Strength of Material

Shear Strain

Dr. Attaullah Shah

Shear Strain

$$\delta_s = \frac{VL}{A_s G} = \frac{\tau L}{G}$$

Poisson's Ratio

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

BIAXIAL DEFORMATION

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} \quad \text{or} \quad \sigma_x = \frac{(\varepsilon_x + \nu\varepsilon_y)E}{1-\nu^2}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} \quad \text{or} \quad \sigma_y = \frac{(\varepsilon_y + \nu\varepsilon_x)E}{1-\nu^2}$$

TRIAXIAL DEFORMATION

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

Relationship Between E, G, and v

$$G = \frac{E}{2(1+\nu)}$$

Bulk Modulus of Elasticity or Modulus of Volume Expansion, K

- The bulk modulus of elasticity K is a measure of a resistance of a material to change in volume without change in shape or form. It is given as:

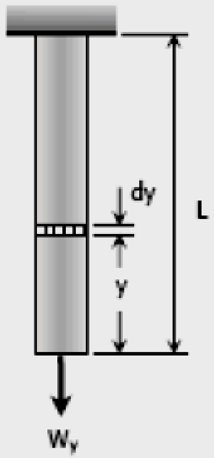
$$K = \frac{E}{3(1-2\nu)} = \frac{\sigma}{\Delta V / V}$$

- Where V is the volume and ΔV is change in volume. The ratio ΔV / V is called volumetric strain and can be expressed as

$$\frac{\Delta V}{V} = \frac{\sigma}{K} = \frac{3(1-2\nu)}{E}$$

- **Problem 205** A uniform bar of length L, cross-sectional area A, and unit mass ρ is suspended vertically from one end. Show that its total elongation is $\delta = \rho g L^2 / 2E$. If the total mass of the bar is M, show also that $\delta = MgL/2AE$.

Solution 205



$$\delta = \frac{PL}{AE}$$

From the figure:
$\delta = d\delta$
$P = W_y = (\rho A y)g$
$L = dy$

$$d\delta = \frac{(\rho A y)g\,dy}{AE}$$

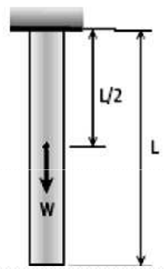
$$\delta = \frac{\rho g}{E}\int_0^L y\,dy = \frac{\rho g}{E}\left[\frac{y^2}{2}\right]_0^L$$

$$\delta = \frac{\rho g}{2E}\left[L^2 - 0^2\right] = \rho g L^2/2E \qquad ok!$$

Given the total mass M:
$\rho = M/V = M/AL$

$\delta = \rho g L^2/2E = (M/AL)(gL^2/2E)$
$\delta = MgL/2AE$ *ok!*

Another Solution:



The weight will act at the center of gravity of the bar:

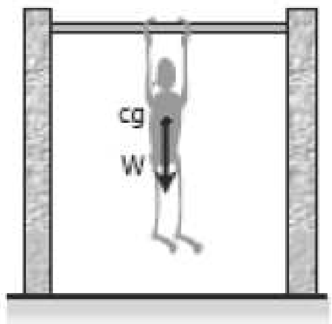
$$\delta = \frac{PL}{AE}$$

Where: $P = W = (\rho A L)g$
$L = L/2$

$$\delta = \frac{[(\rho A L)g](L/2)}{AE}$$

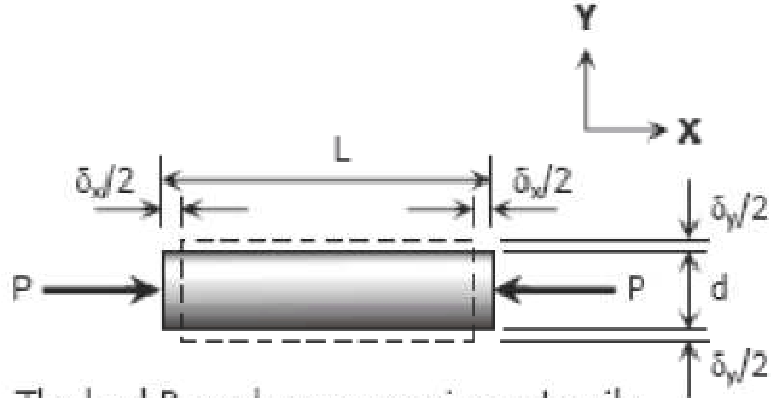
$$\delta = \frac{\rho g L^2}{2E} \qquad ok!$$

For you to feel the situation, position yourself in pull-up exercise with your hands on the bar and your body hang freely above the ground. Notice that your arms suffer all your weight and your lower body fells no stress (center of weight is approximately just below the chest). If your body is the bar, the elongation will occur at the upper half of it.



- **Problem 222** A solid cylinder of diameter d carries an axial load P. Show that its change in diameter is 4Pν / πEd.

Solution 222



The load P can be compressive or tensile

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

$$\varepsilon_y = -\nu\varepsilon_x$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E}$$

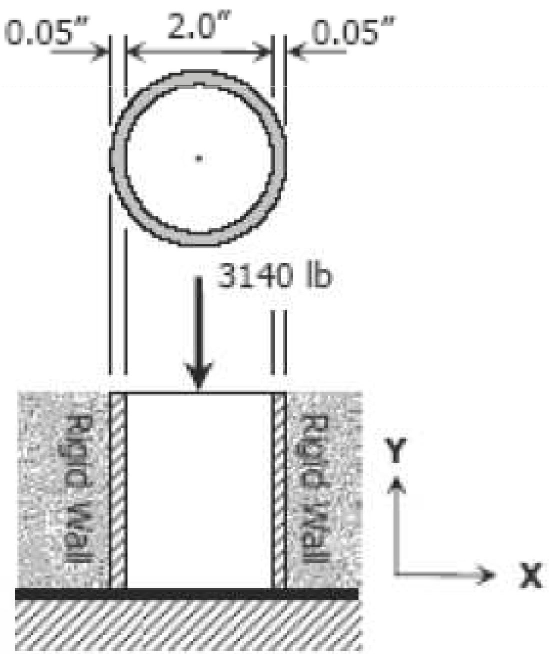
$$\frac{\delta_y}{d} = -\nu\frac{-P}{AE}$$

$$\delta_y = \nu\frac{Pd}{\frac{1}{4}\pi d^2 E}$$

$$\delta_y = \frac{4P\nu}{\pi E d} \qquad ok!$$

Problem 226 A 2-in.-diameter steel tube with a wall thickness of 0.05 inch just fits in a rigid hole. Find the tangential stress if an axial compressive load of 3140 lb is applied. Assume ν = 0.30 and neglect the possibility of buckling.

Solution 226



$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = 0$$

$$\sigma_x = \nu\sigma_y$$

where σ_x = tangential stress

σ_y = longitudinal stress

$$\sigma_y = \frac{P_y}{A} = \frac{3140}{\pi(2)(0.05)}$$

$$\sigma_y = 31400/\pi \text{ psi}$$

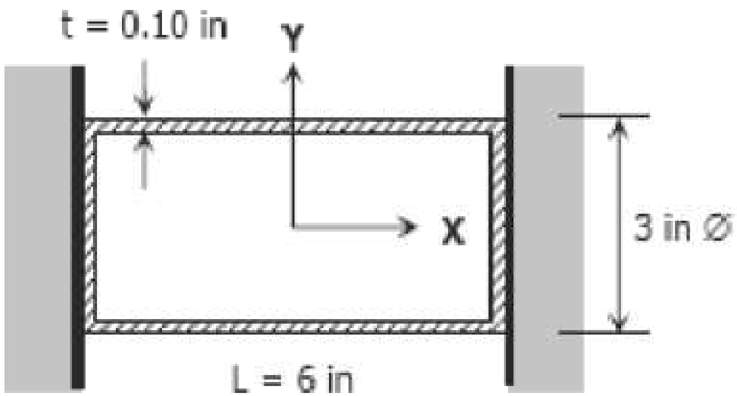
$$\sigma_x = 0.30(31400/\pi)$$

$$\sigma_x = 9430/\pi \text{ psi}$$

$$\sigma_x = 2298.5 \text{ psi}$$

- **Problem 228** A 6-in.-long bronze tube, with closed ends, is 3 in. in diameter with a wall thickness of 0.10 in. With no internal pressure, the tube just fits between two rigid end walls. Calculate the longitudinal and tangential stresses for an internal pressure of 6000 psi. Assume ν = 1/3 and E = 12 × 106 psi.

Solution 228



$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = 0$$

$\sigma_x = \nu\sigma_y = \sigma_l \rightarrow$ longitudinal stress

$\sigma_t = \sigma_y \rightarrow$ tangential stress

$$\sigma_t = \frac{pD}{2t} = \frac{6000(3)}{2(0.10)}$$

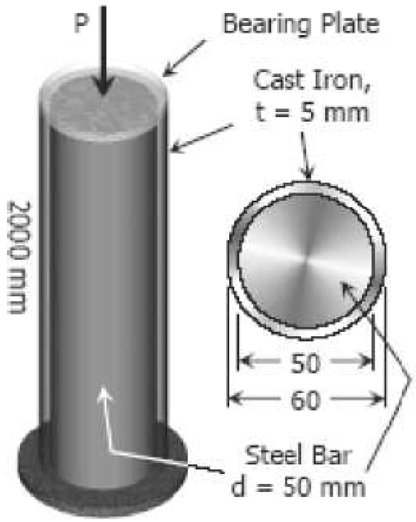
$$\sigma_t = \mathbf{90{,}000\ psi}$$

$$\sigma_l = \nu\sigma_y = \tfrac{1}{3}(90{,}000)$$

$$\sigma_l = \mathbf{30{,}000\ psi}$$

Problem 233 A steel bar 50 mm in diameter and 2 m long is surrounded by a shell of a cast iron 5 mm thick. Compute the load that will compress the combined bar a total of 0.8 mm in the length of 2 m. For steel, E = 200 GPa, and for cast iron, E = 100 GPa.

Solution 233



$$\delta = \frac{PL}{AE}$$

$$\delta = \delta_{cast\ iron} = \delta_{steel} = 0.8\ \text{mm}$$

$$\delta_{cast\ iron} = \frac{P_{cast\ iron}(2000)}{[\frac{1}{4}\pi(60^2 - 50^2)](100\,000)} = 0.8$$

$$P_{cast\ iron} = 11\ 000\pi\ \text{N}$$

$$\delta_{steel} = \frac{P_{steel}(2000)}{[\frac{1}{4}\pi(50^2)](200\,000)} = 0.8$$

$$P_{steel} = 50\ 000\pi\ \text{N}$$

$$\Sigma F_V = 0$$

$$P = P_{cast\ iron} + P_{steel}$$

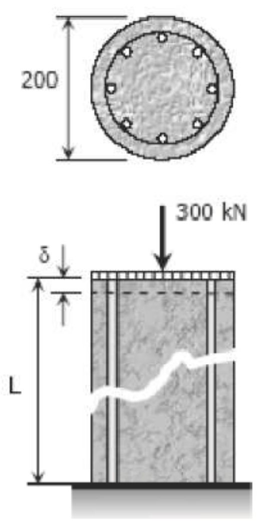
$$P = 11\ 000\pi + 50\ 000\pi$$

$$P = 61\ 000\pi\ \text{N}$$

$$P = \mathbf{191.64\ kN}$$

- **Problem 234** A reinforced concrete column 200 mm in diameter is designed to carry an axial compressive load of 300 kN. Determine the required area of the reinforcing steel if the allowable stresses are 6 MPa and 120 MPa for the concrete and steel, respectively. Use Eco = 14 GPa and Est = 200 GPa.

Solution 234



$$\delta_{co} = \delta_{st} = \delta$$

$$\left(\frac{PL}{AE}\right)_{co} = \left(\frac{PL}{AE}\right)_{st}$$

$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$

$$\frac{\sigma_{co} L}{14000} = \frac{\sigma_{st} L}{200\,000}$$

$$100\sigma_{co} = 7\sigma_s$$

When $\sigma_{st} = 120$ MPa

$$100\sigma_{co} = 7(120)$$

$$\sigma_{co} = 8.4 \text{ MPa} > 6 \text{ MPa (not ok!)}$$

When $\sigma_{co} = 6$ MPa

$$100(6) = 7\sigma_{st}$$

$$\sigma_{st} = 85.71 \text{ MPa} < 120 \text{ MPa (ok!)}$$

Use $\sigma_{co} = 6$ MPa and $\sigma_{st} = 85.71$ MPa

$$\Sigma F_V = 0$$

$$P_{st} + P_{co} = 300$$

$$\sigma_{st} A_{st} + \sigma_{co} A_{co} = 300$$

$$85.71A_{st} + 6[\tfrac{1}{4}\pi(200)^2 - A_{st}] = 300(1000)$$

$$79.71A_{st} + 60\,000\pi = 300\,000$$

$$A_{st} = 1398.9 \text{ mm}^2$$

- **Problem 239** The rigid platform in Fig. P-239 has negligible mass and rests on two steel bars, each 250.00 mm long. The center bar is aluminum and 249.90 mm long. Compute the stress in the aluminum bar after the center load P = 400 kN has been applied. For each steel bar, the area is 1200 mm2 and E = 200 GPa. For the aluminum bar, the area is 2400 mm2 and E = 70 GPa.

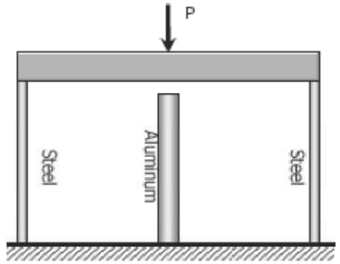
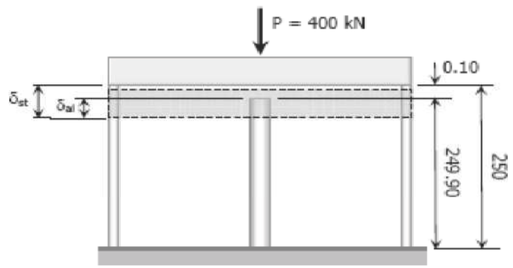


Figure P-239

Solution 239



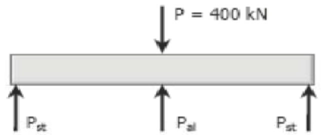
$$\delta_{st} = \delta_{al} + 0.10$$

$$\left(\frac{\sigma L}{E}\right)_{st} = \left(\frac{\sigma L}{E}\right)_{al} + 0.10$$

$$\frac{\sigma_{st}(250)}{200\,000} = \frac{\sigma_{al}(249.90)}{70\,000} + 0.10$$

$$0.00125\sigma_{st} = 0.00357\sigma_{al} + 0.10$$

$$\sigma_{st} = 2.856\sigma_{al} + 80$$



$$\Sigma F_V = 0$$

$$2P_{st} + P_{al} = 400\,000$$

$$2\sigma_{st}A_{st} + \sigma_{al}A_{al} = 400\,000$$

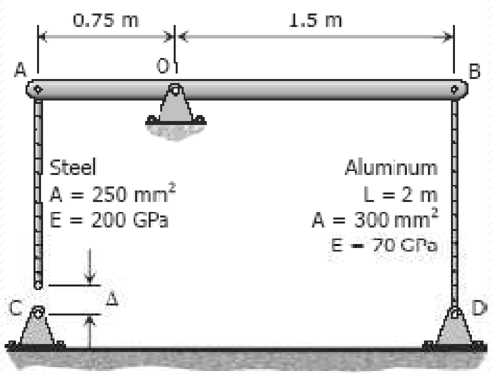
$$2(2.856\sigma_{al} + 80)1200 + \sigma_{al}(2400) = 400\,000$$

$$9254.4\sigma_{al} + 192\,000 = 400\,000$$

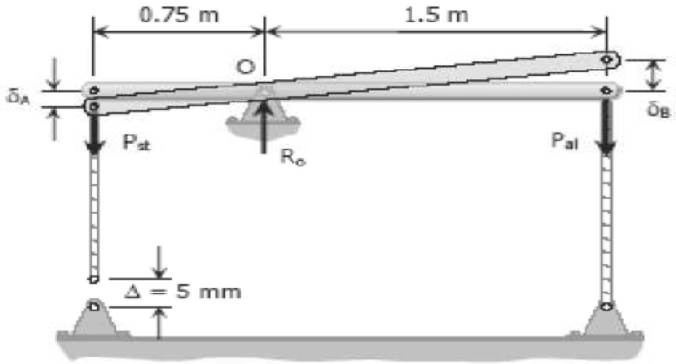
$$\sigma_{al} = 22.48 \text{ MPa}$$

- **Problem 242** The assembly in Fig. P-242 consists of a light rigid bar AB, pinned at O, that is attached to the steel and aluminum rods. In the position shown, bar AB is horizontal and there is a gap, Δ = 5 mm, between the lower end of the steel rod and its pin support at C. Compute the stress in the aluminum rod when the lower end of the steel rod is attached to its support.

Figure P-242



Solution 242



$\sum M_O = 0$

$0.75P_{st} = 1.5P_{al}$

$P_{st} = 2P_{al}$

$\sigma_{st} A_{st} = 2(\sigma_{al} A_{al})$

$$\sigma_{st} = \frac{2(\sigma_{al} A_{al})}{A_{st}}$$

$$\sigma_{st} = \frac{2[\sigma_{al}(300)]}{250}$$

$\sigma_{st} = 2.4\sigma_{al}$

$\delta_{al} = \delta_B$

By ratio and proportion:

$$\frac{\delta_A}{0.75} = \frac{\delta_B}{1.5}$$

$\delta_A = 0.5\delta_B$

$\delta_A = 0.5\delta_{al}$

$\Delta = \delta_{st} + \delta_A$

$5 = \delta_{st} + 0.5\delta_{al}$

$$5 = \frac{\sigma_{st}(2000-5)}{250(200000)} + 0.5\left[\frac{\sigma_{al}(2000)}{300(70000)}\right]$$

$5 = (3.99 \times 10^{-5})\ \sigma_{st} + (4.76 \times 10^{-5})\ \sigma_{al}$

$\sigma_{al} = 105\,000 - 0.8379\sigma_{st}$

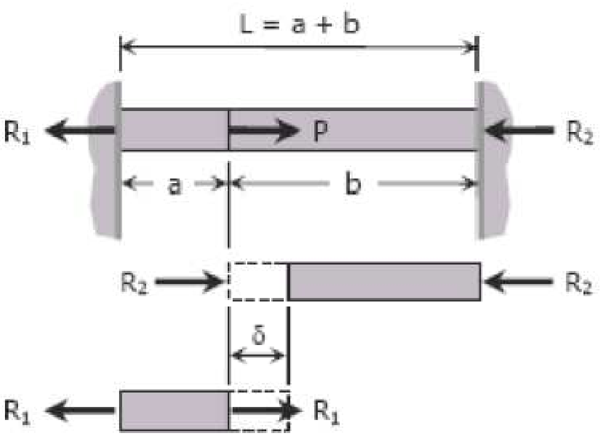
$\sigma_{al} = 105\,000 - 0.8379(2.4\sigma_{al})$

$3.01096\sigma_{al} = 105\,000$

$\sigma_{al} = 34\,872.6\ \text{MPa}$

Problem 243 A homogeneous rod of constant cross section is attached to unyielding supports. It carries an axial load P applied as shown in Fig. P-243. Prove that the reactions are given by $R_1 = Pb/L$ and $R_2 = Pa/L$.

Solution 243



$$\sum F_H = 0$$
$$R_1 + R_2 = P$$
$$R_2 = P - R_1$$

$$\delta_1 = \delta_2 = \delta$$
$$\left(\frac{PL}{AE}\right)_1 = \left(\frac{PL}{AE}\right)_2$$
$$\frac{R_1 a}{AE} = \frac{R_2 b}{AE}$$
$$R_1 a = R_2 b$$

$$R_1 a = (P - R_1) b$$
$$R_1 a = Pb - R_1 b$$
$$R_1 (a + b) = Pb$$
$$R_1 L = Pb$$
$$R_1 = Pb / L \qquad \text{ok!}$$

$$R_2 = P - Pb/L$$
$$R_2 = \frac{P(L-b)}{L}$$
$$R_2 = Pa / L \qquad \text{ok!}$$

- **Problem 247** The composite bar in Fig. P-247 is stress-free before the axial loads P1 and P2 are applied. Assuming that the walls are rigid, calculate the stress in each material if P1 = 150 kN and P2 = 90 kN.

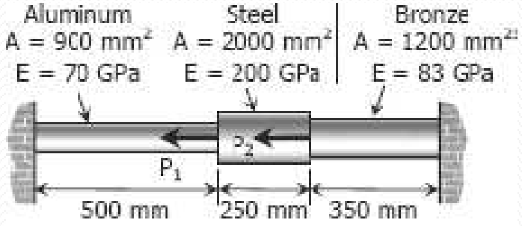
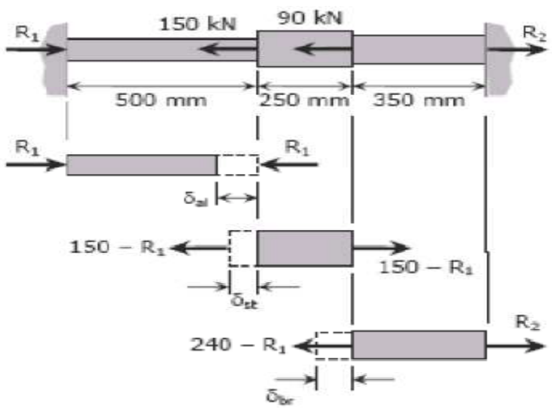


Figure P-247 and P-248

Solution 247



From the *FBD* of each material shown:

δ_{al} is shortening

δ_{st} and δ_{br} are lengthening

$R_2 = 240 - R_1$

$P_{al} = R_1$

$P_{st} = 150 - R_1$

$P_{br} = R_2 = 240 - R_1$

$$\delta_{al} = \delta_{st} + \delta_{br}$$

$$\left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st} + \left(\frac{PL}{AE}\right)_{br}$$

$$\frac{R_1(500)}{900(70000)} = \frac{(150-R_1)(250)}{2000(200000)} + \frac{(240-R_1)(350)}{1200(83000)}$$

$$\frac{R_1}{126000} = \frac{150-R_1}{1600000} + \frac{(240-R_1)7}{1992000}$$

$$\frac{1}{63}R_1 = \frac{1}{800}(150 - R_1) + \frac{7}{996}(240 - R_1)$$

$$\left(\frac{1}{63} + \frac{1}{800} + \frac{7}{996}\right)R_1 = \frac{1}{800}(150) + \frac{7}{996}(240)$$

$R_1 = 77.60$ kN

$P_{al} = R_1 = 77.60$ kN

$P_{st} = 150 - 77.60 = 72.40$ kN

$P_{br} = 240 - 77.60 = 162.40$ kN

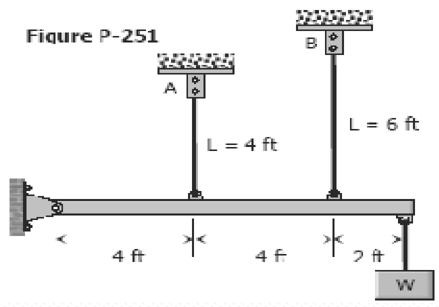
$\sigma = P/A$

$\sigma_{al} = 77.60(1000)/900$
$= \mathbf{86.22\ MPa}$

$\sigma_{st} = 72.40(1000)/2000$
$= \mathbf{36.20\ MPa}$

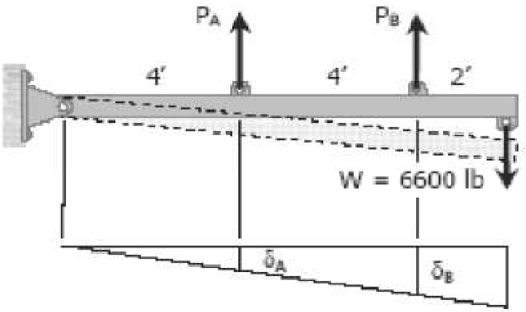
$\sigma_{br} = 162.40(1000)/1200$
$= \mathbf{135.33\ MPa}$

- **Problem 251** The two vertical rods attached to the light rigid bar in Fig. P-251 are identical except for length. Before the load W was attached, the bar was horizontal and the rods were stress-free. Determine the load in each rod if W = 6600 lb.



Solution 251

$$\sum M_{pin\ support} = 0$$
$$4P_A + 8P_B = 10(6600)$$
$$P_A + 2P_B = 16500 \quad \rightarrow (1)$$



$$\frac{\delta_A}{4} = \frac{\delta_B}{8}$$
$$\delta_A = 0.5\delta_B$$
$$\left(\frac{PL}{AE}\right)_A = 0.5\left(\frac{PL}{AE}\right)_B$$
$$\frac{P_A(4)}{AE} = \frac{0.5P_B(6)}{AE}$$
$$P_A = 0.75P_B$$

From equation (1)
$$0.75P_B + 2P_B = 16500$$
$$P_B = \mathbf{6000\ lb}$$

$$P_A = 0.75(6000)$$
$$P_A = \mathbf{4500\ lb}$$

- **Problem 256** Three rods, each of area 250 mm2, jointly support a 7.5 kN load, as shown in Fig. P-256. Assuming that there was no slack or stress in the rods before the load was applied, find the stress in each rod. Use Est = 200 GPa and Ebr = 83 GPa.

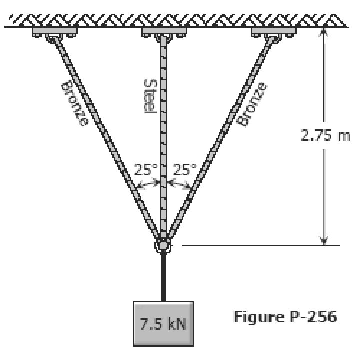
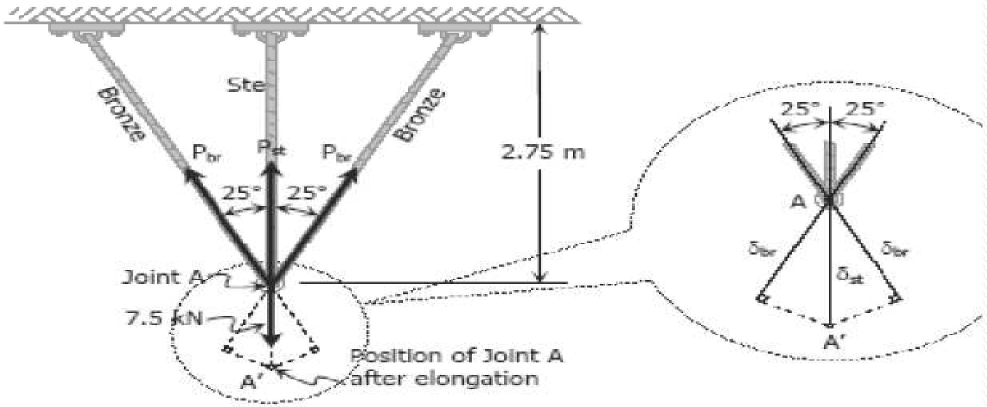


Figure P-256

Solution 256

$$\cos 25° = \frac{2.75}{L_{br}}\ ;\ L_{br} = 3.03 \text{ m}$$



$$\Sigma F_V = 0$$
$$2P_{br} \cos 25° + P_{st} = 7.5(1000)$$
$$P_{st} = 7500 - 1.8126 P_{br}$$
$$\sigma_{st} A_{st} = 7500 - 1.8126 \sigma_{br} A_{br}$$
$$\sigma_{st}\,(250) = 7500 - 1.8126[\sigma_{br}\,(250)]$$

$$\sigma_{st} = 30 - 1.8126\ \sigma_{br} \quad \rightarrow (1)$$

$$\cos 25° = \frac{\delta_{br}}{\delta_{st}}$$
$$\delta_{br} = 0.9063\ \delta_{st}$$
$$\left(\frac{\sigma L}{E}\right)_{br} = 0.9063\left(\frac{\sigma L}{E}\right)_{st}$$
$$\frac{\sigma_{br}(3.03)}{83} = 0.9063\left[\frac{\sigma_{st}(2.75)}{200}\right]$$
$$\sigma br = 0.3414\sigma_{st} \quad \rightarrow (2)$$

From equation (1)

$$\sigma_{st} = 30 - 1.8126(0.3414\sigma_{st})$$
$$\sigma_{st} = 18.53 \text{ MPa}$$

From equation (2)

$$\sigma br = 0.3414(18.53)$$
$$\sigma_{br} = 6.33 \text{ MPa}$$

Thermal Stresses

- Temperature changes cause the body to expand or contract. The amount δT, is given by

$$\delta_T = \alpha L(T_f - T_i) = \alpha L \, \Delta T$$

  - where α is the coefficient of ther… °C,
  - L is the length in meter, and Ti and Tf are the initial and final temperatures, respectively in °C.
  - For steel, $\alpha = 11.25 \times 10^{-6}$ / °C.
- If temperature deformation is permitted to occur freely, no load or stress will be induced in the structure. In some cases where temperature deformation is not permitted, an internal stress is created. The internal stress created is termed as thermal stress. For a homogeneous rod mounted between unyielding supports as shown, the thermal stress is computed as:
- Defc $\delta_T = \alpha L \, \Delta T$ ue to equivalent axial stress are

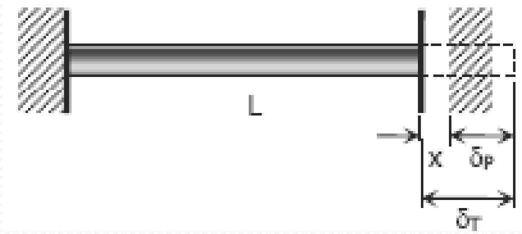
$$\delta_P = \frac{PL}{AE} = \frac{\sigma L}{E}$$

$$\delta_T = \delta_P$$

$$\alpha L \, \Delta T = \frac{\sigma L}{E}$$

$$\sigma = E \, \alpha \, \Delta T$$

- where σ is the thermal stress in MPa and E is the modulus of elasticity of the rod in MPa.
- If the wall yields a distance of x as shown, the following calculations will be made:



- $$\delta_T = x + \delta_P$$
  $$\alpha L\,\Delta T - x\,\frac{\sigma L}{E}$$

- Take note that as the temperature rises above the normal, the rod will be in compression, and if the temperature drops below the normal, the rod is in tension.

- **Problem 261** A steel rod with a cross-sectional area of 0.25 in2 is stretched between two fixed points. The tensile load at 70°F is 1200 lb. What will be the stress at 0°F? At what temperature will the stress be zero? Assume ?? = 6.5 × 10-6 in / (in·°F) and E = 29 × 106 psi.