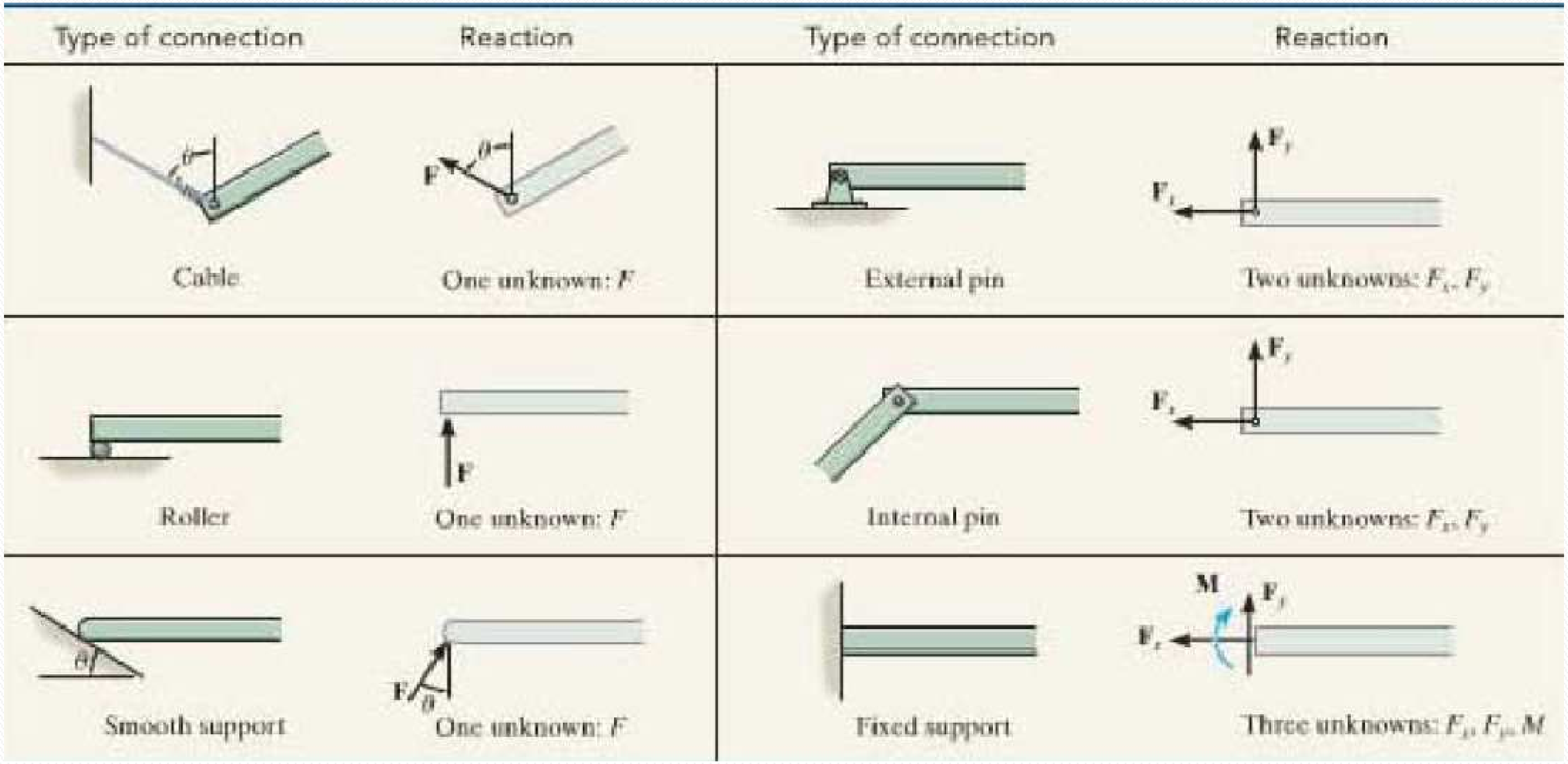
Strength of Material-3

Shear Stress

Dr. Attaullah Shah

Types of supports and reactions:



Units of Stress:

International Standard or SI system, the magnitudes of both normal and shear stress are specified in the basic units of newtons per square meter (N/m^2). This unit, called a pascal ($1\ Pa = 1\ N/m^2$) is rather small, and in engineering work prefixes such as kilo- (10^3), symbolized by k, mega- (10^6), symbolized by M, or giga- (10^9), symbolized by G, are used to represent larger, more realistic values of stress.* Likewise, in the Foot-Pound-Second system of units, engineers usually express stress in pounds per square inch (psi) or kilopounds per square inch (ksi), where 1 kilopound (kip) = 1000 lb.

Important Points

- When a body subjected to external loads is sectioned, there is a distribution of force acting over the sectioned area which holds each segment of the body in equilibrium. The intensity of this internal force at a point in the body is referred to as *stress*.
- Stress is the limiting value of force per unit area, as the area approaches zero. For this definition, the material is considered to be continuous and cohesive.
- The magnitude of the stress components at a point depends upon the type of loading acting on the body, and the orientation of the element at the point.
- When a prismatic bar is made from homogeneous and isotropic material, and is subjected to an axial force acting through the centroid of the cross-sectional area, then the center region of the bar will deform uniformly. As a result, the material will be subjected *only to normal stress*. This stress is uniform or *averaged* over the cross-sectional area.

Procedure for Analysis

The equation $\sigma = P/A$ gives the *average* normal stress on the cross-sectional area of a member when the section is subjected to an internal resultant normal force **P**. For axially loaded members, application of this equation requires the following steps.

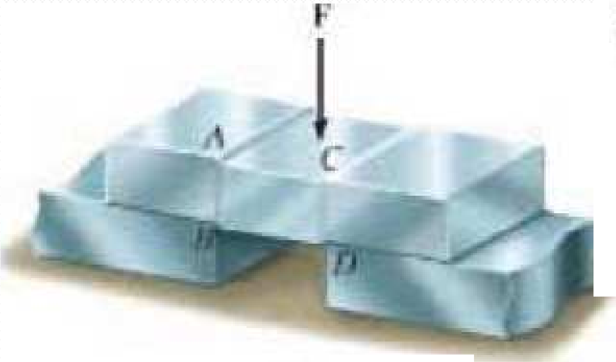
Internal Loading.

- Section the member *perpendicular* to its longitudinal axis at the point where the normal stress is to be determined and use the necessary free-body diagram and force equation of equilibrium to obtain the *internal axial force* **P** at the section.

Average Normal Stress.

- Determine the member's cross-sectional area at the section and calculate the average normal stress $\sigma = P/A$.
- It is suggested that σ be shown acting on a small volume element of the material located at a point on the section where stress is calculated. To do this, first draw σ on the face of the element coincident with the sectioned area A. Here σ acts in the *same direction* as the internal force **P** since all the normal stresses on the cross section develop this resultant. The normal stress σ on the other face of the element acts in the opposite direction.

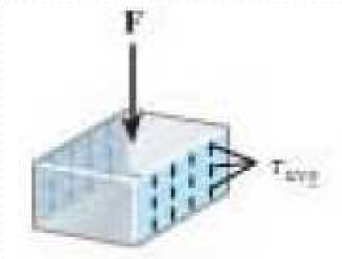
Average Shear Stress



τ_{avg} = average shear stress at the section, which is assumed to be the *same* at each point located on the section

V = internal resultant shear force on the section determined from the equations of equilibrium

A = area at the section



(c)

$$\tau_{avg} = \frac{V}{A}$$

The loading case discussed here is an example of ***simple or direct shear***, since the shear is caused by the *direct action* of the applied load **F**. This type of shear often occurs in various types of simple connections that use bolts, pins, welding material, etc. In all these cases, however, application of Eq. 1–7 is *only approximate*. A more precise investigation of the shear-stress distribution over the section often reveals that much larger shear stresses occur in the material than those predicted by this equation. Although this may be the case, application of Eq. 1–7 is generally acceptable for many problems in engineering design and analysis. For example, engineering codes allow its use when considering design sizes for fasteners such as bolts and for obtaining the bonding strength of glued joints subjected to shear loadings.

Procedure for Analysis

The equation $\tau_{avg} = V/A$ is used to determine the *average shear stress* in the material. Application requires the following steps.

Internal Shear.

- Section the member at the point where the average shear stress is to be determined.
- Draw the necessary free-body diagram, and calculate the internal shear force **V** acting at the section that is necessary to hold the part in equilibrium.

Average Shear Stress.

- Determine the sectioned area A, and determine the average shear stress $\tau_{avg} = V/A$.
- It is suggested that τ_{avg} be shown on a small volume element of material located at a point on the section where it is determined. To do this, first draw τ_{avg} on the face of the element, coincident with the sectioned area A. This stress acts in the *same direction* as **V**. The shear stresses acting on the three adjacent planes can then be drawn in their appropriate directions following the scheme shown in Fig. 1–21.

Allowable Stress

- It is necessary to restrict the stresses in material due to:
  - To have safety against any errors
  - Unknown vibrations, accidental loads and over loads
  - Atmospheric erosions and decays etc.
  - Due to high variability in material properties.
- Factor of Safety

One method of specifying the allowable load for a member is to use a number called the factor of safety. The ***factor of safety*** (F.S.) is a ratio of the failure load F_{fail} to the allowable load F_{allow}. Here F_{fail} is found from experimental testing of the material, and the factor of safety is selected based on experience so that the above mentioned uncertainties are accounted for when the member is used under similar conditions of loading and geometry. Stated mathematically,

$$\text{F.S.} = \frac{F_{fail}}{F_{allow}} \tag{1-8}$$

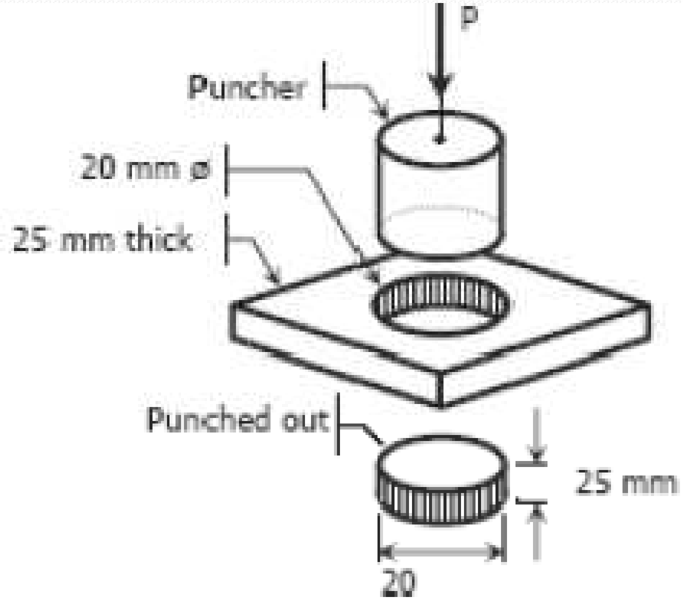
In any of these equations, the factor of safety must be *greater* than 1 in order to avoid the potential for failure. Specific values depend on the types of materials to be used and the intended purpose of the structure or machine. For example, the F.S. used in the design of aircraft or space-vehicle components may be close to 1 in order to reduce the weight of the vehicle. Or, in the case of a nuclear power plant, the factor of safety for some of its components may be as high as 3 due to uncertainties in loading or material behavior. In many cases, the factor of safety for a specific case can be found in design codes and engineering handbooks. These values are intended to form a balance of ensuring public and environmental safety and providing a reasonable economic solution to design.

Shear Stress-Solved Examples

- What force is required to punch a 20-mm-diameter hole in a plate that is 25 mm thick? The shear strength is 350 MN/m².

The resisting area is the shaded area along the perimeter and the shear force V is equal to the punching force P.

$$V = \tau A$$
$$P = 350[\pi(20)(25)]$$
$$= 549\,778.7 \text{ N}$$
$$= 549.8 \text{ kN}$$



As in Fig. 1-11c, a hole is to be punched out of a plate having a shearing strength of 40 ksi. The compressive stress in the punch is limited to 50 ksi. **(a)** Compute the maximum thickness of plate in which a hole 2.5 inches in diameter can be punched. **(b)** If the plate is 0.25 inch thick, determine the diameter of the smallest hole that can be punched.

(a) Maximum thickness of plate:

Based on puncher strength:

$$P = \sigma A$$
$$= 50[\tfrac{1}{4}\pi(2.5^2)]$$
$$= 78.125\pi \text{ kips}$$ → Equivalent shear force of the plate

Based on shear strength of plate:

$$V = \tau A$$ → V = P

$$78.125\pi = 40[\pi(2.5t)]$$
$$t = 0.781 \text{ inch}$$

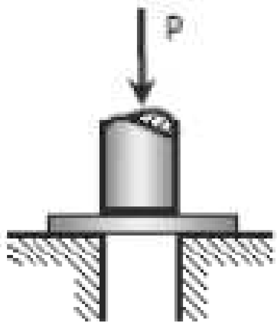


Figure 1-11c

(b) Diameter of smallest hole:

Based on compression of puncher:

$$P = \sigma A$$
$$= 50(\tfrac{1}{4}\pi d^2)$$
$$= 12.5\pi d^2$$ → Equivalent shear force for plate

Based on shearing of plate:

$$V = \tau A$$ → V = P

$$12.5\pi d^2 = 40[\pi d(0.25)]$$
$$d = 0.8 \text{ in}$$

Find the smallest diameter bolt that can be used in the clevis shown in Fig. 1-11b if P = 400 kN. The shearing strength of the bolt is 300 MPa.

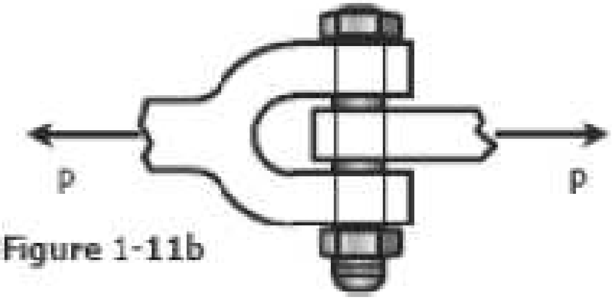


Figure 1-11b

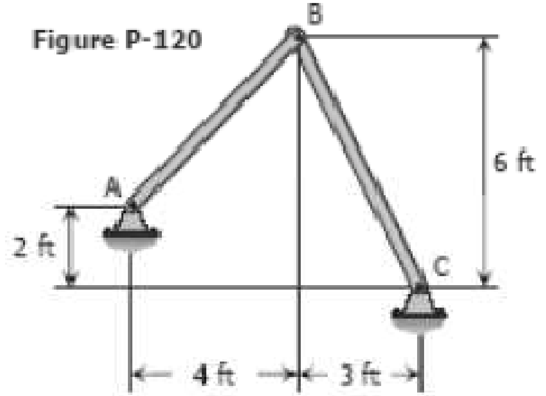
The bolt is subject to double shear.

$V = \tau A$

$400(1000) = 300[2(\frac{1}{4}\pi d^2)]$

$d = 29.13$ mm

The members of the structure in Fig. P-120 weigh 200 lb/ft. Determine the smallest diameter pin that can be used at A if the shearing stress is limited to 5000 psi. Assume single shear.



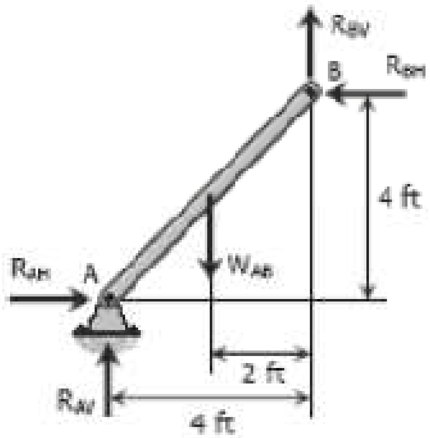
For member *AB*:

Length, $L_{AB} = \sqrt{4^2 + 4^2}$

$= 5.66$ ft

Weight, $W_{AB} = 5.66(200)$

$= 1132$ lb



FBD of member

$\Sigma M_A = 0$

$4R_{BH} + 4R_{BV} = 2W_{AB}$

$4R_{BH} + 4R_{BV} = 2(1132)$

$R_{BH} + R_{BV} = 566 \quad \rightarrow (1)$

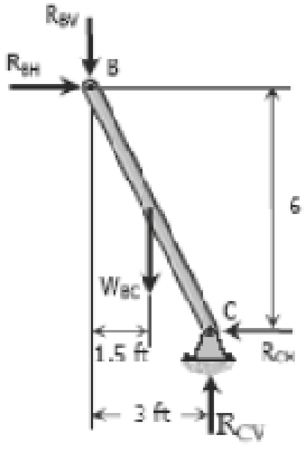
For member *BC*:

Length, $L_{BC} = \sqrt{3^2 + 6^2}$

$= 6.71$ ft

Weight, $W_{BC} = 6.71(200)$

$= 1342$ lb



FBD of member BC

$\sum M_C = 0$

$6R_{BH} = 1.5W_{BC} + 3R_{BV}$

$6R_{BH} - 3R_{BV} = 1.5(1342)$

$2R_{BH} - R_{BV} = 671 \quad \rightarrow (2)$

Add equations (1) and (2)

$R_{BH} + R_{BV} = 566 \quad \rightarrow (1)$

$2R_{BH} - R_{BV} = 671 \quad \rightarrow (2)$

$3R_{BH} = 1237$

$R_{BH} = 412.33$ lb

From equation (1):

$412.33 + R_{BV} = 566$

$R_{BV} = 153.67$ lb

From the FBD of member *AB*

$\sum F_H = 0$

$R_{AH} = R_{BH} = 412.33$ lb

$\sum F_V = 0$

$R_{AV} + R_{BV} = W_{AB}$

$R_{AV} + 153.67 = 1132$

$R_{AV} = 978.33$ lb

$$R_A = \sqrt{R_{AH}^2 + R_{AV}^2}$$

$$= \sqrt{412.33^2 + 978.33^2}$$

$= 1061.67$ lb → shear force of pin at A

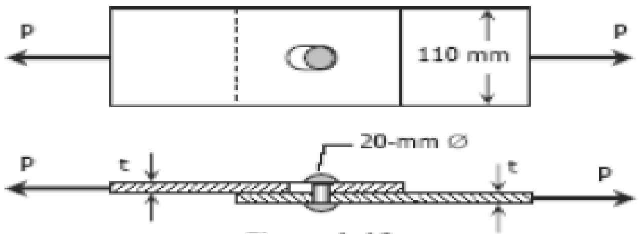
$V = \tau A$

$1061.67 = 5000(\frac{1}{4}\pi d^2)$

$d = 0.520$ in

Bearing Stresses: Solved Examples

In Fig. 1-12, assume that a 20-mm-diameter rivet joins the plates that are each 110 mm wide. The allowable stresses are 120 MPa for bearing in the plate material and 60 MPa for shearing of rivet. Determine (a) the minimum thickness of each plate; and (b) the largest average tensile stress in the plates.



ure 1-12

(a) From shearing of rivet:

$P = \tau A_{rivets}$

$= 60[\frac{1}{4}\pi(20^2)]$

$= 6000\pi$ N

From bearing of plate material:

$P = \sigma_b A_b$

$6000\pi = 120(20t)$

$t = 7.85$ mm

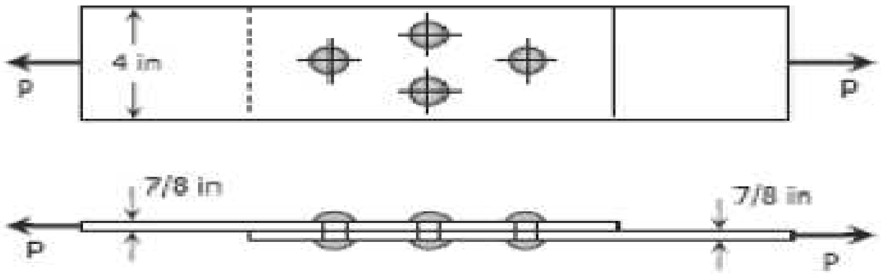
(b) Largest average tensile stress in the plate:

$P = \sigma A$

$6000\pi = \sigma[7.85(110 - 20)]$

$\sigma = 26.67$ MPa

The lap joint shown in Fig. P-126 is fastened by four ¾-in.-diameter rivets. Calculate the maximum safe load P that can be applied if the shearing stress in the rivets is limited to 14 ksi and the bearing stress in the plates is limited to 18 ksi. Assume the applied load is uniformly distributed among the four rivets.



Based on shearing of rivets:

$P = \tau A$

$P = 14[4\,(\frac{1}{4}\pi)(\frac{3}{4})^2\,]$

$P = 24.74$ kips

Based on bearing of plates:

$P = \sigma_b\, A_b$

$P = 18[4\,(\frac{3}{4})(\frac{7}{8})\,]$

$P = 47.25$ kips

Safe load $P = 24.74$ kips

In the clevis shown in Fig. 1-11b, find the minimum bolt diameter and the minimum thickness of each yoke that will support a load P = 14 kips without exceeding a shearing stress of 12 ksi and a bearing stress of 20 ksi.

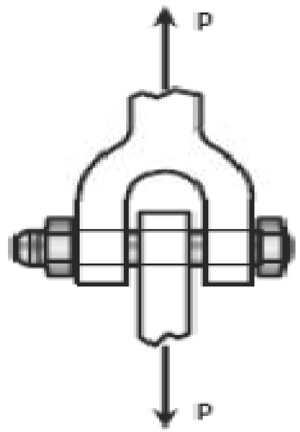
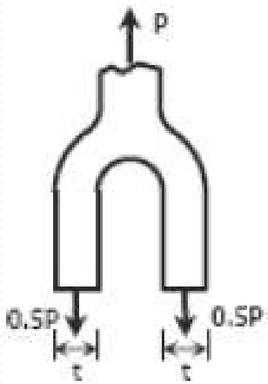


Figure 1-11b



For shearing of rivets (double shear)

$P = \tau A$

$14 = 12[2(\frac{1}{4}\pi d^2)]$

$d = 0.8618$ in → diameter of bolt

For bearing of yoke:

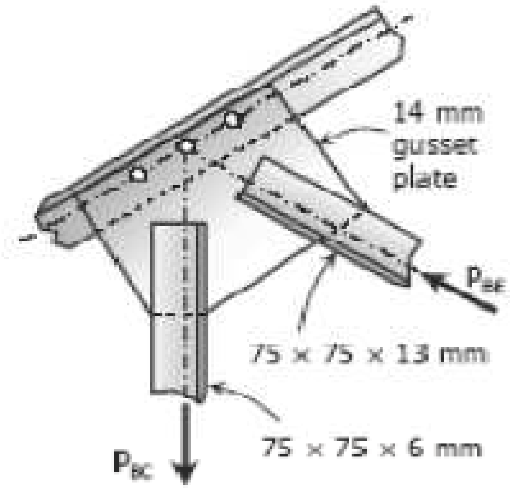
$P = \sigma_b A_b$

$14 = 20[2(0.8618t)]$

$t = 0.4061$ in → thickness of yoke

Problem 128

Figure P-130 shows a roof truss and the detail of the riveted connection at joint B. Using allowable stresses of τ = 70 MPa and σ_b= 140 MPa, how many 19-mm diameter rivets are required to fasten member BC to the gusset plate? Member BE? What is the largest average tensile or compressive stress in BC and BE?



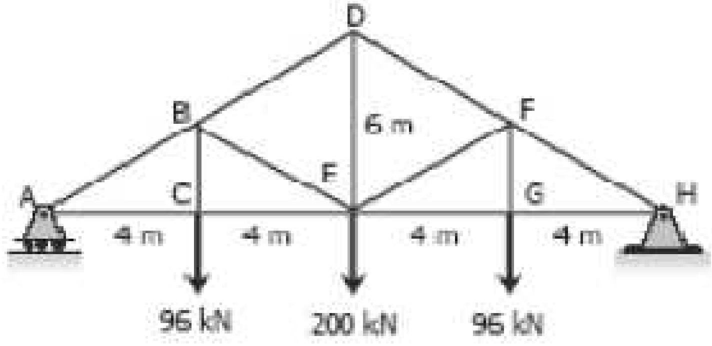
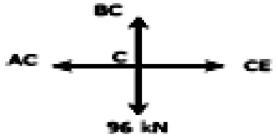
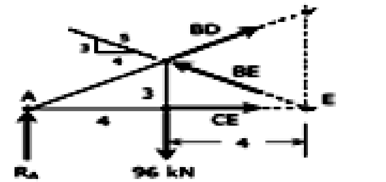


Figure P-130 and P-131



Joint C

At Joint C:
$\Sigma F_V = 0$
$BC = 96$ kN (Tension)



Section through BD, BE, and CE

Consider the section through member BD, BE, and CE:
$\Sigma M_A = 0$
$8(\frac{3}{5}BE) = 4(96)$
$BE = 80$ kN (Compression)

For Member BC:
Based on shearing of rivets:
$BC = \tau A$
Where A = area of 1 rivet × number of rivets, n
$96\,000 = 70[\frac{1}{4}\pi(19^2)n]$
$n = 4.8$ say 5 rivets

Based on bearing of member:
$BC = \sigma_b A_b$
Where A_b = diameter of rivet × thickness of BC × number of rivets, n
$96\,000 = 140[19(6)n]$
$n = 6.02$ say 7 rivets

use 7 rivets for member BC

For member BE:
Based on shearing of rivets:
$BE = \tau A$
Where A = area of 1 rivet × number of rivets, n
$80\,000 = 70[\frac{1}{4}\pi(19^2)n]$
$n = 4.03$ say 5 rivets

Based on bearing of member:
$BE = \sigma_b A_b$
Where A_b = diameter of rivet × thickness of BE × number of rivets, n
$80\,000 = 140[19(13)n]$
$n = 2.3$ say 3 rivets

use 5 rivets for member BE

Relevant data from the table (Appendix B of textbook): *Properties of Equal Angle Sections: SI Units*

Designation	Area
L75 × 75 × 6	864 mm²
L75 × 75 × 13	1780 mm²

t = thickness of member
d = diameter of rivet hole

Note:
A = Area – dt

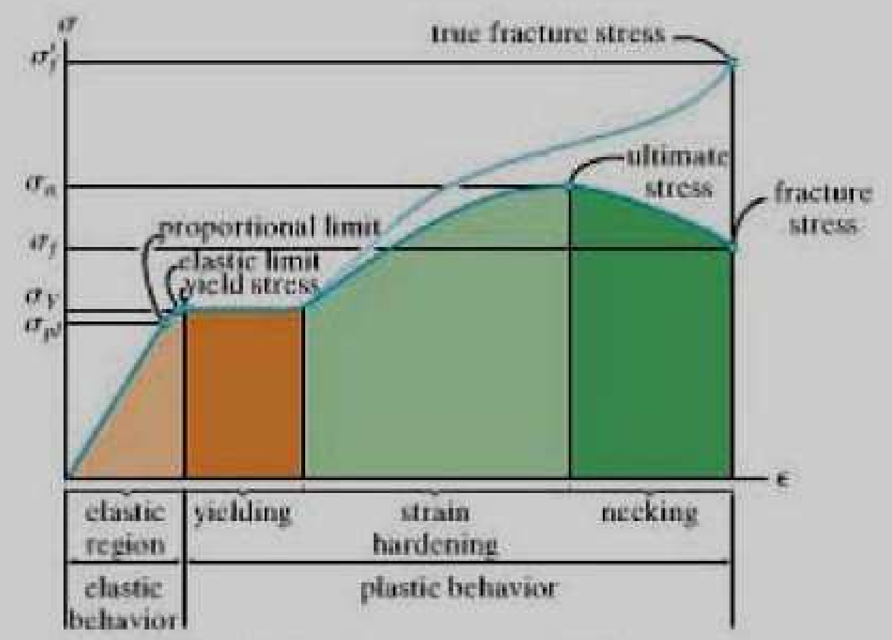
Tensile stress of member BC (L75 × 75 × 6):
$$\sigma = \frac{P}{A} = \frac{96(1000)}{864 - 19(6)}$$
$\sigma = 128$ Mpa

Compressive stress of member BE (L75 × 75 × 13):
$$\sigma = \frac{P}{A} = \frac{80(1000)}{1780}$$
$\sigma = 44.94$ Mpa

Stress strain diagram of material

Elastic Behavior. Elastic behavior of the material occurs when the strains in the specimen are within the light orange region shown in Fig. 3–4. Here the curve is actually a *straight line* throughout most of this region, so that the stress is *proportional* to the strain. The material in this region is said to be *linear elastic*. The upper stress limit to this linear relationship is called the ***proportional limit***, σ_{pl}. If the stress slightly exceeds the proportional limit, the curve tends to bend and flatten out as shown. This continues until the stress reaches the ***elastic limit***. Upon reaching this point, if the load is removed the specimen will still return back to its original shape. Normally for steel, however, the elastic limit is seldom determined, since it is very close to the proportional limit and therefore rather difficult to detect.

Yielding. A slight increase in stress above the elastic limit will result in a breakdown of the material and cause it to *deform permanently*. This behavior is called ***yielding***, and it is indicated by the rectangular dark orange region of the curve. The stress that causes yielding is called the ***yield stress*** or ***yield point***, σ_Y, and the deformation that occurs is called ***plastic deformation***. Although not shown in Fig. 3–4, for low-carbon steels or those that are hot rolled, the yield point is often distinguished by two values. The ***upper yield point*** occurs first, followed by a sudden decrease in load-carrying capacity to a ***lower yield point***. Notice that once the yield point is reached, then as shown in Fig. 3–4, the specimen will continue to elongate (strain) *without any increase in load*. When the material is in this state, it is often referred to as being ***perfectly plastic***.



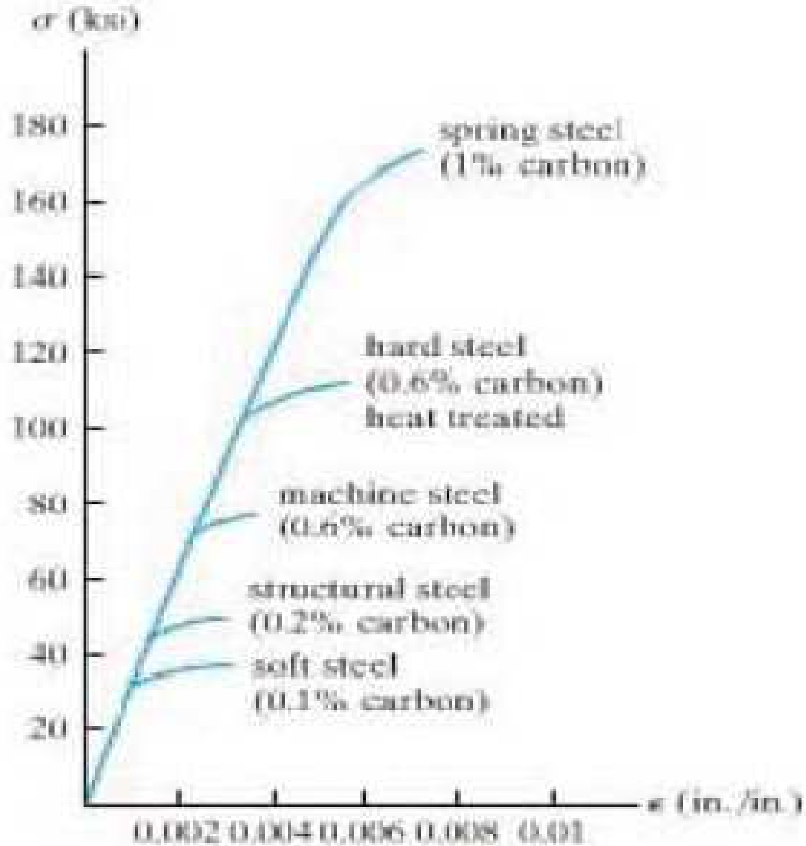
Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

Strain Hardening. When yielding has ended, an increase in load can be supported by the specimen, resulting in a curve that rises continuously but becomes flatter until it reaches a maximum stress referred to as the ***ultimate stress***, σ_u. The rise in the curve in this manner is called ***strain hardening***, and it is identified in Fig. 3–4 as the region in light green.

Necking. Up to the ultimate stress, as the specimen elongates, its cross-sectional area will decrease. This decrease is fairly *uniform* over the specimen's entire gauge length; however, just after, at the ultimate stress, the cross-sectional area will begin to decrease in a *localized* region of the specimen. As a result, a constriction or "neck" tends to form in this region as the specimen elongates further, Fig. 3–5*a*. This region of the curve due to necking is indicated in dark green in Fig. 3–4. Here the stress–strain diagram tends to curve downward until the specimen breaks at the ***fracture stress***, σ_f, Fig. 3–5*b*.

Hook's Law

- In most of the material, the stress strain relationship is linear i.e. Increase in stress leads to proportionate increase in the strain:
- E represents Young's Mod $\sigma = E\epsilon$ steel E is give as



$$E = \frac{\sigma_{pl}}{\epsilon_{pl}} = \frac{35 \text{ ksi}}{0.0012 \text{ in./in.}} = 29(10^3) \text{ ksi}$$

Important Points

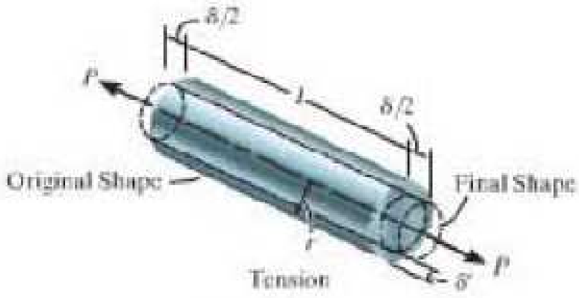
- A *conventional stress–strain diagram* is important in engineering since it provides a means for obtaining data about a material's tensile or compressive strength without regard for the material's physical size or shape.
- *Engineering stress and strain* are calculated using the *original* cross-sectional area and gauge length of the specimen.
- A *ductile material*, such as mild steel, has four distinct behaviors as it is loaded. They are *elastic behavior, yielding, strain hardening*, and *necking*.
- A material is *linear elastic* if the stress is proportional to the strain within the elastic region. This behavior is described by *Hooke's law*, $\sigma = E\epsilon$, where the *modulus of elasticity* E is the slope of the line.
- Important points on the stress–strain diagram are the *proportional limit, elastic limit, yield stress, ultimate stress*, and *fracture stress*.
- The *ductility* of a material can be specified by the specimen's *percent elongation* or the *percent reduction in area*.
- If a material does not have a distinct yield point, a *yield strength* can be specified using a graphical procedure such as the *offset method*.
- *Brittle materials*, such as gray cast iron, have very little or no yielding and so they can fracture suddenly.
- *Strain hardening* is used to establish a higher yield point for a material. This is done by straining the material beyond the elastic limit, then releasing the load. The modulus of elasticity remains the same; however, the material's ductility *decreases*.
- *Strain energy* is energy stored in a material due to its deformation. This energy per unit volume is called *strain-energy density*. If it is measured up to the proportional limit, it is referred to as the *modulus of resilience*, and if it is measured up to the point of fracture, it is called the *modulus of toughness*. It can be determined from the area under the $\sigma-\epsilon$ diagram.

3.6 Poisson's Ratio

When a deformable body is subjected to an axial tensile force, not only does it elongate but it also contracts laterally. For example, if a rubber band is stretched, it can be noted that both the thickness and width of the band are decreased. Likewise, a compressive force acting on a body causes it to contract in the direction of the force and yet its sides expand laterally.

Consider a bar having an original radius r and length L and subjected to the tensile force P in Fig. 3–21. This force elongates the bar by an amount δ, and its radius contracts by an amount δ'. Strains in the longitudinal or axial direction and in the lateral or radial direction are, respectively,

$$\epsilon_{\text{long}} = \frac{\delta}{L} \quad \text{and} \quad \epsilon_{\text{lat}} = \frac{\delta'}{r}$$



In the early 1800s, the French scientist S. D. Poisson realized that within tl *elastic range* the *ratio* of these strains is a *constant*, since the deformatio δ and δ' are proportional. This constant is referred to as ***Poisson's ratio***, ν (nu), and it has a numerical value that is unique for a particular material that is both *homogeneous and isotropic*. Stated mathematically it is

$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}} \qquad (3\text{–}9)$$