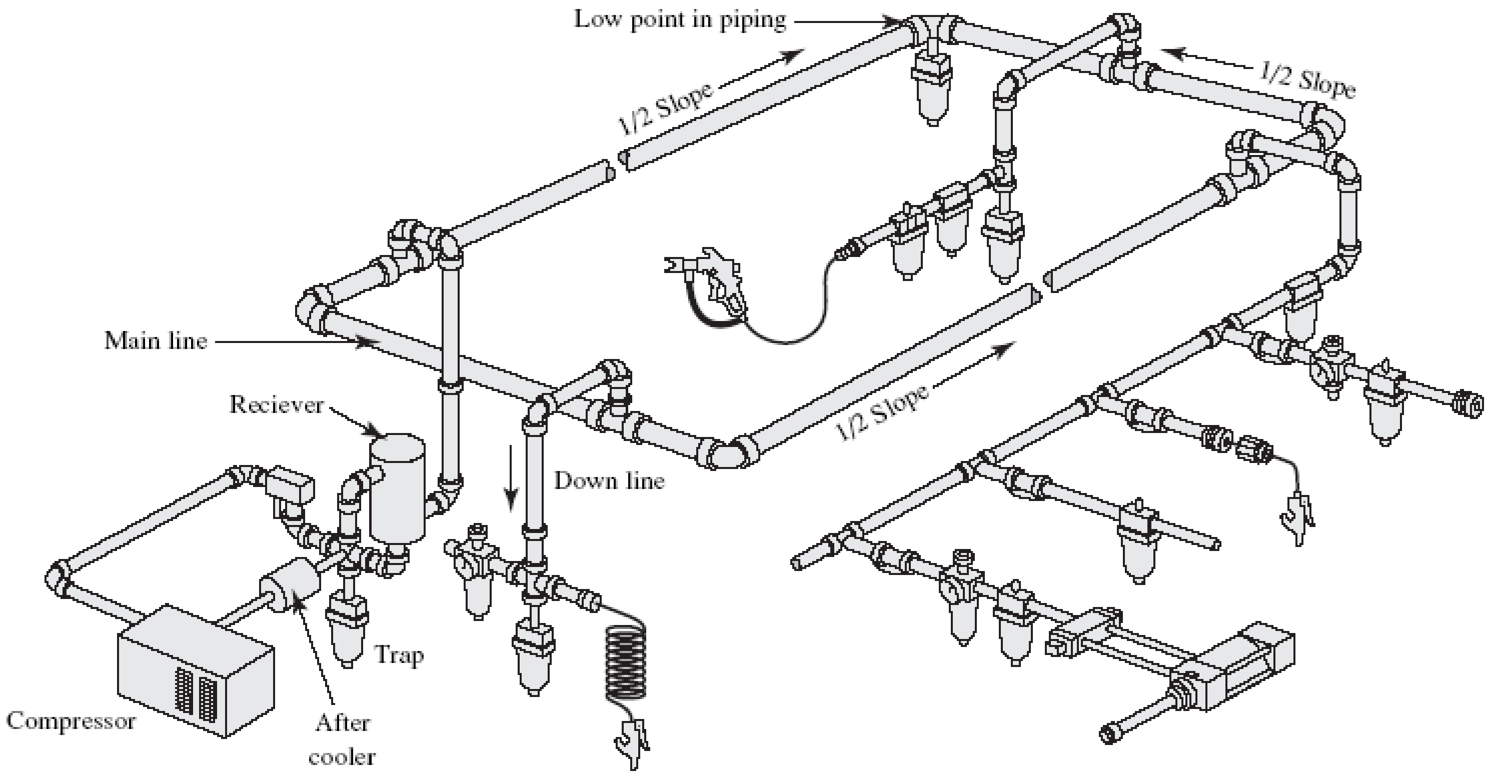
18.4.3 Pipe Size Selection

- Many factors must be considered when specifying a suitable pipe size for carrying compressed air in industrial plants.
- Some of those factors and the parameters involved are as follows:

1. *Pressure drop*
2. *Compressor power requirement*
3. *Cost of piping*
4. *Cost of the compressor*
5. *Installation cost*
6. *Space required*
7. *Future expansion*
8. *Noise*

18.4.3 Pipe Size Selection

- Figure 18.9 shows a sketch of a typical layout of a piping system serving an industrial operation.



Example 18.4

Specify a suitable size of pipe for the delivery of 0.24 m^3/s (500 ctm) (free air) at 690 kPa gage at 27°C (80°F) to an automated machine. The total length of straight pipe required between the compressor and the machine is 42.7 m. The line also contains two fully open gate valves, six standard elbows, and two standard tees, in which the flow is through the run of the tee. Then, analyze the pressure required at the compressor to ensure that the pressure at the machine is no less than 690 kPa gage.

Example 18.4

As a tentative choice, let's consult Table 18.1 and specify a 1.5-inch Schedule 40 steel pipe to carry the air. Then, from Appendix F we find D=40.9 mm and A=1.314 x 10^{-3} m^2. We now check to determine the actual pressure drop through the system and judge its acceptability. The solution procedure is similar to that used in Chapter 11. Special circumstances relative to air will be discussed.

Example 18.4

Maximum Flow Rate (cfm)		
Free Air	Compressed Air (100 psig, 60°F)	Pipe Size (in) (Schedule 40)
4	0.513	1/8
8	1.025	1/4
20	2.563	3/8
35	4.486	1/2
80	10.25	3/4
150	19.22	1
300	38.45	1 1/4
450	57.67	1 1/2
900	115.3	2
1400	179.4	2 1/2
2500	320.4	3
3500	448.6	3 1/2
5000	640.8	4

Note: The sizes listed are the smallest standard Schedule 40 steel pipes that will carry the given flow rate at a pressure of 690 kPa (100 psig) with no more than 34.5 kPa (5.0-psi) pres sure drop in 30.5 m (100 ft) of pipe. See Appendix F for pipe dimensions. (1 cfm = 0.472 L/s, 60°F ≈ 15°C)

Example 18.4

Step 1. Write the energy equation between the outlet from the compressor and the inlet to the machine:

$$\frac{p_1}{\gamma_1} + z_1 + \frac{v_1^2}{2g} - h_L = \frac{p_2}{\gamma_2} + z_2 + \frac{v_2^2}{2g}$$

Note that the specific weight terms have been identified with subscripts for the reference points. Because air is compressible, there could be a significant change in specific weight. However, it is our intention in this design to have a small change in pressure between points 1 and 2.

Example 18.4

The velocity at the two reference points will be the same because we will use the same size of pipe throughout. Then, the velocity head terms can be cancelled from the energy equation.

Step 2. Solve for the pressure at the compressor:

$$p_1 = p_2 + \gamma h_L$$

Step 3. Evaluate the energy loss h_L by using Darcy's equation, and include the effects of minor losses:

$$h_L = f\left(\frac{L}{D}\right)\left(\frac{v^2}{2g}\right) + f_T\left(\frac{L_e}{D}\right)\left(\frac{v^2}{2g}\right)$$

Example 18.4

The *L*/*D* term is the actual ratio of pipe length to flow diameter:

$$\text{Pipe: } L/D = (42.7\ \text{m}/0.0409\ \text{m}) = 1043$$

The equivalent *Le*/*D* values for the valves and fittings are found in Table 10.4:

$$\begin{aligned} \text{2 valves:} \quad & L_e/D = 2(8) = 16 \\ \text{6 elbows:} \quad & L_e/D = 6(30) = 180 \\ \text{2 tees:} \quad & L_e/D = 2(20) = 40 \\ \text{total:} \quad & L_e/D = 236 \end{aligned}$$

Example 18.4

The flow velocity can be computed from the continuity equation. It was determined in Example Problem 18.3 that the flow rate of 0.24 m^3/s of free air at actual conditions of 690 kPa gage and 27°C is 0.031 m^3/s. Then,

$$v = \frac{Q}{A} = \frac{0.031\ \mathrm{m}^3}{\mathrm{s}} \cdot \frac{1}{1.314 \times 10^{-3}\ \mathrm{m}^2} = 23.6\ \mathrm{m/s}$$

The velocity head is

$$\frac{v^2}{2g} = \frac{(23.6)^2\ \mathrm{m}^2/\mathrm{s}^2}{2(9.81\ \mathrm{m/s}^2)} = 28.4\ \mathrm{m}$$

Example 18.4

To evaluate the friction factor *f*, we need the density and the viscosity of the air. Knowing the specific weight of the air, we can compute the density from

$$\rho = \frac{\gamma}{g} = \left(\frac{90.3\ \text{N}}{\text{m}^3}\right)\left(\frac{\text{s}^2}{9.81\ \text{m}}\right) = 9.2\ \text{kg/m}^3$$

The dynamic viscosity of a gas does not change much as pressure changes. So, we can use the data from Appendix E, even though they are for standard atmospheric pressure. The dynamic viscosity is found to be $\mu = 1.83 \times 10^{-5}$ Pa•s.

Example 18.4

It would be incorrect to use the kinematic viscosity listed for air in Appendix E because that value includes the density, which is dramatically different at 690 kPa than it is at atmospheric pressure. We can now compute the Reynolds number:

$$N_R = \frac{vD\rho}{\mu} = \frac{(23.6)(0.0409)(9.2)}{1.83 \times 10^{-5}} = 4.86 \times 10^5$$

The relative roughness is

$$D/\epsilon = 0.0409/4.6 \times 10^{-5} = 895$$

Example 18.4

Then, from the Moody diagram (Fig. 8.6), we read f = 0.021 and

$$(L_e/D)_{total} = 1043 + 236 = 1279$$

Now the energy loss can be computed:

$$h_L = f_T\left(\frac{L_e}{D}\right)_{total}\left(\frac{v^2}{2g}\right) = (0.021)(1279)(28.4) = 762.8 \text{ m}$$

Step 4. Compute the pressure drop in the pipeline:

$$p_1 - p_2 = \gamma h_L = \frac{90.3 \text{ N}}{\text{m}^3} \cdot 762.8 \text{ m} = 68.9 \text{ kPa}$$

Example 18.4

Step 5. Compute the pressure at the compressor:

$$p_1 = p_2 + 68.9\ \text{kPa} = 689\ \text{kPa} + 68.9\ \text{kPa} = 757.9\ \text{kPa}$$

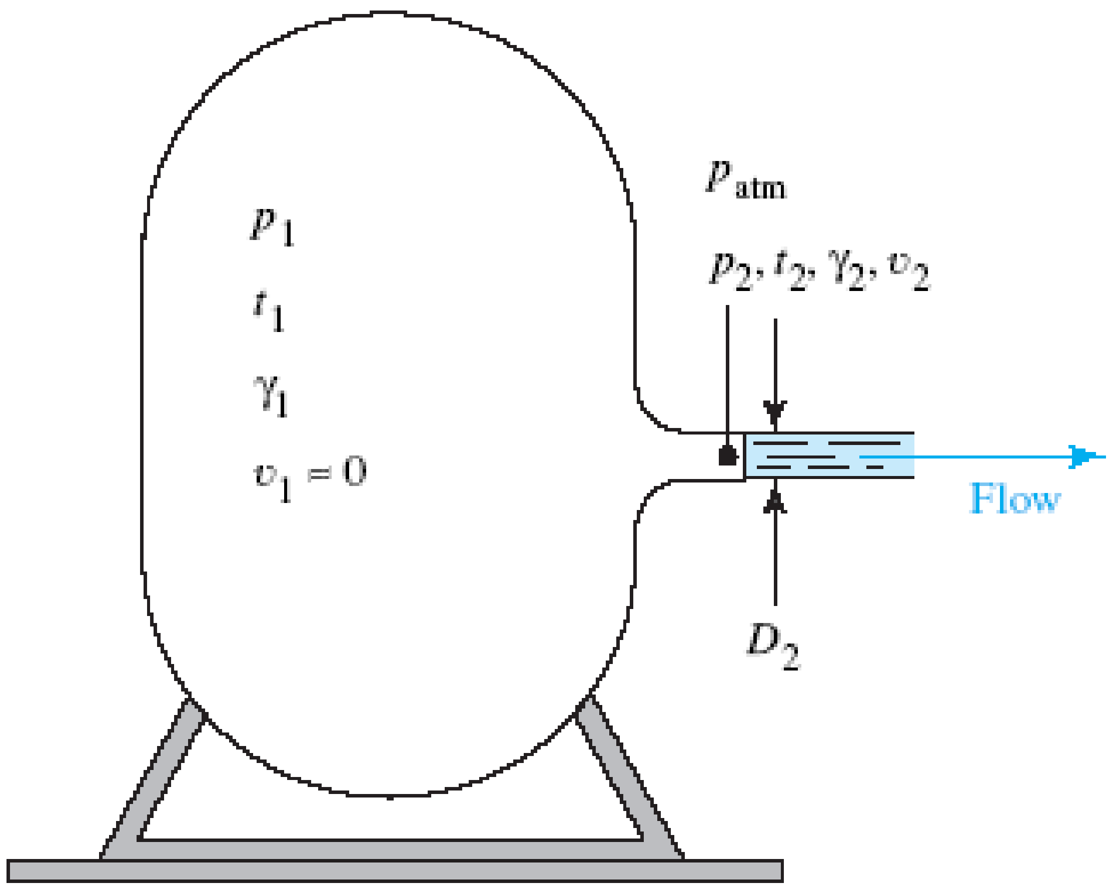
Step 6. Because the change in pressure is less than 10 percent, the assumption that the specific weight of the air is constant is satisfactory. If a larger pressure drop had occurred, we could either redesign the system with a larger pipe size or adjust the specific weight to the average of those at the beginning and end of the system. This system design appears to be satisfactory with regard to pressure drop.

18.5 Flow of Air and Other Gases Through Nozzles

- The typical design of a nozzle is a converging section through which a fluid flows from a region of higher pressure to a region of lower pressure.
- Figure 18.10 shows a nozzle installed in the side of a relatively large tank with flow from the tank to the atmosphere.
- The nozzle shown converges smoothly and gradually, terminating at its smallest section, called the *throat.*

18. Fan, Blowers, Compressors and the Flow of Gases

18.5 Flow of Air and Other Gases Through Nozzles



18.5 Flow of Air and Other Gases Through Nozzles

- When the flow of a gas occurs very slowly, heat from the surroundings can transfer to or from the gas to maintain its temperature constant.
- Such flow is called *isothermal.*
- Under ideal conditions with *no* heat transferred, the flow is called *adiabatic.*
- Real systems behave in some manner between isothermal and adiabatic.

18.5.1 Nozzle Flow for Adiabatic Process

- For an adiabatic process, the equation that describes the relationship between the absolute pressure and the specific weight of the gas is

$$\frac{p}{\gamma^k} = \text{Constant} \qquad (18\text{–}5)$$

- The exponent *k* is called the *adiabatic exponent*, a dimensionless number, and its value for air is 1.40.

18.5.1 Nozzle Flow for Adiabatic Process

- Equation (18–5) can be used to compute the condition of a gas at a point of interest if the condition at some other point is known and if an adiabatic process occurs between the two points.
- That is,

$$\frac{p}{\gamma^k} = \text{Constant} = \frac{p_1}{\gamma_1^k} = \frac{p_2}{\gamma_2^k} \qquad (18\text{–}6)$$

- Stated differently,

$$\frac{p_2}{p_1} = \left(\frac{\gamma_2}{\gamma_1}\right)^k \qquad (18\text{–}7)$$

$$\frac{\gamma_2}{\gamma_1} = \left(\frac{p_2}{p_1}\right)^{1/k} \qquad (18\text{–}8)$$

18.5.1 Nozzle Flow for Adiabatic Process

- The weight flow rate of gas exiting the tank through the nozzle in Fig. 18.10 is

$$W = \gamma_2 v_2 A_2 \qquad (18\text{–}9)$$

- The principles of thermodynamics can be used to show that the velocity of the flow in the nozzle is

$$v_2 = \left\{ \left(\frac{2gp_1}{\gamma_1}\right)\left(\frac{k}{k-1}\right)\left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right]\right\}^{1/2} \qquad (18\text{–}10)$$

18.5.1 Nozzle Flow for Adiabatic Process

- Note that the pressures here are *absolute pressures.*
- The weight rate of flow from the tank in terms of the conditions of the gas in the tank and the pressure ratio

$$W = A_2\sqrt{\frac{2gk}{k-1}(p_1\gamma_1)\left[\left(\frac{p_2}{p_1}\right)^{2/k} - \left(\frac{p_2}{p_1}\right)^{(k+1)/k}\right]} \qquad (18\text{–}11)$$

- Note that a *decreasing* pressure ratio actually indicates an *increasing* pressure difference , and therefore it is expected that the weight flow rate *W* will increase as the pressure ratio is decreased.

18.5.1 Nozzle Flow for Adiabatic Process

- However, it can be shown that the flow rate reaches a maximum at a *critical pressure ratio*, defined as

$$\left(\frac{p'_2}{p_1}\right)_c = \left(\frac{2}{k+1}\right)^{k/(k-1)} \qquad (18\text{–}12)$$

- The speed of sound in the gas is

$$c = \sqrt{\frac{kgp'_2}{\gamma_2}} \qquad (18\text{–}13)$$

18.5.1 Nozzle Flow for Adiabatic Process

- Another name for *c* is the *sonic velocity*, the velocity that a sound wave would travel in the gas.
- *Supersonic velocity*, velocity greater than the speed of sound, can be obtained only with a nozzle that first converges and then diverges.
- The name *Mach number* is given to the ratio of the actual velocity of flow to the sonic velocity. That is,

$$N_M = v/c \qquad (18\text{–}14)$$

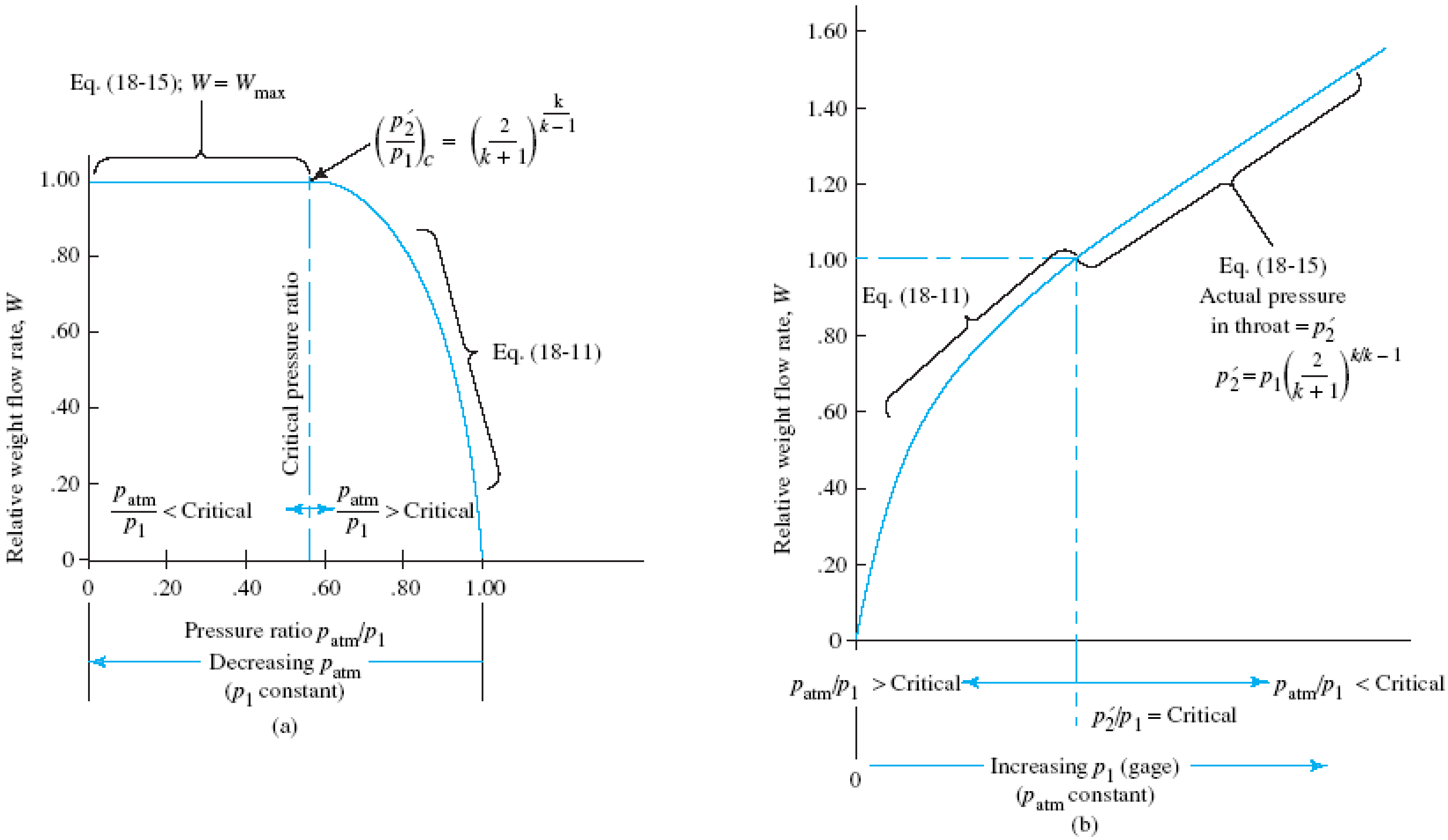
18.5.1 Nozzle Flow for Adiabatic Process

- The maximum weight flow rate of gas from a tank through a converging nozzle,

$$W_{\max} = A_2 \sqrt{\left(\frac{2gk}{k+1}\right)(p_1 \gamma_1)\left(\frac{2}{k+1}\right)^{2/(k-1)}} \qquad (18\text{–}15)$$

- This equation must be used when the pressure ratio is less than the critical ratio.
- Figure 18.11 shows the behavior of gas flow through a nozzle from a relatively large tank, according to Eqs. (18–11) and (18–15).

18.5.1 Nozzle Flow for Adiabatic Process



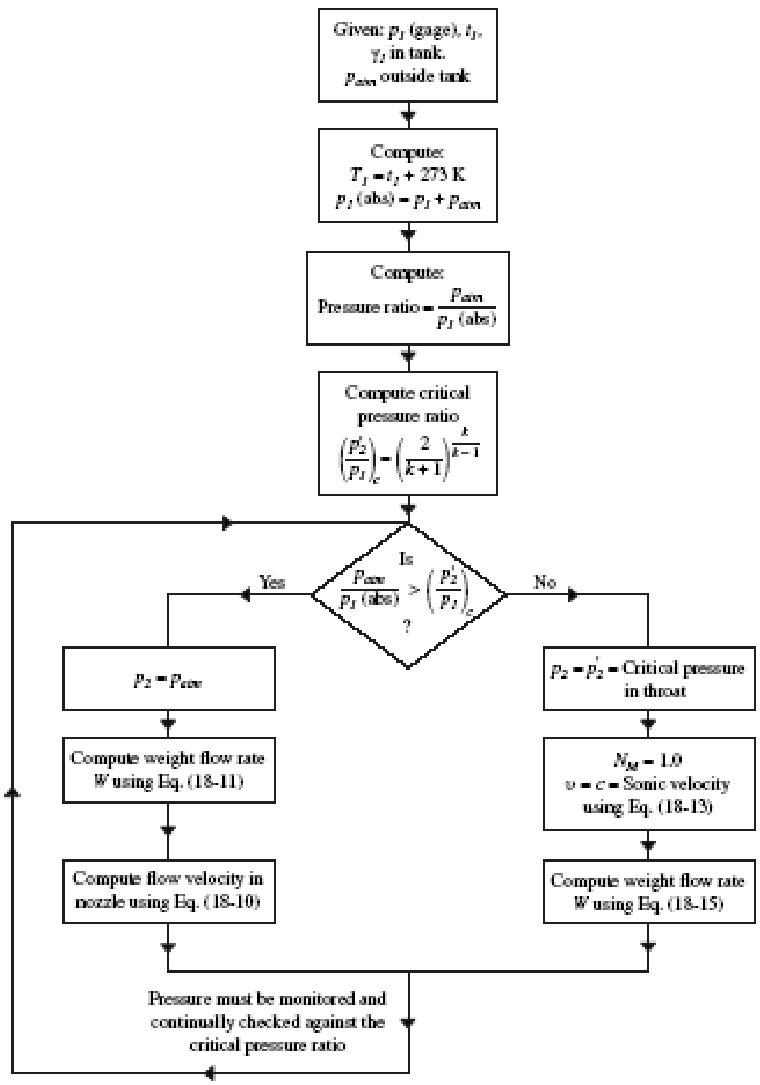
18.5.1 Nozzle Flow for Adiabatic Process

- However, when the critical pressure ratio is reached, the velocity in the throat reaches sonic velocity and the pressure remains at the critical pressure computed from Eq. (18–12). That is,

$$p_2 = p_2' = p_1 \left(\frac{2}{k+1} \right)^{k/(k-1)} \qquad (18\text{–}16)$$

- Figure 18.12 charts the process of calculation.

18.5.1 Nozzle Flow for Adiabatic Process



18.5.1 Nozzle Flow for Adiabatic Process

- Below is the computation of adiabatic flow of a gas through a nozzle:

1. Compute the actual pressure ratio between the pressure outside the nozzle and that in the tank, p_{atm}/p_1.
2. Compute the critical pressure ratio by using Eq. (18–12).

18.5.1 Nozzle Flow for Adiabatic Process

3. If the actual pressure ratio is greater than the critical pressure ratio, use Eq. (18–11) to compute the weight flow rate through the nozzle with $p_2 = p_{atm}$. If desired, the velocity of flow can be computed by using Eq. (18–10).
4. If the actual pressure ratio is less than the critical pressure ratio, use Eq. (18–15) to compute the weight flow rate through the nozzle. Also recognize that the velocity of flow in the throat of the nozzle is equal to the sonic velocity, computed from Eq. (18–13), and that the pressure at the throat is that called p'_2 in Eq. (18–16). The gas then expands to p_{atm} as it leaves the nozzle.

Example 18.5

For the tank with a nozzle in its side, shown in Fig. 18.10, compute the weight flow rate of air leaving the tank for the following conditions:

$$p_1 = 69 \text{ kPa gage} = \text{Pressure in the tank}$$

$$p_{atm} = 101.3 \text{ kPa abs} = \text{Atmospheric pressure outside the tank}$$

$$t_1 = 27°\text{C} = \text{Temperature of the air in the tank}$$

$$D_2 = 2.5 \text{ mm} = \text{Diameter of the nozzle at its outlet}$$

Example 18.5

1. Actual pressure ratio:

$$\frac{p_{atm}}{p_1} = \frac{101.3 \text{ kPa abs}}{(69 + 101.3) \text{ kPa abs}} = 0.59$$

2. Determine the critical pressure ratio from Appendix N. For air, it is 0.528.

3. Because the actual ratio is greater than the critical ratio, Eq. (18–11) is used for the weight flow rate. We must compute the nozzle throat area :

$$A_2 = \pi(D_2)^2/4 = \pi(2.5)^2/4 = 4.91 \text{ mm}^2 = 4.91 \times 10^{-6} \text{ m}^2$$

Example 18.5

Equation (18–2) can be used to compute

$$\gamma_1 = \frac{p_1}{RT_1} = \frac{170.3 \times 10^3 \text{ N/m}^2 \text{ NK}}{(29.2) \text{ Nm } (300 \text{ K})} = 19.4 \text{ N/m}^3$$

Then, using Eq. (18–11), we find the result for *W* in N/s. For these conditions, $p_2 = p_{atm}$:

$$\dot{W} = A_2 \sqrt{\frac{2gk}{k-1}(p_1\gamma_1)\left[\left(\frac{p_2}{p_1}\right)^{2/k} - \left(\frac{p_2}{p_1}\right)^{(k+1)/k}\right]} \qquad (18\text{–}11)$$

$$\dot{W} = (4.91 \times 10^{-6})\sqrt{\frac{2(9.81)(1.4)(170.3 \times 10^3)(19.4)}{(1.4 - 1)} \bullet [(0.59)^{2/1.4} - (0.59)^{2.4/1.4}]}$$

$$= 0.0189 \text{ N/s}$$

Example 18.6

For the conditions used in Example Problem 18.5, compute the velocity of flow in the throat of the nozzle and the Mach number for the flow.

Equation (18–10) must be used to compute the velocity in the throat.

$$\upsilon_2 = \left\{ \frac{2gp_1}{\gamma_1} \left(\frac{k}{k-1} \right) \left[1 - \left(\frac{p_2}{p_1} \right)^{(k-1)/k} \right] \right\}^{1/2} \quad (18\text{–}10)$$

$$\upsilon_2 = \left\{ 2(9.81\ \text{m/s}^2)\, \frac{170.3 \times 10^3\ \text{N/m}^2}{19.4} \left(\frac{1.4}{1.4-1} \right) (1 - (0.59)^{0.4/1.4}) \right\}^{1/2}$$

$$\upsilon_2 = 290.4\ \text{m/s}$$

Example 18.6

To compute the Mach number, we need to compute the speed of sound in the air at the conditions existing in the throat by using Eq. (18–13):

$$c = \sqrt{\frac{kgp_2}{\gamma_2}} \qquad (18\text{–}13)$$

The pressure $p_2 = p_{atm}$ = 101.3 kPa abs.

The specific weight can be computed from Eq. (18–8):

$$\frac{\gamma_2}{\gamma_1} = \left(\frac{p_2}{p_1}\right)^{1/k} \qquad (18\text{–}8)$$

Example 18.6

Knowing that γ_1=19.4 N/m^3, we get

$$\gamma_2 = \gamma_1 \left(\frac{p_2}{p_1} \right)^{1/k}$$

$$\gamma_2 = (19.4 \text{ N/m}^3)(0.59)^{1/1.4} = 13.3 \text{ N/m}^3$$

Then the speed of sound is

$$c = \sqrt{\frac{kgp_2}{\gamma_2}}$$

$$c = \sqrt{\frac{(1.4)(9.81)(101.3 \times 10^3)}{13.3}} = 323.4 \text{ m/s}$$

Example 18.6

Now we can compute the Mach number:

$$N_M = \frac{v}{c} = \frac{290.4 \text{ m/s}}{323.4 \text{ m/s}} = 0.90$$

Example 18.7

Compute the weight flow rate of air from the tank through the nozzle shown in Fig. 18.10 if the pressure in the tank is raised to 137.4 kPa gage. All other conditions are the same as in Example Problem 18.5.

1. Actual pressure ratio:

$$\frac{p_{\text{atm}}}{p_1} = \frac{101.3 \text{ kPa abs}}{(137.4 + 101.3) \text{ kPa abs}} = 0.42$$

Example 18.7

2. The critical pressure ratio is again 0.528 for air.

3. Because the actual pressure ratio is less than the critical pressure ratio, Eq. (18–15) should be used:

$$\dot{W}_{\max} = A_2 \sqrt{\frac{2gk}{k+1}(p_1\gamma_1)\left(\frac{2}{k+1}\right)^{2/(k-1)}} \qquad (18\text{–}15)$$

Computing γ_1 for p_1=238.7 kPa abs, we get

$$\gamma_1 = \frac{p_1}{RT_1} = \frac{238.7 \times 10^3}{(29.2)300} = 27.25\ \mathrm{N/m^3}$$

Example 18.7

Then the weight flow rate is

$$W_{max} = (4.91 \times 10^{-6})\sqrt{\frac{2(9.81)(1.4)(238.7 \times 10^3)27.25}{2.4}\left(\frac{2}{1.4+1}\right)^{2/(1.4-1)}}$$

$$W_{max} = 0.0268 \text{ N/s}$$

The velocity of the air flow at the throat will be the speed of sound at the throat conditions. However, the pressure at the throat must be determined from the critical pressure ratio, Eq. (18–12):

$$\left(\frac{p_2'}{p_1}\right)_c = \left(\frac{2}{k+1}\right)^{k/(k-1)} = 0.528 \qquad (18\text{–}12)$$

$$p_2' = p_1(0.528) = 238.7(0.528) = 126.03 \text{ kPa}$$

Example 18.7

Knowing that γ_1=27.25 N/m^3, we find

$$\gamma_2 = \gamma_1\left(\frac{p_2}{p_1}\right)^{1/k}$$

$$\gamma_2 = 27.25(0.528)^{1/1.4} = 17.26 \text{ N/m}^3$$

Then, the speed of sound and also the velocity in the throat is

$$c = \sqrt{\frac{kgp_2}{\gamma_2}} = \sqrt{\frac{(1.4)(9.81)(126.03 \times 10^3)}{17.26}} = 316.6 \text{ m/s}$$

Of course, the Mach number in the throat is then 1.0.