# DESIGN OF STRUCTURES 

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## RETAINING WALLS

## Function of retaining wall

Retaining walls are used to hold back masses of earth or other loose material where conditions make it impossible to let those masses assume their natural slopes.
$>$ Such conditions occur when the width of an excavation, cut, or embankment is restricted by conditions of ownership, use of the structure, or economy. For example, in railway or highway construction the width of the right of way is fixed, and the cut or embankment must be contained within that width.
$>$ Similarly, the basement walls of the buildings must be located within the property and must retain the soil surrounding the base.


## Types of retaining walls

Free standing retaining walls, as distinct from those that form parts of structures, such as basement walls, are of various types.
$>$ The gravity retaining wall retains the earth entirely by its own weight and generally contains no reinforcement. It is used up to 10 to 12 ft height.
> The reinforced concrete cantilever retaining wall consists of the vertical arm that retains the earth and is held in position by a footing or base slab. In this case, the weight of the fill on top of the heel, in addition to the weight of the wall, contributes to the stability of the structure. Since the arm represents a vertical cantilever, its required thickness increase rapidly, with increasing height. It is used in the range of 10 to 25 ft height.


# Types of retaining walls 

 In the counterfort wall the stem and base slab are tied together by counterforts which are transverse walls spaced at intervals and act as tension ties to support the stem wall. Counterforts are of half or larger heights. Counterfort walls are economical for heights over 25 ft .Property rights or other restrictions sometimes make it necessary to place the wall at the forward edge of the base slab, i.e. to omit the toe. Whenever it is possible, toe extensions of one-third to one-fourth of the width of the base provide a more economical solution.

## Types of retaining walls

A buttress wall is similar to a counterfort wall except that the transverse support walls are located on the side of the stem opposite to the retained material and act as compression struts. Buttress, as compression elements, are more efficient than the tension counterforts and are
 economical in the same height range.

A counterfort is more widely used than a buttress because the counterfort is hidden beneath the retained material, whereas the buttress occupies what may otherwise be usable space in front of the wall.

## Types of retaining walls

This is an free standing wall category. A wall type bridge abutment acts similarly to a cantilever retaining wall except that the bridge deck provides an additional horizontal restraint at the top of the stem. Thus this abutment is designed as a beam fixed at the bottom and simply supported or partially restrained at the top.


Bridge abutment

## Earth Pressure

$\Rightarrow$ For liquid $P=w_{w} h, w_{w}$ is the unit weight of water.
$>$ Soil retaining structure $\mathrm{P}_{\mathrm{h}}=\mathrm{C}_{0}$ wh
$>\mathrm{w}$ is unit weight of the soil

$>\mathrm{C}_{0}$ is a constant known as the coefficient of earth pressure at rest According to Rankin, the coefficient for active and passive earth pressure are

$$
\begin{aligned}
& \mathrm{C}_{\mathrm{a}}=\operatorname{Cos} \delta \frac{\operatorname{Cos} \delta-\sqrt{\operatorname{Cos}^{2} \delta-\operatorname{Cos}^{2} \phi}}{\operatorname{Cos} \delta+\sqrt{\operatorname{Cos}^{2} \delta-\operatorname{Cos}^{2} \phi}} \\
& \mathrm{C}_{\mathrm{p}}=\operatorname{Cos} \delta \frac{\operatorname{Cos} \delta+\sqrt{\operatorname{Cos}^{2} \delta-\operatorname{Cos}^{2} \phi}}{\operatorname{Cos} \delta-\sqrt{\operatorname{Cos}^{2} \delta-\operatorname{Cos}^{2} \phi}}
\end{aligned}
$$

> For the case of horizontal surface $\delta=0$

$$
C_{a h}=\frac{1-\sin \phi}{1+\sin \phi} \quad C_{p h}=\frac{1+\sin \phi}{1-\sin \phi}
$$

## Earth pressure for common condition of loading



$$
\begin{aligned}
y & =\frac{h}{3} \\
P & =\frac{1}{2} C_{a h} w h^{2}
\end{aligned}
$$

$$
\begin{aligned}
& y=\frac{h}{3} \\
& p=\frac{1}{2} C a w h^{2}
\end{aligned}
$$

For $\delta=\phi \mathrm{C}_{\mathrm{a}}=\operatorname{Cos} \phi$

## Earth pressure for common condition of loading



$$
\begin{aligned}
& y=\frac{h^{2}+3 h h^{\prime}}{3\left(h+2 h^{\prime}\right)} \\
& P=\frac{1}{2} C_{a h} w h\left(h+2 h^{\prime}\right)
\end{aligned}
$$

1. Individual parts should be strong enough to resist the applied forces
2. The
wall as
a whole should be stable against (i)

Settlem ent
Sliding
(iii)

Overtur ning


## Stability Requirement

## Settlement

It is necessary to ensure that the pressure under the footing does not exceed the "permissible bearing pressure" for the particular soil.
By the formula $\quad \mathrm{q}_{\substack{\max \\ \min }}=\frac{\mathrm{N}}{\mathrm{A}} \pm \frac{\mathrm{MC}}{\mathrm{l}}$
> If $\mathrm{a}>\frac{\ell}{3}$, compression will act throughout the section


## Stability Requirement

## Settlement

$$
\begin{aligned}
& \mathbf{a}_{1,2}=\frac{\mathbf{R}_{v}}{\ell} \pm \frac{\left(\frac{\ell}{2}-\mathbf{a}\right) \mathbf{R}_{v} \frac{\ell}{2}}{\frac{\ell^{3}}{12}} \\
& \mathrm{a}_{1,2}=\frac{\mathbf{R}_{v}}{\ell} \pm \frac{\frac{\mathbf{R}_{v} \ell^{2}}{4}-\frac{\mathbf{R}_{v} \ell \mathbf{a}}{2}}{\frac{\ell^{3}}{12}}=\frac{\mathbf{R}_{v}}{\ell} \pm \frac{\frac{\mathbf{R}_{v} \ell^{2}-2 \mathbf{R}_{v} \ell \mathbf{a}}{4}}{\frac{\ell^{3}}{12}}
\end{aligned}
$$

$$
\mathbf{a}_{1,2}=\frac{\mathbf{R}_{v}}{\ell} \pm\left[\frac{3 \mathbf{R}_{v}}{\ell}-\frac{6 \mathbf{R}_{v} \mathbf{a}}{\ell^{2}}\right]=\frac{\mathbf{R}_{v}}{\ell^{2}}[\ell \pm(\mathbf{3} \ell-6 \mathbf{a})]
$$

$$
\mathbf{a}_{1}=\frac{\mathbf{R}_{v}}{\ell^{2}}[4 \ell-6 \mathbf{a}] \quad \& \quad \mathbf{a}_{2}=\frac{\mathbf{R}_{v}}{\ell^{2}}[6 \mathbf{a}-2 \ell]
$$

$$
\text { when } \mathrm{a}=\frac{\ell}{2} \quad \mathrm{a}_{1}=\mathrm{a}_{2}=\frac{\mathrm{R}_{\mathrm{v}}}{\ell}
$$

Unit dimension in the direction perpendicular to the paper.

## Stability Requirement Settlement



$$
\mathrm{a}_{1,2}=\frac{\mathrm{R}_{\mathrm{v}}}{\ell} \pm \frac{\left(\frac{\ell}{2}-\frac{\ell}{3}\right) \mathrm{R}_{\mathrm{v}} \times \frac{\ell}{2}}{\frac{\ell^{3}}{1}} \quad \mathrm{a}_{1}\left[\uparrow \uparrow \hat{N+\uparrow+1} \mathrm{a}_{2}\right.
$$

$$
\mathrm{a}_{1,2}=\mathrm{R}_{\mathrm{v}}\left[\frac{1}{\ell} \pm \frac{12\left(\frac{3 \ell-2 \ell}{6}\right) \mathrm{R}_{\mathrm{v}} \times \frac{\ell}{2}}{\ell^{3}}\right]=\mathrm{R}_{\mathrm{v}}\left[\frac{1}{\ell} \pm \frac{12\left(\frac{\ell}{6}\right) \mathrm{R}_{\mathrm{v}} \times \frac{\ell}{2}}{\ell^{3}}\right]
$$

$$
\mathrm{a}_{1,2}=\mathrm{R}_{\mathrm{v}}\left(\frac{1}{\ell} \pm \frac{1}{\ell}\right)
$$

$$
\mathrm{a}_{1}=\frac{2 \mathrm{R}_{\mathrm{v}}}{\ell} \quad \& \quad \mathrm{a}_{2}=0
$$

## Stability Requirement

Settlement


$$
\begin{aligned}
& R_{v}=\left(\frac{1}{2}\right)(q)(3 a) \\
& q=\frac{2 R_{v}}{3 a}
\end{aligned}
$$

## Stability Requirement Sliding

$$
\begin{aligned}
& F=\mu R_{v} \\
& \frac{F}{P_{h}} \geq 1.5
\end{aligned}
$$

## Overturning

Stabilizing moment
Overturning moment

## Basis of Structural Design

1. Lateral earth pressure will be considered to be live loads and a factor of 1.7 applied.
2. In general, the reactive pressure of the soil under the structure at the service load stage will be taken equal to 1.7 times the soil pressure found for service load conditions in the stability analysis.
3. For cantilever retaining walls, the calculated dead load of the toe slab, which causes moments acting in the opposite sense to those produced by the upward soil reaction, will be multiplied by a factor of 0.9.

## Basis of Structural Design

4. For the heel slab, the required moment capacity will be based on the dead load of the heel slab itself, plus the earth directly above it, both multiplied by 1.4.
5. Surcharge, if resent, will be treated as live load with load factor of 1.7.
6. The upward pressure of the soil under the heel slab will be taken equal to zero, recognizing that for severe over load stage a non linear pressure distribution will probably be obtained, with most of the reaction concentrated near the toe.

## Drainage

$>$ Reduction in bearing capacity.
$>$ Hydrostatic pressure.
> Ice pressure
Drainage can be provided in various ways
i. Weep holes, 6 to 8 in. 5 to 10 ft horizontally spaced. $1 \mathrm{ft}^{3}$ stone at the bottom weep holes to facilitate drainage and to prevent clogging.
ii. Longitudinal drains embedded in crushed stone or gravel, along rear face of the wall.
iii. Continuous back drain consisting of a layer of gravel or crushed stone covering the entire rear face of the wall with discharge at the ends.

## Problem

- A gravity retaining wall consisting of plain concrete $\mathrm{w}=144$ $\mathrm{lb} / \mathrm{ft}^{3}$ is shown in fig. The bank of supported earth is assumed to weigh $110 \mathrm{lb} / \mathrm{ft}^{3} \varnothing=30^{\circ}$ and to have a coefficient of friction against sliding on soil of 0.5 .


## Solution

$C_{a}=\frac{1-\sin \phi}{1+\sin \phi} \Rightarrow \frac{1-\sin 30}{1+\sin 30}=0.333$
$C_{p}=\frac{1+\sin \phi}{1-\sin \phi} \Rightarrow \frac{1+\sin 30}{1-\sin 30}=3$


$$
P_{\mathrm{ah}}=\frac{1}{2} \mathrm{C}_{\mathrm{a}} \mathrm{wh}^{2}=\left(\frac{1}{2}\right)(0.333)(110)(12)^{2}=2637.36 \mathrm{lb}
$$

Overturning moment $=2637.36 \times \frac{12}{3}=10549.44 \mathrm{lb} . \mathrm{ft}$

## Solution

> Restoring moment:
Moment about toe

Component Force Weights
(6)(1)(144)

W2

$$
(1)(11)(144)
$$

W3

$$
\left(\frac{1}{2}\right)(4)(144)(11)
$$

$$
\left(\frac{1}{2}\right)(4)(11)(110)
$$

$$
(0.5)(11)(110)
$$

Moment arm

| $864 \times 3$ | $2592 \mathrm{lb} . \mathrm{ft}$ |
| :--- | :--- |
| $1584 \times 1$ | $1584 \mathrm{lb} . \mathrm{ft}$ |

$3168 \times\left(1.5+\frac{1}{3} \times 4\right) \quad 8976 \mathrm{lb} . \mathrm{ft}$
$2420 \times\left(1.5+\frac{2}{3} \times 4\right) \quad 10083.33 \mathrm{lb} . \mathrm{ft}$
$605 \times 5.75$
Rv $=8641 \mathrm{lb}$


2592 lb.ft
$1584 \mathrm{lb} . \mathrm{ft}$
$3478.75 \mathrm{lb} . \mathrm{ft}$
M=26714.08 lb.ft

Safety factor against overturning $=\frac{26714.08}{10549.44}=2.53>2$ O.K

## Solution

> Distance of resultant from the toe

$$
\mathrm{a}=\frac{26714.08-10549.44}{8641}=1.87 \mathrm{ft}
$$

$>$ The max. soil pressure will be $\mathrm{q}=\frac{2 \mathrm{R}_{\mathrm{v}}}{3 \mathrm{a}} \Rightarrow \frac{(2)(8641)}{(3)(1.87)}$
$>\mathrm{q}=3080.57 \mathrm{lb} / \mathrm{t}^{2}$

## Solution Sliding

$>$ Assuming that soil above footing toe has eroded and thus the passive pressure is only due to soil depth equal to footing thickness.

$$
P_{p h}=\frac{1}{2} C_{p} w h^{2}=\left(\frac{1}{2}\right)(3)(110)(1)^{2}=165 \mathrm{lb}
$$

$>$ Friction force between footing concrete and soil.

$$
\mathrm{F}=\mu \mathrm{R}_{\mathrm{v}}=(0.5)(8641)=4320.5 \mathrm{lb}
$$

$>$ F.O.S. against sliding $=\frac{4320.5+165}{2637.36}=1.7>1.5 \quad$ O.K

## Estimating size of cantilever retaining wall

## Height of Wall

> The base of footing should be below frost penetration about 3' or 4'.

## Stem Thickness.

Stem is thickest at its base. They have thickness in the range of 8 to $12 \%$ of overall height of the retaining wall. The minimum thickness at the top is 10 " but preferably 12 ".

## Base Thickness

> Preferably, total thickness of base fall between 7 and $10 \%$ of the overall wall height. Minimum thickness is at least 10 " to 12 " used.

## Estimating size of cantilever retaining wall



## Base Length

> For preliminary estimates, the base length can be taken about 40 to $60 \%$ of the overall wall height.
$>$ Another method refer to fig. W is assumed to be equal to weight of the material within area abcd.
> Take moments about toe and solve for x .

## Problem

> Design a cantilever retaining wall to support a bank of earth of 16 ft height above the final level of earth at the toe of the wall. The backfill is to be level, but a building is to be built on the fill.
$>$ Assume that an 8' surcharge will approximate the lateral earth pressure effect.
Weight of retained material $=130 \mathrm{lb} / \mathrm{ft}^{3}$
Angle of internal friction $=35^{\circ}$
Coefficient of friction $\mathrm{b} / \mathrm{w}$
concrete and soil $=0.4$ $\mathrm{f}_{\mathrm{c}}{ }^{\prime}=3000 \mathrm{psi}$
$\mathrm{f}_{\mathrm{y}}=40,000 \mathrm{psi}$
Maximum soil pressure
$=5 \mathrm{k} / \mathrm{ft}^{2}$


## Solution

## Height of Wall

$>$ Allowing 4' for frost penetration to the bottom of footing in front of the wall, the total height becomes.
$>\mathrm{h}=16+4=20 \mathrm{ft}$.
Thickness of Base
$>$ At this stage, it may be assumed 7 to $10 \%$ of the overall height $h$.
$>$ Assume a uniform thickness $t=2^{\prime}(10 \%$ of $h)$
Base Length
$>\mathrm{h}=20^{\prime} \quad \mathrm{h}^{\prime}=8 \prime$

## Solution

$$
\begin{aligned}
\mathrm{P} & =\frac{1}{2} C_{a h} w h\left(\mathrm{~h}+2 \mathrm{~h}^{\prime}\right) \\
& =\left(\frac{1}{2}\right)\left(\frac{1-\sin \phi}{1+\sin \phi}\right)(120)(20)(20+2 \times 8) \\
& =(0.5)(0.271)(120)(20)(36) \\
& =11707.2 \mathrm{lb} \\
\mathrm{y} & =\frac{\mathrm{h}^{2}+3 \mathrm{hh}^{\prime}}{3\left(\mathrm{~h}+2 \mathrm{~h}^{\prime}\right)}=\frac{(20)^{2}+(3)(20)(8)}{3(20+2 \times 8)} \\
& =8.148 \mathrm{ft}
\end{aligned}
$$

## Solution

> Moments about point a
> $\mathrm{W}=(120)(\mathrm{x})(20+8)=3360 \mathrm{xlb}$
$\Rightarrow \quad(W)\left(\frac{x}{2}\right)=P \times y$
$>(3360 x)\left(\frac{x}{2}\right)=(11707.2)(8.148)$
> $\mathrm{x}=7.54 \mathrm{ft}$
$>$ So base length $=1.5 \times x=11.31 \mathrm{ft}$
Use $11 \mathrm{ft} 4^{\prime \prime}$ with $x=7^{\prime}-8{ }^{\prime \prime}$ and $3^{\prime}-8$ ' toe length

## Solution

## Stem Thickness

> Prior computing stability factors, a more accurate knowledge of the concrete dimensions is necessary.
> The thickness of the base of the stem is selected with the regard for bending and shear requirements.
$>P$ for 18 ' height and $h^{\prime}=8 \prime$

$$
\begin{aligned}
P & =\left(\frac{1}{2}\right) C_{a h} w h\left(h+2 h^{\prime}\right) \\
& =\left(\frac{1}{2}\right)(0.271)(120)(18)(18+2 \times 8) \\
& =9951.12 \mathrm{lb}
\end{aligned}
$$

$$
\begin{aligned}
y & =\frac{h^{2}+3 h h^{\prime}}{3\left(h+2 h^{\prime}\right)}=\frac{(18)^{2}+(3)(18)(8)}{3(18+2 \times 8)} \\
& =7.412 \mathrm{ft}
\end{aligned}
$$

## Solution

$$
\begin{aligned}
M_{u} & =(1.7) P y=(1.7)(9951.12)(7.412) \\
& =125388.09 \mathrm{lb} . \mathrm{ft} \\
\rho_{\mathrm{b}} & =\frac{(0.85) \beta_{\mathrm{f}} \mathrm{f}_{\mathrm{c}}^{\prime}}{\mathrm{f}_{\mathrm{y}}}\left(\frac{87000}{87000+\mathrm{f}_{\mathrm{y}}}\right) \\
& =\frac{(0.85)(0.85)(3000)}{40,000}\left(\frac{87000}{87000+40,000}\right) \\
& =0.03712 \\
\rho_{\max } & =0.75 \rho_{\mathrm{b}}=0.02784
\end{aligned}
$$

For adequate deflection control, choose $\rho=\frac{1}{2} \rho_{\max }=0.01392$
then $R_{n}=\rho f_{y}\left(1-\frac{1}{2} \rho m\right)$

$$
\begin{aligned}
\mathrm{m} & =\frac{\mathrm{f}_{\mathrm{y}}}{0.85 \mathrm{f}_{\mathrm{c}}^{\prime}}=\frac{40,000}{0.85 \times 3000} \\
& =15.686
\end{aligned}
$$

Solution

$$
\begin{aligned}
R_{n} & =\rho f_{y}\left(1-\frac{1}{2} \rho m\right) \\
& =(0.01392)(40,000)\left(1-\frac{1}{2} \times 0.01392 \times 15.686\right) \\
& =496
\end{aligned}
$$

Required $b d^{2}=\frac{\text { Required } M_{n}}{R_{n}}$

$$
\begin{aligned}
& \frac{M_{n}}{\phi} \\
= & \frac{125388.09 \times 12}{R_{n}}=\frac{10.9)(496)}{(0.2} \\
= & 3370.65 \\
d & =16.76^{\prime \prime}
\end{aligned}
$$

$>$ Total thickness $=16.7+0.5+3=20.26 "$
Try 21 " thickness of base of stem and select $12^{\prime \prime}$ for top of the wall

## Shear at d

d used now $=17.5^{\prime \prime}=1.458^{\prime}$
$>$ At $18^{\prime}-1.458^{\prime}=16.542^{\prime}$ from top

$$
\begin{aligned}
\mathrm{P} & =\left(\frac{1}{2}\right) \mathrm{C}_{\mathrm{ah}} \mathrm{wh}\left(\mathrm{~h}+2 \mathrm{~h}^{\prime}\right) \\
& =\left(\frac{1}{2}\right)(0.271)(120)(16.542)(16.542+2 \times 8) \\
& =8752.92 \mathrm{lb} \\
\mathrm{~V}_{\mathrm{u}} & =1.7 \mathrm{P} \\
& =17879.96 \mathrm{lb} \\
\phi \mathrm{~V}_{\mathrm{u}} & =\phi \times 2 \sqrt{\mathrm{f}_{\mathrm{c}}^{\prime} \mathrm{bd}} \\
& =0.85 \times 2 \times \sqrt{3000} \times 12 \times 17.5 \\
& =19553.7 \mathrm{lb}
\end{aligned}
$$



Since $\phi V_{u}>V_{u}$, So no shear reinf orcement is required.

## Solution

## F.O.S Against Overturning

Component

| W1 | $(5.92)(18)(120)=12787.2$ | $3.67+21^{1}+\frac{5.92}{2}=8.38$ | 107156.74 |
| :---: | :---: | :---: | :---: |
| W2 | $\left(\frac{1}{2}\right)(0.75)(18)(150)=1012.5$ | $3.67+1+0.25=4.92$ | 4981.50 |
| W3 | (18)(1)(150)=2700 | $3.67+0.5=4.17$ | 11259 |
| W4 | $(11.33)(2)(150)=3399.00$ | $\frac{11.33}{2}=5.665$ | 19255.34 |
| W5 | $\left(\frac{\mathbf{1}}{\mathbf{2}}\right)(18)(0.75)(120)=810$ | $3.67+1+0.5=5.17$ | 4187.7 |
| W6 | $(6.67)(8)(120)=6403.2$ | $3.67+1+\frac{6.67}{2}=8.005$ | 51257.62 |
| Total | 27111.9 |  | 198097. |

Moment
107156.74
4981.50

11259
19255.34
4187.7
51257.62
198097.9

## Solution

> $\mathrm{P}=11707.2 \mathrm{lb} \quad \mathrm{y}=8.148 \mathrm{ft}$
> Overturning Moment $=11707.2 \times 8.148=95390.27 \mathrm{lb} . \mathrm{ft}$
F.O.S. against overturning

$$
=\frac{198097.9}{95390.27}=2.077>20 . \mathrm{K}
$$

## Location of Resultant \& Footing Soil Pressure

$>$ Distance of the resultant from the front edge is
$\mathrm{a}=\underline{\text { Righting moment }- \text { Overturning moment }}$
Total load (righting)
$\mathrm{a}=\frac{198097.9-95390.27}{27111.9}=3.7883$
Middle third $=3.7778 \mathrm{ft}$, So resultant is within the middle third.

$$
\begin{aligned}
& \mathrm{q}_{1}=\frac{\mathrm{R}_{\mathrm{v}}}{\ell^{2}}(4 \ell-6 \mathrm{a}) \\
& \mathrm{q}_{1}=\frac{27111.9}{(11.33)^{2}}(4 \times 11.33-6 \times 3.7883)
\end{aligned}
$$



## Solution

$$
\begin{aligned}
& \mathrm{a}_{1}=4771.12 \mathrm{lb} / \mathrm{ft}^{2}<5 \mathrm{k} / \mathrm{ft}^{2} \\
& \mathrm{a}_{2}=\frac{\mathrm{R}_{\mathrm{v}}}{\ell^{2}}(6 \mathrm{a}-2 \ell)=14.74 \mathrm{lb} / \mathrm{ft}^{2}
\end{aligned}
$$

So O.K against bearing pressure.

## Solution

## F.O.S. against sliding

$>$ force causing sliding $=P=11707.2 \mathrm{lb}$
$>$ Frictional resistance $=\mu \mathrm{R}$

$$
\begin{aligned}
& =(0.4)(27111.9) \\
& =10844.76 \mathrm{lb}
\end{aligned}
$$

> Passive earth pressure against 2 ' height of footing $=\frac{1}{2} \mathrm{wh}^{2} \mathrm{C}_{\mathrm{aph}}$ $=\left(\frac{1}{2}\right)(120)(2)\left[\frac{1+\sin \phi}{1-\sin \phi}\right]$
$=442.82 \mathrm{lb}$
F.O.S. $=\frac{10844.76+442}{11707.2}=0.964<1.5$ So key is required.

## Solution

$>$ The front of key is 4 " in front of back face of the stem. This will permit anchoring the stem reinforcement in the key..

$$
\begin{aligned}
& \frac{x}{6.243}=\frac{4756.38}{11.33} \\
& x=2620.84 \mathrm{lb} / \mathrm{ft}
\end{aligned}
$$

Total ordinate $2635.58 \mathrm{lb} / \mathrm{ft}$

$>$ Frictional resistance between soil to soil $=\mu \mathrm{R}$
$=(\tan \phi)\left(\frac{4771.12+2635.58}{2}\right)(5.087)$
$=13191.17 \mathrm{lb}$

## Solution

Frictional resistance between heel concrete to soil $=\mu \mathrm{R}$

$$
\begin{aligned}
& =(0.4)\left(\frac{2635.58+14.74}{2}\right)(6.243) \\
& =3309.19 \mathrm{lb}
\end{aligned}
$$

Passive earth pressure $=\frac{1}{2} w h^{2} C_{p h}$

$$
=\left(\frac{1}{2}\right)(120)(h)^{2}(3.69)=221.4 \mathrm{~h}^{2} \mathrm{lb}
$$

F.O.S. against sliding $=1.5$

$$
\begin{aligned}
& 1.5=\frac{13191.17+3309.19+221.4 h^{2}}{11707.2} \\
& \mathrm{~h}=2.19 \mathrm{ft}
\end{aligned}
$$

So use key of height $=2^{\prime}-3^{\prime \prime}=2.25^{\prime}$

## Solution

## Design of Heel Cantilever

$$
\mathrm{W}_{\mathrm{u}}=(1.7)(120)(8)+(1.4)[18 \times 120+2 \times 150]
$$

$$
=5076 \mathrm{lb} / \mathrm{ft}
$$

$$
M_{u}=\frac{W}{2}
$$

$$
=\left(\frac{1}{2}\right)(5.76)(5.92)^{2}
$$

$=88947.76 \mathrm{lb} . \mathrm{ft}$

$\mathrm{V}_{\mathbf{u}}=$ Factored shear a joint of stem and heel
When the support reaction introduces compression into the end region, then critical shear is at a distance d from face of support. However, the support is not producing compression, therefore, critical shear is at joint of stem and heel.

## Solution

## Design of Heel Cantilever

$V_{u}=$ Factored shear a joint of stem and heel
$=(5.92)(5076)$
$=30049.92 \mathrm{lb}$

$$
\begin{aligned}
\phi \mathrm{V}_{\mathrm{c}} & =\phi 2 \sqrt{\mathrm{f}_{\mathrm{c}}^{\prime} \mathrm{bd}} \\
& =0.85 \times 2 \times \sqrt{3000} \times 12 \times 21.5 \\
& =24023.11<\mathrm{V}_{\mathrm{u}}
\end{aligned}
$$

$>$ So depth is required to be increased.

$$
d=\frac{V_{u}}{\phi 2 \sqrt{f_{c}^{\prime} b}}=\frac{30049.92}{(0.85)(2)(\sqrt{3000})(12)}=26.89^{\prime \prime}
$$

> Therefore heel thickness 30",
$d=27.5^{\prime \prime}$

## Solution

## Design of Heel Cantilever

Now $\mathrm{W}_{\mathrm{u}}=(1.7)(120)(8)+(1.4)[(17.5)(120)+(2.5)(150)]=5097 \mathrm{lb} / \mathrm{ft}$

$$
M_{u}=\left(\frac{1}{2}\right)(5097)(5.92)^{2}=89315.75 \mathrm{lb} . \mathrm{ft} .
$$

Required $R_{n}=\frac{M_{u}}{\phi b d^{2}}=\frac{89315 \times 12}{(0.9)(12)(27.5)^{2}}=131.23 \mathrm{psi}$
$\rho=\frac{1}{m}\left[1-\sqrt{1-\frac{2 \mathrm{mR}_{\mathrm{n}}}{\mathrm{f}_{\mathrm{y}}}}\right]=0.00337$
$\rho_{\text {min }}=\frac{200}{f_{y}}=0.005$

## Solution

$A_{s}=\rho_{\text {min }} b d=1.8 \mathrm{in}^{2}$
> Use \# 8 @ 5 " c/c $\left(\mathrm{A}_{\mathrm{s}}=1.88 \mathrm{in}^{2}\right)$
$>$ Dev. Length required $=23^{\prime \prime}$ top bars so $23 \times 1.3=29.9$ "
> Available $=5.92$ ' -3 " $=68.04$ " O.K
Design of Toe Slab $\xrightarrow{\text { P.67 }} \xrightarrow{7.66^{\prime}}$

$$
\begin{aligned}
& \frac{x}{4756.38}=\frac{7.66}{11.33} \\
& x=3215.7
\end{aligned}
$$

$3215.7+14.74=3230.44$


Self load=(0.9) $(1 \times 2 \times 150)=270 \mathrm{lb} / \mathrm{ft}$
$\mathrm{W}_{\mathrm{u}}=(1.7)\left[\frac{3230.44+4771.12}{2}\right]$
Overload factor $=0.9$

$$
d=24 "-3.5 "=20.5 "
$$

(6801.33)lb/ft
$\mathrm{W}_{\mathrm{u}}=3801.33 \mathrm{lb} / \mathrm{ft}-270=6531.33$

## Solution

$$
\begin{aligned}
& M_{u}=\frac{W_{u} \ell^{2}}{2}=\frac{(6531.33)(3.67)^{2}}{2}=43984.92 \mathrm{lb} . \mathrm{ft} \\
& \text { Required } R_{n}=\frac{M_{u}}{\phi b d^{2}}=\frac{43984.92 \times 12}{(0.9)(12)(20.5)^{2}}=116.3
\end{aligned}
$$

So $\rho_{\text {min }}$ will control
$\mathrm{A}_{\mathrm{s}}=(0.005)(12)(20.5)=1.23 \mathrm{in}^{2}$
Use \# 8 @ $7 \frac{1}{2} \mathrm{c} / \mathrm{c}\left(\mathrm{As}=1.26 \mathrm{in}^{2}\right)$
> Available dev. Length $=3.76^{\prime}-3^{\prime \prime}=41.01^{\prime \prime}$ Required $=23^{\prime \prime}$
At a distance $d=20.5^{\prime \prime}=1.71^{\prime}$
$\frac{\mathrm{x}}{4756.38}=\frac{9.37^{3.67}}{11.33}-1.71^{\prime}=1.96^{\prime}$
$\mathrm{x}=3933.56$


$$
3933.56+14.74=3948.3
$$

## Solution

Earth pressure $=\left(\frac{3948.3+4771.12}{2}\right)(1.96)(1.7)=14526.55 \mathrm{lb}$
$\mathrm{V}_{\mathrm{u}}=14526.55-270 \times 1.96=13997.37 \mathrm{lb}$
$\phi V_{c}=\phi\left(2 \sqrt{f_{c}^{\prime}}\right) b d$
$=0.85 \times 2 \times \sqrt{3000} \times 12 \times 20.5=22905.76 \mathrm{lb}>\mathrm{V}_{\mathrm{u}}$
$>$ So no shear reinforcement is required
Reinforcement for stem

$$
\begin{aligned}
P & =\frac{1}{2} C_{a h} w h\left(h+2 h^{\prime}\right) \\
& =\left(\frac{1}{2}\right)(0.271)(120)(17.5)(17.5+2 \times 8) \\
& =9532.43 \mathrm{lb}
\end{aligned}
$$

$$
y=\frac{h^{2}+3 h h^{\prime}}{3\left(h+2 h^{\prime}\right)}=\frac{(17.5)^{2}+3 \times 17.5 \times 8}{3(17.5+2 \times 8)}=\frac{726.25}{100.5}=7.23^{\prime}
$$

## Solution

Reinforcement for stem

$$
M_{u}=(1.7)(9532.43)(7.23)=117163.1 \mathrm{lb} . \mathrm{ft}
$$

$$
R_{n}=\frac{M_{u}}{\phi b d^{2}}=\frac{117163.1 \times 12}{0.9 \times 12 \times(17.5)^{2}}=425.08 \mathrm{psi}
$$

$$
\rho=\frac{1}{m}\left[1-\sqrt{1-\frac{2 m R_{n}}{f_{y}}}\right]
$$

$$
\mathrm{m}=\frac{\mathrm{f}_{\mathrm{y}}}{0.85 \mathrm{f}_{\mathrm{c}}^{\prime}}=\frac{40,000}{0.85 \times 3000}=15.686
$$

$$
\rho=\frac{1}{15.686}\left[1-\sqrt{1-\frac{2 \times 15.686 \times 425.08}{40,000}}\right]
$$

$$
=0.0117
$$

$$
A_{s}=\rho b d=0.0117 \times 12 \times 17.5=2.457{i n^{2}}^{2}
$$

Use \#9 bars @ $4 \frac{1}{2}^{\prime \prime} \mathrm{c} / \mathrm{c} \quad\left(\mathrm{A}_{\mathrm{s}}=2.67 \mathrm{in}^{2}\right)$

# At 5' from top $\quad y=2.302, \quad M_{u}=6681.34 \mathrm{lb} . \mathrm{ft}$ <br> $\mathrm{P}=1707.3 \mathrm{lb}$ <br> $=6.68 \mathrm{k} . \mathrm{ft}$ 



At 15' from top $\quad y=6.29, \quad M_{u}=80848.7 \mathrm{lb} . \mathrm{ft}$
$\mathrm{P}=7560.9 \mathrm{lb}$
$=80.85 \mathrm{k} . \mathrm{ft}$
$M_{u}$ at base $117163.1 \mathrm{lb} . \mathrm{ft}=1405957.2 \mathrm{lb} . \mathrm{in}$ $=117.16 \mathrm{k} . \mathrm{ft}$

## Solution

With Full Reinforcement
$>\mathrm{C}=0.85 \mathrm{f}_{\mathrm{c}}{ }^{\prime} \mathrm{ba}=0.85 \times 3000 \times 12 \times \mathrm{a}=30600 \mathrm{a} \mathrm{lb}$
$\Rightarrow \mathrm{T}=\mathrm{A}_{\mathrm{s}} \mathrm{f}_{\mathrm{y}}=2.67 \times 40000=106800$
> $\mathrm{a}=3.49 \mathrm{in}$
> At top of wall $\quad d=8.5$ "

$$
\begin{aligned}
\phi \mathrm{M}_{\mathrm{n}} & =0.9 \mathrm{~A}_{\mathrm{s}} \mathrm{f}_{\mathrm{y}}\left(\mathrm{~d}-\frac{\mathrm{a}}{2}\right)=(0.9)(106800)\left(8.5-\frac{3.49}{2}\right) \\
& =649290.6 \mathrm{lb} . \mathrm{in} .
\end{aligned}
$$

At base of stem $\quad d=17.5$ "

$$
\begin{aligned}
\phi \mathrm{M}_{\mathrm{n}} & =(0.9)(106800)\left(17.5-\frac{3.49}{2}\right) \\
& =1514370.6 \mathrm{lb} . \mathrm{in} .=126.2 \mathrm{k} . \mathrm{ft}
\end{aligned}
$$

## Solution

## With half Reinforcement

$>\mathrm{C}=0.85 \mathrm{f}_{\mathrm{c}}{ }^{\prime} \mathrm{ba}=0.85 \times 3000 \times 12 \times \mathrm{a}=30600 \mathrm{a} \mathrm{lb}$
$>\mathrm{T}=\mathrm{A}_{\mathrm{s}} \mathrm{f}_{\mathrm{y}}=1.335 \times 40,000=53400$ for $\# 9 @ 9 " \mathrm{c} / \mathrm{c}\left(\mathrm{A}_{\mathrm{s}}=1.33 \mathrm{in}^{2}\right)$
> $\mathrm{a}=1.745 \mathrm{in}$
> At top of wall $\quad d=8.5$ "

$$
\begin{aligned}
\phi \mathrm{M}_{\mathrm{n}} & =(0.9)(53400)\left(8.5-\frac{1.745}{2}\right) \\
& =366577.65 \mathrm{lb} . \mathrm{in} .=30.55 \mathrm{k} . \mathrm{ft}
\end{aligned}
$$

At base of stem $\quad d=17.5^{\prime \prime}$

$$
\begin{aligned}
\phi \mathrm{M}_{\mathrm{n}} & =(0.9)(53400)\left(17.5-\frac{1.745}{2}\right) \\
& =799117.65 \mathrm{lb} . \mathrm{in} .=66.59 \mathrm{k} . \mathrm{ft}
\end{aligned}
$$

## Solution

## With one-fourth Reinforcement

$>\mathrm{C}=0.85 \mathrm{f}_{\mathrm{c}}{ }^{\prime} \mathrm{ba}=0.85 \times 3000 \times 12 \times \mathrm{a}=30600 \mathrm{a} \mathrm{lb}$
$>\mathrm{T}=\mathrm{A}_{\mathrm{s}} \mathrm{f}_{\mathrm{y}}=0.67 \times 40,000=26800$ for $\# 9 @ 18$ " $\mathrm{c} / \mathrm{c}\left(\mathrm{A}_{\mathrm{s}}=0.67 \mathrm{in}^{2}\right)$
> $\mathrm{a}=0.88$ in
> At top of wall $d=8.5$ "

$$
\begin{aligned}
\phi \mathrm{M}_{\mathrm{n}} & =(0.9)(26800)\left(8.5-\frac{0.88}{2}\right) \\
& =194407.2 \mathrm{lb} . \mathrm{in} .=16.2 \mathrm{k} . \mathrm{ft}
\end{aligned}
$$

At base of stem $\quad d=17.5^{\prime \prime}$

$$
\begin{aligned}
\phi \mathrm{M}_{\mathrm{n}} & =(0.9)(26800)\left(17.5-\frac{0.88}{2}\right) \\
& =411487.2 \mathrm{lb} . \mathrm{in} .=34.29 \mathrm{k} . \mathrm{ft}
\end{aligned}
$$

## Solution

> Thus half bars should be cut at 4'-8" distance from bottom and further half bars should be cut 8 '-8" theoretically from bottom.

The actual termination point is found by extending beyond the intersection of capacity moment line with the factored moment diagram a distance of either the effective depth d or 12 bar diameters, whichever is greater.

Solution


Solution

$$
\begin{aligned}
& \frac{x}{12 \times y}=\frac{9}{17.5 \times 12} \\
& x=\frac{9}{17.5} \times y=6.63^{\prime \prime}
\end{aligned}
$$

d at 4.6' from bottom ( $\mathrm{y}=12.9^{\prime}$ )
$=8.5+6.63=15.13^{\prime \prime}$

$$
\cong 15^{\prime \prime}
$$

$$
x=\frac{9}{17.5} \times y=4.58^{\prime \prime}
$$

d at $8.6^{\prime}$ from bottom ( $\mathrm{y}=8.9^{\prime}$ )
$=8.5+4.58=13.08^{\prime \prime}$ $\cong 13 "$


## Solution

> bar used \# 9 of diameter = 1.128"
$>12 \mathrm{~d}_{\mathrm{b}}=13.54^{\prime \prime}=14^{\prime \prime}$
$>$ Therefore half bars should be terminated actually at $4^{\prime}-8$ " $+15^{\prime \prime}=15^{\prime}-11^{\prime \prime}$ from bottom
$>$ and further half bars should be terminated at $8^{\prime}-88^{\prime \prime}+14^{\prime \prime}=9^{\prime} 10^{\prime \prime}$

F For tension bars to be terminated in the tension zone, one of the following condition must be satisfied.

1. $V_{u}$ at the cut-off point must not exceed two-thirds of the shear strength $\varnothing \mathrm{V}_{\mathrm{n}}$.
2. Continuing bars must provide at last twice the area required for bending moment at the cut off point.
3. Excess shear reinforcement is provided.
$>\phi V_{c}=\phi\left(2 \sqrt{\mathrm{f}_{\mathrm{c}}^{\prime}}\right) \mathrm{bd} \quad$ at $12.9^{\prime}$ from top

## Solution

$$
\begin{aligned}
& \phi \mathrm{V}_{\mathrm{c}}= 0.85 \times 2 \times \sqrt{3000} \times 12 \times 15 \times \frac{1}{1000} \\
&= 16.76 \mathrm{kips} \\
& \frac{2}{3} \times \phi \mathrm{V}_{\mathrm{c}}=11.17 \mathrm{kips} \\
& \mathrm{~V}_{\mathrm{u}}= 1.7\left[\frac{1}{2} \mathrm{C}_{\mathrm{ah}} \mathrm{wh}\left(\mathrm{~h}+2 \mathrm{~h}^{\prime}\right)\right] \times \frac{1}{1000} \\
&= 10.31 \mathrm{kips}
\end{aligned}
$$

Condition(1) is satisfied.

$$
\begin{aligned}
\phi V_{c} & =0.85 \times 2 \times \sqrt{3000} \times 12 \times 13 \times \frac{1}{1000} \\
& =14.53 \mathrm{kips} \\
& \frac{2}{3} \times \phi V_{c}=9.68 \mathrm{kips}
\end{aligned}
$$

## Solution

$$
\begin{aligned}
V_{u} & =1.7\left[\frac{1}{2} C_{a h} w h\left(h+2 h^{\prime}\right)\right] \times \frac{1}{1000} \\
& =1.7\left[\frac{1}{2} \times 0.271 \times 120 \times 8.9 \times(8.9+2 \times 8)\right] \times \frac{1}{1000} \\
& =6.13 \mathrm{kips}
\end{aligned}
$$

$>$ Condition (1) satisfied so bars can be terminated.
The above condition are imposed as a check stress concentration.
$>$ Shear at bottom $=\mathrm{V}_{\mathrm{u}}=1.7 \times 9.53243=16.21 \mathrm{kips}$

$$
\phi V_{c}=0.85 \times 2 \times \sqrt{3000} \times 12 \times 17.5 \times \frac{1}{1000}=19.56 \mathrm{kips}
$$

Since $\varnothing V_{c}>V_{u}$ so no need of shear reinforcement.

## Solution

> The $\rho$ used should not be less than $\frac{200}{f_{y}}$ at any point. This minimum limit, strictly speaking, does nott apply to retaining walls. However, because the integrity of retaining wall depends absolutely on the vertical walls, it appear prudent to use this limit un such cases.
> First termination point is $5^{\prime}-11^{\prime \prime}$ from bottom where $d=14.46$ " $\quad A_{s}=1.335 \mathrm{in}^{2}$

$$
\rho=0.0077>\frac{200}{f_{y}}=0.005
$$

> Second termination point is $9^{\prime}-10^{\prime \prime}$ form bottom where $\mathrm{d}=12.44^{\prime \prime} \mathrm{A}_{\mathrm{s}}=0.6675 \mathrm{in}^{2}$
$\rho=0.0045 \approx 0.005$ Therefore the above condition is satisfied.

## Solution

> Another requirement is that maximum spacing of the primary flexural reinforcement exceed neither 3 times the wall thickness nor 18 in. These restrictions are satisfied as well.
For splices of deformed bars in tension, at sections where the ratio of steel provided to steel required is less than 2 and where no more than $50 \%$ of the steel is spliced, the ACl code requires a class-B splice of length $1.3 \ell_{\mathrm{d}}$.
$>\ell_{\mathrm{d}}$ for $\# 9$ bars $=29$ "
Splice length $=1.3 \times 29=37.7^{\prime \prime}$ or $3^{\prime}-2^{\prime \prime} \quad$ O.K

## Solution

## Temperature \& shrinkage reinforcement

> Total amount of horizontal bars ( h is average thickness)

$$
\mathrm{A}_{\mathrm{s}}=0.002 \mathrm{bh}=0.002 \times 12 \times \frac{12+20.50}{2}=0.39 \mathrm{in}^{2} / \mathrm{ft}
$$

$>$ Since front face is more exposed for temperature changes therefore two third of this amount is placed in front face and one third in rear face.
Accordingly $\frac{2}{3} A_{s}=0.26 \mathrm{in}^{2} / \mathrm{ft} \quad \# 4 @ 9 \mathrm{in} . \mathrm{c} / \mathrm{c} \quad \mathrm{A}_{\mathrm{s}}=0.26 \mathrm{in}$.
$>\frac{1}{3} \mathrm{~A}_{\mathrm{s}}=0.13 \mathrm{in}^{2} / \mathrm{ft} \quad$ Use \# $3 @ 10 \mathrm{in} . \mathrm{c} / \mathrm{c} \quad \mathrm{A}_{\mathrm{s}}=0.13 \mathrm{in}^{2}$.
> For vertical reinforcement on the front face, use any nominal amount. Use \# 3 @ 18 in. c/c
Since base is not subjected to extreme temperature changes, therefore \# 4@ 12" c/c just for spacers will be sufficient.



VERTICAL


LONGITUDINAL SECTION

