Moving Loads

Theory of Simple Bending

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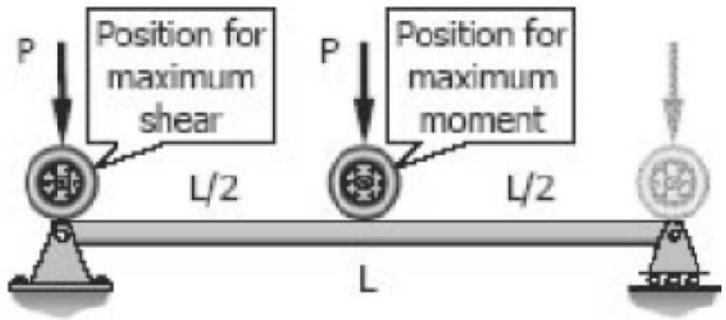
Moving Loads

- We know that:
  - The maximum moment occurs at a point of zero shears.
  - For beams loaded with concentrated loads, the point of zero shears usually occurs under a concentrated load and so the maximum moment.
  - Beams and girders such as in a bridge or an overhead crane are subject to moving concentrated loads, which are at fixed distance with each other.
  - The problem here is to determine the moment under each load when each load is in a position to cause a maximum moment. The largest value of these moments governs the design of the beam.

Single Moving Load:

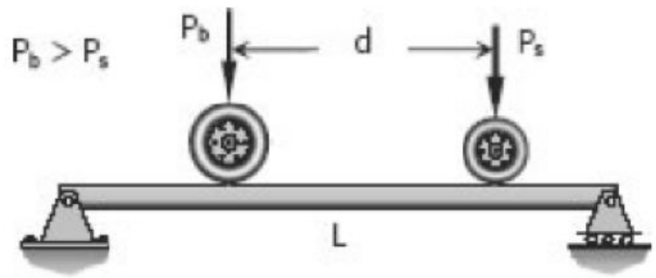


$$M_{max} = \frac{PL}{4} \text{ and } V_{max} = P$$

Single Moving Load:

The maximum moment occurs when the load is at the midspan and the maximum shear occurs when the load is very near the support (usually assumed to lie over the support).

TWO MOVING LOADS



$$M_{max} = \frac{(PL - P_s d)^2}{4PL}$$

- For two moving loads, the maximum shear occurs at the reaction when the larger load is over that support.
- The max moment is given as above.
- Where Ps is the smaller load, Pb is the bigger load, and P is the total load (P = Ps + Pb).

Three moving load

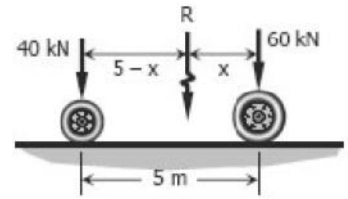
- In general, the bending moment under a particular load is a maximum when the center of the beam is midway between that load and the resultant of all the loads then on the span.
- With this rule, we compute the maximum moment under each load, and use the biggest of the moments for the design. Usually, the biggest of these moments occurs under the biggest load.
- The maximum shear occurs at the reaction where the resultant load is nearest. Usually, it happens if the biggest load is over that support and as many a possible of the remaining loads are still on the span.
- In determining the largest moment and shear, it is sometimes necessary to check the condition when the bigger loads are on the span and the rest of the smaller loads are outside.

- **Problem 453** A truck with axle loads of 40 kN and 60 kN on a wheel base of 5 m rolls across a 10-m span. Compute the maximum bending m
shearing force.

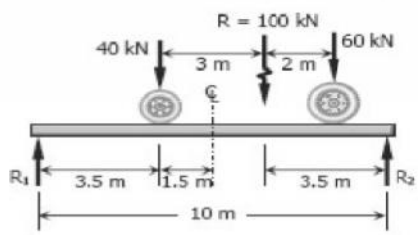
Solution:

$R = 40 + 60 = 100$ kN

$xR = 40(5)$
$x = 200/R$
$x = 200/100$
$x = 2$ m



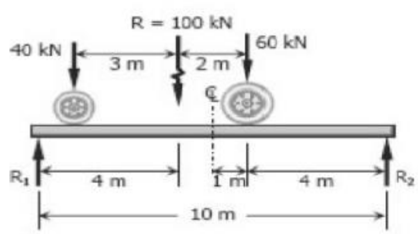
For maximum moment under 40 k



$\Sigma M_{R2} = 0$
$10R_1 = 3.5(100)$
$R_1 = 35$ kN

$M_{\text{To the left of 40 kN}} = 3.5R_1$
$M_{\text{To the left of 40 kN}} = 3.5(35)$
$M_{\text{To the left of 40 kN}} = 122.5$ kN·m

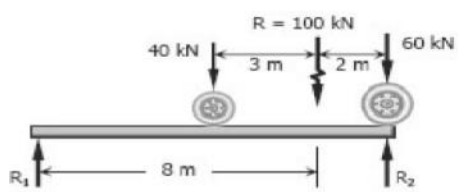
For maximum moment under 60 kN wheel:



$\Sigma M_{R1} = 0$
$10R_2 = 4(100)$
$R_2 = 40$ kN

$M_{\text{To the right of 60 kN}} = 4R_2$
$M_{\text{To the right of 60 kN}} = 4(40)$
$M_{\text{To the right of 60 kN}} = 160$ kN·m

Thus, $M_{max} = 160$ kN·m



The maximum shear will occur when the 60 kN is over a support.
$\Sigma M_{R1} = 0$
$10R_2 = 100(8)$
$R_2 = 80$ kN

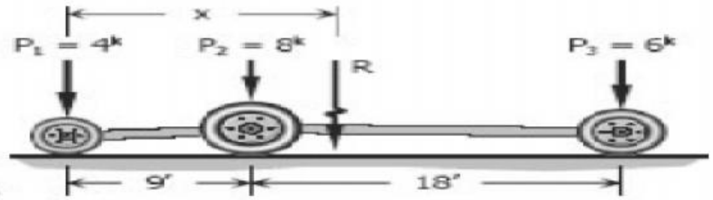
Thus, $V_{max} = 80$ kN

- **Class Example:**
- **Problem 455** A tractor weighing 3000 lb, with a wheel base of 9 ft, carries 1800 lb of its load on the rear wheels. Compute the maximum moment and maximum shear when crossing a 14 ft-span.

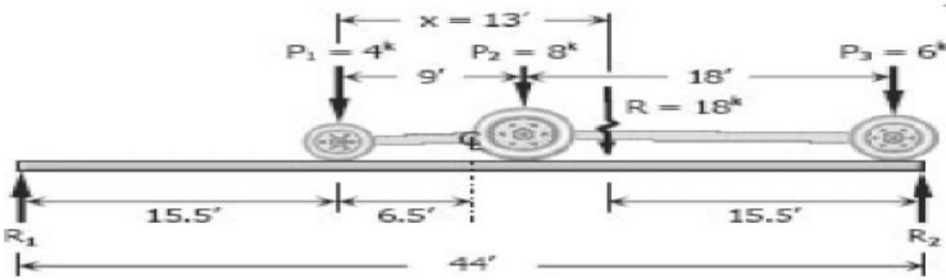
- **Problem 456** Three wheel loads roll as a unit across a 44-ft span. The loads are P1 = 4000 lb and P2 = 8000 lb separated by 9 ft, and P3 = 6000 lb at 18 ft from P2. Determine the maximum moment and maximum shear in the simply supported span.

Solution 456

$R = P_1 + P_2 + P_3$
$R = 4^k + 8^k + 6^k$
$R = 18$ kips
$R = 18{,}000$ lbs



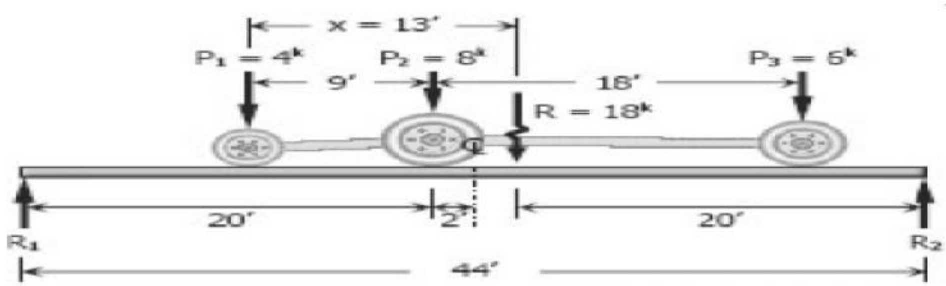
$xR = 9P_2 + (9 + 18)P_3$
$x(18) = 9(8) + (9 + 18)(6)$
$x = 13$ ft → the resultant R is 13 ft from P_1



Maximum moment under P_1

$\Sigma M_{R2} = 0$
$44R_1 = 15.5R$
$44R_1 = 15.5(18)$
$R_1 = 6.34091$ kips
$R_1 = 6{,}340.91$ lbs

$M_{\text{To the left of } P1} = 15.5R_1$
$M_{\text{To the left of } P1} = 15.5(6340.91)$
$M_{\text{To the left of } P1} = 98{,}284.1$ lb-ft

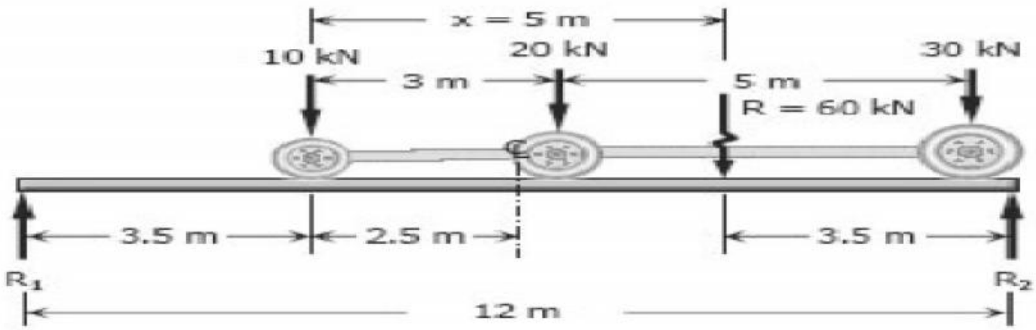


Maximum moment under P_2

$\Sigma M_{R2} = 0$
$44R_1 = 20R$
$44R_1 = 20(18)$
$R_1 = 8.18182$ kips
$R_1 = 8{,}181.82$ lbs

$M_{\text{To the left of } P2} = 20R_1 - 9P_1$
$M_{\text{To the left of } P2} = 20(8{,}181.82) - 9(4000)$
$M_{\text{To the left of } P2} = 127{,}636.4$ lb-ft

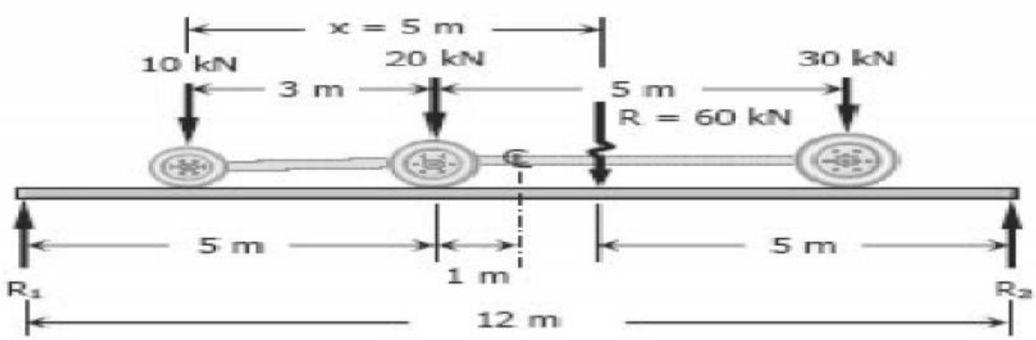
Maximum moment under 10 kN



$\Sigma M_{R2} = 0$
$12R_1 = 3.5R$
$12R_1 = 3.5(60)$
$12R_1 = 210$
$R_1 = 12.7$ kN

$M_{\text{To the left of 10 kN}} = 3.5R_1$
$= 3.5(12.7)$
$= 61.25$ kN·m

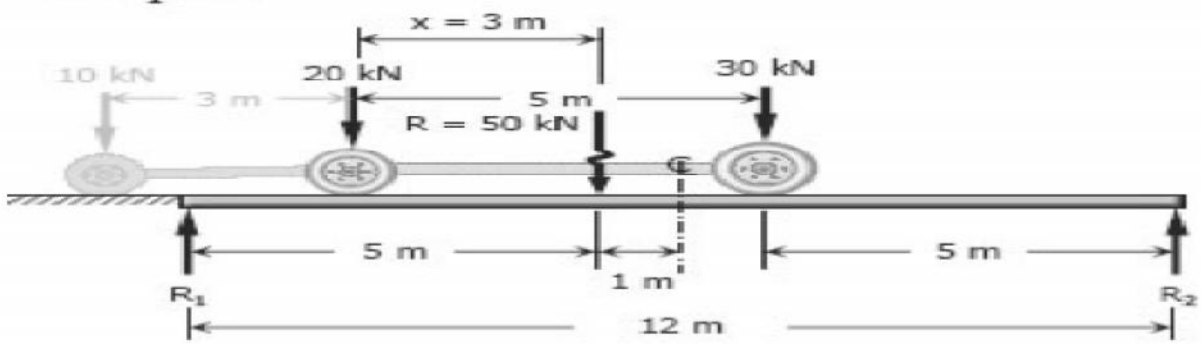
Maximum moment under 20 kN



$\Sigma M_{R2} = 0$
$12R_1 = 5R$
$12R_1 = 5(60)$
$R_1 = 25$ kN

$$M_{\text{To the left of 20 kN}} = 5R_1 - 3(10)$$
$$= 5(25) - 30$$
$$= 95 \text{ kN-m}$$

When the centerline of the beam is midway between reaction R = 60 kN and 30 kN, the 10 kN comes off the span.



$R = 20 + 30$
$R = 50 \text{ kN}$

$xR = 5(30)$
$x(50) = 150$
$x = 3 \text{ m}$ from 20 kN

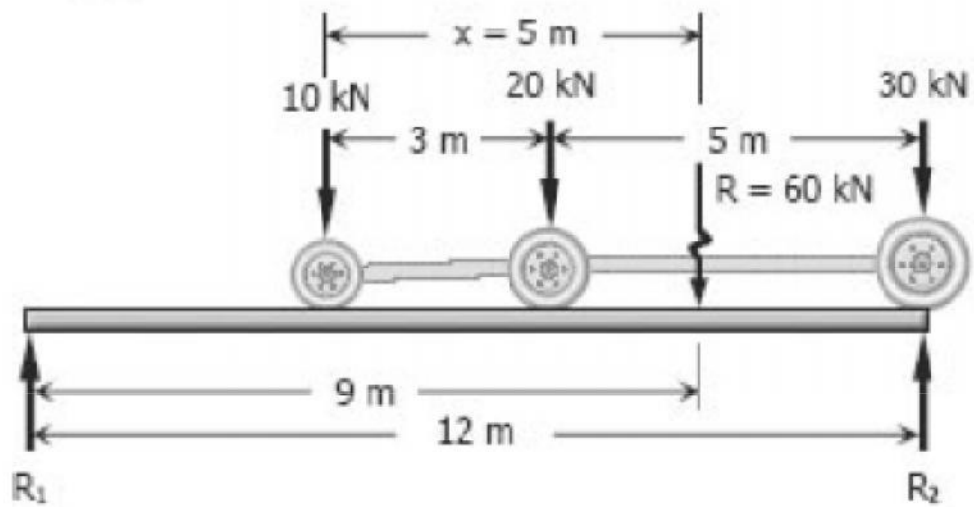
$\Sigma M_{R1} = 0$
$12R_2 = 5R$
$12R_2 = 5(50)$
$R_2 = 20.83 \text{ kN}$

$$M_{\text{To the right of 30 kN}} = 5R_2$$
$$= 5(20.83)$$
$$= 104.17 \text{ kN-m}$$

Thus, the maximum moment will occur when only the 20 and 30 kN loads are on the span.

$M_{max} = M_{\text{To the right of 30 kN}}$
$M_{max} = 104.17 \text{ kN-m}$

The maximum shear will occur when the three loads are on the span and the 30 kN load is directly over the support.



$\Sigma M_{R1} = 0$
$12R_2 = 9R$
$12R_2 = 9(60)$
$R_2 = 45$ kN

Thus, $V_{max} = 45$ **kN**

Deflection of beams

The Elastic Curve

Before the slope or the displacement at a point on a beam (or shaft) is determined, it is often helpful to sketch the deflected shape of the beam when it is loaded, in order to "visualize" any computed results and thereby partially check these results. The deflection curve of the longitudinal axis that passes through the centroid of each cross-sectional area of a beam is called the *elastic curve*. For most beams the elastic curve can be sketched without much difficulty. When doing so, however, it is necessary to know how the slope or displacement is restricted at various types of supports. In general, supports that resist a *force*, such as a pin, restrict *displacement*, and those that resist a *moment*, such as a fixed wall, restrict *rotation* or *slope* as well as displacement. With this in mind, two typical examples of the elastic curves for loaded beams (or shafts), sketched to an exaggerated scale, are shown in Fig. 12–1.

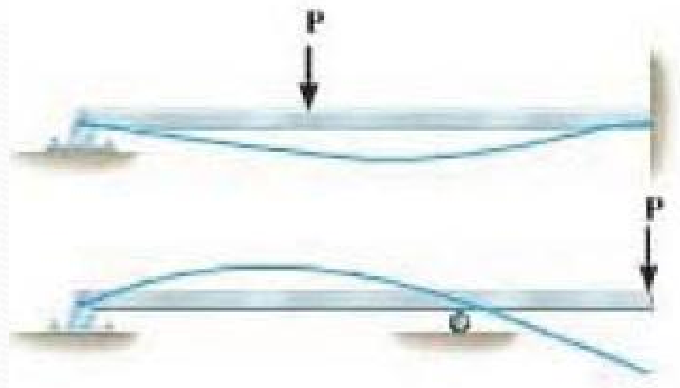
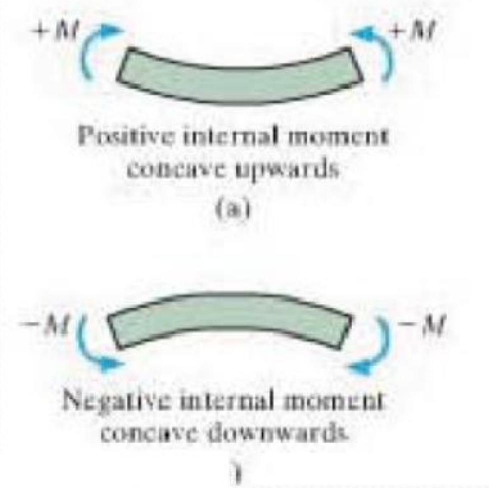
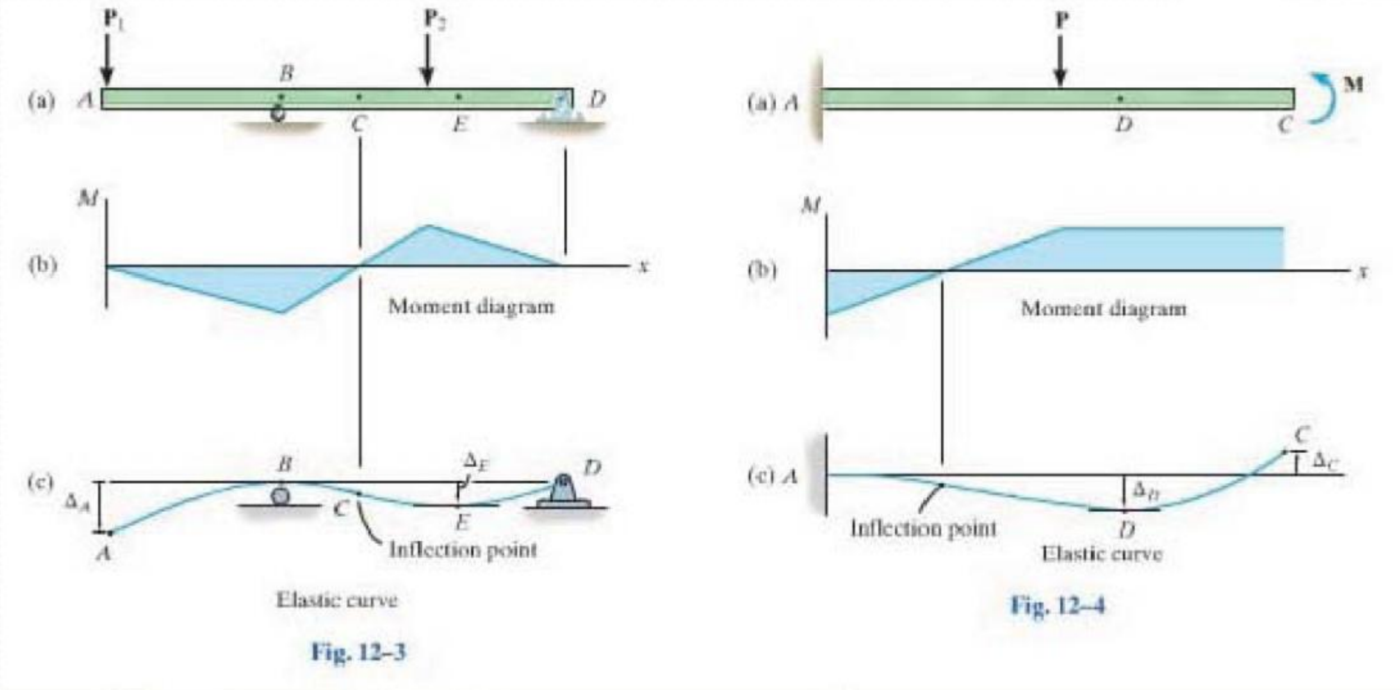


Fig. 12–1

Sketching of Elastic Curve with Moment diagram





Assignment

1. A truck with axle loads of 100 kN and 80 kN on a wheel base of 6 m rolls across a 15-m span bridge. Compute the maximum bending moment and the maximum shearing force.

2. Three wheel loads roll as a unit across a 60-ft span. The loads are P_1 = 10000 lb and P_2 = 15000 lb separated by 10 ft, and P_3 = 8000 lb at 15 ft from P_2. Determine the maximum moment and maximum shear in the simply supported span.